EC5.406 – Signal Detection and Estimation Theory – Quiz 2

Date: 18th October, 2024 | Maximum marks: 18
Instructor: Santosh Nannuru | Exam duration: 45 minutes

Instructions:

a) Mention any additional assumptions you make that is not given in the question.

b) Clearly show the steps used to arrive at the solutions.

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1. [10 marks]

A single observation $x = A + w$ is available from which the parameter A is to be estimated. The noise w has a uniform distribution in the range $[-5, 5]$. The unknown parameter A is assumed to have a uniform prior distribution in the range $[0, 4]$. The noise w is independent of A. In an experiment, the value of x is observed to be -3.

(a) [4] Find and sketch the exact posterior PDF, $p(A|x)$, for this observation.

(b) [1] Find the MMSE estimate of A based on this observation.

(c) [2] Find the range of observations x such that the posterior PDF of A is same as its prior.

(d) [3] Instead of uniform prior, assume that the parameter A has a Gaussian prior distribution with mean 2 and unit variance. Plot the posterior $p(A|x)$ and find the MAP estimate of A for the same observation.

2. [8 marks]

It is known that the random variable x follows an exponential distribution given by,

$$p(x) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0 \\ 0, & x < 0. \end{cases}$$

The parameter λ of the distribution is not exactly known, but it has been hypothesized that either $\lambda = 1$ or $\lambda = 2$.

(a) [3] What is the form of the Neyman Pearson detector for this binary hypothesis testing problem given a single sample of x? Clearly sketch the decision regions.

(b) [2] Given false alarm rate of α, find the probability of detection.

(c) [3] If two i.i.d. samples of x are available, find the corresponding detector and decision regions.