

1. The cosmic microwave background (CMB) photons are in the frequency range between 10^9 Hz to 10^{12} Hz . Meanwhile, the electromagnetic spectrum ranges from radio-waves (frequency $\sim 10^4 \text{ Hz}$) to gamma rays (frequency $\sim 10^{24} \text{ Hz}$). When the background photons first decoupled after the recombination, in which region of the electromagnetic spectrum did their frequency lie? Justify your answer. [2]

photons freq is between 10^9 Hz to 10^{12} Hz ,
after decoupling the cosmic microwaves are redshifted hence:

$$\frac{\nu}{\nu_0} = (1+z)^3; \text{ the } n\nu = n\nu_0(1+z)^3; \nu = \nu_0(1+z)^3$$

so the redshift waves lie in ~~microwave~~ gamma rays.

2. If the scattering rate of photons with electrons is $5 \times 10^{-6} / \text{s}$ and the Hubble constant is $2 \times 10^{-10} / \text{s}$, then are the photons still coupled to the electrons? Give reasons. [1]

$$H = 2 \times 10^{-10} / \text{s} \text{ and Thomson's scattering} = 5 \times 10^{-6} / \text{s}$$

The photons are ~~not~~ coupled because of the huge number density and high temperature.

and since $T < T_{\text{freeze}}$ the reaction rate is higher and

hence $H + \nu \rightleftharpoons p + e$ reaction is in equilibrium

4. The number density of photons in the present universe is $n_{\gamma 0} = 400/\text{cm}^3$. A photon emitted from a star is observed with redshift $z = 500$. What was the number density n_{γ} when the photon was emitted. [4]

$$n_{\gamma 0} = 400/\text{cm}^3$$

$$z = 500$$

$$n_{\gamma}$$

$$\frac{n_{\gamma}}{n_{\gamma 0}} = (1+z)^3$$

$$\therefore n_{\gamma} = 400 \times (501)^3$$

$$= n_{\gamma} = 400 \times 8.01 \times 10^8$$

$$= 3.204 \times 10^{11} / \text{cm}^3$$

photon emitted from a star is
photon was emitted. [4]

5. Calculate the redshift of the universe at the transition from radiation to matter domination. Assume that $\Omega_M^0 = 0.3$, $\Omega_R^0 = 3 \times 10^{-5}$, and $\Omega_\Lambda = 0$, $\Omega_K^0 = 0$. [8]

at transition,
 $\rho_R = \rho_M$

$$\rho_R = \rho_M$$

$$\rho_R \propto \frac{1}{a^4} \quad \rho_M \propto \frac{1}{a^3}$$

$$\rho_R \propto \frac{1}{a^4} \quad \rho_M \propto \frac{1}{a^3}$$

$$\rho_R = \frac{\rho_R}{a^4}$$

$$\rho_M = \frac{\rho_M}{a^3}$$

$$\rho_R \times (1+z)^4 = \rho_M (1+z)^3$$

$$1+z = \frac{\rho_M}{\rho_R}$$

$$= \frac{0.3}{3 \times 10^{-5}}$$

$$= \frac{3 \times 10^4}{3}$$

$$1+z = 10000$$

$$z = 999$$