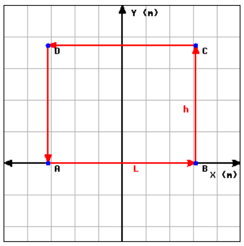
Assignment FINAL_EXAM_EGC16 due 01/02/2023

Problem 1.

1. (40 points)



An object travels along a path shaped as an $L \times h$ rectangle that is centered to y-axis and the bottom side is on the x-axis, as shown in the figure. The force applied on the object is given as follows:

$$\mathbf{F}(x,y) = by\,\mathbf{i} + cx^2\,\mathbf{j}$$

where the height of the triangle is $h = 3.72$ m, the base, $L = 6.21$ m, and the constants are $b = 17.4\ \frac{\mathrm{N}}{\mathrm{m}}$ and $c = 19.6\ \frac{\mathrm{N}}{\mathrm{m}^2}$.

Part 1:
For an applied constant force

$$\mathbf{F} = F_x\,\mathbf{i} + F_y\,\mathbf{j}$$

and a displacement vector

$$\Delta\mathbf{r} = \Delta x\,\mathbf{i} + \Delta y\,\mathbf{j}$$

Write the expression of the work done, using Δx as **Dx**, Δy as **Dy**, F_x as **Fx** and F_y as **Fy** in the expression

$W =$ ________________ (formula)

Part 2:
Write the expression of the work done on path AB in terms of b, c, L and h.

$W_{AB} =$ ________________ (formula)

Calculate the total work

$W_{AB} =$ ________________ (numeric result with units)

Part 3:
Write the expression of the work done on path BC in terms of b, c, L and h.

$W_{BC} =$ ________________ (formula)

Calculate the total work

$W_{BC} =$ ________________ (numeric result with units)

Part 4:
Write the expression of the work done on path CD in terms of b, c, L and h.

$W_{CD} =$ ________________ (formula)

Calculate the work

$W_{CD} =$ ________________ (numeric result with units)

Part 5:
Write the expression of the work done on path DA in terms of b, c, L and h.

$W_{DA} =$ ________________ (formula)

Calculate the work

$W_{DA} =$ ________________ (numeric result with units)

Part 6:
Write the expression of the total work done on the complete path AB+BC+CD+DA in terms of b, c, L and h.

$W_{total} =$ ________________ (formula)

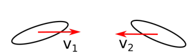
Calculate the total work using the values given in the question:

$W_{total} =$ ________________ (numeric result with units)

Correct Answers:

- `(Fx*i+Fy*j).(Dx*i+Dy*j)`
- `(b*0*i+c*x^2*j).(L*i+0*j)`
- `0 J`
- `[b*0*i+c*(L/2)^2*j].(0+h*j)`
- `702 J`
- `(b*h*i+c*x^2*j).(-L*i+0*j)`
- `-401 J`
- `[b*y*i+c*(-L/2)^2*j].[0+(-h)*j]`
- `-702 J`
- `(b*0*i+c*x^2*j).(L*i+0*j)+[b*0*i+c*(L/2)^2*j].(0+h*j)+(b*h`
- `-401 J`

Problem 2. **2.** (40 points)



In astronomy and astrophysics, the masses are measured by **solar mass unit**, $M_\odot$ or just "sun", which is the mass of our sun. The distances are measured by **light-year** or **ly**, that is, the distance that light travels in one year.

Our home galaxy, the Milkyway galaxy, which is as massive as $M_1 = 1.16 \times 10^{12}$ sun, is roaming with a velocity of $\vec{v}_1 = \left(50.4\ \frac{\mathrm{km}}{\mathrm{s}}\right)\hat{\mathbf{i}}$. On the other hand, our nearest galactic neighbor, the Andromeda galaxy which is $M_2 = 1.52 \times 10^{12}$ sun has a velocity of $\vec{v}_2 = \left(-56\ \frac{\text{km}}{\text{s}}\right)\hat{\mathbf{i}}$. Both Milkyway and Andromeda are on the same collision axis.

Now, the two galaxies are on a collision course into each other, and after approximately 3 billion years, they will collide and merge to become a single galaxy, called "Milkdromeda". The resulting galaxy will be spherical and it is expecteed to have a velocity of v'.

So, write down the formula for the moment of inertia of each galaxy, in terms of radius and mass. Then calculate the numeric value with units.

- You don't need to convert the units into kg, meters and seconds, but the units should be consistent.
- Write the exponent as, for example, **1.23 10^(-38)** or **1.23E-38** for 1.23×10^{-38}.
- Write units without paranthesis and use only one "/". For example **m^3 / kg s^2** means $\text{m}^3/(\text{kg} \cdot \text{s}^2)$

Choose the name and then write down the formula of the conservation law related to the problem:

The name of the conservation law:
- `< Select >`
- Law of Inertia
- `Murphy's Law`
- Second Law of Thermodynamics
- Conservation of Angular Momentum
- `Newton's Second Law of Motion`
- Heisenberg Uncertainty Principle
- Conservation of Linear Momentum
- Gauss Law
- Conservation of Energy

Define the variable in the conservation law:
- `< Select >`
- momentum, p
- energy, E
- time, t
- angular momentum, L
- moment of inertia, I
- magnetic field, B
- lagrangian, L
- position, r

$\equiv$__________ (formula)

Write the formula of that variable for Milkyway galaxy:
____________________ (formula)

Calculate the numeric value with units:
____________________ (numeric result with units)

Write the formula of that variable for Andromeda galaxy:
____________________ (formula)

Calculate the numeric value with units:
____________________ (numeric result with units)

Write the formula of that variable for Milkdromeda galaxy:
____________________ (formula)

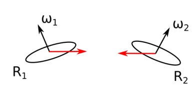
The equation of the law in terms of the variables in the question:
________________________ (equation with a single '=' in terms of M1, M2, v1, v2, and v)

Calculate the **speed** of Milkdromeda galaxy:
$v' =$__________________________ (numeric result with units)

Correct Answers:

- `Conservation of Linear Momentum`
- `momentum, p`
- `M*v`
- `M1*v1`
- `5.85E+13 sun*km/s`
- `M2*v2`
- `-8.51E+13 sun*km/s`
- `(M1+M2)*v`
- `M1*v1+M2*v2 = (M1+M2)*v`
- `38.5 km/s`

Problem 3. 3. (40 points)



In astronomy and astrophysics, the masses are measured by **solar mass unit**, $M_\odot$ or just "sun", which is the mass of our sun. The distances are measured by **light-year** or **ly**, that is, the distance that light travels in one year.

Our home galaxy, the Milkyway galaxy, which is as massive as $M_1 = 1.17 \times 10^{12}$ sun, is rotating with an angular velocity of $\vec{\omega}_1 = \left(7840\ \frac{\text{rad}}{\text{s}}\right)\hat{\mathbf{i}}$. On the other hand, our nearest galactic neighbor, the Andromeda galaxy which is $M_2 = 1.66 \times 10^{12}$ sun is rotating with an angular velocity of $\vec{\omega}_2 = \left(9260\ \frac{\text{rad}}{\text{s}}\right)\hat{\mathbf{j}}$. Both Milkyway and Andromeda are spiral **disk galaxies**.

Now, the two galaxies are on a collision course into each other, and after approximately 3 billion years, they will collide and merge to become a single galaxy, called "Milkdromeda". The resulting galaxy will be spherical and it is expecteed to rotate at an angular velocity of ω'.

One can make an idealization such that the colliding galaxies are homogeneous disks with average radii $R_1 = 44000$ ly and $R_2 = 75500$ ly, repectively. Similarly, Mikdromeda can be thought of a homogeneous sphere with an average radius, $R = 88900$ ly.

So, write down the formula for the moment of inertia of each galaxy, in terms of radius and mass. Then calculate the numeric value with units.

- You don't need to convert the units into kg, meters and seconds, but the units should be consistent.
- Write the exponent as, for example, **1.23 10^(-38)** or **1.23E-38** for 1.23×10^{-38}.
- Write units without paranthesis and use only one "/". For example **m^3 / kg s^2** means $\mathrm{m}^3/(\mathrm{kg}\cdot\mathrm{s}^2)$

Formula of the moment of inertia for Milkyway galaxy:
$I_1 =$________________ (formula)

Calculate the numeric value with units:
$I_1 =$________________ (numeric result with units)

Formula of the moment of inertia for Andromeda galaxy:
$I_2 =$________________ (formula)

Calculate the numeric value with units:
$I_2 =$________________ (numeric result with units)

Formula of the moment of inertia for Milkdromeda galaxy:
$I' =$________________ (formula)

Calculate the numeric value with units:
$I' =$________________ (numeric result with units)

Choose the name and then write down the formula of the conservation law related to the problem:

The name of the conservation law:
- < Select >
- Law of Inertia
- Murphy's Law
- Second Law of Thermodynamics
- Conservation of Angular Momentum
- Newton's Second Law of Motion
- Heisenberg Uncertainty Principle
- Conservation of Linear Momentum
- Gauss Law
- Conservation of Energy

Define the variable in the conservation law:
- < Select >
- momentum, p
- energy, E
- time, t
- angular momentum, L
- moment of inertia, I
- magnetic field, B
- lagrangian, L
- position, r

$\equiv$__________ (formula)

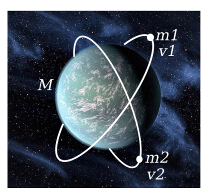
The equation of the law in terms of the variables in the question:
____________________ (equation with a single '=' in terms of M1, M2, R1, R2, R, omega1, omega2, omega)

Calculate the **angular speed** of Milkdromeda galaxy:
$\omega' =$____________________ (numeric result with units)

Correct Answers:

- `0.5*M1*R1^2`
- `1.13E+21 sun*ly^2`
- `0.5*M2*R2^2`
- `4.73E+21 sun*ly^2`
- `0.5*(M1+M2)*R^2`
- `8.07E+21 sun*ly^2`
- `Conservation of Angular Momentum`
- `angular momentum, L`
- `I*omega`
- `0.5*M1*R1^2*omega1+0.5*M2*R2^2*omega2 = 0.6*(M1+M2)*R^2*om`
- `3330 rad/s`

Problem 4. 4. (30 points)



Two satellites are in circular orbits around a planet that has a mass M. One satellite has mass $m_1 = 60$ kg, orbital radius $R_1 = 3.6 \times 10^7$ m, and orbital speed $v_1 = 4750\ \frac{\mathrm{m}}{\mathrm{s}}$. The second satellite has mass $m_2 = 80$ kg and orbital radius $R_2 = 5.6 \times 10^7$ m.

Part A: Orbital Motion
Write the **gravitational force** pulling satellite 1 on the orbit around the planet.
$\vec{F}_1 =$________________ (formula using $\hat{\mathbf{r}}$, G, M, R_1 as **R1** and m_1 as **m1**, only)

Write the **gravitational force** pulling satellite 2 on the orbit around the planet.

$\vec{F}_2 =$______ (formula using $\hat{\mathbf{r}}$, G, M, R_2 as **R2** and m_2 as **m2**, only)

Part B: Accelerations of the satellites
Write the **radial acceleration** of satellite 1 on the orbit around the planet.

$a_1 =$______ (formula using v_1 as **v1** and R_1 as **R1**, only)

Write the **radial acceleration** of satellite 1 on the orbit around the planet.

$a_2 =$______ (formula using v_2 as **v2** and R_2 as **R2**, only)

Part C: Velocity of the satellites
Write the formula for velocity of satellite 1 on the orbit using the formulas on Part A and laws of motion.

$v_1 =$______ (formula using G, M, R_1 as **R1** and m_1 as **m1**, only)

Write the formula for velocity of satellite 2 on the orbit using the formulas on Part A and laws of motion.

$v_2 =$______ (formula using G, M, R_2 as **R2** and m_2 as **m2**, only)

Part D: Mass of the planet
Given $v_1 = 4750\ \frac{\mathrm{m}}{\mathrm{s}}$ and $G = 6.67 \times 10^{-11}\ \mathrm{m}^3\,\mathrm{kg}^{-1}\,\mathrm{s}^{-2}$, calculate the mass of the planet:

$M =$______ (numeric result with unit)

Part E: Velocity of the second satellite
Given $v_1 = 4750\ \frac{\mathrm{m}}{\mathrm{s}}$, calculate the velocity of the secon d satellite:

$v_2 =$______ (numeric result with unit)

Correct Answers:

- `-G*M*m1/(R1^2)*r`
- `-G*M*m2/(R2^2)*r`
- `v1^2/R1`
- `v2^2/R2`
- `sqrt(G*M/R1)`
- `sqrt(G*M/R2)`
- `1.22E+25 kg`
- `5920 m/s`