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# ANALYSIS AND SIMULATION OF A QPSK SYSTEM
# EQ2310 Digital Communications

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# Analysis and Simulation of a QPSK System

EQ2310 Digital Communications, Project Assignment

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## Abstract

In this project, we looked into different aspects on analysis and simulation of a QPSK system over an additive white Gaussian noise (AWGN) channel. Aspects such as bit error probability (BER), synchronization, carrier phase estimation, signal constellation, spectral properties, and realistic implications were studied. By designing MATLAB functions according to relevant principles, in combination with the codes provided, we implemented different transmitter and receiver algorithms to simulate the communication system and investigated their performance.

## 1 Background and Problem Formulation

The QPSK system in this project is considered to consist of five modules: a bit source, transmitter, channel, receiver, and a bit sink. The bits to be transmitted are generated randomly at the bit source, multiplexed with the training sequence. Then, they are mapped onto corresponding QPSK symbols using Gray Coding, followed by an optional pulse shaping filter. In this project, we take both the rectangular pulse shape and the (truncated) root raised cosine pulse into account. The subsequent carrier modulation converts this complex valued baseband signal to a real valued passband signal, through multiplying the baseband signal with a complex carrier of frequency $f_c$ and taking the real part of the product. This process is also known as up-conversion. The output of the transmitter is fed through the AWGN channel. The AWGN channel is the simplest condition. At the receiver, the output of the channel will be filtered by a bandpass filter, down-converted, and sampled to form a discrete-time signal. The discrete signal will be processed by the baseband part which consists of matched filtering (MF), phase estimation, synchronization, sampling, phase correction, and decision. The synchronization and phase estimation are crucial to the demodulation. Finally, the output bits are fed into the bit sink, which will count the bit errors and thus calculate the BER.

We will focus on the following aspects:

(1) the BER performance with perfect phase estimation and synchronization;

(2) the BER performance with realistic phase estimation and synchronization;

(3) the signal constellation with different noise levels at the receiver and the implications from an error in the phase estimation;

(4) the PSD of the transmitted signal with different pulse shapes;

(5) the performance of phase estimation and synchronization for different lengths of the training sequence;

(6) the BER performance of differential QPSK (DQPSK).

# 2 Methodology

## 2.1 BER Performance with Perfect Phase Estimation and Synchronization

We design the functions to achieve perfect synchronization and phase estimation. The *sync*() function determines when to sample the matched filter outputs. The algorithm is based on cross-correlating a replica of the training sequence with the output from the matched filter. The sampling time can be found as

$$t_{samp} = \arg \max_{t_{samp} \in [t_{start}, t_{end}]} |\sum_{k=0}^{L-1} r(kQ + t_{samp}) * c(k)|, \tag{1}$$

where $[t_{start}, t_{end}]$ represents the search window, $L$ is the length of the QPSK training sequence, $r(n)$ is the output of the matched filter, $Q$ is the number of samples per symbol, and $c(n)$ is the symbol-spaced replica of the training sequence.

The phase estimation function *phase_estimation*() is obtained by minimizing the norm of the difference between the QPSK-modulated training sequence and the received training sequence.

We simulate the QPSK system and investigate its BER performance for different SNR ($E_b/N_0$). Then, we compare the simulated result with the theoretical BER, which can be expressed as

$$\text{BER}_{theoretical} = \text{Q}(\sqrt{\frac{2E_b}{N_0}}) - \frac{1}{2}[\text{Q}(\sqrt{\frac{2E_b}{N_0}})]^2. \tag{2}$$

## 2.2 BER Performance with Realistic Phase Estimation and Synchronization

In this problem, we utilize non-ideal synchronization and phase estimation algorithms to simulate. Firstly, we set a range for both sampling time and phase estimation. In the range, we perform simulations to obtain the curves of the system BER variation as a function of sampling time and phase estimation for each SNR, respectively. Then, we select and apply the optimal sampling time and phase estimate corresponding to the lowest BER to obtain the curve of the system BER vs. SNR. Finally, we compare it with the result of perfect estimation. Especially, when we study synchronization, perfect phase estimation is ensured, and vice versa.

## 2.3 Signal Constellation at the Receiver

We plot the constellation of the complex valued received signal for different noise levels, i.e., SNR. Also, we will investigate the effect of an error in phase estimation on the signal constellation. Here, we set the phase estimation to a fixed appropriate value according to the previous problem.

## 2.4 Differential Modulation/Demodulation

Differential modulation means that the phase of the transmitting symbol is generated by the phase of the symbol before it and $\Delta\phi$ -depending on the transmitting bits.

In this section, we choose one of the differential mapping schemes shown as figure below.

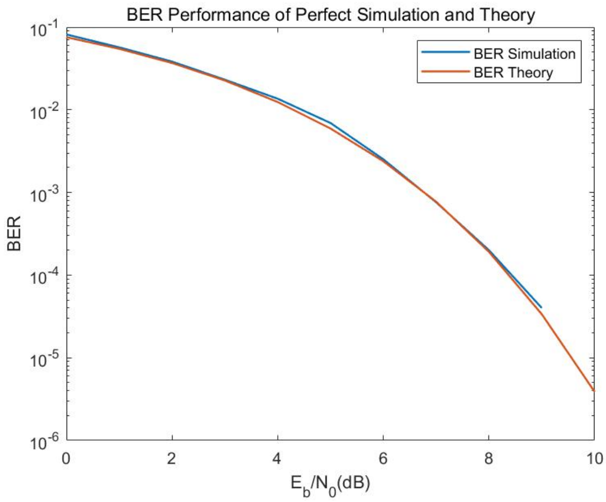



Figure 2: BER Performance of Perfect Simulation and Theory

| transmitting bits this moment | $\Delta\varphi$ $(phase\ difference)$ |
|---|---|
| 00 | $-\frac{3\pi}{4}$ |
| 01 | $\frac{3\pi}{4}$ |
| 10 | $-\frac{\pi}{4}$ |
| 11 | $\frac{\pi}{4}$ |

Figure 1: Differential Modulation

# 3 Results

## 3.1 BER Performance with Perfect Phase Estimation and Synchronization

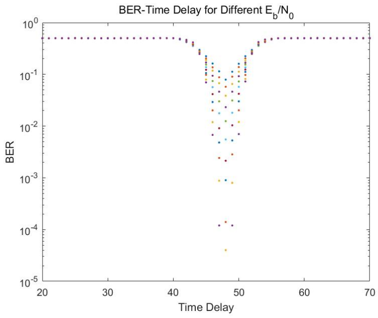
The simulated BER performance with perfect phase estimation and synchronization and the theoretical BER are shown in Figure 2.

We can see that the simulation results are in good agreement with the theoretical results. As the SNR increases, the BER of the communication system decreases. This is because the reliability of the system increases and errors decrease as the signal energy is enhanced.

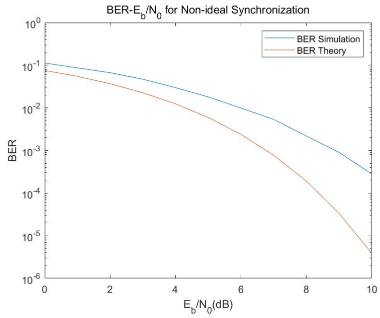
## 3.2 BER Performance with Realistic Phase Estimation and Synchronization

The BER vs. time delay for different SNR with perfect phase estimation is shown in Figure 3(a). We can see that the optimal delay occurs at 48. For non-ideal synchronization, we set the delay to 47 and obtain the curve of the system BER vs. SNR as shown in Figure 3(b), with the phase estimation being perfect.

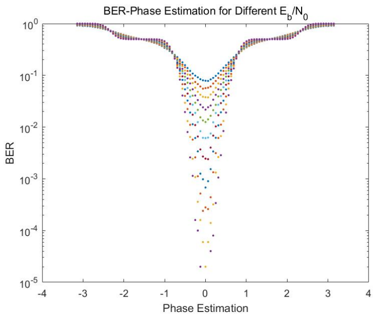
The BER vs. phase estimation for different SNR with perfect synchronization is shown in Figure 3(c). We can see that the optimal phase estimation is around zero. For non-ideal phase estimation, we set the phase error to $0.1\pi$ and



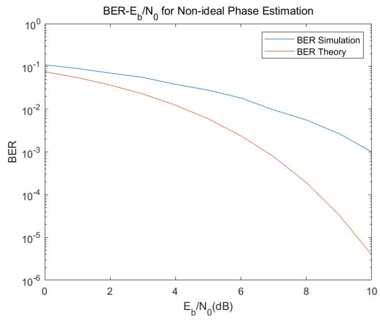

(a) BER-Time Delay for Different SNR




(b) BER-SNR for Non-ideal Synchronization




(c) BER-Phase Estimation for Different SNR




(d) BER-SNR for Non-ideal Phase Estimation

Figure 3: Non-ideal Synchronization and Phase Estimation

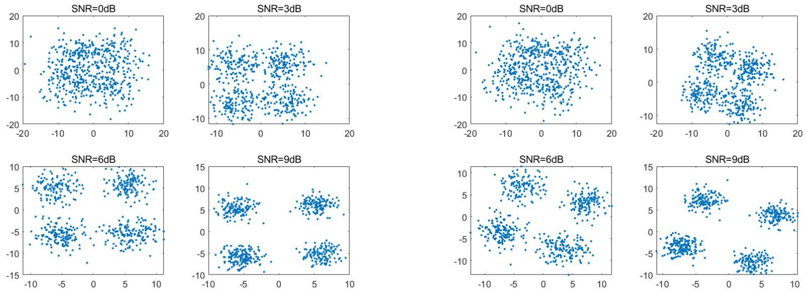
obtain the curve of the system BER vs. SNR as shown in Figure 3(d), in the case of perfect synchronization.

We can see that imperfect synchronization and phase estimation deteriorate the BER performance of the communication system. In Figure 3(a) and Figure 3(c), each curve represents the BER of the corresponding SNR. We can see that the BER minimum points corresponding to each SNR for phase estimation are more dispersed. Therefore, phase estimation is more sensitive to noises. This is also consistent with our theory that the distortion of the phase will cause the rotation of the received signal, which will affect the decision and lead to more errors. The application of DQPSK can avoid this problem, which will be investigated later.

## 3.3 Signal Constellation at the Receiver

The constellations of the received signal with perfect and imperfect phase estimation are shown in Figure 4(a) and Figure 4(b).

We can see that the received symbols are mainly concentrated in the four corners of the constellation. As the SNR increases, the symbols in each corner become more and more concentrated, the noise interferes less and less with the decision, and it becomes easier and easier to decide correctly with smaller and smaller BER. When there is an error in phase estimation, the constellation of the signal is rotated, which makes the decision more prone to errors and leads to an increase in BER. This is because the non-ideal phase estimation is not able to counteract the rotation of the signal in the transmission process, and thus the signal cannot be restored to its original phase.




(a) Constellation for Perfect Phase Estimation

(b) Constellation for Imperfect Phase Estimation

Figure 4: Signal Constellation

## 3.4 Power Density Spectrum with Different Pulse Shapes

The power density spectrum of transmitted signal with a rectangular pulse and a root cosine pulse are shown in Figure 5. We can see from the following figure that compared to the rectangular pulse shape, root cosine pulse shape leads to smaller frequency bandwidth. Also, the amplitude of high frequency components is lower when using the root cosine pulse, which means that the high frequency components are filtered out by the root cosine pulse, unlike the rectangular pulse.

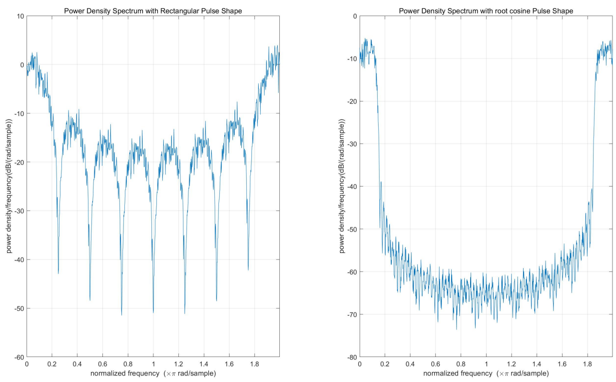
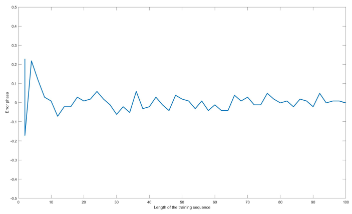


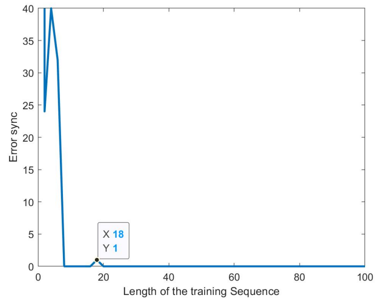

Figure 5: Power Density Spectrum with Different Pulse Shapes

## 3.5 Performance of Phase Estimation and Synchronization for Different Lengths of the Training Sequence

We estimate the phase error and time error for training length from 2 to 100 when SNR is equal to 10dB. From the figure below, we can see that when the training sequence is larger than 14, the phase estimation error becomes stable to a small value. And when the training sequence is larger than 18, the synchronization error becomes 0. Hence, we can conclude that when SNR is 0-20 dB, a training seuquence with length 20 can achieve better phase and synchronization performance and the rest length larger than 20 is useless.



(a) Phase Error for Different Lengths of the Training Sequence




(b) Synchronization Error for Different Lengths of the Training Sequence

Figure 6: Performance of Phase Estimation and Synchronization for Different Lengths of the Training Sequence

### 3.6 Performance of Differential Modulation/Demodulation

We can see from the Figure 7 that the differential modulation/demodulation has worse BER performance in different SNRs compared to the demodulator using an explicit phase estimate shown in Figure 2.

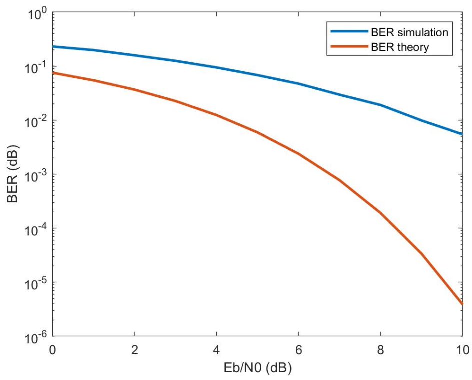



Figure 7: BER performance of the differential demodulator in different SNRs

## 4 Conclusions

In this project, A QPSK communication system is simulated using MATLAB. Aspects such as bit error probability, synchronization, carrier phase estimation, signal constellation, spectral properties, training sequence and practical imperfections due to filtering have been studied. For future studies, we can focus on different modulation formats not only on QPSK. We can investigate the BER performance of 16-QAM, BPSK or ASK.

## 5 References

[1] Upamanyu Madhow, Fundamentals of Digital Communication, Cambridge University Press .