EC5.406 – Signal Detection and Estimation Theory – Quiz 1

Date: 27th August, 2024
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Maximum marks: 18
Exam duration: 45 minutes

Instructions:

a) Mention any additional assumptions you make that is not given in the question.

b) Clearly show the steps used to arrive at the solutions.

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1. [10 marks] A sinusoidal signal is observed in presence of white Gaussian noise:

$$x[n] = \sin(2\pi f n) + w[n], \; n = 0, 1, \ldots, N-1,$$

where $w[n]$ are i.i.d. Gaussian random variables with mean 0 and variance σ^2 (known). The pdf of the observation $x[n]$ depends on the frequency parameter f and is denoted by $p(x[n]; f)$.

(a) For $f = \frac{1}{4}$, plot the pdf of $x[0], x[1], x[2]$, and $x[3]$.

(b) For $x[0] = 1$, plot the likelihood function $L(f) = p(x[0]; f)$ for $f \in [-1, 1]$.

(c) If $N = 3$, compute the CRLB for estimating f when $f = \frac{1}{4}$ and $f = \frac{1}{2}$. Give simplified answers.

(d) If σ^2 is also unknown, find the Fisher information matrix (FIM) corresponding to the parameters f and σ^2.

2. [8 marks] We wish to estimate the parameter A which influences the observations as

$$x[n] = A(-1)^n + w[n], \; n = 0, 1, \ldots, N-1,$$

where $w[n]$ are i.i.d. Gaussian random variables with mean 0 and variance σ^2.

(a) Verify that regularity conditions are satisfied when N observations are available.

(b) Find the Fisher information for the parameter A.

(c) Find the Cramer-Rao lower bound for A.

(d) Find MVU estimator for A when $N = 2$. Is this an efficient estimator? Justify.