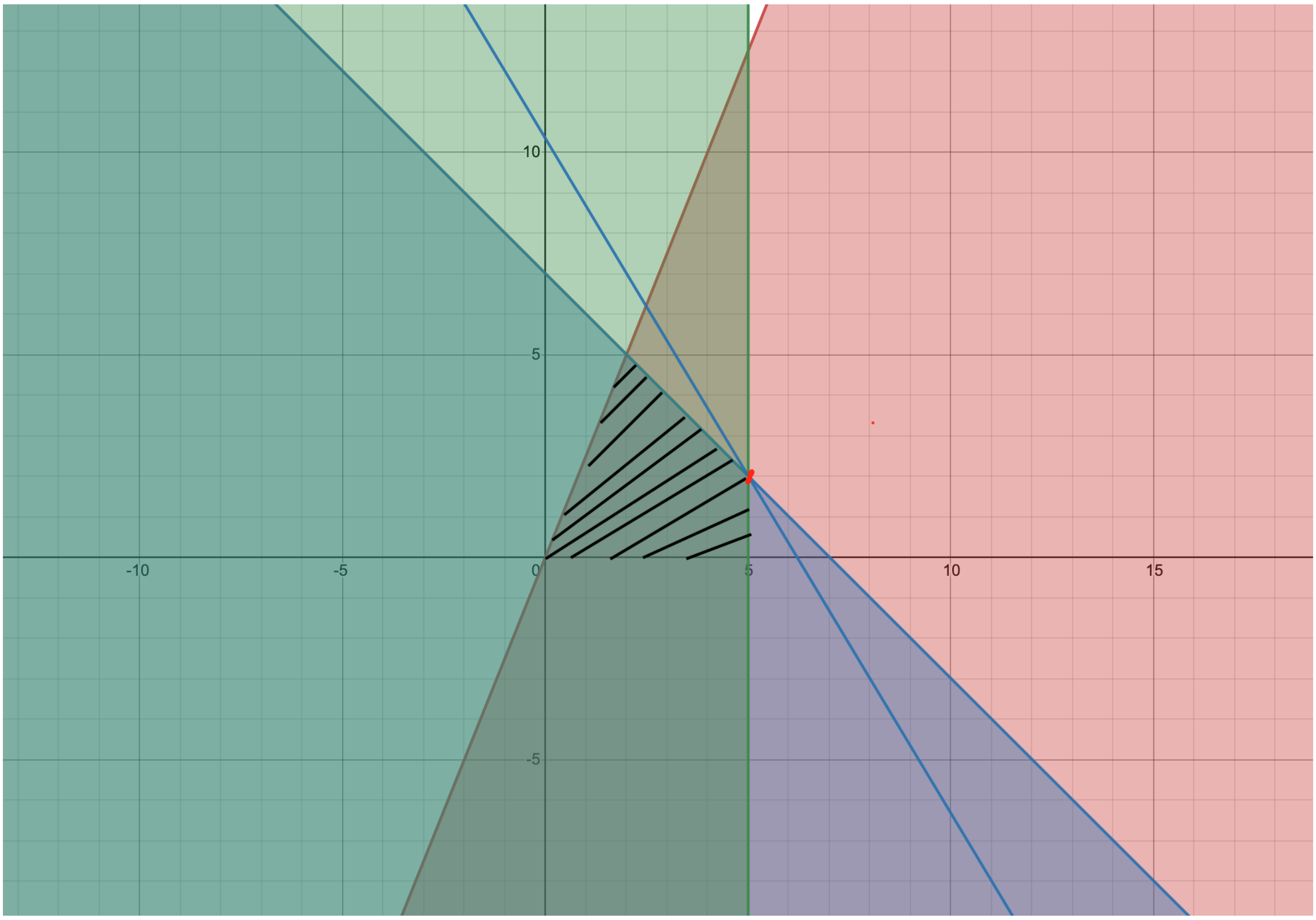
7.1.

Consider the following linear program.

> maximize 5x+3y
>
> 5x − 2y ≥ 0
>
> x+y≤7
>
> x≤5
>
> x≥0
>
> y≥0

Plot the feasible region and identify the optimal solution.



Z=5x+3y, if we view z as a constant, Z=5x+3y is a line whose slope is -3/5.Our line can only go through the black shaded part.We need to find a point in the black shaded part that maximizes the y transverse coordinate of the line. So I find (5,2), When the line passes through (5, 2), the line passes through the boundary point of the shaded part, and makes 5x+3y get a maximum value of 31.

7.3

A cargo plane can carry a maximum weight of 100 tons and a maximum volume of 60 cubic meters. There are three materials to be transported, and the cargo company may choose to carry any amount of each, upto the maximum available limits given below. • Material 1 has density 2 tons/cubic meter,maximum available amount 40 cubic meters,and revenue $1,000 per cubic meter.

• Material 2 has density 1 ton/cubic meter, maximum available amount 30 cubic meters, and revenue $1,200 per cubic meter.

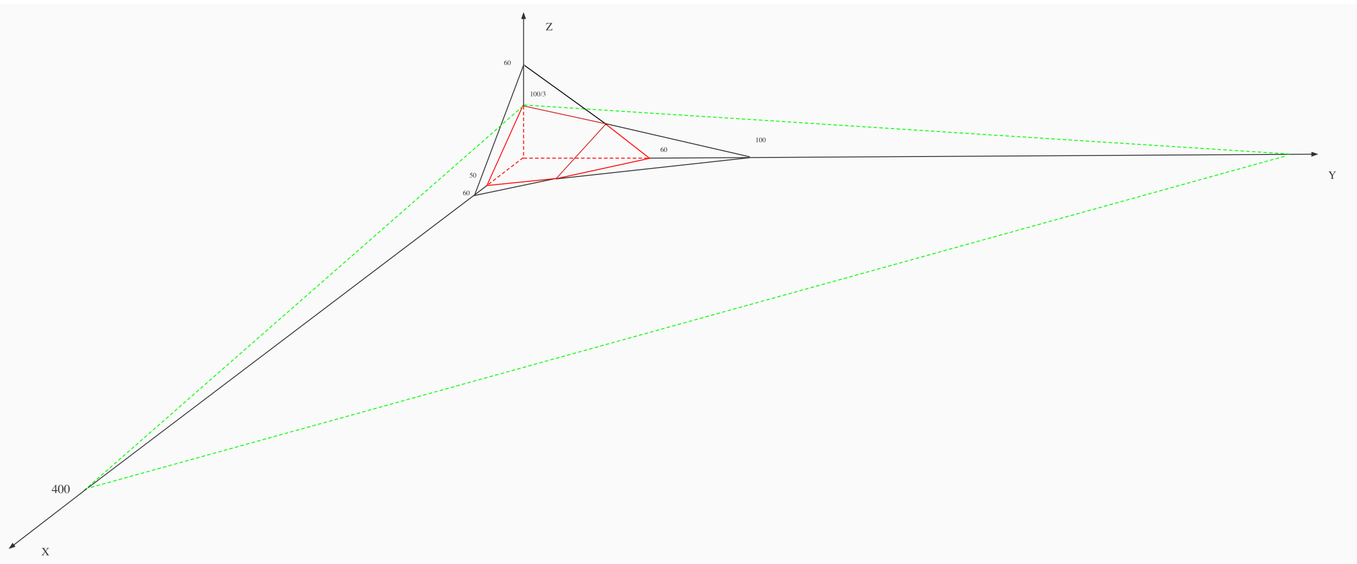
• Material 3 has density 3 tons/cubic meter,maximum available amount 20 cubic meters,and revenue $12,000 per cubic meter.

Write a linear program that optimizes revenue within the constraints.

- According to the definition I wrote following linear program
- x represents the cubic meters of Material 1, y represents the cubic meters of Material 2, z represents the the cubic meters of Material 3

> maximize $1000x+1200y+12000z$
>
> $x+y+z\leq60$
>
> $2x+y+3z\leq100$
>
> $x\geq0$
>
> $y\geq0$
>
> $z\geq0$

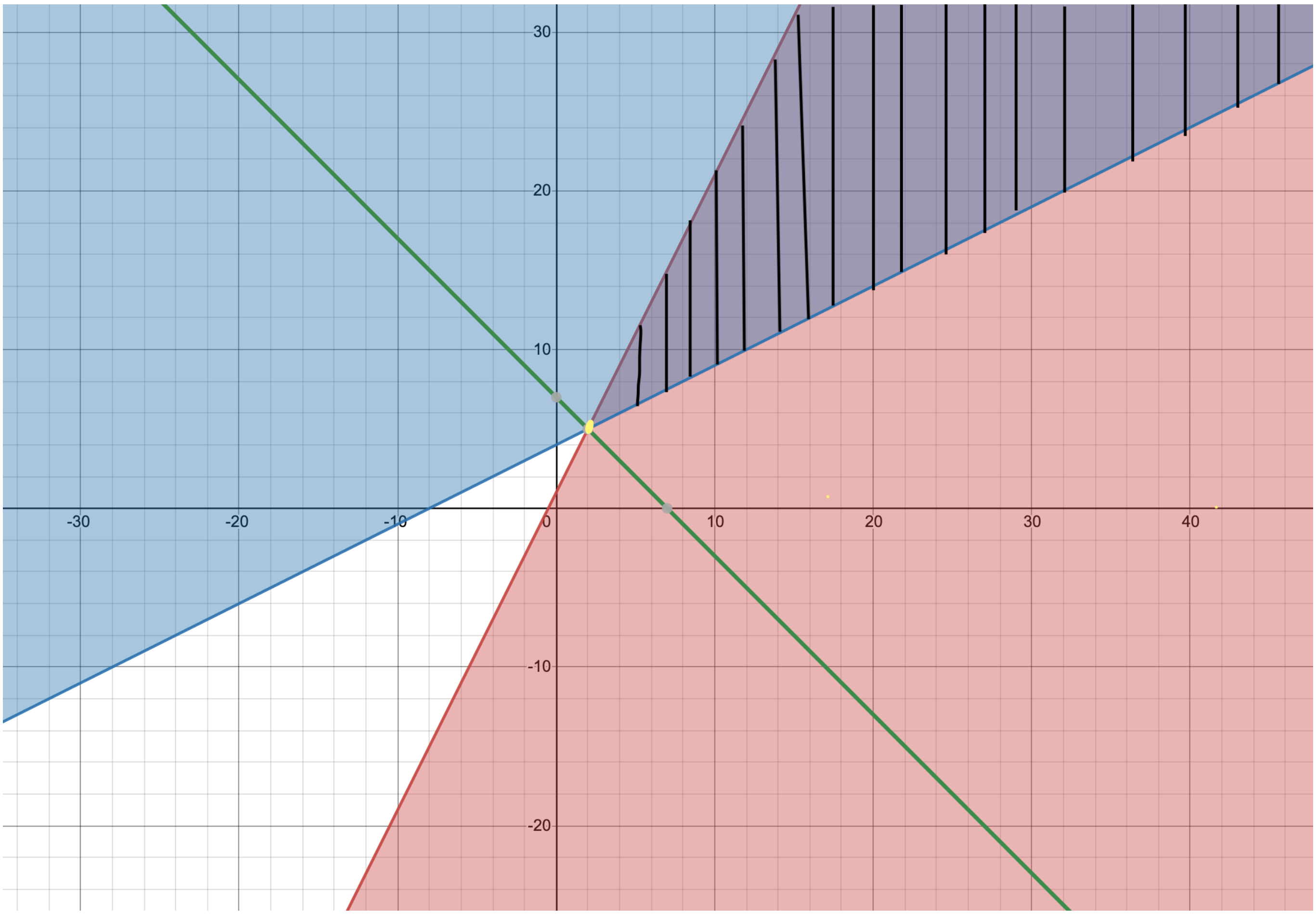
$V=1000x+1200y+12000z$, if we view V as a constant, then $V=1000x+1200y+12000z$ is a plane. And this plane must have at least one point in the space drawn by the red line. I drew this plane with a green dashed line.In theory when it crosses the point (0,0,1), it can make $V=1000x+1200y+12000z$ take the maximum value. But x,y,z can only take integers, so when it crosses the point (0,1,33), the maximum value will be 397200.



7.6.

Give an example of a linear program in two variables whose feasible region is infinite, but such that there is an optimum solution of bounded cost.

> minimize x+y
>
> y≤2x+1
>
> y≥1/2x+4
>
> x≥0
>
> y≥0



feasible region is a two-dimensional space besieged by y≤2x+1 and y≥1/2x+4, it is infinite. But if we want to minimize. There is an optimum solution of bounded cost.Z=x+y, if we view z as a constant, Z=x+y is a line whose slope is -3/5.Our line can only go through the black shaded part.We need to find a point in the black shaded part that minimizes Z=x+y. Apparently, when it go through(2,5), we can get the smallest z, which is 7.

7.16

A salad is any combination of the following ingredients: (1) tomato, (2) lettuce, (3) spinach, (4) carrot, and (5) oil. Each salad must contain: (A) at least 15 grams of protein, (B) at least 2 and at most 6 grams of fat, (C) at least 4 grams of carbohydrates, (D) at most 100 milligrams of sodium. Furthermore, (E) you do not want your salad to be more than 50% greens by mass. The nutritional contents of these ingredients (per 100 grams) are

ingredient	energy (kcal)	protein (grams)	fat (grams)	carbohydrate (grams)	sodium (milligrams)
tomato	21	0.85	0.33	4.64	9.00
lettuce	16	1.62	0.20	2.37	8.00
spinach	371	12.78	1.58	74.69	7.00
carrot	346	8.39	1.39	80.70	508.20
oil	884	0.00	100.00	0.00	0.00

find a linear programming applet on the web and use it to make the salad with the fewest calories under the nutritional constraints. describe your linear programming formulation and hte optimal solution(the quantity of each ingredient and the value). Cite the web resource you used

Solution:

Find solution using Simplex method (BigM method)
MIN Z = 21x1 + 16x2 + 371x3 + 346x4 + 884x5
subject to
0.85x1 + 1.62x2 + 12.78x3 + 8.39x4 >= 15
0.33x1 + 0.2x2 + 1.58x3 + 1.39x4 + 100x5 <= 6
0.33x1 + 0.2x2 + 1.58x3 + 1.39x4 + 100x5 >= 2
4.64x1 + 2.37x2 + 74.69x3 + 80x4 >= 4
9x1 + 8x2 + 7x3 + 508.20x4 <= 100
x2 + x3 <= 50
and x1,x2,x3,x4,x5 >= 0

Solution:
Problem is

Min $Z = 21x_1 + 16x_2 + 371x_3 + 346x_4 + 884x_5$

subject to

$$\begin{array}{l} 0.85x_1 + 1.62x_2 + 12.78x_3 + 8.39x_4 \geq 15 \\ 0.33x_1 + 0.2x_2 + 1.58x_3 + 1.39x_4 + 100x_5 \leq 6 \\ 0.33x_1 + 0.2x_2 + 1.58x_3 + 1.39x_4 + 100x_5 \geq 2 \\ 4.64x_1 + 2.37x_2 + 74.69x_3 + 80x_4 \geq 4 \\ 9x_1 + 8x_2 + 7x_3 + 508.2x_4 \leq 100 \\ x_2 + x_3 \leq 50 \end{array}$$

and $x_1, x_2, x_3, x_4, x_5 \geq 0;$

The problem is converted to canonical form by adding slack, surplus and artificial variables as appropiate

1. As the constraint-1 is of type ' $\geq$ ' we should subtract surplus variable S_1 and add artificial variable A_1
2. As the constraint-2 is of type ' $\leq$ ' we should add slack variable S_2
3. As the constraint-3 is of type ' $\geq$ ' we should subtract surplus variable S_3 and add artificial variable A_2
4. As the constraint-4 is of type ' $\geq$ ' we should subtract surplus variable S_4 and add artificial variable A_3
5. As the constraint-5 is of type ' $\leq$ ' we should add slack variable S_5

6. As the constraint-6 is of type ' $\le$ ' we should add slack variable S_6

After introducing slack,surplus,artificial variables

Min $Z = 21x_1 + 16x_2 + 371x_3 + 346x_4 + 884x_5 + 0S_1 + 0S_2 + 0S_3 + 0S_4 + 0S_5 + 0S_6 + MA_1 + MA_2 + MA_3$

subject to

$$0.85x_1 + 1.62x_2 + 12.78x_3 + 8.39x_4 - S_1 + A_1 = 15$$
$$0.33x_1 + 0.2x_2 + 1.58x_3 + 1.39x_4 + 100x_5 + S_2 = 6$$
$$0.33x_1 + 0.2x_2 + 1.58x_3 + 1.39x_4 + 100x_5 - S_3 + A_2 = 2$$
$$4.64x_1 + 2.37x_2 + 74.69x_3 + 80x_4 - S_4 + A_3 = 4$$
$$9x_1 + 8x_2 + 7x_3 + 508.2x_4 + S_5 = 100$$
$$x_2 + x_3 + S_6 = 50$$

and $x_1, x_2, x_3, x_4, x_5, S_1, S_2, S_3, S_4, S_5, S_6, A_1, A_2, A_3 \ge 0$

Iteration-1		C_j	21	16	371	346	884	0	0	0	0	0	0	M	M	M	
B	C_B	X_B	x_1	x_2	x_3	x_4	x_5	S_1	S_2	S_3	S_4	S_5	S_6	A_1	A_2	A_3	MinRatio $\frac{X_B}{x_5}$
A_1	M	15	0.85	1.62	12.78	8.39	0	-1	0	0	0	0	0	1	0	0	---
S_2	0	6	0.33	0.2	1.58	1.39	100	0	1	0	0	0	0	0	0	0	$\frac{6}{100} = 0.06$
A_2	M	2	0.33	0.2	1.58	1.39	**(100)**	0	0	-1	0	0	0	0	1	0	$\frac{2}{100} = 0.02 \rightarrow$
A_3	M	4	4.64	2.37	74.69	80	0	0	0	0	-1	0	0	0	0	1	---
S_5	0	100	9	8	7	508.2	0	0	0	0	0	1	0	0	0	0	---
S_6	0	50	0	1	1	0	0	0	0	0	0	0	1	0	0	0	---
$Z = 21M$		Z_j	$5.82M$	$4.19M$	$89.05M$	$89.78M$	$100M$	$-M$	0	$-M$	$-M$	0	0	M	M	M	
		$Z_j - C_j$	$5.82M - 21$	$4.19M - 16$	$89.05M - 371$	$89.78M - 346$	$100M - 884 \uparrow$	$-M$	0	$-M$	$-M$	0	0	0	0	0	

Positive maximum Z_j - C_j is $100M$ - 884 and its column index is 5. So, the entering variable is x_5.

Minimum ratio is 0.02 and its row index is 3. So, the leaving basis variable is A_2.

$\therefore$ The pivot element is 100.

Entering $= x_5$, Departing $= A_2$, Key Element $= 100$

+ R_3(new) = R_3(old) ÷ 100

+ R_1(new) = R_1(old)

+ R_2(new) = R_2(old) - $100R_3$(new)

+ R_4(new) = R_4(old)

+ R_5(new) = R_5(old)

+ R_6(new) = R_6(old)

Iteration-2		C_j	21	16	371	346	884	0	0	0	0	0	0	M	M	
B	C_B	X_B	x_1	x_2	x_3	x_4	x_5	S_1	S_2	S_3	S_4	S_5	S_6	A_1	A_3	MinRatio $\frac{X_B}{x_4}$
A_1	M	15	0.85	1.62	12.78	8.39	0	-1	0	0	0	0	0	1	0	$\frac{15}{8.39} = 1.7878$
S_2	0	4	0	0	0	0	0	0	1	1	0	0	0	0	0	---
x_5	884	0.02	0.0033	0.002	0.0158	0.0139	1	0	0	-0.01	0	0	0	0	0	$\frac{0.02}{0.0139} = 1.4388$
A_3	M	4	4.64	2.37	74.69	**(80)**	0	0	0	0	-1	0	0	0	1	$\frac{4}{80} = 0.05 \rightarrow$
S_5	0	100	9	8	7	508.2	0	0	0	0	0	1	0	0	0	$\frac{100}{508.2} = 0.1968$
S_6	0	50	0	1	1	0	0	0	0	0	0	0	1	0	0	---
$Z = 19M + 17.68$		Z_j	$5.49M + 2.9172$	$3.99M + 1.768$	$87.47M + 13.9672$	$88.39M + 12.2876$	884	$-M$	0	-8.84	$-M$	0	0	M	M	
		Z_j - C_j	$5.49M$ - 18.0828	$3.99M$ - 14.232	$87.47M$ - 357.0328	$88.39M$ - 333.7124 $\uparrow$	0	$-M$	0	-8.84	$-M$	0	0	0	0	

Positive maximum Z_j - C_j is $88.39M$ - 333.7124 and its column index is 4. So, the entering variable is x_4.

Minimum ratio is 0.05 and its row index is 4. So, the leaving basis variable is A_3.

$\therefore$ The pivot element is 80.

Entering $= x_4$, Departing $= A_3$, Key Element $= 80$

+ R_4(new) = R_4(old) $\div$ 80

+ R_1(new) = R_1(old) - $8.39R_4$(new)

+ R_2(new) = R_2(old)

+ R_3(new) = R_3(old) - $0.0139R_4$(new)

+ R_5(new) = R_5(old) - $508.2R_4$(new)

+ R_6(new) = R_6(old)

Iteration-3		C_j	21	16	371	346	884	0	0	0	0	0	0	M	
B	C_B	X_B	x_1	x_2	x_3	x_4	x_5	S_1	S_2	S_3	S_4	S_5	S_6	A_1	MinRatio $\frac{X_B}{x_3}$
A_1	M	14.5805	0.3634	1.3714	4.9469	0	0	-1	0	0	0.1049	0	0	1	$\frac{14.5805}{4.9469} = 2.9474$
S_2	0	4	0	0	0	0	0	0	1	1	0	0	0	0	---
x_5	884	0.0193	0.0025	0.0016	0.0028	0	1	0	0	-0.01	0.0002	0	0	0	$\frac{0.0193}{0.0028} = 6.8394$
x_4	346	0.05	0.058	0.0296	(0.9336)	1	0	0	0	0	-0.0125	0	0	0	$\frac{0.05}{0.9336} = 0.0536 \rightarrow$
S_5	0	74.59	-20.4756	-7.0554	-467.4682	0	0	0	0	0	6.3525	1	0	0	---
S_6	0	50	0	1	1	0	0	0	0	0	0	0	1	0	$\frac{50}{1} = 50$
$Z = 14.5805M + 34.3656$		**Z_j**	**$0.3634M + 22.2725$**	**$1.3714M + 11.6542$**	**$4.9469M + 325.5294$**	**346**	**884**	**$-M$**	**0**	**-8.84**	**$0.1049M - 4.1714$**	**0**	**0**	**M**	
		$Z_j - C_j$	$0.3634M + 1.2725$	$1.3714M - 4.3458$	$4.9469M - 45.4706 \uparrow$	0	0	$-M$	0	-8.84	$0.1049M - 4.1714$	0	0	0	

Positive maximum Z_j - C_j is $4.9469M$ - 45.4706 and its column index is 3. So, the entering variable is x_3.

Minimum ratio is 0.0536 and its row index is 4. So, the leaving basis variable is x_4.

$\therefore$ The pivot element is 0.9336.

Entering $= x_3$, Departing $= x_4$, Key Element $= 0.9336$

+ R_4(new) = R_4(old) $\div$ 0.9336

+ R_1(new) = R_1(old) - $4.9469 R_4$(new)

+ R_2(new) = R_2(old)

+ R_3(new) = R_3(old) - $0.0028 R_4$(new)

+ R_5(new) = R_5(old) + $467.4682 R_4$(new)

+ R_6(new) = R_6(old) - R_4(new)

Iteration-4		C_j	21	16	371	346	884	0	0	0	0	0	0	M	
B	**C_B**	**X_B**	**x_1**	**x_2**	**x_3**	**x_4**	**x_5**	**S_1**	**S_2**	**S_3**	**S_4**	**S_5**	**S_6**	**A_1**	**MinRatio** $\frac{X_B}{x_2}$
A_1	M	14.3156	0.0561	1.2145	0	-5.2986	0	-1	0	0	0.1711	0	0	1	$\frac{14.3156}{1.2145} = 11.7874$
S_2	0	4	0	0	0	0	0	0	1	1	0	0	0	0	---
x_5	884	0.0192	0.0023	0.0015	0	-0.003	1	0	0	-0.01	0.0002	0	0	0	$\frac{0.0192}{0.0015} = 12.7807$
x_3	371	0.0536	0.0621	**(0.0317)**	1	1.0711	0	0	0	0	-0.0134	0	0	0	$\frac{0.0536}{0.0317} = 1.6878 \rightarrow$
S_5	0	99.6251	8.5651	7.7779	0	500.7023	0	0	0	0	0.0937	1	0	0	$\frac{99.6251}{7.7779} = 12.8088$
S_6	0	49.9464	-0.0621	0.9683	0	-1.0711	0	0	0	0	0.0134	0	1	0	$\frac{49.9464}{0.9683} = 51.5832$
$Z = 14.3156M + 36.8008$		**Z_j**	**$0.0561M + 25.0973$**	**$1.2145M + 13.0971$**	**371**	**$-5.2986M + 394.7032$**	**884**	**$-M$**	**0**	**-8.84**	**$0.1711M - 4.7802$**	**0**	**0**	**M**	
		$Z_j - C_j$	$0.0561M + 4.0973$	$1.2145M - 2.9029 \uparrow$	0	$-5.2986M + 48.7032$	0	$-M$	0	-8.84	$0.1711M - 4.7802$	0	0	0	

Positive maximum Z_j - C_j is $1.2145M$ - 2.9029 and its column index is 2. So, the entering variable is x_2.

Minimum ratio is 1.6878 and its row index is 4. So, the leaving basis variable is x_3.

$\therefore$ The pivot element is 0.0317.

Entering $= x_2$, Departing $= x_3$, Key Element $= 0.0317$

+ R_4(new) = R_4(old) $\div$ 0.0317

+ R_1(new) = R_1(old) - $1.2145R_4$(new)

+ R_2(new) = R_2(old)

+ R_3(new) = R_3(old) - $0.0015R_4$(new)

+ R_5(new) = R_5(old) - $7.7779R_4$(new)

+ R_6(new) = R_6(old) - $0.9683R_4$(new)

Iteration-5		C_j	21	16	371	346	884	0	0	0	0	0	0	M	
B	C_B	X_B	x_1	x_2	x_3	x_4	x_5	S_1	S_2	S_3	S_4	S_5	S_6	A_1	**MinRatio** $\frac{X_B}{S_4}$
A_1	M	12.2658	-2.3216	0	-38.2739	-46.2935	0	-1	0	0	**(0.6835)**	0	0	1	$\frac{12.2658}{0.6835} = 17.9444 \rightarrow$
S_2	0	4	0	0	0	0	0	0	1	1	0	0	0	0	---
x_5	884	0.0166	-0.0006	0	-0.0472	-0.0536	1	0	0	-0.01	0.0008	0	0	0	$\frac{0.0166}{0.0008} = 19.7$
x_2	16	1.6878	1.9578	1	31.5148	33.7553	0	0	0	0	-0.4219	0	0	0	---
S_5	0	86.4979	-6.6624	0	-245.1181	238.1578	0	0	0	0	3.3755	1	0	0	$\frac{86.4979}{3.3755} = 25.625$
S_6	0	48.3122	-1.9578	0	-30.5148	-33.7553	0	0	0	0	0.4219	0	1	0	$\frac{48.3122}{0.4219} = 114.5$
$Z = 12.2658M + 41.7003$		Z_j	**$-2.3216M + 30.7807$**	**16**	**$-38.2739M + 462.4854$**	**$-46.2935M + 492.6927$**	**884**	**$-M$**	**0**	**-8.84**	**$0.6835M - 6.0051$**	**0**	**0**	**M**	
		$Z_j - C_j$	$-2.3216M + 9.7807$	0	$-38.2739M + 91.4854$	$-46.2935M + 146.6927$	0	$-M$	0	-8.84	$0.6835M - 6.0051 \uparrow$	0	0	0	

Positive maximum $Z_j - C_j$ is $0.6835M - 6.0051$ and its column index is 9. So, the entering variable is S_4.

Minimum ratio is 17.9444 and its row index is 1. So, the leaving basis variable is A_1.

$\therefore$ The pivot element is 0.6835.

Entering $= S_4$, Departing $= A_1$, Key Element $= 0.6835$

+ R_1(new) = R_1(old) $\div 0.6835$

+ R_2(new) = R_2(old)

+ R_3(new) = R_3(old) - $0.0008R_1$(new)

+ R_4(new) = R_4(old) + $0.4219R_1$(new)

+ R_5(new) = R_5(old) - $3.3755R_1$(new)

+ R_6(new) = R_6(old) - $0.4219R_1$(new)

Iteration-6		C_j	21	16	371	346	884	0	0	0	0	0	0	
B	**C_B**	**X_B**	**x_1**	**x_2**	**x_3**	**x_4**	**x_5**	**S_1**	**S_2**	**S_3**	**S_4**	**S_5**	**S_6**	**MinRatio**
S_4	0	17.9444	-3.3965	0	-55.9933	-67.7257	0	-1.463	0	0	1	0	0	
S_2	0	4	0	0	0	0	0	0	1	1	0	0	0	
x_5	884	0.0015	0.0023	0	0	0.0035	1	0.0012	0	-0.01	0	0	0	
x_2	16	9.2593	0.5247	1	7.8889	5.179	0	-0.6173	0	0	0	0	0	
S_5	0	25.9259	4.8025	0	-56.1111	466.7679	0	4.9383	0	0	0	1	0	
S_6	0	40.7407	-0.5247	0	-6.8889	-5.179	0	0.6173	0	0	0	0	1	
$Z = 149.4578$		**Z_j**	**10.3846**	**16**	**126.2419**	**85.9953**	**884**	**-8.7852**	**0**	**-8.84**	**0**	**0**	**0**	
		$Z_j - C_j$	-10.6154	0	-244.7581	-260.0047	0	-8.7852	0	-8.84	0	0	0	

Since all $Z_j - C_j \le 0$

Hence, optimal solution is arrived with value of variables as :
$x_1 = 0, x_2 = 9.2593, x_3 = 0, x_4 = 0, x_5 = 0.0015$

Min $Z = 149.4578$

I use the this calculator . https://cbom.atozmath.com/CBOM/Simplex.aspx?q=sm&q1=5%606%60MIN%60Z%60x1%2cx2%2cx3%2cx4%2cx5%6021%2c16%2c371%2c346%2c884%600.85%2c1.62%2c12.78%2c8.39%2c0%3b0.33%2c0.2%2c1.58%2c1.39%2c100%3b0.33%2c0.2%2c1.58%2c1.39%2c100%3b4.64%2c2.37%2c74.69%2c80%2c0%3b9%2c8%2c7%2c508.20%2c0%3b0%2c1%2c1%2c0%2c0%60%3e%3d%2c%3c%3d%2c%3e%3d%2c%3e%3d%2c%3c%3d%2c%3c%3d%6015%2c6%2c2%2c4%2c100%2c50%60%60D%60false%60true%60false%60true%60false%60false%60true&do=1#PrevPart