LPDE Endsem 2025

Full marks: 100 (3 hours)

1. Consider two engines are connected in series between two heat baths with temperature T_h and T_c ($T_h > T_c$) respectively. If the efficiency of two heat engines are η_1 and η_2 respectively, then prove that the efficiency of the total heat engine is

$$\eta = \eta_1 + \eta_2 - \eta_1\eta_2.$$

(10)

2. Find the work done for adiabatic process considering n moles of ideal gas. (10)

3. Find the temperature $u(x,t)$ in a homogeneous bar of heat conducting material of length L with its ends kept at zero temperature and initial temperature given by $ax(L-x)/L^2$ (a is a constant). You can directly use any formula without deriving it. (10)

4. The equation of motion for one dimensional wave equation for a string of length L attached rigidly on both ends is given by

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}.$$

Find a general solution of this equation (25)

5. Construct the Euler-Lagrangian equation from variational principle. (25)

6. Construct the Hamilton's canonical equations (You can use any methods) (20)