Q1 consider the signal

$$x[n] = a^n u[n] \quad |a|<1$$

spectrum of this signal is sampled at $\omega = \frac{2\pi k}{N}$.

Determine reconstructed spectra at $N = 5, 50$. comment trend of this wrt N.

Q2 prove the Identity

$$\sum_{l=-\infty}^{\infty} \delta[n + lN] = \frac{1}{N} \sum_{k=0}^{N-1} e^{\frac{2\pi j k n}{N}}$$

Q3 If $X(k)$ is the N point DFT of $x[n]$, what is the N point DTFT of $s(n) = X(n)$, $0 \le n \le N-1$. Derive

Q4 compute 8 point DFT of sequence

$$x(n) = \begin{cases} 1, & 0 \le n \le 7 \\ 0, & \text{otherwise} \end{cases}$$

By decimation in frequency FFT. Also comment on number of multiplication and addition used.

$x[n] = n a^n u[n]$

$X(z) = az^{-1} + 2a^2 z^{-2} + 3 a^3 z^{-3} + \cdots$

$az^{-1} X(z) = a^2 z^{-2} + 2a^3 z^{-3} + 3a^4 z^{-4} \cdots$

$\cancel{az^{-1}X(z) - X(z)}$

$X(z)(1 - az^{-1}) = az^{-1} + a^2 z^{-2} + a^3 z^{-3} + \cdots$

$X(z)(1 - az^{-1}) = \dfrac{az^{-1}}{(1-az^{-1})^2}$

1) $x[n] = \underline{n a^n u[n]}$

2) $X(z) = \log(1 + az^{-1})$

$\dfrac{dX(z)}{dz} = \dfrac{\quad z^{-2}}{1+az}$

3) Auto correlation seq

$x[n] = a^n u(n)$

$-1 < a < 1$

find ROC.

3) Autocorrelation seq of $x[n] = a^n u[n]$

$\sum x_1(\ldots) x$

$R_{xx}(x) = \sum_{k=-\infty}^{\infty} x_1[k]\, x_2[n-k] = X_1(z) X_2(z)$

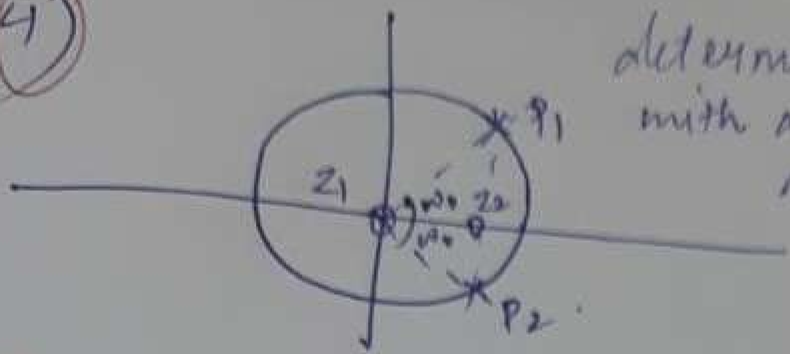
$R_{xx}[x] = \sum_{k=-\infty}^{\infty} a^k u[k]\, a^{n-k} u[n-k]$

$\sum_{k=0}^{n} a^n \qquad \sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} x_1[k]\, x_1[n-k] z^{-n}$

$n - k = t$

$\sum^{\infty} x_1[n-k] z^{-n}$

4)



determine $X(z)$ with a scaling factor of 4. also find $x[n]$.

5) $$H(z) = \frac{3-4z^{-1}}{1-3.5z^{-1}+1.5z^{-2}}$$

Specify the ROC for $H(z)$ & determine $h[n]$ for.

1) System is stable
2) System is causal.
3) System is anticausal.

1) a) $\left(\frac{1}{2}\right)^{n-1} u[n-1]$ b) $\left(\frac{1}{2}\right)^{|n-1|}$.

c) $\delta[n-1] + \delta[n+1]$ d) $\delta[n+2] - \delta[n-2]$

2) $$|X(e^{j\omega})| = \begin{cases} 1 & 0 \le |\omega| < \pi/4 \\ 0 & \frac{\pi}{4} \le |\omega| \le \pi \end{cases}$$

$\measuredangle X(e^{j\omega}) = \frac{-3\omega}{2}$.

→ at what values of n, $x[n] = 0$.

1) $x[n] = (-1)^{|n|+1} a^{|n|}$

$x[n] = x_1[n] + x_2[n]$

a) compute DTFT $X_1(e^{j\omega})$ & $X_2(e^{j\omega})$

b) plot the magnitude & the phase spectrum

c) compare the spectrums of $X_1(e^{j\omega})$ & $X_2(e^{j\omega})$ find a mathematical relation between them.

2) If $x_1[n] \xleftrightarrow{F} X_1(\omega)$

& $x_2[n] \longleftrightarrow X_2(\omega)$

then prove $\Rightarrow$ $\sum_{n=-\infty}^{\infty} x_1[n] x_2^*[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X_1(\omega) X_2^*(\omega) \, d\omega$

(Q.)

$$x(n) = \begin{cases} (-1)^n, & -\infty < n < \infty \\ 0, & \text{otherwise} \end{cases}$$

find DTFT.

Values of ω at which $X(e^{j\omega})$ is periodic.

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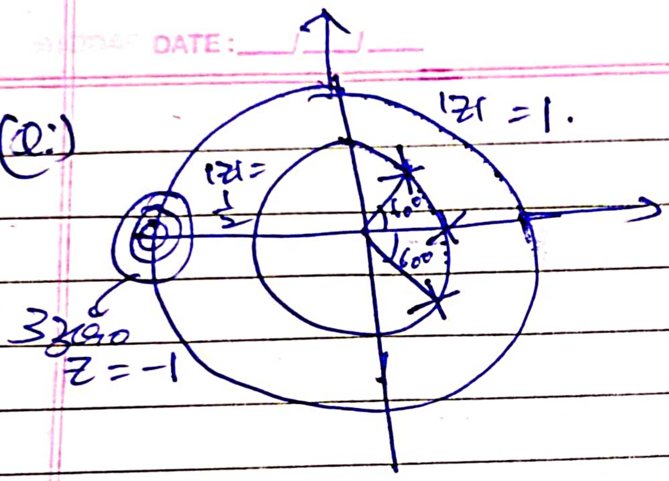
(Q:)



Is this FIR filter
Is it a linear phase
Give a direct and
linear phase
~~realiz~~ realization.

(Q:) An LTI system is given by

$$h(n) = 5\delta(n) - 7\delta(n-1) + 7\delta(n-3) - 5\delta(n-4)$$

what type of filter is this (LP, BP, BS, HP)?.

(Q:) Design a symmetric FIR lowpass filter where

$$H_d(\omega) = \begin{cases} e^{-j\omega 4}; & |\omega| \le \omega_c \\ 0, & \text{otherwise} \end{cases} \qquad M = 7, \quad \omega_c = 1\ \text{rad/s}.$$

using Rectangular window [can leave Answer in terms of numerical sinusoids].