M532 **Spring 2018**
Problem Set 2. Due Tuesday, February 27

Part A: Theory and Foundations

Problem 1

Consider the optimization problem

$$\text{maximize } u^T A v$$

subject to the constraints

$$\|u\|_2 = \|v\|_2 = 1$$

Show that a necessary condition for the solutions is given by the SVD equations.

Problem 2

Let

$$H = I - ee^T/n$$

where e is the column vector of n ones. Let A be an $n \times n$ matrix.

a) Show that $\frac{1}{n}ee^T$ is a projection and hence H is a complementary projection.

b) Provide a geometric interpretation of the action of each of the projections in a).

c) What is the result of applying H to A from the left, i.e., $B = HA$. Show explicitly.

d) What is the result of applying H to A from the right, i.e., $B = AH$. Show explicitly.

e) Let $B = HAH$. Does multiplying HA by H from the right undo the result described in c)? Give a mathematical argument.

Problem 3.

Show that the vector of ones is an eigenvector of B. What is the associated eigenvalue?

Problem 4.

Show that the columns of $\tilde{V}$ where

$$B = \tilde{V}\tilde{V}^T$$

have the property

$$\tilde{V}^T e = 0$$

where e is the vector of ones.

Problem 5.

Using the result from Problem 4., show that the mean value of the configuration of points coming from MDS is the origin.

Part B: Computing

Problem 1.

Download the city data consisting of pairwise distances from Canvas. Make a map using MDS to determine a configuration of points in 2-dimensions. Verify that the mean of the configuration is zero. What can you conclude from the eigenvalues of B?

Problem 2.

Add a new UK city after the fact and put it on your map in red. Comment on the quality of your map both visually and quantitatvely, by comparing the original distance matrix with the one resulting from the one computed from your configuration.