EC5.406 - Signal Detection and Estimation Theory
Mid Exam

Date: 23rd September, 2024 | Maximum marks: 40
Instructor: Santosh Nannuru | Exam duration: 90 minutes

Instructions:

a) Mention any additional assumptions you make that is not given in the question.

b) Clearly show the steps used to arrive at the solutions.

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1. [10 marks] **A line and a sine**

   You have recently figured out that the stock price $x[n]$ of a company X on the nth day can be accurately modeled using a sinusoidal variation superimposed on a linear trend in presence of some WGN $w[n]$ as,

   $$x[n] = A_0 + A_1 n + \sin(\omega n) + w[n], n = 0, 1, 2 \ldots N-1,$$

   where the parameters A_0, A_1, ω are all unknown while the noise variance σ^2 is assumed to be known.

   (a) [4] Find the Fisher information matrix corresponding to the parameters A_0, A_1, ω.

   (b) [6] Later you have realized that the linear trend is in fact only a constant ($A_1 = 0$) and the frequency of the sinusoid is very small allowing the approximation, $\sin(\omega n) \approx \omega n$, to hold. Also, the noise is not Gaussian but has zero mean and variance σ^2. With this modification, answer the following:

      i. [5] How would you estimate the BLUE for the parameters A_0, ω? Give the relevant expressions and analysis.

      ii. [1] Is the above BLUE efficient? Explain.

2. [10 marks] **Channel gain**

   Shiva and Madhuri are trying to estimate the channel gain by transmitting sinusoidal signals of known frequency f. They set up the experiment as follows: Shiva transmits a sampled sinusoid $\sin(2\pi f n)$. The signal after passing through an additive white Gaussian noise channel with a gain of $g \in \mathbb{R}$ is received by Madhuri and can be written as,

   $$x[n] = g \sin(2\pi f n) + w[n], n = 0, 1, 2 \ldots N-1,$$

   where $w[n]$ is the additive white Gaussian noise with known variance σ^2. A total of N samples are received.

(a) [4] Derive the MVU estimator for g.

(b) [4] What is the mean, variance, and PDF of this estimator?

(c) [2] If Madhuri assumes no model for noise, find the LSE of the channel gain g.

3. [10 marks] **Noisy machine**

An object is being weighed using a noisy weighing machine. The true weight of the object is A and the weighing machine adds uniform noise in the range $[0, \alpha]$. The noise is IID across measurements. Both A and α are positive and are unknown to us. Our objective is to find maximum likelihood estimates of them using N measurements of the object obtained using the weighing machine.

(a) [4] What is the likelihood function? Note that it is a function of two variables – A and α. Clearly indicate and sketch the domain where the likelihood function is non-zero.

(b) [4] Find the maximum likelihood estimates of A and α. Explain your answer.

(c) [2] In an experiment, $N = 5$ measurements are obtained as follows – $\{50.1, 49, 55.7, 56, 52.9\}$. Using these measurements, calculate the maximum likelihood estimate of the signal to noise ratio defined as $\frac{A^2}{\alpha^2}$.

4. [10 marks] **Symmetry and Asymmetry**

A company manufactures two types of sensors – S_1 and S_2 with varying noise characteristics.

(a) [4] Sensor S_1 is known to have Laplacian noise distribution with PDF given as,

$$p(w) = \frac{\lambda_1}{2} e^{-\lambda_1 |w|},$$

where $\lambda_1 > 0$. Find the MLE of the parameter λ_1 given N IID observations of this sensor noise.

(b) [6] Sensor S_2 has an asymmetric noise distribution given as,

$$p(w) = \begin{cases} \frac{\lambda_2}{2} e^{\lambda_2 w}, & w < 0, \\ \frac{3\lambda_2}{2} e^{-3\lambda_2 w}, & w > 0, \end{cases}$$

where $\lambda_2 > 0$.

i. [1] Verify that this is a valid PDF and plot it.

ii. [5] Find the MLE of the parameter λ_2 given N IID observations of this sensor noise.