Numerical Calculation of Compositeness

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I. CALCULATION ON DEUTERON

In this section, we will calculate compositeness with a new perspective as an application of our theoretical consideration. To calculate the compositeness of deuteron, we need to take some experiment values. We take binding energy as 2.2MeV, and effective range expansion factors $a = 5.424$ fm, $r_0 = 1.759$ fm, $r_1 = 0.040\text{fm}^3$ [1]. Even though we have much more accurate values on deuteron binding energy, we take 2.2MeV for simplicity because it will not influence the analysis since we are trying to account for the problem of deuteron compositeness above one. We consider a model with isospin symmetry and nucleon mass is defined to be 938.9185 MeV.

We choose the loop function to be relativistic dimensional regularized loop function [2], with the form of

$$\begin{aligned} G &= -i(4M^2)\int \frac{d^4q}{(2\pi)^4}\frac{1}{(P-q)^2-M^2+i\epsilon}\frac{1}{q^2-M^2+i\epsilon} \\ &= \frac{4M^2}{16\pi^2}\{a_\mu + \frac{q}{E}(2ln(E^2+2qE)-2ln(-E^2+2qE))\} \end{aligned} \tag{1}$$

The substraction constant a_μ is determined through a mapping with 3-momentum cutoff scheme at threshold

$$a_\mu = -\frac{M}{2\pi^2}\Lambda \tag{2}$$

We choose the cutoff to be 150 MeV. We need to note that the loop function in 3-momentum cutoff scheme is non-relativistic since a relativistic one has to be described by elliptic functions. However, we expect no difference between them at threshold where we perform the comparision.

A. Low Energy Expansion

The first model we consider is low energy expansion model. To get the low energy constants, we perform a mapping between the theoretical potential and experimental effective range expansion.

$$1 - \frac{qM}{2\pi}\frac{1}{\frac{1}{v^{eff}} - G} = S = 1 + \frac{2iq}{qcot\delta - iq} \tag{3}$$

, in which $v^{\text{eff}} = c_0 + c_1q^2 + ...$ as model input and $qcot\delta = -\frac{1}{a} + \frac{1}{2}r_0q^2 + r_1q^4 + ...$ from effective range expansion.

We can first look at the outcome of the 0th order, namely the constant interaction. At this order, as long as the sign of the interaction is correctly chosen, a pole will always exist. Due to mathematical structure, this pole is always of compositeness one. By fixing this constant to scattering length from the experiment, the binding energy is 1.477 MeV compared with 2.2 MeV from experiment. Our pole has a lower binding energy, but with a reasonable compositeness.

Since the attraction is not enough for reasonable deuteron energy at the 0th order, the 1st order also needs to be attractive, which aligns with the 1st order interaction factor coming from effective range. By adding this order, the binding energy is modified to 2.232 MeV, which is close to the experiment. However, with the improvement in binding energy, the compositeness turns out to be 1.479. Compositeness is defined as the proportion of a state composed by composite states and thus should be lower than one to be reasonable. This outcome can be interpreted by considering a theory with an explicit deuteron pole. An explicit deuteron pole can be related with positive elementariness, and the effective range and deuteron energy indicate a similar potential structure but with negative factor, which will result in negative elementariness, and thus higher than one compositeness.

B. One Pion Exchange Potential

We then calculated One Pion Exchange Potential(OPEP), which is normally written as,

$$\begin{aligned} V_\pi(r) = \frac{m_\pi^2}{12\pi}(\frac{g_A}{\sqrt{2}f_\pi})^2(\tau_1\cdot\tau_2)[S_{12}(1+\frac{3}{m_\pi r}+\frac{3}{(m_\pi r)^2}) \\ +\sigma_1\cdot\sigma_2]\frac{e^{-m_\pi r}}{r} - \frac{4\pi}{3}\sigma_1\cdot\sigma_2\delta^3(r) \end{aligned} \tag{4}$$

in coordinate space. In this equation, g_A and f_π are constants about interaction, m_π is pion mass, and τ is pauli matrix representing isospin and σ is pauli matrix for spin. We need to note that S_{12} is tensor force term with interaction only between channels with $\Delta L = 2$, and thus is irrelevant in deuteron which is dominated by

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S wave component. Besides, in the case of deuteron, we have already known that $S=1$ and $T=0$, thus $\sigma_1 \cdot \sigma_2 = 1, \tau_1 \cdot \tau_2 = -3$. Additionally, the term with $\delta^3(r)$ will finally result in a constant in momentum space, and can be absorbed in the low energy constant since the leading order of Chiral purterbation theory is constant+OPEP [3], we can safely ignore this term. Simplifying the OPEP potential with the information above, we have

$$V_\pi(r) = c_{opep}\frac{e^{-m_\pi r}}{r} \tag{5}$$

in which $c_{opep} = -\frac{m_\pi^2}{4\pi}(\frac{g_A}{\sqrt{2}f_\pi})^2$. performing Fourier transformation, we have

$$V_\pi(q_\pi) = c_{opep}\frac{4\pi}{m_\pi}\frac{1}{q_\pi^2+m_\pi^2} \tag{6}$$

Transform it into S wave componet, since S wave dominates in deuteron,

$$V_\pi(q) = c_{opep}\frac{\pi}{2q^2}ln\frac{4q^2+m_\pi^2}{m_\pi^2} \tag{7}$$

Since c_{opep} is negative, this interaction is always attractive, and can result in a higher binding energy. By combining it with the constant term, we will get compositeness $X = 1.870$.

C. Yamaguchi Potential

We need to note that the potential in the previous subsection is not strictly a one pion exchange potential in a field theory sense. It is because we projected it into the S wave and the potential expresses a apure momentum transfer. On the other hand, the pion exchange should finally generate pion loops. The pion momentums ought to be integrated through, and the case calculated here is a low momentum approximation instead of an on-shell approximation from field theory point of view. It, however, shows a possibility that can be exhibited from the t channel dynamics.

To further fix this problem, we considered Yamaguchi Potential, which is defined as a modification of form factor as

$$f(q) = \frac{\beta^2}{q^2+\beta^2} \tag{8}$$

in which, q is COM frame momentum, and β is an energy scale related with finite range interaction.

By fixing scattering length and effective range to experiment in a $V = c_0 + c_1 q^2$ theory, we find that compositeness increases as binding energy increases. The compositeness will reach 1 at $\beta = 336MeV$ with binding energy $B = 2.01MeV$. Either further increasing β or change the theory will result in a higher than 1 compositeness according to the calculation above. Since we are aiming at a finite range interaction, we will include further terms $V = c_0 + c_1 q^2 + c_2 q^2$. To reproduce the binding energy, the compositeness turns to be $X = 1.21$. Thus, finite range interaction can be a reason of unreasonable compositeness.

D. Coupled Channels

We then go to two channel coupling case, at next to next to leading order (NNLO), the interaction is given by[4]

$$V = \begin{pmatrix} c_0 + c_1 q^2 + c_2 q^4 & c_{SD,1}q^2 + c_{SD,2}q^4 \\ c_{SD,1}q^2 + c_{SD,2}q^4 & c_{DD}q^4 \end{pmatrix} \tag{9}$$

Since we don't expect any elementariness in crossing channels, i.e. no energy dependence should exist in crossing channels, we transform the T matrix, G matrix and potential through

$$\overset{*}{T} = P^{-1}TP^{-1} \tag{10}$$

$$\overset{*}{V} = P^{-1}TP^{-1} \tag{11}$$

$$\overset{*}{G} = PGP \tag{12}$$

in which

$$P = \begin{pmatrix} 1 & 0 \\ 0 & q^2 \end{pmatrix} \tag{13}$$

We have more parameters than we need, but when the pole gets deeper, the compositeness in the S channel will get higher for any change parameter. Examples of outcome are shown in the Table. Thus, we can not achieve reasonable compositeness in a 2 channel LEC model up to NNLO.

Finally, we tried to use square well potential. The potential is defined as

$$V(r) = \begin{cases} A & r < R \\ 0 & r > R \end{cases} \tag{14}$$

The fourier transformation of it is

$$V(q) = \frac{4\pi A}{q}(-\frac{1}{q}Rcos(Rq) + \frac{1}{q^2}sin(Rq)) \tag{15}$$

The parameters are fixed as $r_{well} = 1fm$. If we fix the scattering length to experimental value, we will have binding energy $B = 1.472MeV$ and compositeness $X = 1.046$. By changing the parameter of this potential, the 0 th order expansion which is related with scattering length is also influenced. We further include a core with $r_{core} = 1fm$, and fix the scattering length. We have a free parameter to move, but we found that as long as the binding energy increases, the compositeness will increase. It is shown that square well potential can not give a reasonable compositeness.

E. Discussion on Outcome of Numerical Calculation

We found that as long as the scattering length, or scattering property at threshold, is fixed, all of these models shares the same behavior that any attempt to deepen the deuteron pole will result in a higer compositeness than 1. This outcome aligns with the outcome from weak binding limit which is 1.67. Besides, the expansion is made with regard to momentum, and higher-order terms are expected to have much less influence on the state. Any model will have a projection on this LEC model, which means that any model, as long as it follows an analytical expansion around threshold, should not be able to reproduce deuteron pole with a reasonable compositeness.

Since all of the interactions discussed up to now except for OPEP potential comes from S channel, this energy dependece can be accounted for by an inclusion of t channel interaction like meson exchange interaction or u channels.

In the calculation of OPEP potential, we noticed an increase in compositeness. It means that unphysical compositeness can be a result of finite range interaction. However, this interaction ought to be a dynamical component inside this channel without generating any new states, and thus should not induce any change in compositeness and elementariness.

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