Q3. **[12 marks]** Siva and Madhuri are observing a star to study its phase behaviour. When the star is in Phase-1, it emits a signal of frequency f_1 and when it transitions to Phase-2, it emits a signal of frequency f_2. During the observation period of N days, it is known that the phase change has occurred from Phase-1 to Phase-2 but the exact time of the phase change is not known. The measured signal on the nth day is,

$$\begin{aligned}\text{Phase-1}: x[n] &= \sin(2\pi f_1 n) + w[n], n = 0, 1, \ldots, L-1 \\ \text{Phase-2}: x[n] &= \sin(2\pi f_2 n) + w[n], n = L, L+1, \ldots, N-1.\end{aligned}$$

The observation noise $w[n]$ is zero-mean white Gaussian with unit variance. Assume that the frequencies f_1 and f_2 are known to Siva and Madhuri.

(a) Madhuri plans to use the maximum likelihood estimate for the time of phase change L. **How does the the simplified MLE look like? Assume that the total transmission energy** over the entire observation period is constant.

(b) Siva plans to use the least squares estimate for L. How does the estimator look like under same assumption on signal energy?

(c) A third observer, Kalyan, has knowledge of the phase change timing L but is unaware of the frequencies f_1 and f_2. Formulate and simplify the equations to be solved by Kalyan to estimate the frequencies if MLE is performed. You don't have to solve the equations.

Q4. **[10 marks]** The observed data $x[n]$ is known to have the following form,

$$\begin{aligned}x[n] &= 2\theta_1 + \theta_2 + w[n], \quad n = 0, 1, \ldots, N-1 \\ x[n] &= \theta_1 - 2\theta_2 + w[n], \quad n = N, N+1, \ldots, 2N-1\end{aligned}$$

where θ_1 and θ_2 are two unknown parameters and $w[n]$ is white Gaussian with mean zero and known variance σ^2.

(a) Find the minimum variance unbiased estimator (MVUE) for $\underline{\theta} = [\theta_1\ \theta_2]^T$ using the observations $x[n]$ from $n = 0, 1, \ldots, 2N-1$. **Is this estimator efficient? Justify.**

(b) Simplify and provide explicit formula for these MVU estimators of θ_1 and θ_2 in terms of the data $x[n]$.

(c) What is the variance of these estimators?

EC5.406 - Signal Detection and Estimation Theory
Mid Exam

Date: 25th September, 2025
Instructor: Santosh Nannuru

Maximum marks: 40
Exam duration: 90 minutes

Instructions:

a) Mention any additional assumptions you make that is not given in the question.

b) Clearly show the steps used to arrive at the solutions.

Q1. **[8 marks]** The cumulative internet usage at Vijay's home on nth day can be modeled to be minimum of 'An' **units with additional fluctuation** $w[n]$ given by,

$$x[n] = An + w[n]\,,\ n = 0, 1, \ldots, N-1,$$

where $w[n]$ are i.i.d. exponential random variables with parameter $\lambda > 0$. The exponential PDF is given as,

$$p(w[n]) = \begin{cases} \lambda e^{-\lambda w[n]}, & w[n] \geq 0 \\ 0, & w[n] < 0. \end{cases}$$

Based on the internet connected devices at home, Vijay knows the value of A.

(a) Given A, what is the CRLB for the parameter λ? Verify the regularity conditions.

(b) From the observed cumulative usage $x[n]$, **Vijay wishes to find the MLE of** λ. How can he do this? What is the asymptotic distribution of this estimator?

Q2. ***[10 marks]*** The radiation emitted by a certain radioactive material on the nth day is known to be $A_0 r^n$ where $A_0 > 0$ is the radiation emitted on 0th day and $r \in (0, 1)$ is the decay rate. The measured radiation on nth day is given by,

$$x[n] = A_0 r^n + w[n]\,, n = 0, 1, 2 \ldots N-1\,,$$

where the noise $w[n]$ depends on the measuring device. Answer the following.

(a) A measuring device D1 has white noise with $w[n]$ uniform in $[0, 1]$. Find the BLUE estimator for A_0 assuming that r is known.

(b) A measuring device D2 also has white noise but with $w[n]$ uniform in $[0, A_0]$. Find the BLUE estimator for A_0 assuming that r is known.

(c) Given $N = 2$ samples, which device would you prefer to use? How does your answer depend on A_0 and r?