1. Please answer the following questions:

   a. [1+1 points] Please write down the formula for bias ? Briefly explain the terms of the formula ?

   2

   b. [1+1 points] Please write down formula for Variance ? Briefly explain the terms of the formula ?

   2

   c. [1+1+1 points] Please explain in words what the Bias-Variance tradeoff means ? Please write down the formula that captures the tradeoff and mention the assumption being made regarding the noise in data ?

   3

d. [3 points] A hacker breaks into your system and corrupts the data that you are using to train your ML model. In particular, the hacker adds a Gaussian noise with mean 0 and variance which is triple the variance of the original noise (as mentioned in (c)) for each of the data points. How would the Bias-Variance tradeoff you mentioned in (c) change due to this ? Please provide detailed explanation along with some mathematical basis.

3

e. [3 points] If the goal of a regularization function is to reduce the bias component specifically, please provide insights for what maybe the design of such a regularization function along with an example that supports your design ?

3

2. **[3+4 points]** Consider a coin that comes up heads with probability p and tails with probability 1-p. Let q_n be the probability, that after n independent tosses, there have been an even number of heads. Derive a recursion that relates q_n to q_{n-1}. Please solve this recursion to establish the formula $q_n = (1+(1-2p)^n)/2$. Please show all the steps involved.

3. Probability represents degree of belief in a hypothesis. For a True or False question, the belief gets mentioned as <1,0> for True or <0,1> for False. However, you could specify that your belief of True to be x while False to be (1-x) – here x would be a real number between 0 to 1. Let's say the question has 2 points for right answer and -2 points for wrong answer. If the right answer is True, you would receive (2*x -2*(1-x)) while you would receive (-2*x+2*(1-x)) if the actual answer is False. Please answer the following questions based on this information:

   a. **[2 points]** If you believe the answer for a question is True, what would your estimate be, of the marks that your friend would obtain if you believe your friend would answer <0.4, 0.6> for this question ? Please show all steps of computation.

b. **[2 points]** If you believe the answer for a question is True, what would your estimate be, of the marks your friend would obtain if you believe your friend would answer <0.4, 0.6> with 0.6 probability while would answer <0.3, 0.7> with 0.4 probability ? Please show all steps of computation

c. **[3 points]** If you believe the answer for a question is <0.3, 0.7>, what would your estimate be, of the marks your friend would obtain if you believe your friend would answer <0.4, 0.6> with 0.6 probability while would answer <0.3, 0.7> with 0.4 probability ? Please show all steps of computation.

d. **[2+2 points]** If you believe the answer for a question is <0.6, 0.4>, what would your estimate be, of the marks your friend would obtain if you believe your friend has no clue of this question i.e., does not have any particular belief ? How would your estimate change (for the marks your friend would obtain) if you also do not have any clue of this question i.e., do not have any particular belief ?

4. You are part of a game show where the amount of money associated with each successive question doubles. The first question is valued 2 rupees, second question 4 rupees and so on till the last question which is valued 1024 rupees. If you answer a question correctly, you accumulate money corresponding to the question and move on to the next question. For example, if you answered question 3 correctly you will have accumulated (2 + 4 + 8) rupees and will be presented question 4. If a question is answered incorrectly, you are out

of the game and lose all the money accumulated so far i.e. your accumulated rupees become 0.

Each question has 4 choices (1), (2), (3) and (4). You have 1 minute to either answer the question or drop out of the game. If you exceed the 1 minute without specifying an option, it means you answered incorrectly. If you choose to drop out you will retain the money accumulated so far. Hence, if you choose to drop out at question 4 you will receive the 14 rupees accumulated so far i.e. till question 3.

Your utility for money is as follows: If x is the amount of money, your utility is x/2 till 100 rupees, x from 101 to 300 rupees and (x*2) thereafter.

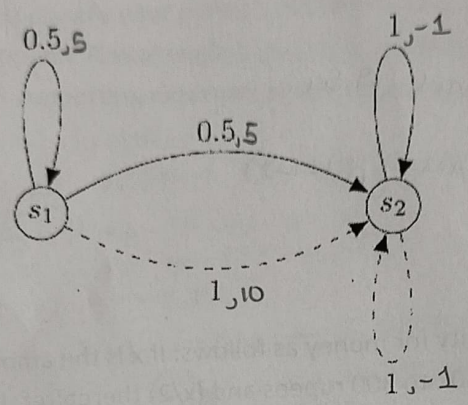
a) **[2 points]** How do you classify your utility function -- risk averse, risk neutral, risk seeking or a combination of them ? Please explain your reasoning for the classification in detail. Classification without supporting explanation will receive 0 points.

b) **[3 points]** You are at question number 6. Out of the 4 choices available for the question, you know (1) and (3) are wrong choices but are unsure about whether the answer is (2) or (4). However, you think the correct answer is (2) with probability 0.8 and (4) with 0.2. Please show the steps to compute what your expected utility would be if you pick (2), for all the possibilities of ground truth (i.e. (2) is the right answer and (2) is the wrong answer) ?

c) **[2 points]** For the question in (b), what will your game plan be i.e. pick option (2), pick option (4) or drop out ? Please explain your assumptions and reasoning in detail.

d) **[4 points]** Your friend has a utility for money as follows: If x is the amount of money, utility is (x*2) till 100 rupees, x from 101 to 300 rupees and (x/2) thereafter. If your friend faces the situation in (b), what would the game plan be for your friend i.e. pick option (2), pick option (4) or drop out ?

5. Consider the following simple Markov Decision Process. There are two states s1 and s2 and two actions a1, indicated by solid line and a2 indicated by dashed line. The diagram shows the transition probabilities. The rewards for the two actions are:
r(s1, a1) = 5, r(s2, a1) = -1, r(s1, a2) = 10 and r(s2, a2) = -1.



Calculate the values for the following questions with no discounting i.e. gamma = 1. Any of the two states can be a start state and can consider both the possibilities when needed.

[Useful formula: $ar^0+ar^1+ar^2+\ldots+ar^{n-1}=\frac{a(1-r^n)}{1-r}$]

a) **[3 points]** Calculate the value of the policy P1 that always takes action a1 for a finite time horizon T. Your answer should be derived as an expression in terms of T. Show all steps of the derivation.
[Interpretation for T: T = 1 => one action taken, T = 2 => two actions taken, one at T = 1 and the second at T = 2 and so on]

b) **[3 points]** Calculate the value of the policy P2 that always takes action a2 for a finite time horizon T. Your answer should be derived as an expression in terms of T. Show all steps of the derivation.

c) **[3 points]** Between the policies P1 and P2, which is the better policy at T = 1? At what time horizon do the policies switch order? Please show the computation.

d) **[4 points]** Please compute the A matrix for the above MDP (that would be needed as part of a Linear Programming solution).

6. Consider the game of Tic-Tac-Toe (noughts and crosses). Define X_n as the number of rows, columns or diagonals having exactly n X's and no O's. Similarly, O_n is the number of rows, columns or diagonals with just n O's. The utility function thus assigns +1 to any position with $X_3 = 1$ and -1 to any position with $O_3 = 1$. All other terminal positions have utility 0. We will use a linear evaluation function defined as

$$\text{Eval} = 2X_2 + X_1 - (2O_2 + O_1)$$

(a) Draw the whole game tree starting from an empty board down to depth 2, (i.e., one X and one O on the board), taking symmetry into account. You should have 3 board positions at level 1 and 12 at level 2.

[4 points]

(b) Mark on your tree the evaluations of all the positions at level 2.

[4 points]

(c) Mark on your tree the backed-up values for the positions at levels 1 and 0, using the minimax algorithm, and use them to choose the best starting move. <u>Underline</u> the node to which the first move should be made.

[4 points]

Briefly describe your answers.

7. Consider a dataset containing six one-dimensional points: {2, 4, 7, 8, 12, 14}. After three iterations of Hierarchical Agglomerative Clustering using Euclidean distance between points, we get the 3 clusters: C1 = {2, 4}, C2 = {7, 8} and C3 = {12, 14}. [10 points]

(a) Briefly describe single linkage. What is the distance between clusters C1 and C2 using Single Linkage?

(b) Briefly describe complete linkage. What is the distance between clusters C1 and C2 using Complete Linkage?

(c) Briefly describe hierarchical agglomerative clustering using euclidean distance. What clusters are merged at the next iteration using Single Linkage?

8. Use the table below that defines a training set containing 4 examples. The two attributes, X1 and X2 , and the class label, Y, are all binary. [10 points]

X1	0	0	1	1
X2	0	1	0	1
Y	1	1	1	0

(a) What is entropy? What is the entropy of Y, i.e., H(Y)? Use the convention that 0 log 0 = 0.

(b) Using the above training set, a Decision Tree is built that contains only 3 nodes: the root and its 2 children. Each leaf node is assigned the majority class of its associated set of examples; break ties in favour of Y = 0. What is the classification accuracy of this Decision Tree on the training set of 4 examples? Explain your answers briefly.

9. Tick *all* correct answers; Cross *all* wrong answers. Negative marks of 25% will be awarded for each wrongly marked answer.

(i) If Ram took the bus (B) or drove in his own car (D), then he arrived late (A) and missed the first session (M).

✓ ✗ (a) $\neg B \vee \neg D \vee A \vee M$

✓ ✗ (b) $\neg B \wedge \neg D \vee A \vee M$

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(c) $(B \vee D) \rightarrow (A \wedge M)$ [4 points]

(d) $B \vee D \vee A \vee M$

(ii) If the king does not castle (K) and the pawn advances (P) then either the bishop is blocked or the rook is pinned (R).

(a) $(\neg K \wedge P) \rightarrow (B \vee R)$

(b) $K \vee P \vee B \vee R$

(c) $\neg K \vee \neg P \vee B \vee R$

(d) $K \vee \neg P \vee B \vee R$ [4 points]

Briefly describe your answers.