Science-II (SC1.111)

End-semester re-examination

(Spring 2025)

Total Marks: 24

Time: 2hrs.

Q1. Consider the Compartmental models (Susceptible-Recovery-Infection) of epidemiology. If at any time t, the density of the susceptible, infected, and recovered population is captured by s, i, and r respectively, construct the associated differential equation of each compartment. Explain each term in one or two sentences. Here the rate of infection is α, rate of recovery is β, and take $s + i + r = 1$. In which condition will the disease spread (assume at $t \sim 0, i \sim 0$, and $s \sim 1$)?

(3+2=5)

Q2a. $\frac{dx}{dt} = x - y^2; \frac{dy}{dt} = 2x + y;$

Find out the fixed points. Check if the fixed points are stable or not. Also, find out the eigenvectors. (1+2+2=5)

(You have to calculate the Jacobian matrix and try to find the eigenvalues and eigenvectors of the Jacobian at the fixed points. No need to proof of the construction of Jacobian).

Q2b. Give an algorithm to numerically solve the above coupled differential equations. **(2)**

Q3a. The Predator-prey system consists of two kinds of animals. One of which preys on the other. If S_1 symbolize prey, S_2 the predator, and A the food of the prey,

$S_1 + A \rightarrow 2S_1$ *(rate constants: C_1)*

$S_2 + S_1 \rightarrow 2S_2$ *(rate constants: C_2)*

$S_2 \rightarrow B$ *(rate constants: C_3)*

Write the master equations. Explain the terms.

Q3b. Write down the associated mean-field equations (coupled ODE model).

(3+3=6)

Q4a. The simple random walk problem in one dimension.

Let n_1 denotes the number of steps to the right, n_1 the number of steps to the left, and N $(n_1 + n_2)$ the total number of steps. Assuming successive steps are independent of each other, p = probability that the step is to the right, and $q = 1 - p$ = probability that the step is to the left. Calculate the probability $W(n_1)$ of taking (in a total of N steps) n_1 steps to the right and $n_2 = N - n_1$ steps to the left, in any order. Explain the terms.

(2)

Q4b. Calculate the mean number of steps to the right. (2)

Q5. Let $S = \{s_1, s_2, s_3, \dots \dots s_\infty\}$ be an infinitely long sequence generated by a Random Number Generator. Also assume that each s_i is between [0,1]. What is the arithmetic mean of the sequence? Here RNG is *uniform* over [0,1]. (2)