Renormalizing Galaxy Bias

for Robust Galaxy Large-Scale Structure Models

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1. What is galaxy bias?

- **Galaxy bias** reflects the statistical relationship between observed galaxy distribution and the underlying dark matter.
- It enables **full-shape galaxy statistics** modeling without needing detailed knowledge of galaxy formation processes.

bias parameter — stochasticity

$$\delta_g(\boldsymbol{x},\tau) = \sum_{O} b_O(\tau) O(\boldsymbol{x}) + \varepsilon(\boldsymbol{x},\tau) + \varepsilon_O(\boldsymbol{x},\tau) O(\boldsymbol{x})$$

bias operator

(all the **local gravitational observables** up to a given order, can be computed via perturbation theory)

2. Problem Statement

- **Challenge:** Higher-order operators in the galaxy bias expansion are unregulated, and contributing significantly on large scales (***Fig. 1***).
- **Goal**: Develop a renormalization method to regulate these contributions, ensuring the bias expansion convergent.

3. Key Principle of Renormalization

- On sufficiently large scales, galaxy statistics must be dominated by the **leading-order terms**. Contributions from **higher-order operators** must vanish.

4. Renormalized Operator

- We add **counterterms** to the original operators to remove the large-scale divergences:

$$[\![O^{(n)}]\!] = O^{(n)} - \sum_{\tilde{O}} Z^{O}_{\tilde{O}} \tilde{O}^{(n-2)} \quad \text{for } n \geq 3,$$

such that the $(n-1)$-point correlation function

$$\lim_{k_i \to 0} \left\langle [\![O^{(n)}]\!](\boldsymbol{k}) \delta^{(1)}(\boldsymbol{k}_1) \cdots \delta^{(1)}(\boldsymbol{k}_{n-2}) \right\rangle \to 0$$

- We compute all the coefficients $Z^{O}_{\tilde{O}}$ up to **fourth order** with **Standard Perturbation Theory**, sufficient for one-loop **galaxy power spectrum** and **bispectrum** predictions.

$Z^{O}_{\tilde{O}}$		$\tilde{O}$: δ	δ^2	$\mathcal{G}_2$
O	δ^2	$\frac{68}{21}\sigma^2 - \frac{2}{3}{\sigma_2}^2$	$\frac{8126}{2205}\sigma^2 - \frac{283}{105}{\sigma_2}^2 + \frac{2}{5}{\sigma_4}^2$	$\frac{254}{2205}\sigma^2 - \frac{34}{35}{\sigma_2}^2 + \frac{4}{15}{\sigma_4}^2$
	δ^3	$3\sigma^2$	$\frac{68}{7}\sigma^2 - 2\sigma_2^2$	
	$\delta\mathcal{G}_2$	$-\frac{4}{3}\sigma^2$	$-\frac{376}{105}\sigma^2 + \frac{8}{15}\sigma_2^2$	$\frac{116}{105}\sigma^2 - \frac{8}{15}\sigma_2^2$
	δ^4		$6\sigma^2$	
	$\delta^2\mathcal{G}_2$		$-\frac{8}{3}\sigma^2$	σ^2
	$(\mathcal{G}_2)^2$		$\frac{32}{15}\sigma^2$	$\frac{8}{15}\sigma^2$
	$\delta\mathcal{G}3$			$-\sigma^2$
	$\delta\Gamma_3$		$\frac{16}{15}\sigma^2$	$-\frac{2}{5}\sigma^2$

Table 1: The matrix of scale-independent coefficients for the counterterms in the renormalized operators. These results are valid in Eulerian perturbation theory up to fourth order. Empty entries represent zero values.

Table 1 is all you need for implementing *renormalized galaxy bias expansion*!

5. Perturbative Forward Model of Galaxy Large-scale Structure

$\{\Omega_m, \Omega_\Lambda, h, \sigma_8, \cdots\}$ cosmological parameter

$\delta^{(1)}$ initial linear matter density perturbation

↓ GridSPT [1,2,3]

$\{\delta^{(1)}(z), \cdots, \delta^{(n)}(z)\}$ evolved matter density perturbation field at each order

↓ ***Table 1***

$[\![O^{(n)}]\!] = O^{(n)} - \sum_{\tilde{O}} Z^{O}_{\tilde{O}} \tilde{O}^{(n-2)}$ renormalized bias operator at each order

↓

$\delta_g^{(n)} = \sum_{[\![O]\!]} b_O [\![O^{(n)}]\!]$ galaxy number density perturbation field at each order

↓

P_{gg}, B_{ggg} galaxy power spectrum and bispectrum

6. Numerical Validation of Renormalized Operators

- We generate original and renormalized operators from the **perturbative forward model** and measure their contribution to **galaxy-matter power spectrum** up to one-loop order.

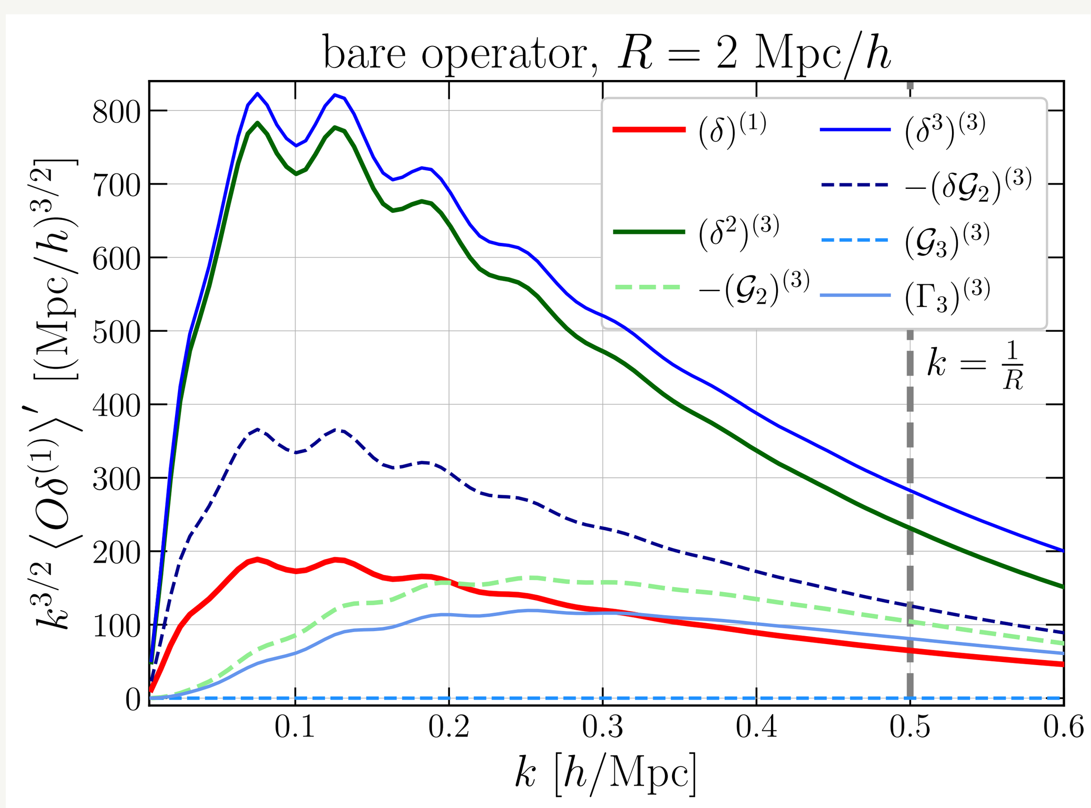

Fig.1 Before Renormalization

- The power spectrum receives significant contributions from both **leading-order** and higher-order bias operators (**green** and **blue**).
- **Unregulated higher-order operators** could bypass the leading-order operators on large scales.

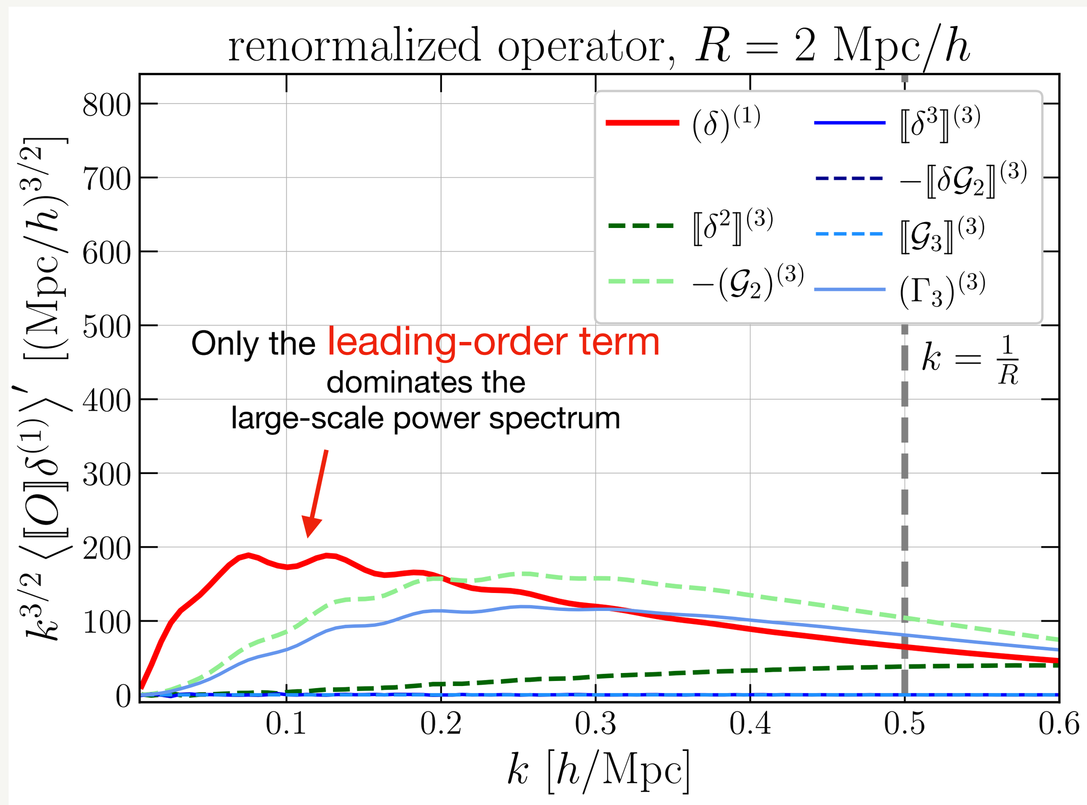

Fig. 2 After Renormalization

- Only the **leading-order term** dominates the large-scale power spectrum.
- **Higher-order operators** (**green** and **blue**) become subdominant on large scales but remain relevant on smaller scales.

We also numerically validate the well-regulate behavior of **bispectrum** with renormalized operators.

7. Discussion & Conclusion

- **Counterterms** systematically remove unregulated contributions from original bias operators, ensuring convergence of galaxy statistics from renormalized operators.
- We **validate** the counterterms by confirming that renormalized operators exhibit **vanishing statistics** in the large-scale limit.
- The results in ***Table 1*** differ from previous work[4] due to the proper treatment of the **smoothing filter** in constructing bias operators.
- By applying this **perturbative forward modeling tool**, we enable **field-level inference** in upcoming galaxy surveys, allowing us to better constrain cosmological parameters and address fundamental questions in cosmology.

References

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