# Assignment-I

**Q1**

(a) Nope, No point exist for which sequence is symmetric since for a sequence to be symmetric the left and right should be completely same, clearly no such points exist

(b) goals[n] =
$2b[n-1] + b[n-3] + b[n-4] + b[n-7] + b[n-10] + b[n-14] + b[n-15] + 2b[n-17]$

(c) Average number of goals $= \frac{5}{10} = 0.5$

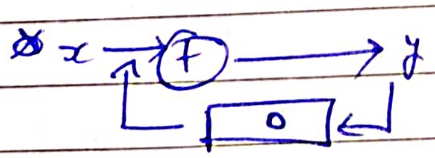
All matches $= \frac{10}{20} = 0.5$

(d) If $h[n] = u[n]$

$y[n] = x[n] * h[n]$

↳ accumulator

$y[n] = x[n] + y[n-1]$




If

$Z[n] \to$ average function

then clearly

$$z[n] = \frac{y[n]}{n}$$

where

$y[n] = x[n] + y[n-1]$, $n \geq 0$

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~~Q2~~ $x(n) = u(n) = \delta(n) - \delta(n-1)$

Q2

(a) $y(n) = 1, \quad n \leq 0$
$y(n) = 0$, otherwise

clearly

$$\boxed{y(n) = u(-n) = x(-n)}$$

(b) $y(n) = -1$, odd values of $n$
$y(n) = 1$, even values of $n$.

$$\boxed{y(n) = \begin{cases} 1 & n = 0 \\ (-1)^n[u(n) + u(-n)], & \text{otherwise} \end{cases}}$$

(c) $y(n) = \delta(n)$

$= u(n) - u(n-1)$

$$\boxed{y(n) = x(n) - x(n-1)}$$

(d) $y(n) = n$ for $n \geq 0$, 0 otherwise

$$\boxed{y(n) = n\,u(n) = n\,x(n)}$$

(e) $y(n) = 1$ for $4 \leq n \leq 10$ and $y(n) = -3$ for $11 \leq n \leq 16$

$$\boxed{y(n) = u(n-4) - u(n-10) - 3(u(n-11) - u(n-16))}$$

replace $u(n-d) \to x(n-d)$

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Q3

(a) $x[2n-3]$

→ first compress
By half then
shift right by 3.

draw the graph

(b) $x[1-2n]$

↳ ~~compress By Half~~
~~Reverse~~
shift left By 1

(b) ~~$x[1-2n] = x[-(2n-1)]$~~
$= x[-2(n-\frac{1}{2})]$

(b) $x(1-2n)$
↳ shift By 1 to right side
↳ compress By 2
↳ Reverse.

(c) $x[n-1]\delta[n-2]$

↳ clearly
only
exist at $n=2$
↳ $x[1]$

(d) $-0.5x[n+4]$
↳ left. shift By 4
scale By $-0.5$.

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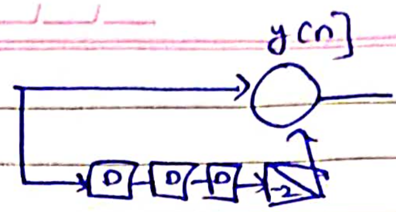
Q4




(a) $y(n] = x(n] + (-2)\, x(n-3]$

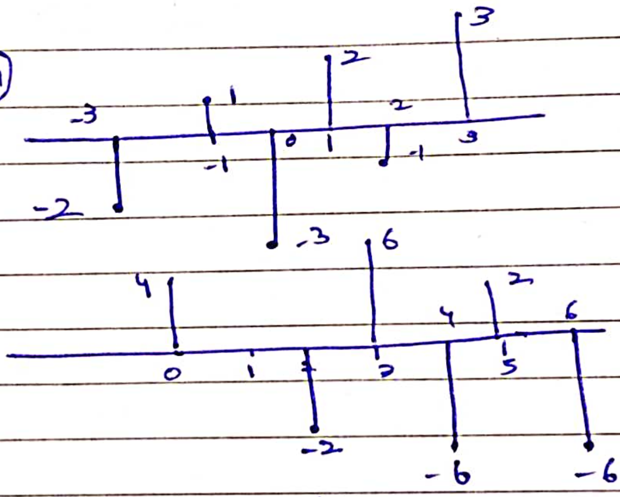
(b) $Y = X - 2XR^3$

$$\boxed{\frac{Y}{X} = 1 - 2R^3}$$

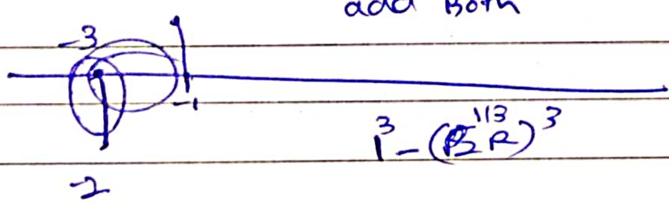
(c) (i) $x(n) = \delta(n]$

$y(n] = \delta(n] - 2\delta(n-3)$

(ii)




add Both




$1^3 - (2^{1/3}R)^3$

(d) $\frac{Y}{X} = (1 - 2R^3) = \cancel{(1 - 2^{1/3}R)}(1 +$

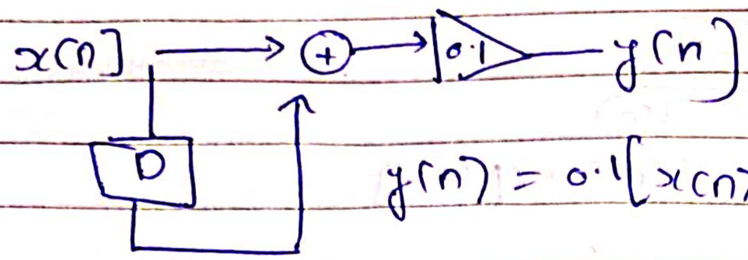
$= (1 - 2^{1/3}R)(1 + 2^{2/3}R^2 + 2^{1/3}R)$

System can be represented as multiplication of Both.

Advantage: some terms are redundant.

## Assignment - 2

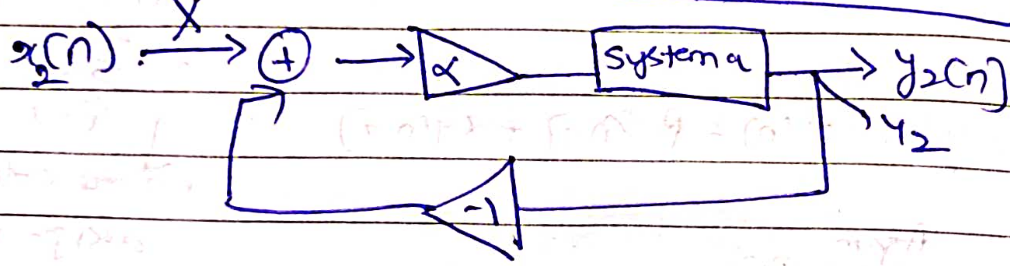
Q1




$$y(n) = 0.1[x(n) + x(n-1)]$$

$$Y(R) = 0.1[X(R) + X(R)R]$$

$$\boxed{\frac{Y(R)}{X(R)} = 0.1(1+R)}$$




$$Y_2 = (X_2 - Y_2)(\alpha)(0.1)(1+R)$$

$$Y_2[1 + 0.1\alpha(1+R)] = \alpha(0.1)(1+R)(X_2)$$

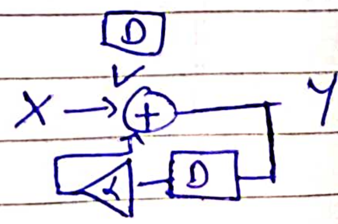
$$\boxed{\frac{Y_2}{X_2} = \frac{\alpha(1+R)}{\alpha(1+R)+10}}$$

$$\boxed{(\alpha+10)\, y_2(n] + \alpha\, y_2(n-1] = \alpha x_2(n) + \alpha x_2(n-1]}$$

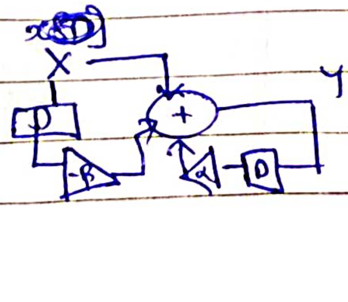
Q2 (i) $Y = \frac{XR}{1-\alpha R}$. $Y = \left(\frac{1-\beta R}{1-\alpha R}\right)X$

(a) $y(n] - y(n-1](\alpha) = x(n-1]$

(b)




$y(n] - \alpha y(n-1] = x(n) - \beta x(n-1]$



(c). (i) $y(n) = x(n-1] + \alpha y(n-1]$

If we assume causalty.

$x(n) = \delta(n)$

$h(n] = \delta(n-1) + \alpha h(n-1]$.

$h[0] = 0$

$h(1] = 1$

$h(2] = \alpha$

$h(3] = \alpha^2$

$$h(n] = \begin{cases} \alpha^{n-1}, & n \geq 1 \\ 0, & n \leq 0. \end{cases}$$

$$y(n) = \sum_{k=1}^{\infty} h(k)\, x(n-k) = \sum_{k=1}^{\infty} \alpha^{k-1} x(n-k)$$

we are able to assign higher or lower weights to subsequent samples.

(ii) $y(n) = x(n) - \beta x(n-1] + \alpha y(n-1]$

Again assuming causalty

$x(n) = \delta(n]$

$h(n) = \delta(n] - \beta \delta(n-1] + \alpha h(n-1]$

$h[0] = 1$

$h(1] = -\beta + \alpha$.

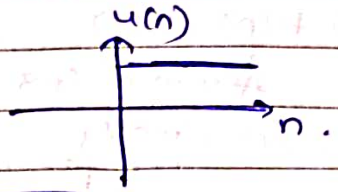
$h(2] = \alpha(\alpha - \beta)$

$h(3] = \alpha^2(\alpha - \beta)$

$$\boxed{h(n] = \begin{cases} 1, & n = 0 \\ \alpha^{n-1}(\alpha - \beta) & n > 0. \\ 0. & n < 0 \end{cases}}$$

Q3

a) $x(n) = h(n) = u(n)$

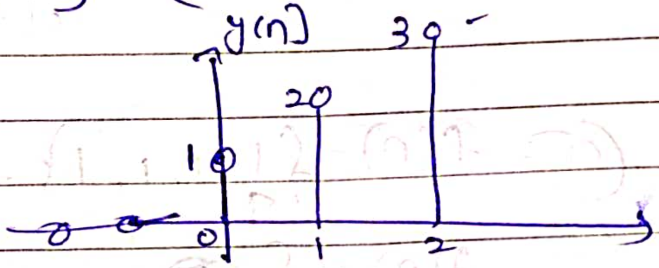



$y(n) = x(n) * h(n)$

$= \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$ → $k \geq 0$

$= \sum_{k=0}^{\infty} h(n-k)$

$= \sum_{k=0}^{n} u(n-k) = (n+1)u(n)$ → only works when $n \geq 0$.




b) $x(n) = \left(\frac{1}{2}\right)^n u(n) \quad h(n) = u(n)$

$y(n) = x(n) * h(n)$ → $k \geq 0$.

$= \sum_{k=0}^{\infty} u(k) \left(\frac{1}{2}\right)^{(n-k)} u(n-k)$

$= \sum_{k=0}^{n} \left(\frac{1}{2}\right)^{(n-k)} u(n-k)$

If $n < 0$. then automatically zero.

If $n \geq 0$.

$= \left(\frac{1}{2}\right)^n + \left(\frac{1}{2}\right)^{n-1} + \dots + 1$

$y(n) = \dfrac{1\left(\left(\frac{1}{2}\right)^{n+1} - 1\right)}{-\frac{1}{2}} = 2 - \left(\frac{1}{2}\right)^n = \dfrac{2^{n+1} - 1}{2^n}\, u(n)$

functions

(a) $h(n) = u(n)$ → Accumulator
specifically adds previous samples.

(b) $h(n) = u(n)$ → Accumulator
Adds to give GP series sum.

(c) moving average.

Q4. $r_{f,g}[l] = \sum_{m=-\infty}^{\infty} f(l+m)\, g(m)$

$l$ → lag factor.

(a) $f(n) = \{\underset{\uparrow}{1}, 1, 1, 1\}$.

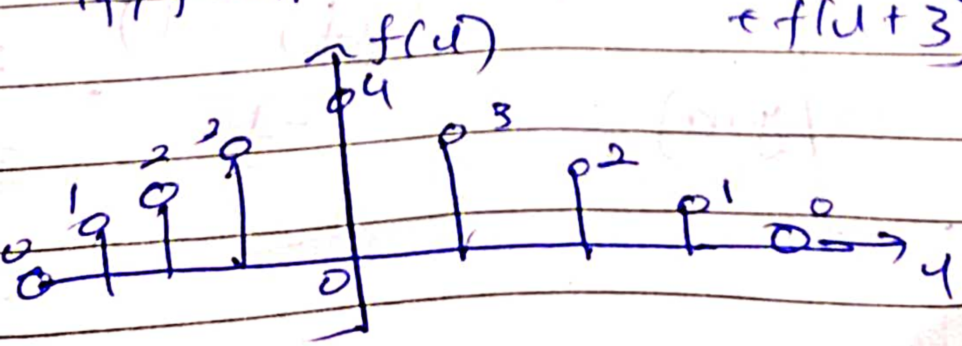
$g(n) = \{1, \underset{\uparrow}{-2}, 1\}$

$r_{f,g}(l) = f[l-1] - 2f(l] + f[l+1]$.

(b) $r_{f,f}(l) = \sum_{m=-\infty}^{\infty} f[l+m]\, f(m)$

$r_{g,g}(l) = \sum_{m=\infty}^{\infty} g(l+m)\, g(m)$

$r_{f,f}[l] = f(l) + f(l+1) + f(l+2) + f(l+3)$



(c) Autocorrelation:

might help to extract periodic pattern in signal.

Peaks in autocorrelation function at specific lags may indicate repeating pattern or cycle in data.

Cross correlation:

↳ can be used to find delay between signal. lag at which cross correlation is maximum is time shift.

~~(d)~~ $\cancel{r_{f,g}(u) = \sum_{m=-\infty}^{\infty} f(u+m) g(m)}$

$\cancel{u+m = p}$

$\cancel{m = p-u}$

$\cancel{= \sum f(p) g(p-u)}$

(d) $r_{f,g}(u) = \sum_{m=-\infty}^{\infty} g(m) f[m+u]$

$c_{f,g}(u) = \sum_{m=-\infty}^{\infty} g(m) f[u-m]$

$m = -p$

$r_{f,g}(u) = \sum_{p=-\infty}^{\infty} g[-p] f(u-p)$.

$\boxed{r_{f,g}(u) = g[-u] * f(u)}$

Q1 $X(z) = \dfrac{1}{1+0.5z^{-1}}$

note that

$x[n-k] \xrightarrow{z\ \text{transform}} z^{-k}X(z)$

$x[-n] \xrightarrow{z\ \text{transform}} X\left(\frac{1}{z}\right)$

(a) $y_1[n] = x[3-n] + x[n-3]$

consider

~~$y_2[n] = x[n-3]$~~

$$\boxed{Z(y_2[n]) \Rightarrow z^{-3}\left(\frac{1}{1+0.5z^{-1}}\right)}$$

$y_3[n] = x[n+3]$

$Z(y_3(n)) = z^{3}\left(\dfrac{1}{1+0.5z^{-1}}\right)$

$$\boxed{Z(y_3(-n)) = \left(\frac{1}{z}\right)^{3}\left(\frac{1}{1+0.5z}\right)}$$

$$\boxed{Z(y_1(n)) = Z(y_2(n)) + Z(y_3(-n))}$$

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(b) $y_2[n] = \left(\frac{1}{2}\right)^n x(n-2]$

In general

$x[n] \xrightarrow{Z \text{ transform}} X(z)$

$a^n x[n] \xrightarrow{Z \text{ transform}} \sum_{n=-\infty}^{\infty} a^n x(n] z^{-n}$

$= \sum_{n=-\infty}^{\infty} x[n] \left(\frac{z}{a}\right)^{-n}$

$$\boxed{a^n x(n] \rightarrow = X\left(\frac{z}{a}\right)}$$

$y_2[n] = \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^{n-2} x(n-2]$

Now

$\left(\frac{1}{2}\right)^n x[n] \xrightarrow{Z \text{ transform}} X(2z)$

$\left(\frac{1}{2}\right)^{n-2} x(n-2) \xrightarrow{Z \text{ transform}} z^{-2} X(2z)$

$$\boxed{\left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^{n-2} x(n-2) \xrightarrow{Z \text{ transform}} \left(\frac{1}{2}\right)^2 z^{-2} X(2z)}$$

(c) $y_4[n] = x[n-2] * x[2-n]$

convolution In time domain $\rightarrow$ multiplication in z domain.

$Y_4(z) \xrightarrow{Z \text{ transform}} Z(x(n-2]) \times Z(x(2-n))$

$z^{-2} X(z) \times \cdot \left(\frac{1}{2}\right)^2 X(z)$

$= z^{-4} (X(z))^2$

(d) $X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$

$$\frac{d}{dz}(X(z)) = \sum_{n=-\infty}^{\infty} x(n](-n) z^{-n-1}$$

$$\Rightarrow -z^{-1} \sum_{n=-\infty}^{\infty} n x(n] z^{-n}$$

Z transform $(n x(n]) = -z \frac{d}{dz}(X(z))$

$$\sum_{n=-\infty}^{n=\infty} n x(n] z^{-n} = -z^{1} X'(z)$$

differentiating again

$$\sum_{n=-\infty}^{\infty} n x[n] z^{-n-1}(-n) = -\left[z X''(z) + X'(z)\right]$$

from here

$y(n] = n x(n]$

$$Z(n y(n]) = -z \frac{d}{dz}(Y(z))$$

$$= +z \frac{d}{dz}\left(+z \frac{d}{dz}(X(z))\right)$$

$$\Rightarrow z\left[\frac{d}{dz}(X(z)) + z \frac{d^2}{dz^2} X(z)\right]$$

$$Z(n^2 x(n]) = z^2 \frac{d^2}{dz^2} X(z) + z \frac{d}{dz} X(z)$$

$$Z(n^2 x(n] + n x(n] + x(n])$$
$$= z^2 X''(z) + z X(z) - z X(z) + X(z)$$
$$= z^2 X''(z) + X(z)$$

Q2 $x[n] = \left(\frac{1}{2}\right)^n u[n]$

$X(z) = U(2z)$ [ $U(z) \to$ z transform of $u(n)$ ]

$U(z) = \sum_{n=0}^{\infty} z^{-n} \Rightarrow \frac{1}{1-z^{-1}}$

$|z^{-1}| < 1$

$|z| > 1$

$= \frac{z}{z-1}$

$X(z) = \frac{2z}{2z-1}$

(a) $y_1(z) = z\,X\left(\frac{1}{z}\right)$

↳ replace

$X\left(\frac{1}{z}\right) = P(z) \to P[n] = x[-n]$

$y_1(z) = z\,P(z)$

$y_1[n] = P(n+1)$

$= x[-(n+1)]$

$\boxed{y_1[n] = x[-n-1]}$

(b) $y_2(z) = \left(\frac{z-1}{z}\right) X(z)$

$= X(z) - \frac{X(z)}{z}$

Inverse ↓ $x(n)$ ↳ $x[-n]$

$y_2[n] = x[n] - x[-n]$

(c) $y_3(z) = X(z) \times X\left(\frac{1}{z}\right)$

$X(z) = \frac{z}{z-1}$

multiplication in z domain ≡ convolving in Time domain

$X\left(\frac{1}{z}\right) = \frac{\frac{1}{z}}{\frac{1}{z}-1}$

$= \frac{\frac{1}{z}}{\frac{1-z}{z}}$

$= \frac{1}{1-z}$

$\boxed{y_3[n] = x[n] * x[-n]}$

$X(z)\,X\left(\frac{1}{z}\right) = \frac{-z}{(z-1)^2}$

$= \frac{-z}{z^2-2z+1}$

(d) $y_1(z) = z^2 \frac{d\,X(z)}{dz} = z\left(z\frac{d\,X(z)}{dz}\right) = z\,P(z)$

$n^2 x[n] \xrightarrow{z\ \text{transform}} z^2 \ldots +$

$y_1[n] = p[n+1]$

$n\,x[n] \xrightarrow{z\ \text{transform}} \underbrace{-z\frac{d}{dz}X(z)}_{P(z)}$

$-n\,x[n] \xrightarrow{z\ \text{transform}} P(z)$

$p[n] = -n\,x[n]$

$\boxed{y_1[n] = -(n+1)\,x[n+1]}$

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Q3) $X(z) \to$ zero at origin

$z = 3$ and $\frac{1}{3}$

$$X(z) = \frac{Gz}{(z-3)(z-\frac{1}{3})}$$

$$= G\left[\frac{\frac{3}{3-\frac{1}{3}}}{z-3} + \frac{\frac{-\frac{1}{3}}{3-\frac{1}{3}}}{z-\frac{1}{3}}\right]$$

$$\frac{1}{8}\left[\cancel{9z-3} - z + 3\right]$$

$$\Rightarrow \left(\frac{9}{8(z-3)} + \frac{-1}{8(z-\frac{1}{3})}\right)G$$

Since $x(n)$ is double sided $\Rightarrow$

$$\boxed{\frac{9}{8}\left[-3^{n-1}u[-n]\right] - \frac{1}{8}\left[\left(\frac{1}{3}\right)^{n-1}u[n-1]\right]} \times G.$$

$\frac{1}{3} < |z| < 3$

$$a^n u(n) \xrightarrow{z\text{ transform}} \frac{1}{1-az^{-1}} \quad \text{if } |z| > |a|$$

$$-a^n u[-n-1] \to \frac{1}{1-az^{-1}} \quad \text{if } |z| < |a|$$

utilising

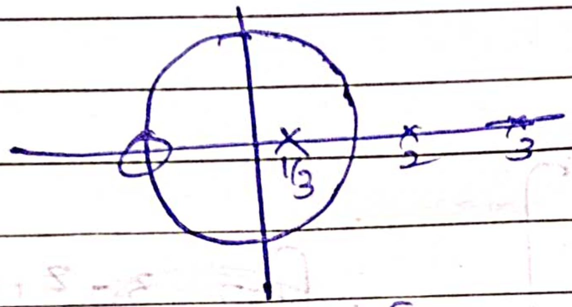
$x(n) \to X(z)$

$$\frac{1}{z}\left(\frac{1}{1-az^{-1}}\right) = \frac{1}{z-a} \qquad x(n-1) \to z^{-1}X(z)$$

$$\underbrace{\text{Inverse.} \qquad \text{Inverse}}_{x[n-1]}$$

generalize
and do the
math
It's simple :)

Q4



$$X(z) = \frac{(z+1)}{(z-\frac{1}{3})(z-2)(z-3)}$$

(a) ROC →

$\frac{1}{3} < |z| < 2$

→ double sided signal

(b) (i)

All possible ROC

(i) $|z| < \frac{1}{3}$

(ii) $\frac{1}{3} < |z| < 2$

(iii) $2 < |z| < 3$

(iv) $|z| > 3$

(i) stable and causal

→ for stable outside of outermost pole

~~$|z| > 3$~~ → obviously contains unit circle

(ii) Stable
But
not
causual

ROC → $\frac{1}{3} < |z| < 2$

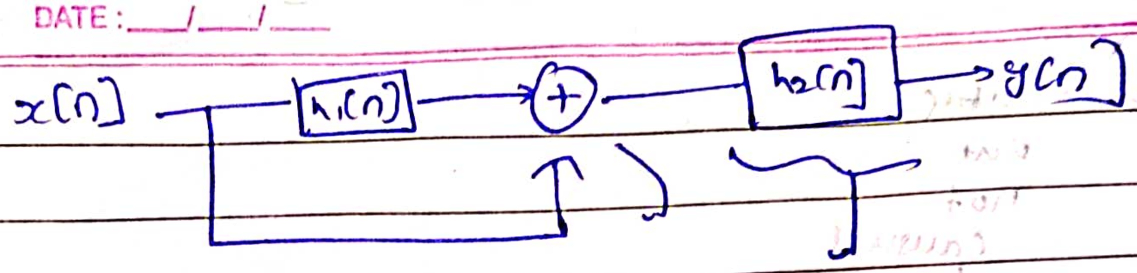
(iii) Causual but
unstable

→ $|z| > 3$

unstable [since
doesnt
contain unit
circle]

Causual [outside
outermost
pole]

Q5




$$\underbrace{x(n) * h_1(n) + x[n]} \quad H_2(z)$$

$$X(z)H_1(z) + X(z)$$

$$\Rightarrow X(z)(1 + H_1(z))$$

$$Y(z) = X(z)(1 + H_1(z))(H_2(z))$$

$$h_1[n] = \frac{\delta(n-2)}{4} - \delta(n-1)$$

$$H_1(z) = \frac{1}{4}\left[z^{-2}\right] - z^{-1}$$

$$H_2(z) = \frac{1}{1 - 0.5z^{-1}}$$

$\Sigma \left(\frac{1}{2}\right)^0 z^0 + \left(\frac{1}{2}\right)^1 z^{-1}$

$\left|\frac{1}{2} z^{-1}\right| < 1$

$|z^{-1}| < 2$

$|z| > \frac{1}{2}$

$$1 + H_1(z) = 1 - z^{-1} + \frac{1}{4} z^{-2}$$

$$= \left(1 - \frac{z^{-1}}{2}\right)^2$$

$$1 + H_1(z) = (1 - 0.5z^{-1})^2$$

$$Y(z) = X(z)(1 - 0.5z^{-1})$$

$$y(n) = x(n) - 0.5x[n-1]$$

$$\boxed{h(n) = \delta[n] - \frac{1}{2}\delta[n-1]} \rightarrow (a)$$

(b) $H(z) = 1 - 0.5z^{-1}$

$\Rightarrow 1 - \frac{1}{2z}$

$H(z) = \frac{2z-1}{2z}$

Poles $\rightarrow$ $\boxed{z=0}$

zero $\rightarrow$ $\boxed{z=\frac{1}{2}}$

ROC is atleast $|z| > \frac{1}{2}$

[$\because$ ROC of second system is this]

system is stable and causual

[$\because$ ROC is outside of outermost pole and contains unit circle]

(c) gives $x[n] - \frac{1}{2}x[n-1]$

$\rightarrow$ Reduces effect of variations in input signal. $\rightarrow$ simple high pass filter $\rightarrow$ attenuates dc or low frequency component at $n=0$.

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Q1

(a) Time shifting

$\rightarrow x[n] \xrightarrow{DTFT} X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$

$x[n-k] \xrightarrow{DTFT} \sum_{n=-\infty}^{\infty} x[n-k] e^{-j\omega n}$

$n-k=p$

$n=p+k$

$\sum_{n=-\infty}^{\infty} x[p] e^{-j\omega(p+k)}$

$= e^{-j\omega k} \sum_{n=-\infty}^{\infty} x[p] e^{-j\omega p}$

$= e^{-j\omega k} X(e^{j\omega})$

(b) frequency shifting

$x[n] \xrightarrow{DTFT} X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$

$e^{-j\omega_0 n} x[n] \xrightarrow{DTFT} \sum_{n=-\infty}^{\infty} e^{-j\omega_0 n} x[n] e^{-j\omega n}$

$= \sum_{n=-\infty}^{\infty} x[n] e^{-jn(\omega+\omega_0)}$

$= X\left(e^{j(\omega+\omega_0)}\right)$

(c) convolution

$x_1[n] \rightarrow X_1(e^{j\omega}) \quad x_2[n] \rightarrow X_2[e^{j\omega}]$

$x_1[n] * x_2[n] = \sum_{k=-\infty}^{\infty} x_1[k] x_2[n-k] \xrightarrow{DTFT} \sum_{n=-\infty}^{\infty} x \cdot \sum_{k=-\infty}^{\infty} x_1[k] x_2[n-k] e^{-j\omega n}$

$\left(\sum_{k=-\infty}^{\infty} x_1[k] e^{-j\omega k}\right) X_2(e^{j\omega}) = X_1(e^{j\omega}) X_2(e^{j\omega})$

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(4) multiplication

$$\cancel{x_1[n] x_2[n] \to \sum_{n=-\infty}^{\infty} x_1[n]\, x_2(n)\, e^{-j\omega n}} \quad \cancel{\frac{e^{-j\omega(n-k)}}{e^{-j\omega(n-k)}}}$$

$$= \sum_{n=-\infty}^{\infty} \cancel{x_1[n]\, e^{-j\omega n}}\, x_2[n]\, e^{-j\omega(n-k)}$$

$$x_1[n] = \frac{1}{2\pi} \int_{2\pi} X(e^{j\omega})\, e^{j\omega n}\, d\omega$$

$$x_1[n]\, x_2[n] \xrightarrow{DTFT} \sum_{n=-\infty}^{\infty} \left( \frac{1}{2\pi} \int_{2\pi} X_1(e^{j\omega_1})\, e^{j\omega_1 n}\, d\omega_1 \right) x_2[n]\, e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} \frac{1}{2\pi}\, x_2[n] \int_{2\pi} X_1(e^{j\omega_1})\, e^{-jn(\omega-\omega_1)}\, d\omega_1$$

$$= \frac{1}{2\pi} \int_{2\pi} X_1(e^{j\omega_1}) \left( \sum_{n=-\infty}^{\infty} x_2[n]\, e^{-jn(\omega-\omega_1)} \right) d\omega_1$$

$$= \frac{1}{2\pi} \int_{2\pi} X_1(e^{j\omega_1})\, X_2(e^{j(\omega-\omega_1)})$$

$$= \cancel{X_1(e^{j\omega_1}) * X_2(e^{j(\omega-\omega_1)}}$$

$$X_1(e^{j\omega}) * X_2(e^{j\omega})$$

(5) Differentiation in frequency

$$x[n] \to X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n}$$

$$n\, x[n] \to \sum_{n=-\infty}^{\infty} n\, x[n]\, e^{-j\omega n}$$

$$\frac{d}{d\omega}(X(e^{j\omega})) = -j \sum_{n=-\infty}^{\infty} n\, x[n]\, e^{-j\omega n}$$

$$\sum_{n=-\infty}^{\infty} n\, x[n]\, e^{-j\omega n} = j \frac{d}{d\omega}(X(e^{j\omega}))$$

$= X(\omega_0)\, \delta(\omega-\omega_0)$

$X(\omega)$ * $X(\omega_0)$

$$\left[\frac{e^{j\omega(m-n)}}{j(m-n)}\right]_{-\pi}^{\pi} = \frac{1}{j(m-n)}\left[\cos\pi(m-n) + j\sin\pi(m-n) - [\cos\pi(m-n) - j\sin\pi(m-n)]\right]$$

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$$\Rightarrow \frac{2\sin\pi(m-n)}{m-n}$$

(5) parseval relation

$$\sum_{n=-\infty}^{\infty} |x(n)|^2 = \frac{1}{2\pi}\int_{2\pi} |X(e^{j\omega})|^2 \, d\omega$$

$$x(n) = \frac{1}{2\pi}\int_{2\pi} X(e^{j\omega}) e^{j\omega n}$$

RHS:

$$\frac{1}{2\pi}\int_{2\pi} |X(e^{j\omega})|^2 d\omega = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega}) X^c(e^{j\omega}) \, d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n} \left(\sum_{m=-\infty}^{\infty} x(m) e^{-j\omega m}\right)^c d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n} (x(m))^c e^{j\omega m} d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} \sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty} x(n) x^*(m) e^{j\omega(m-n)} d\omega$$

$$\Rightarrow \frac{1}{2\pi}\sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty} x(n) x^*(m) \int_{-\pi}^{\pi} e^{j\omega(m-n)} d\omega$$

$$= \frac{1}{\cancel{2\pi}}\left(\sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty} x(n) x^*(m)\right) \delta(n-m) \cdot \cancel{2\pi}$$

$$\Rightarrow \sum_{n=-\infty}^{\infty} |x[n]|^2$$

$= LHS$

Hence LHS = RHS

$\int_{2\pi} \delta(\omega) d\omega = 1$

$\delta(\Delta)\Delta = 1$

$\delta(\Delta) = \frac{1}{\Delta}$

$2\frac{\sin n\pi}{n\pi}$

$2\pi$

Q2) $\delta(\omega) = 0, \ \omega \neq 2\pi k$

$$\int_{2\pi} \delta(\omega)\, d\omega = 1$$

(a)

$$\int_{2\pi} \delta(\omega) X(\omega)\, d\omega = \lim_{\Delta \to 0^+} \int_0^{\Delta} \delta(\omega) X(\omega)\, d\omega + \int_{\Delta}^{2\pi} \delta(\omega) X(\omega)\, d\omega$$

$$= X(\Delta) \lim_{\Delta \to 0^+} \underbrace{\int_0^{\Delta} \delta(\omega)\, d\omega}_{1}$$

$$= X(0)\,(1) = X(0)$$

(b)

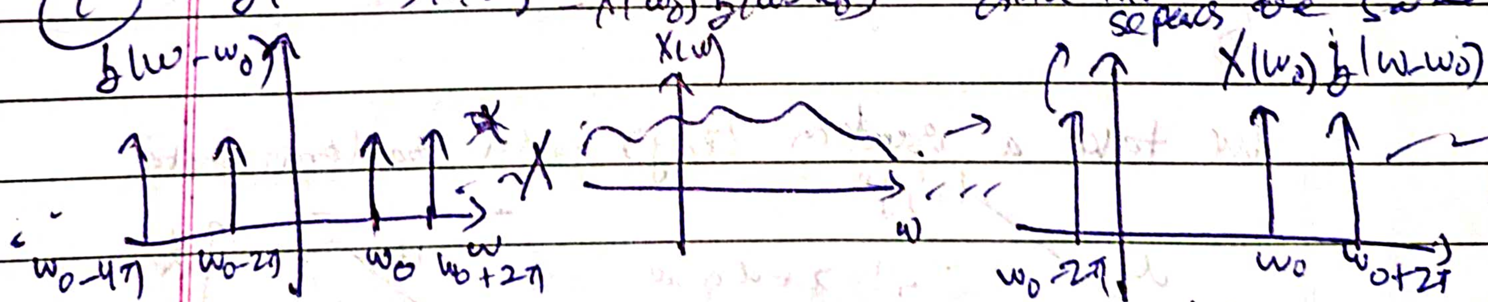
$$\int_{2\pi} \delta(\omega - \omega_0) X(\omega)\, d\omega = \lim_{\Delta \to 0^+} \int_0^{\Delta} \delta(\omega - \omega_0) X(\omega)\, d\omega + \cancelto{0}{\int_{\Delta}^{2\pi} \delta(\omega) X(\omega)\, d\omega}$$

$$= \lim_{\Delta \to 0^+} \int_{\omega_0}^{\Delta + \omega_0} \delta(P) X(P + \omega_0)\, d\omega$$

$$= \lim_{\Delta \to 0^+} \int_0^{\Delta} \delta(P) X(P + \omega_0)\, d\omega$$

$$= \lim_{\Delta \to 0^+} X(\Delta + \omega_0)\,(1) = X(\omega_0)$$

(c) $\delta(\omega - \omega_0) X(\omega) = X(\omega_0)\, \delta(\omega - \omega_0)$ since $X(e^{j\omega})$ is period repeats the same




(a) and (b) can be solved using (c).

$$\int_{2\pi} \delta(\omega) X(\omega)\, d\omega = \int_{2\pi} \delta(\omega) X(0)\, d\omega = X(0)$$

$$\int_{2\pi} \delta(\omega - \omega_0) X(\omega)\, d\omega = \int_{2\pi} \delta(\omega - \omega_0) X(\omega_0)\, d\omega = X(\omega_0)\,(1) = X(\omega_0)$$

$\sum \delta[n+3] e^{-j\omega n}$ $\quad n = p-3$ $\quad e^{-j\omega(p-3)}$

$n+3=p$ $\quad \delta(p)$

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Q3) Causal signal $\longrightarrow x[n]$

yes, the DTFT of a causal signal may have zero phase spectrum [ for eg: signal $x[n] = \begin{cases} 0, & n<0 \\ K, & n=0 \\ 0, & n>0 \end{cases}$ → such signals ]

Take for eg:

$x[n] = \delta[n]$

~~X(z)~~ $X(e^{j\omega}) = 1$, clearly zero phase

In general for $X(e^{j\omega}) = K$ [where K is some constant]

will have zero phase spectrum, we can more generalise

$K = \sum_{n=0}^{\infty} x[n] e^{j\omega n}$

signals like this have zero phase spectrum

This too signals with Real part zero for eg when $\omega = 2\pi K$

$\sum \delta[n-k] e^{-j\omega n}$ $x(n)$ $\sum e^{-j\omega n}$

$\sum \delta(n+1) e^{-j\omega n}$ $n$ $n+5=p$ $n=k+p$

$n+1=p$ $n=p-5$

$n=p-1$

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Q4 $x(n) = \delta(n+3) - \delta(n+1) + 2\delta(n) + 3\delta(n-1)$

$$X(e^{j\omega}) = e^{3j\omega} - e^{j\omega} + 2 + 3e^{-j\omega}$$

$$\boxed{\int_{2\pi} X(e^{j\omega})\,d\omega = 2\pi x(0) = 2\pi(2) = 4\pi}$$

$$X(e^{j\omega}) = \cos 3\omega + j\sin 3\omega - (\cos\omega + j\sin\omega) + 2 + 3(\cos\omega - j\sin\omega)$$

$$= \cos 3\omega + 2\cos\omega + 2 + j\sin 3\omega - 4j\sin\omega$$

$$X_R(e^{j\omega}) = \cos 3\omega + 2\cos\omega + 2$$

$$X_I(e^{j\omega}) = \sin 3\omega - 4\sin\omega$$

$$X_R(e^{j\omega})\, e^{2j\omega} = \left(e^{3j\omega} + e^{-3j\omega} + 2(e^{j\omega} + e^{-j\omega}) + 2\right) e^{2j\omega}$$

$$\Rightarrow \frac{e^{5j\omega}+e^{-j\omega}}{2} + e^{3j\omega} + e^{j\omega} + 2e^{2j\omega}$$

IDFT

$$x_R[n] = \frac{\delta[n+5] + \delta(n-1]}{2} + \delta[n+3] + \delta[n+1] + 2\delta(n+2]$$

$$jX_I(e^{j\omega}) = j\left(\frac{e^{3j\omega} - e^{-3j\omega}}{2} - 4\left(\frac{e^{j\omega} - e^{-j\omega}}{2}\right)\right)$$

IDFT

$$x_I[n] = \frac{\delta(n+3] - \delta(n-3)}{2} - 2(\delta(n] - \delta$$

$$X_R(e^{j\omega})e^{2j\omega} = \frac{e^{5j\omega}+e^{-j\omega}}{2} + e^{3j\omega} + e^{j\omega} + 2e^{2j\omega}$$

Now

$x(n) = \delta(n-k)$

$X(e^{j\omega}) = \sum \delta(n-k)\, e^{-j\omega n}$ $\quad n-k=p \quad n=p+k \quad \sum \delta(p)\, e^{-j\omega p} \cdot e^{-j\omega k}$

$= e^{-j\omega k}$

$x_R(n) =$

$$\frac{\delta(n+5] + \delta(n-1]}{2} + \delta[n+3] + \delta(n+1] + 2\delta(n+2]$$

$x_I[n]$

$$jX_i(e^{j\omega}) = j\left(\frac{e^{3j\omega} - e^{-3j\omega}}{2j} - 4\frac{(e^{j\omega} - e^{-j\omega})}{2j}\right)$$

$$= \frac{e^{3j\omega} - e^{-3j\omega}}{2} - 2(e^{j\omega} - e^{-j\omega})$$

$$x_I[n] = \frac{\delta[n+3] - \delta(n-3)}{2} - 2\left[\delta[n+1] - \delta(n-1)\right]$$

$$\boxed{\begin{aligned} &x_R[n] \\ &+x_I[n] = \frac{\delta(n+3)}{2}(3) - \delta[n+1] + \frac{\delta[n+5]}{2} \\ &\quad + \frac{5}{2}\delta[n-1] + 2\delta[n+2] - \frac{1}{2}\delta[n-3] \end{aligned}}$$

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Q1 $x_1(n) = a^n u(n)$ $x_2(n) = (-1)^n a^n u(n)$.

(a)

$$X_1(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x_1(n) e^{-j\omega n}$$

$$= \sum_{n=0}^{\infty} a^n e^{-j\omega n}.$$ $|ae^{-j\omega}| < 1$

$$= \sum_{n=0}^{\infty} (ae^{-j\omega})^n.$$ $|a| < 1$

$$= \frac{1}{1 - ae^{-j\omega}}$$

$$X_2(e^{j\omega}) = \sum a^n e^{-j\omega n} \cdot e^{j\pi n}.$$

$$= \sum (ae^{j(\pi-\omega)})^n.$$

$$= \frac{1}{1 - ae^{j(\pi-\omega)}}$$ $|a| < 1$

$$X_2(e^{j\omega}) \Rightarrow \frac{1}{1 + ae^{-j\omega}}.$$

$$\boxed{X_2(e^{j\omega}) = X_1(e^{j\omega + j\pi})}$$

plot the magnitude and phase plots.

$$\boxed{\frac{1}{X_1(e^{j\omega})} + \frac{1}{X_2(e^{j\omega})} = 2}$$

property to deduce:

$$DTFT(e^{-j\omega_0 n} x(n)) = X(e^{j(\omega+\omega_0)})$$

# Assignment-6

(Q1) See Prakis theory

(Q2) So while calculating radix 2 FFT.
consider $P(N)$ represents the number of multiplication and $Q(N)$ represent number of Addition.

After dividing the problem into subproblem of finding odd and even signal DFT and combining them we can say, $P(1) = 0 \quad Q(1) = 0$.
Base cases.

$P(N) = \underbrace{2P(N/2)}_{\text{solving 2 problem each of length } N/2} + \underbrace{N/2}_{N/2 \text{ multiplication new required to compute the result}}$

well known reduces to $\rightarrow \frac{N}{2}\log_2 N$.

$Q(N) = 2Q(N/2) + N$.
reduces to $N \log N$

$\boxed{\text{Normal convolution } O(N^2)}$

To compute linear convolution using circular.

$x_1(n) \xrightarrow[L_1+L_2-1]{\text{pad}} x_1''(n) \xrightarrow{\text{FFT}} X_1$

$\frac{N}{2}\log N$ mult
$N\log N$ Addi

$x_2(n) \xrightarrow[L_1+L_2-1]{\text{pad}} x_2''(n) \xrightarrow{\text{FFT}} X_2$ multiply $\xrightarrow{\text{IDFT}}$

FFT: $\frac{N}{2}\log N$ mult, $N\log N$ Addi
multiply: $N$ mult
IDFT: $\frac{N}{2}\log N$ mult, $N$ Addi

$\boxed{\text{Total Add} \rightarrow 3N\log N \quad \text{Total mult} = \frac{3N}{2}\log N + N}$

(Q3) · Read DIT and DIF theory.

one is dividing the input into odd even.

one is dividing the output

complexity and calculation computation all are same.

To prove:

(Q4) · $W_N^{kn} = W_n^{k} \; W_N^{k(n-1)}$ recursively.

It is sufficient to show this relation holds

$$W_N^{kn} = e^{\frac{2\pi jkn}{N}} = e^{\frac{2\pi jk}{N}} \cdot e^{\frac{2\pi j(nk-k)}{N}}$$

$$= W_N^{k} \cdot W_N^{k(n-1)}$$

since this relation is true hence you can calculate recursively.

(Q5) - $x(n) \to 24$ length sequence.

$$X[k] = \sum_{n=0}^{23} x(n) W_{24}^{kn}$$

$$= \sum_{n=0}^{7} x[3n] W_{24}^{3kn} + \sum_{n=0}^{7} x(3n+1) W_{24}^{(3n+1)k}$$

$$+ \sum_{n=0}^{7} x(3n+2) W_{24}^{(3n+2)k}.$$

$$= \sum_{n=0}^{7} x(3n) W_{24}^{3nk} + W_{24}^{k}\left[\sum_{n=0}^{7} x(3n+1) W_{24}^{3nk}\right]$$

$$+ \left(\sum_{n=0}^{7} x(3n+2) W_{24}^{3nk}\right) W_{24}^{2k}$$

$$= \sum_{n=0}^{7} x(3n) W_8^{nk} + a \sum_{n=0}^{7} x(3n+1) W_8^{nk} + b \sum_{n=0}^{7} x(3n+2) W_8^{nk}$$

This problem effectively reduces to computing three 8 point ~~dft~~ DFTs where sequences are

$\{x[3n]\}$ $\{x[3n+1]\}$ $\{x[3n+2]\}$

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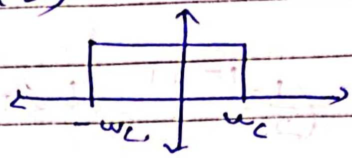
# Assignment-7

**Q1** Assume a low pass filter $H(z)$

Let the new filter be $H_1(z)$




(a) $H_1(z) = H(-z)$

considering

$\omega \in (-\pi, \pi]$

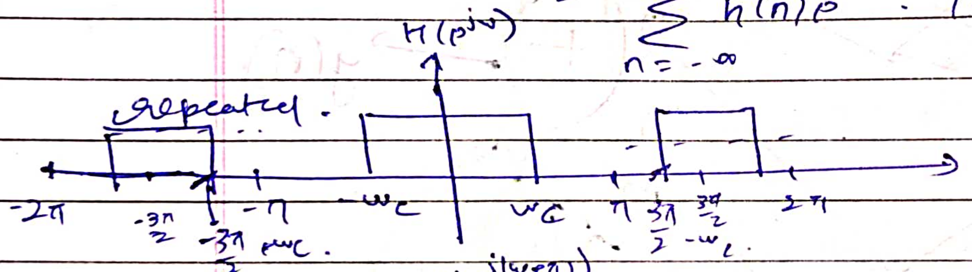
$H_1(e^{j\omega}) = H(-e^{j\omega})$

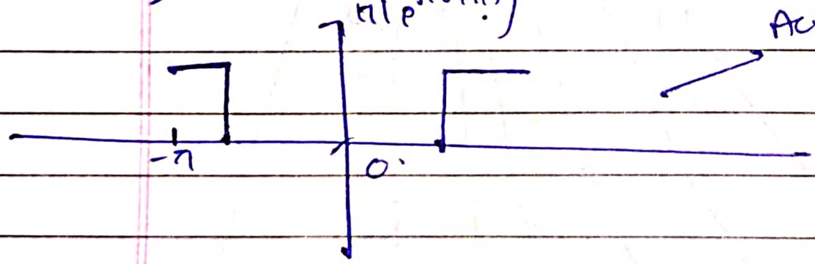
$= H(e^{j(\omega+\pi)})$ → shift of $\pi$

$H(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h(n)\, e^{-j\omega n}$

$H(e^{j(\omega+\pi)}) = \sum_{n=-\infty}^{\infty} h(n)\, e^{-j(\omega+\pi)n}$

$= \sum_{n=-\infty}^{n=\infty} h(n) e^{-j\omega n} \cdot (-1)^n$




Acts as a high pass filter.

(b) $H_1(z) = H(z^{-1})$

considering

$\omega \in (-\pi, \pi]$.

$H_1(e^{j\omega}) = H(e^{-j\omega})$

↳ symmetric

$= H(e^{j\omega})$

so same band is allowed to pass.

(c) $H_1(z) = H(z^2)$

from the assumed Ideal

$H_1(e^{j\omega}) = H(e^{2j\omega})$

at $\omega = \frac{\omega_c}{2}$

$H_1(e^{j\omega}) = 0$

↓

new cutoff. ← still LPF

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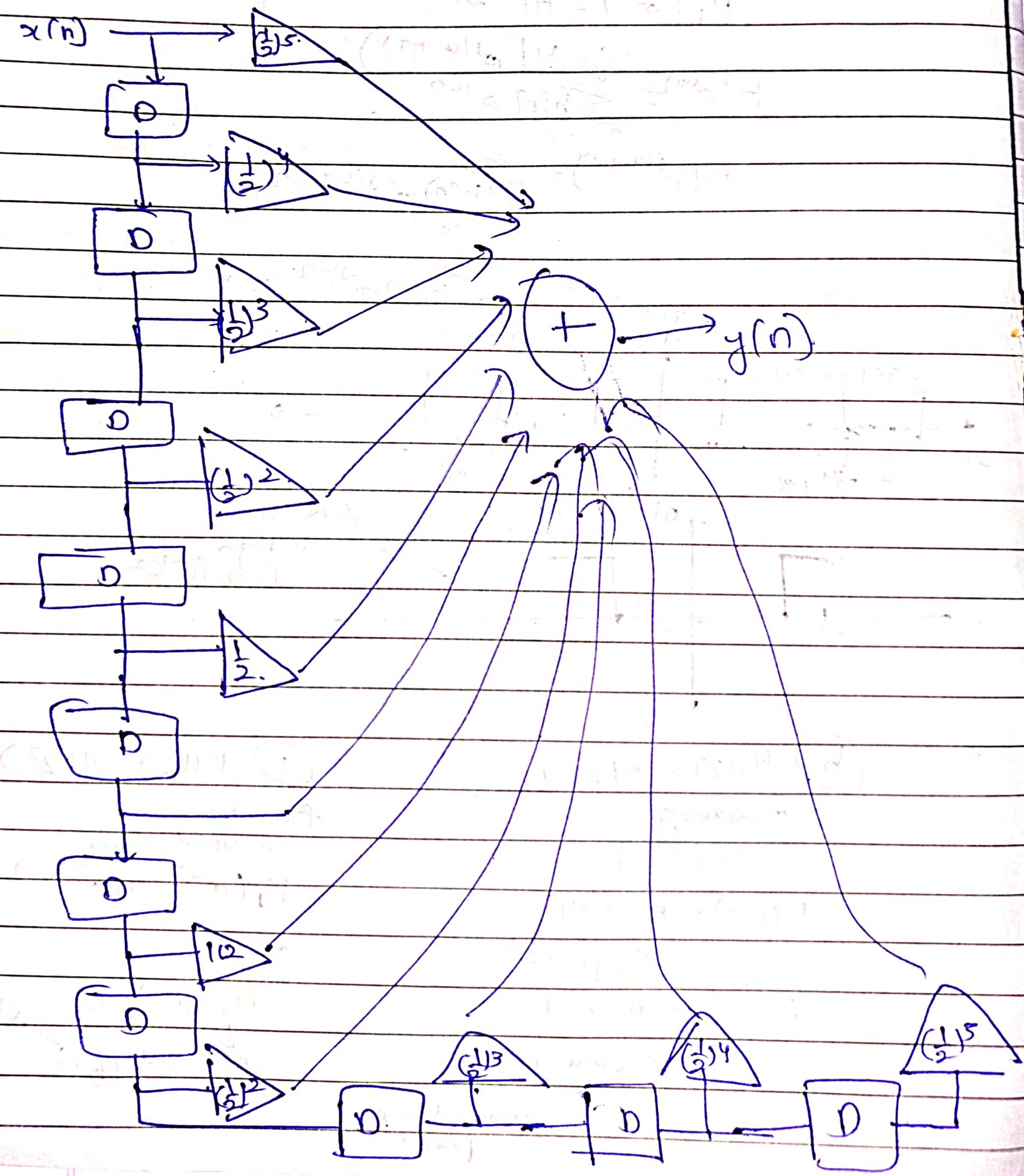
Q2

$$y(n) = \sum_{k=0}^{10} \left(\frac{1}{2}\right)^{|5-k|} x(n-k)$$

Clearly

$$h(k) = \left(\frac{1}{2}\right)^{|5-k|}, \; 0 \le k \le 10$$

Direct →



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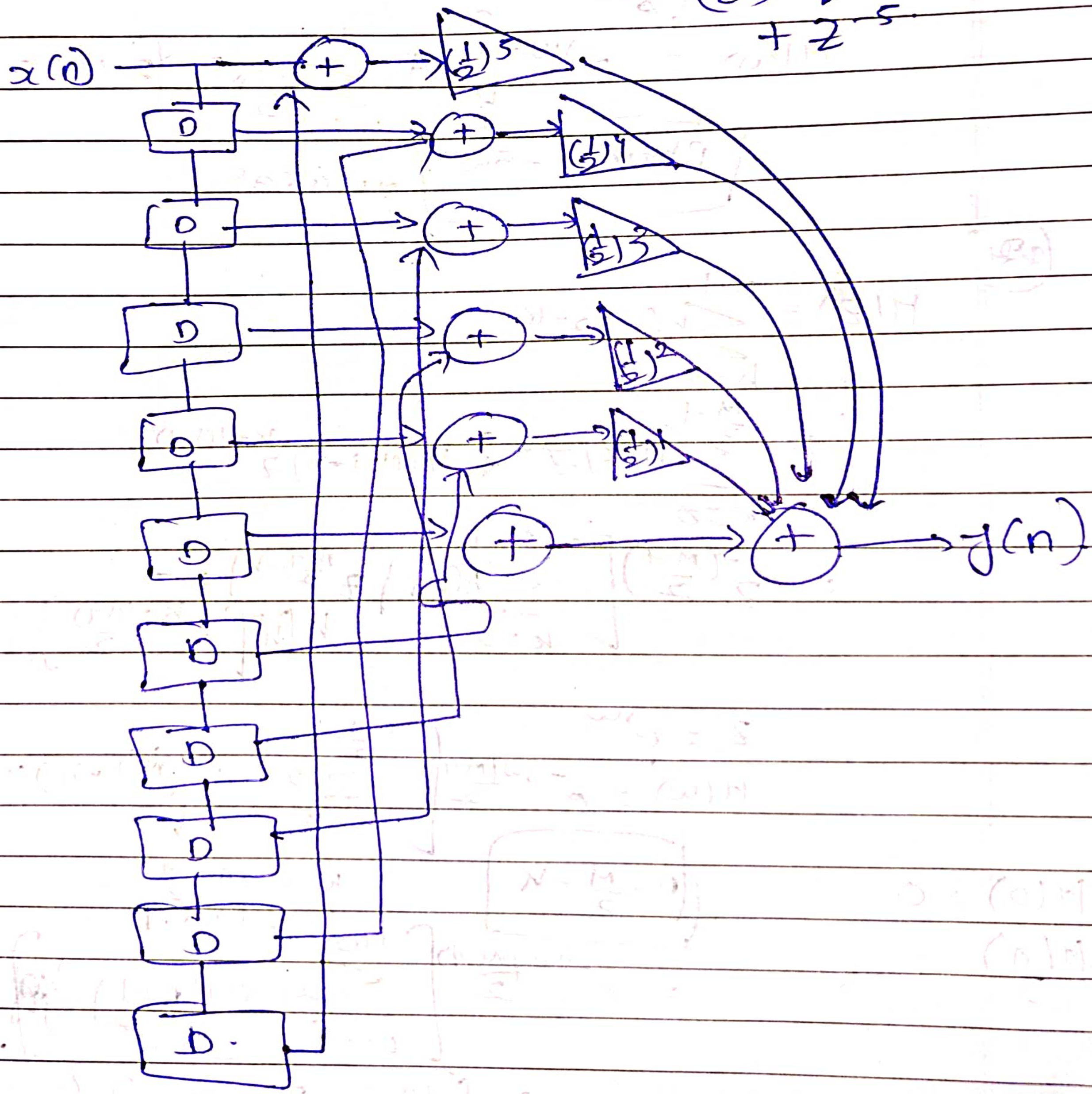
Indirect

group up
the common
scales

$$H(z) = \left(\frac{1}{2}\right)^5 \left[1 + z^{-10}\right] + \left(\frac{1}{2}\right)^4 \left[z^{-1} + z^{-9}\right] + \left(\frac{1}{2}\right)^3 \left[z^{-2} + z^{-8}\right] + \left(\frac{1}{2}\right)^2 \left[z^{-3} + z^{-7}\right] + \left(\frac{1}{2}\right)^1 \left[z^{-4} + z^{-6}\right] + z^{-5}$$



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(c) $H(z) = z^{-5}\left[\sum_{k=0}^{10} \left(\frac{1}{2}\right)^{|5-k|} z^{5-k}\right]$

$= z^{-5}\left[1 + \sum_{k=0}^{4}\left(\frac{1}{2}\right)^{(5-k)} z^{5-k} + \sum_{k=6}^{10}\left(\frac{1}{2}\right)^{k-5} z^{5-k}\right].$

$= z^{-5}\left\{1 + \sum_{k=0}^{4}\left(\frac{1}{2}\right)^{(5-k)}\left[z^{5-k} + z^{k-5}\right]\right\}$

$z = e^{j\omega}.$

$H(\omega) = e^{-j\omega 5}\left[1 + \sum_{k=0}^{4}\left(\frac{1}{2}\right)^{(5-k)}\left[2\cos(5-k)\omega\right]\right]$

$\boxed{\text{phase} = -5\omega} \rightarrow$ linear.

(Q4)

$H(z) = \sum_{k=0}^{M-1} h(k) z^{-k}$

$= \sum_{k=0}^{\frac{M}{2}-1} h(k) z^{-k} + h(M-1-k) z^{k-(M-1)}$

$= z^{-\left(\frac{M-1}{2}\right)}\left[\sum_{k=0}^{\frac{M}{2}-1} h(k)\left(z^{\frac{M-1}{2}-k}\right) - h[k]\left(z^{k-\left(\frac{M-1}{2}\right)}\right)\right].$

$z = e^{j\omega}$

$H(\omega) = e^{-j\omega\left(\frac{M-1}{2}\right)}\left[\sum_{k=0}^{\frac{M}{2}-1} 2j\sin\left(\left(\frac{M-1}{2}-k\right)\omega\right) h(k)\right]$

$H(0) = 0$

$\boxed{n = \frac{M}{2} - k}$

$k = 0 \rightarrow n = \frac{M}{2}$

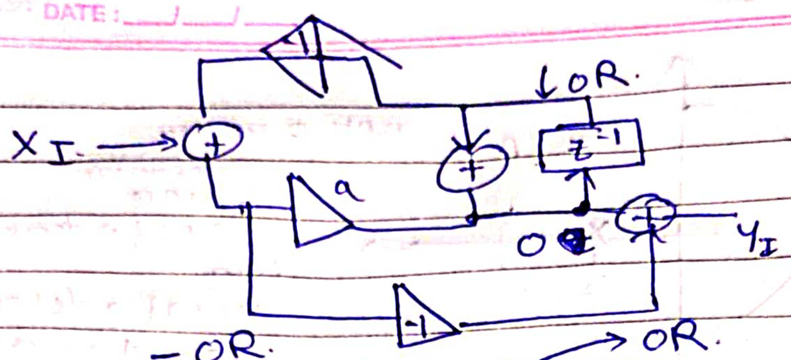
$k = \frac{M}{2} - 1 \rightarrow n = 1$

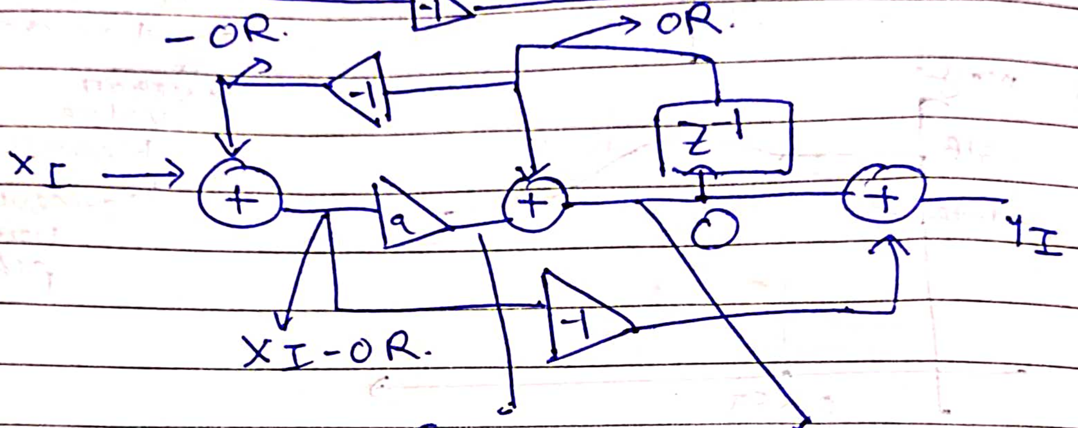
$H\left(\frac{\pi}{2}\right)$

$= e^{-j\omega\left(\frac{M-1}{2}\right)}\left[\sum_{n=1}^{M/2} 2j\sin\left(\left(n - \frac{1}{2}\right)\omega\right) \cdot h\left[\frac{M}{2} - n\right]\right]$

so $\boxed{d(n) = 2h\left(\frac{M}{2} - n\right)}$

Q4




$(X_I - OR)a.$ $(X_I - OR)a + OR = O$

assume

$H = \frac{y_I}{x_I} = \frac{z^{-1} + \frac{1}{2}}{\frac{z^{-1}}{2} - 1}$

$= \frac{\frac{1}{z} + \frac{1}{2}}{\frac{1}{2z} - 1} = \frac{\frac{z+2}{2z}}{\frac{1-2z}{2z}}$

$= \frac{z+2}{1-2z}.$

$X_I a = O(Ra + 1 - R)$

$O = \frac{X_I a}{Ra + 1 - R}.$

$y_I = O + OR - X_I$

$= \frac{X_I a}{Ra+1-R} + \frac{X_I a R}{Ra+1-R} - X_I$

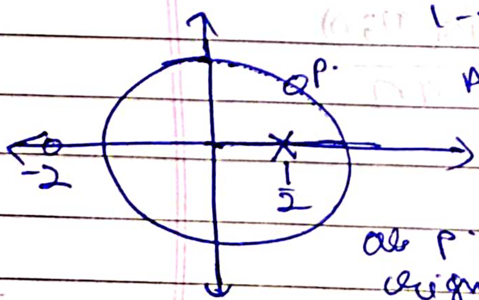
$= \frac{X_I a + \cancel{X_I a R} - \cancel{X_I a R} - X_I - X_I R}{Ra+1-R}.$

$= X_I \frac{[a - 1 - R]}{Ra + 1 - R},$

$\boxed{a = \frac{1}{2}}$

$= X_I \frac{\left[-R - \frac{1}{2}\right]}{1 - \frac{R}{2}}$

$\boxed{y_I = X_I \frac{\left[R + \frac{1}{2}\right]}{\frac{R}{2} - 1}.}$




As you move around the circle.

as P rotates right to left [first half)
$|z+2| \to$ decreas
$|1-2z| \to$ increase.
↳ value will decrease.

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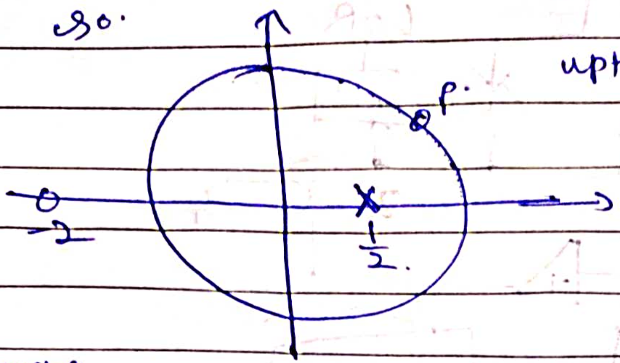
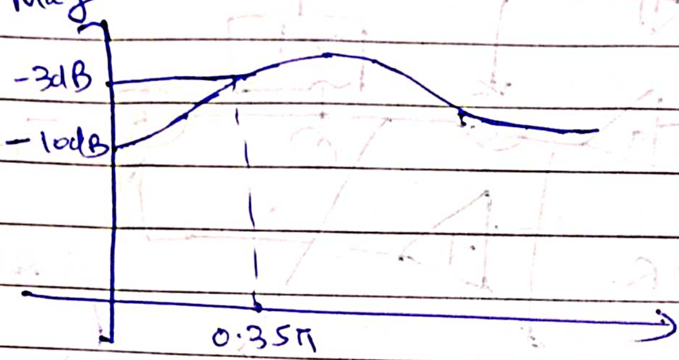
so.




upti 0 to $w_c$
the numerator distance
$|z+2| \to$ dominates
then
$|z+2| \to$ decreases
$|2z-1| \to$ increases
overall value becomes small.
somewhat low pass filter




(Q5) $|H(e^{j\Omega})| = 2, \; 0 \le |\Omega| < \frac{\pi}{6}$
$\qquad 1, \; \frac{\pi}{6} \le |\Omega| \le \frac{\pi}{3}$

$$h(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H(e^{j\Omega}) e^{j\Omega n} \, d\Omega$$

$$= \frac{1}{2\pi} \left[ 2 \left[ 2 \int_{0}^{\pi/6} e^{j\Omega n} \, d\Omega + \int_{\pi/6}^{\pi/3} e^{j\Omega n} \, d\Omega \right] \right]$$

$$= \frac{\sin\left(\frac{\pi}{6} n\right)}{\pi n} + \frac{\sin(\pi/3 \, n)}{\pi n}$$

(b) multiply a hamming window