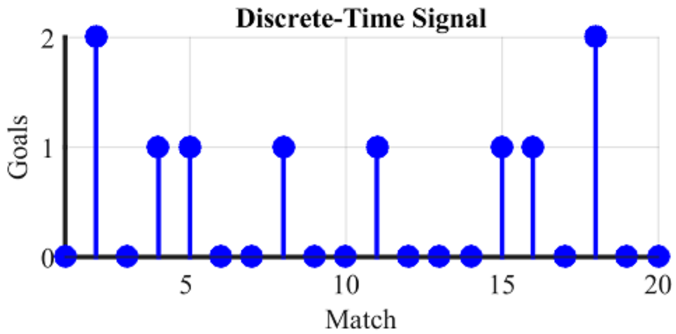
Signal Processing Assignment- 1 Monsoon 2023

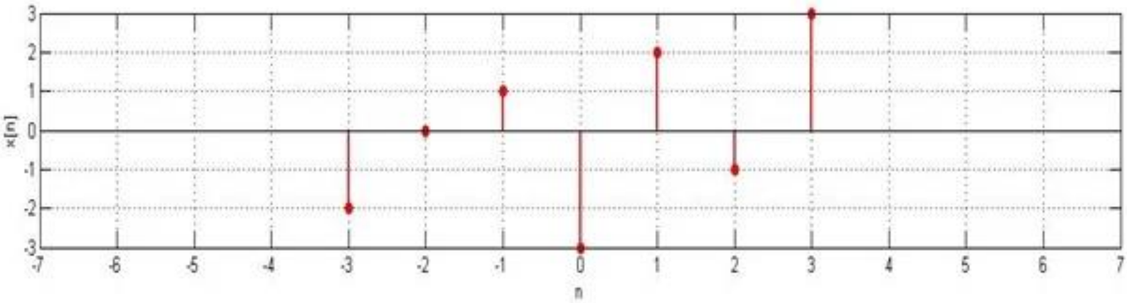
1. Consider the following sequence which is the number of goals scored in 20 matches played by a team.



   a. Is there a time point over which this sequence is symmetric? Explain.
   b. Write the mathematical representation for the “Goals” sequence in terms of unit sample functions.
   c. What is the average number of goals scored by this team over first 10 matches and all matches?
   d. Can the answers to part c be found using a system whose unit sample response is a unit step function? Explain.

2. A signal y[n] is the output of a system whose input is x[n] = u[n]. Write y[n] in terms of u[n] if
   a. y[n] = 1 for $n \leq 0$ and y[n] = 0 otherwise.
   b. y[n] = -1 for odd values of n and y[n] = 1 for even values of n.
   c. y[n] = δ[n].
   d. y[n] = n for $n \geq 0$ and y[n] = 0 otherwise.
   e. y[n] = 1 for $4 \leq n \leq 10$ and y[n] = -3 for $11 \leq n \leq 16$

3. Given the sequence x[n] below find the result of the following operations.



   a. x[2n-3]

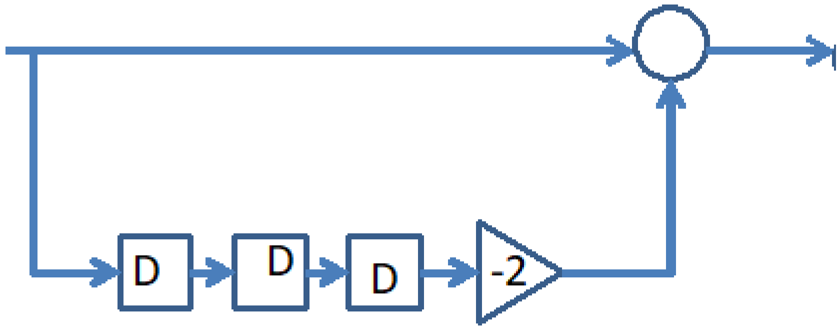
   b. x[1-2n]

   c. x[n-1]δ[n-2]

   d. -0.5x[n+4]

4. In the block diagram shown below the square blocks labelled as D indicate unit Delay elements and triangle indicates a scalar and the circle denotes a summer.

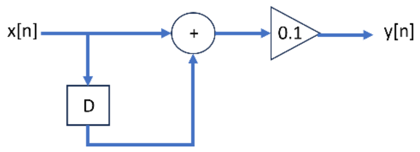
Note: this notation is slightly different from the one used in the class.

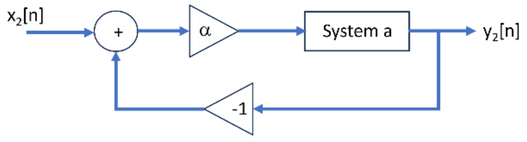


a. Write the difference equation for this system.
b. Use the operator notation and express the relation between Y and X.
c. Find y[n] if x[n] is (i) $\delta[n]$ and (ii) as given in Question 3.
d. Find an alternate implementation for this system and state its advantages over the given one.

Signal Processing **Assignment- 2** Monsoon 2023

1. For the 2 block diagrams given below, write the corresponding ordinary difference equations (ODE) and operator forms for each of them.





System a is the system in the block diagram on the left.

2. Consider these first order systems: (i) $Y = \frac{R}{1-\alpha R}X$, (ii) $Y = \frac{1-\beta R}{1-\alpha R}X$
   a. Write the ODE for these systems.
   b. Draw the block diagram for these systems.
   c. Find and sketch the impulse response of these systems.
   d. Summarise your learning from the above answers. Specifically, state the advantage, if any, between these systems and the standard *accumulator* in terms of system behaviour.

3. Using convolution, find and sketch the response of the following systems.
   a. $x[n] = h[n] = u[n]$
   b. $x[n] = (\frac{1}{2})^n u[n]; h[n] = u[n]$.
   c. $x[n] = (\frac{1}{2})^n u[n]; h[n] = sin\frac{n\pi}{25}(u[n] - u[n-10])$.
   d. Identify the function performed by the systems in parts a through c.

4. Correlation is a popular signal processing operation. Given two real sequences $f[m]$, $g[m]$ the cross-correlation function $r_{f,g}[l]$ is defined as

$$r_{f,g}[l] = \sum_{m=-\infty}^{\infty} f[l+m]g[m];\ l \text{ is called the lag parameter}$$

   a. Find the cross-correlation function if $f[n]$ is a unit height sequence starting at n = 0 and is of length 4, and $g[n] = \delta[n+1]-2\delta[n]+\delta[n-1]$.
   b. When f[m] = g[m] in the above equation the operation is called autocorrelation. Find the autocorrelation functions $r_{f,f}[l]$ and $r_{g,g}[l]$. What can you observe from the results?
   c. Since correlation is a measure of similarity, what information do the cross-correlation and autocorrelation functions help extract?
   d. The correlation operation is very similar to the convolution operation. Derive the relation between the two.

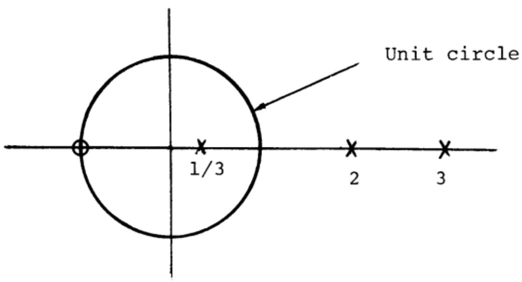
Signal Processing **Assignment- 3** Monsoon 2023

1. If $X(z) = \frac{1}{1+0.5z^{-1}}$; $|z| \geq 0.5$, then find the Z-transform of the following signals.
   a. $y_1[n] = x[3-n] + x[n-3]$
   b. $y_2[n] = (\frac{1}{2})^n x[n-2]$
   c. $y_4[n] = x[n-2] * x[2-n]$
   d. Derive the Z-transform of $n\, x[n]$ and use it to find the Z-transform of the signal $y_3[n] = (1 + n + n^2)x[n]$, where X(z) is as given above.

2. A signal $x[n] = (\frac{1}{2})^n u[n]$ has a Z-transform X(z). Then find the signals with the following Z-transforms.
   a. $Y_1(z) = z\, X(\frac{1}{z})$
   b. $Y_2(z) = (\frac{z-1}{z})X(z)$
   c. $Y_3(z) = X(z)\, X(\frac{1}{z})$
   d. $Y_1(z) = z^2 \frac{dX(z)}{dz}$

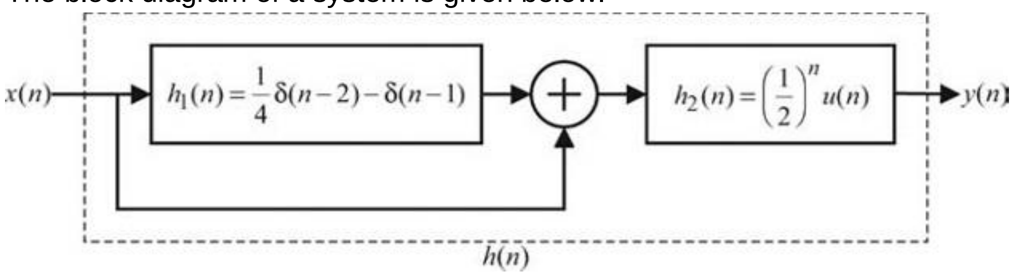
3. $X(z)$ has a zero at origin, and poles at $z = 3\ and\ \frac{1}{3}$. If x[n] is known to be a double-sided signal find X(z), its ROC, and its inverse x[n]. Can you generalize this result?

4. The pole -zero locations of a system is as shown below.



   a. If the system function H(z) is known to converge for |z| =1 find the ROC and state if h[n] is left/right/double sided.
   b. It is unknown if H(z) converges for |z| =1. How many different ROCs are possible, in this case? Pick one, if any, that results in (i) a stable and causal system, (ii) a stable but not causal and (iii) a causal but unstable system.

5. The block diagram of a system is given below.



a. Find the impulse response h[n] for the overall system using Z-transforms.
b. Locate the poles and zeros of H(z) and based on their location state if this system is stable and / or causal.
c. What is the signal processing function performed by this system?

Assignment-4

Q1. Let the signals $x_1[n] = a^n u[n]$ and $x_2[n] = (-1)^n a^n u[n]$ then,

1. Compute DTFT $X_1(e^{j\omega})$ and $X_2(e^{j\omega})$ of the signals.
2. Plot the magnitude and phase spectrums of $X_1(e^{j\omega})$ and $X_2(e^{j\omega})$.
3. Compare the spectrums of $X_1(e^{j\omega})$ and $X_2(e^{j\omega})$ and obtain a mathematical relationship between $X_1(e^{j\omega})$ and $X_2(e^{j\omega})$.
4. From the relationship, please deduce a property of DTFT.

Hint: Express $(-1)^n$ in terms of complex exponential signal form.

Q2. Prove the following DTFT properties.

- **Time shifting:** $DTFT(x[n-k]) = e^{-j\omega k} X(e^{j\omega})$
- **Frequency shifting**: $DTFT\left(e^{-j\omega_0 n} x[n]\right) = X(e^{j(\omega+\omega_0)})$
- **Convolution**: $DTFT(x_1 * x_2[n]) = X_1(e^{j\omega}) X_2(e^{j\omega})$
- **Multiplication**: $DTFT(x_1[n] x_2[n]) = X_1(e^{j\omega}) * X_2(e^{j\omega})$
- **Differentiation in frequency**: $DTFT(nx[n]) = j\frac{dX(e^{j\omega})}{d\omega}$
- **Parseval's relation**: $\sum_{n=-\infty}^{\infty} |x[n]|^2 = \frac{1}{2\pi}\int_{2\pi} |X(e^{j\omega})|^2\, d\omega$

Q3. Let $\delta(\omega) = 0\ for\ \omega \neq 2\pi k, k\ is\ an\ integer$ and $\int_{2\pi} \delta(\omega) d\omega = 1$ then prove the following properties.

- $\int_{2\pi} \delta(\omega) X(\omega) d\omega = X(0)$
- $\int_{2\pi} \delta(\omega - \omega_0) X(\omega) d\omega = X(\omega_0)$
- $\delta(\omega - \omega_0) X(\omega) = X(\omega_0)\delta(\omega - \omega_0)$

Q4. Can the DTFT of arbitrary causal signal result zero phase spectrum? Please explain it briefly.

Q5. Let $x[n] = \delta(n+3) - \delta(n+1) + 2\delta(n) + 3\delta(n-1)$ with DTFT as $X(e^{j\omega}) = X_R(e^{j\omega}) + jX_I(e^{j\omega})$ then

- Compute $\int_{-\pi}^{\pi} X(e^{j\omega})\, d\omega$
- Find the signal whose DTFT is $X_R(e^{j\omega}) e^{j2\omega} + jX_I(e^{j\omega})$

Assignment-6

Q1. State and prove the following DFT properties.

- **Time reversal**
- **Frequency shifting**
- **Complex conjugate**
- **Multiplication**
- **Symmetry properties**
- **Parseval's relation**

Q2. Prove the identity: $\sum_{l=-\infty}^{\infty} \delta[n + lN] = \frac{1}{N}\sum_{k=0}^{N-1} e^{j\frac{2\pi}{N}kn}$

Q3. Let $x[n] = \{\underset{\uparrow}{1}, \ 2, 3, 6\}$ then

- Compute 6-point DFT of $x[n]$ and is represented as $X(k)$. Comment on the relation between $X(1)$ and $X(5)$; $X(2)$ and $X(4)$
- Compute 6-point DFT of $x[n-10]$ and is represented as $Y(k)$; What is relation between $Y(k)$ and $X(k)$;
- Obtain $y[n]$ by computing IDFT of $Y(k)$; What is the relation between $y[n]$ and $x[n]$?
- Find DFT of $x[n]\cos\left(\frac{2\pi k_0 n}{N}\right)$ in terms of $X(k)$; here k_0 is an integer constant.

Q4. $X(k)$ is DFT of $x[n]$, whose values are non-zero $0 \le n \le N-1$ else zero. Let $Y(k) = X(k), 0 \le k \le L, N-L \le k \le N-1$ and zero $L < k < N-L$; $y[n]$ is IDFT of $Y(k)$ then how $y[n]$ can be obtained directly from $x[n]$, explain it clearly.

Q5. Let $x[n]$ values are non-zero $0 \le n \le N-1$ else zero, $y[n] = x[n] + x\left[n + \frac{N}{2}\right], 0 \le n \le N-1$ else zero and $Y(k)$ is $\frac{N}{2}$ point DFT of $y[n]$ then what is the relation between $Y(k)$ and $X(k)$?

<u>Assignment-6</u>

Q1. Describe the overlap-save method of performing linear convolution of long data sequences. Compare its merits and demerits with overlap-add method.

Q2. Considering the complex multiplications and complex additions resulted from radix-2 FFT algorithm, discuss the effectiveness of computing linear convolution using circular convolution. Please provide necessary quantitative number of computations in the discussion.

Q3. Describe and derive the radix-2 decimation-in-frequency (DIF) FFT algorithm and compare it with radix-2 decimation-in-time (DIT) FFT algorithm.

Q4. Show that the W_N^{kn} can be computed recursively as follows: $W_N^{kn} = W_N^k W_N^{k(n-1)}$

Q5. A designer has eight-point FFT chips. Show explicitly how three such chips can be interconnected to compute a 24-point DFT.

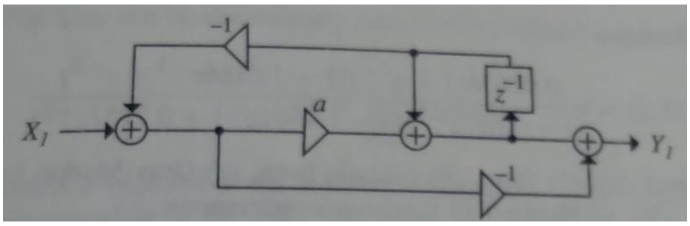
Signal Processing **Assignment- 7** Monsoon 2023

1. Given a *lowpass* FIR filter H(z) determine what happens if
    a. z is replaced by -z
    b. z is replaced by z^{-1}
    c. z is replaced by z^2

2. An FIR filter is described by the difference equation below.
   $y[n] = \sum_{k=0}^{10} (\frac{1}{2})^{|5-k|} \, x[n-k]$
   a. Draw the block diagram for a *direct* implementation of this filter.
   b. Draw the block diagram for a *linear phase* implementation of this filter.
   c. Derive H(Ω) for part b and prove that the implementation is linear phase.

3. An FIR filter is antisymmetric, i.e. $h[n]$ – $h[M\text{-}1\text{-}n]$, $0 \le n \le M\text{-}1$, where M is <u>even</u>.
   a. Show that the amplitude response of this filter is given as
      $H(\Omega) = \sum_{n=1}^{M/2} d[n] \sin\left[\left(n - \frac{1}{2}\right)\Omega\right]$ where d[n] is related to h[n].
   b. Find $H\left(e^{j\frac{\pi}{2}}\right)$ *and* $H(0)$. Give an application for this filter.

4. A digital filter is shown below.



   a. Derive the filter transfer function H(z). Plot the poles and zeros, assuming a = 0.5
   b. What kind of filter is this?

5. The ideal magnitude response of a lowpass filter is given below.

$$\left|H\left(e^{j\Omega}\right)\right| = 2 \quad 0 \le \Omega < \frac{\pi}{6}$$
$$= 1 \quad \frac{\pi}{6} \le \Omega < \frac{\pi}{3}$$

   a. Find the impulse response of this filter.
   b. Design a practical filter of order M = 10 using a Hann window. Plot the magnitude response and compare against the ideal response.

Signal Processing Monsoon 2023
Assignment- 8

1. Given an analog prototype filter $H(s) = \frac{A}{s+\alpha}$, show that the digital IIR filter obtained using the impulse invariance method is given by $H(z) = \frac{A}{1-e^{-\alpha T}z^{-1}}$. Are these filters causal?

2. The following digital filter was obtained from an analog prototype using the impulse invariance method. What is the corresponding analog filter H(s)?
$$H(z) = \frac{2z}{z - e^{-0.9}} + \frac{3z}{z - e^{-1.2}}$$

3. Aliasing can be exploited to realise interesting frequency response characteristics. An ideal, causal analog LPF has a cutoff frequency Ω_c. Two digital filters (H_1 and H_2) are designed from this using $t = nT; T = \frac{3\pi}{2\Omega_c}$ and $\frac{\pi}{\Omega_c}$ respectively. Assume the filters to be normalized so that $H_1(\Omega=0) = H_2(\Omega=0) = 1$. Consider a new filter G which is constructed by connecting H_1 and H_2 in parallel. Find $G(z)$ and identify if it is LPF/HPF, etc.

4. A first order LPF (normalized) analog filter is given as $H(s) = \frac{1}{s+1}$. Design a digital *bandpass* filter using this analog prototype, *bilinear* and LP to BP filter transformations to meet the following specifications: Passband 200-300 Hz; Sampling frequency 2 kHz; Filter order M = 2.
Draw the pole-zero plot for your design.

5. IIR filters can also be designed by placing zeros and poles at suitable locations as the frequency response will vanish (equal 0) at the frequencies where zeros are located while the response peaks at the locations of the poles
    a. If it is true that closer the pole is to the unit circle, the larger the peak in the response, what is the equivalent statement for zero placement?
    b. Assume sampling frequency to be 500Hz and design a BPF to meet these specifications: total signal blocking at dc and 250 Hz; narrow passband centred at 125 Hz; 3 dB bandwidth of 10 Hz. Determine H(Z) and draw a block diagram of your filter.