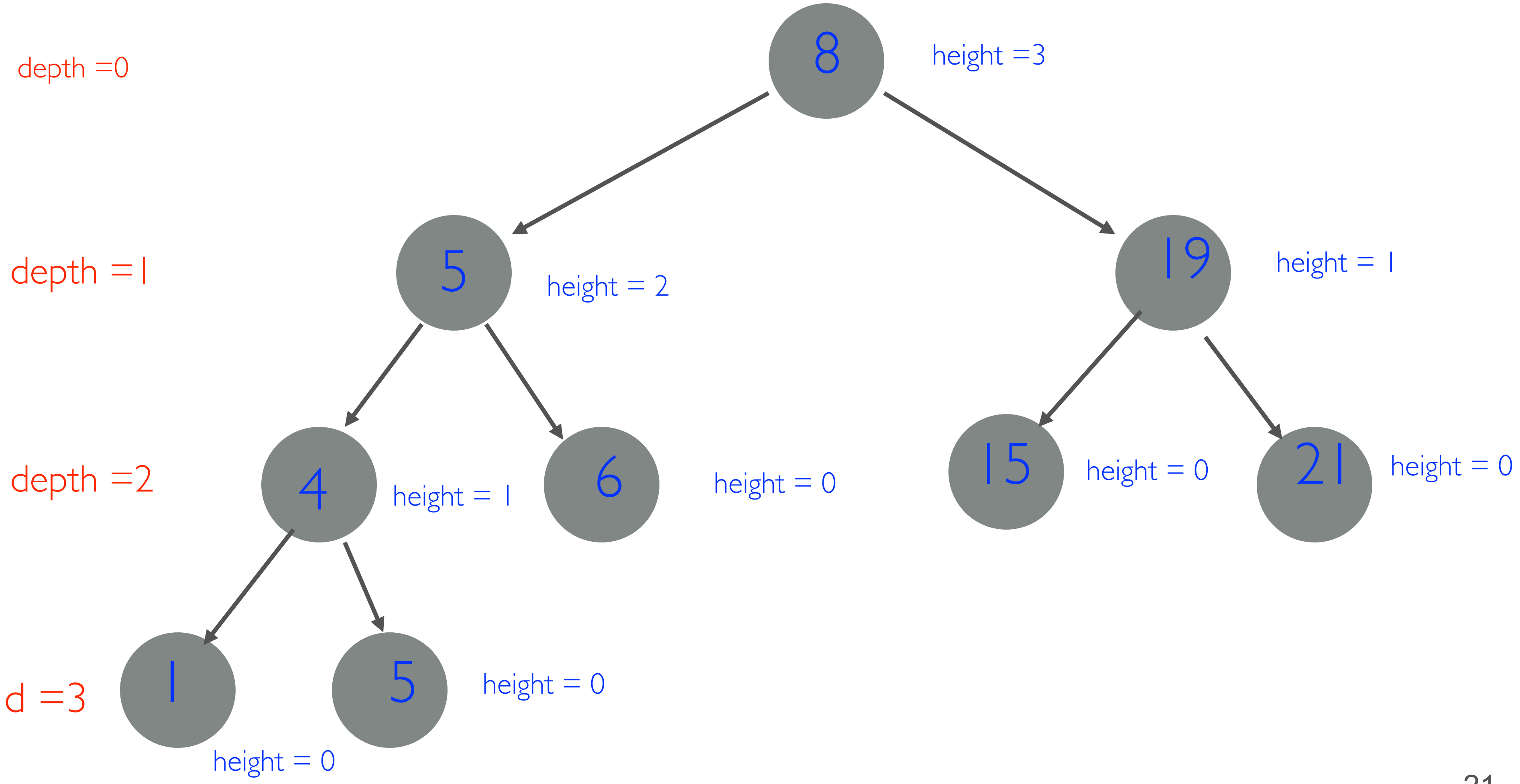
AVL: BST WITH $|H_L - H_R| = 0, 1$



FIBONACCI SERIES & RABBITS

- 2 rabbit beget 2 rabbit every month after one month!
  - $F_K = (F_{K-1} - F_{K-2}) + 2\ F_{K-2} = F_{K-1} + F_{K-2}$
  - with $F_0 = F_1 = 1$
- 1,1 ,2 ,3 ,5 ,8 ,13 ,21 ,.... Fibonacci: $F_k = F_{k-1} + F_{k-2}$
- Bad Recursion: $\Omega(C^N)$ with C = 1.61...
  - Exponential $T(N) = T(N-1) + T(N-2) + 1$
  - $T(N) > F_N > (2)^{N/2}$
- Iteration: $\Theta(N)$
- Math $\Theta(1)$! Try $F_k = x^k$ find two homogeneous solutions

  to characteristic equation.

$$F_k = \frac{1}{\sqrt{5}}[(\frac{1+\sqrt{5}}{2})^{k+1} - (\frac{1-\sqrt{5}}{2})^{k+1}]$$

$$\frac{1+\sqrt{5}}{2} \simeq 1.61803$$

WORST CASE HEIGHT H(N) FOR AVL

- Minimum # of Nodes (see Fig 4.33):

$$N(H) = N(H\text{-}1) + N(H\text{-}2) + 1 > N(H\text{-}1) + N(H\text{-}2)$$

- Almost Fibonacci: $F_k = F_{k-1} + F_{k-2}$

  - So $N(H) > F_H \sim c^H$ with $c = (1 + 5^{1/2})/2 = 1.618034$

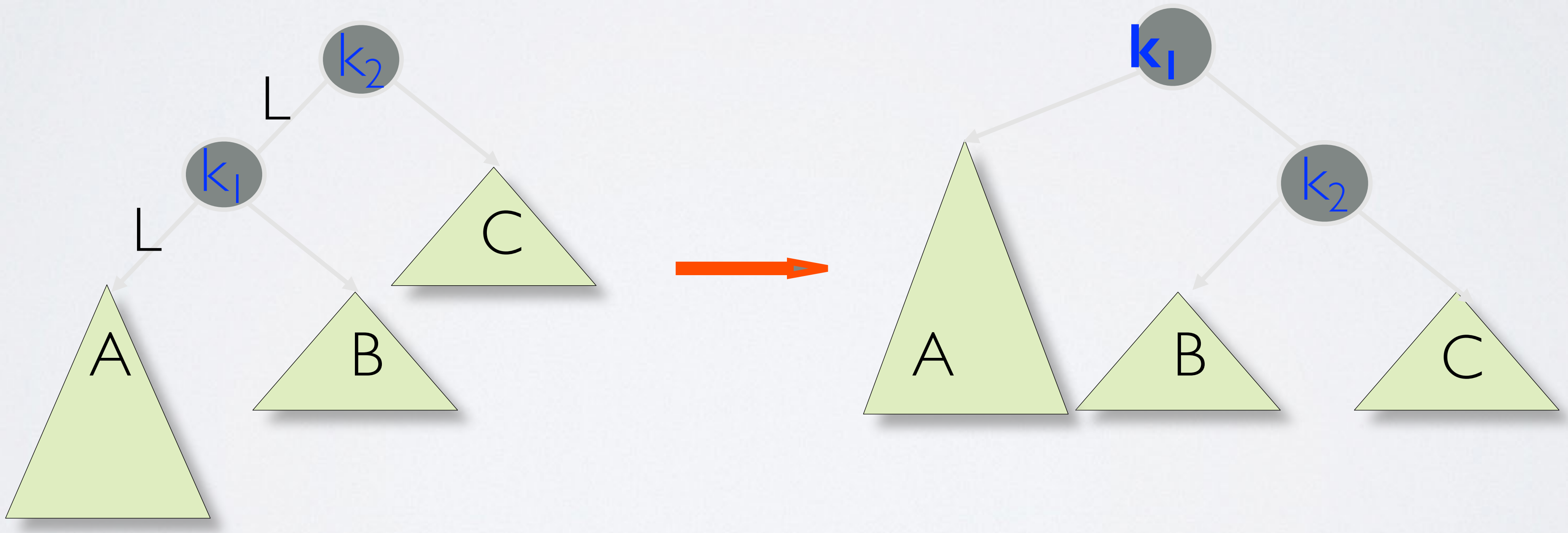
  - Or $H < \log(N)/\log(c) \quad 1.440420 \log_2(N) = 2.078 \ln(N)$

    $= 4.784 \log_{10}(N)$

(Better estimate: $H = 1.44 \log_2(N+2) - 0.328$

ZIG-ZIG INSERTION FOR LL OR RR:

- *Insert New Key along path going Left and Left again into A:*
- *This cause violation of AVL balance.*
- *k_2 is lowest node failing AVL balance.*
- *Single rotation of k_1 ➔ k_2 restores AVL balance*



ZIG-ZAG INSERTION FOR LR

- *Insert New Key along path going Left and then Right into B:*
- *This cause violation of AVL balance.*
- *k_3 is lowest node failing AVL balance.*
- *Double rotation of $k_1 \rightarrow k_2 \rightarrow k_3$ restores AVL blance*