Calculation of Fcl, Fqm and S_entangle explained

Classical Staggered Flippability

$F_{cl} = \frac{1}{N_{pl}} \sum_{p_1,p_2} \langle f_{p_1} f_{p_2} \rangle (-1)^{p_1+p_2}$

Quamtum Staggered Flippability

$F_{QM} = \frac{1}{N_{pl}} \sum_{p_1,p_2} \langle o_{p_1} o_{p_2} \rangle (-1)^{p_1+p_2}$ with $o_p = \sigma_1^+ \sigma_2^- \sigma_3^+ \sigma_4^- + \sigma_1^- \sigma_2^+ \sigma_3^- \sigma_4^+$

Measuring entanglement entropy

We need to rearrange the coefficients(in another word, wave function) of the ground state to a matrix. With the rows of the matrix represent states of the sub system, columns of the matrix represent states of the environment.

To put it explicitly, originally we write every state in the system as

$$|GS\rangle = \sum_i C_i |\psi_i\rangle \tag{original}$$

We separate out a small part (part A) of the system, and treat the rest (part B) of the system as environment.

$$|GS\rangle = \sum_i C_{a,b} |\psi_a\rangle \otimes |\psi_b\rangle \tag{part A seperated}$$

Then we could do a SVD decomposition of the coefficient matrix $C_{a,b}$,

$$C = U^+ D V \tag{SVD}$$

With D being a diagonal matrix. The diagonal matrices are "singular values" λ_i.

The entanglement Entropy can be calculated as:

$$S = -\sum_i \lambda_i^2 \log \lambda_i^2 \tag{Entanglement Entropy}$$

Setting up correspondance between a state number i and the a,b number pair

a is the number the state of the sub system corresponds to. b is the number the state of the environment corresponds to.

As an example: suppose A is site 1,2; environment is site 3-8:

state $|11; 110000\rangle$ corresponds to $i = 15$ and also $a = 3, b = 3$

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