

1. The cosmic microwave background (CMB) photons are in the frequency range between 10^9 Hz to 10^{12} Hz. Meanwhile, the electromagnetic spectrum ranges from radio-waves (frequency $\sim 10^4$ Hz) to gamma rays (frequency $\sim 10^{24}$ Hz). When the background photons first decoupled after the recombination, in which region of the electromagnetic spectrum did their frequency lie? Justify your answer. [2]

*photons freq is between 10^9 Hz to 10^{12} Hz,
after decoupling the cosmic microwaves are redshifted hence:*

$$\frac{n\bar{v}}{n\bar{v}_0} = (1+z)^3; \text{ thus } n\bar{v} = n\bar{v}_0(1+z)^3; \bar{v} = \bar{v}_0(1+z)^3$$

so the redshift waves lie in gamma rays.

✓

✓

2. If the scattering rate of photons with electrons is 5×10^{-6} /s and the Hubble constant is 2×10^{-10} /s, then are the photons still coupled to the electrons? Give reasons. [1]

$$H = 2 \times 10^{-10}/s \text{ and Thomson's scattering} = 5 \times 10^{-6}/s$$

The photons are ~~not~~ coupled because of the huge number density and high temperature.

and since $T < T_{freeze}$ the reaction rate is high and hence $H + e^- \rightleftharpoons p + \bar{\nu}$ reaction is in equilibrium

✓

4. The number density of photons in the present universe is $n_{\gamma 0} = 400/\text{cm}^3$. A photon emitted from a star is observed with redshift $z = 500$. What was the number density n_{γ} when the photon was emitted. [4]

$$n_{\gamma 0} = 400/\text{cm}^3$$

$$z = 500$$

$$n_{\gamma}$$

$$\frac{n_{\gamma}}{n_{\gamma 0}} = (1+z)^3$$

$$\therefore n_{\gamma} = 400 \times (501)^3$$

$$n_{\gamma} = 400 \times 1.25 \times 10^8$$

$$= 5 \times 10^{10}/\text{cm}^3$$

X

photon emitted from a star is
photon was emitted. [4]

5. Calculate the redshift of the universe at the transition from radiation to matter domination. Assume that $\Omega_M^0 = 0.3$, $\Omega_R^0 = 3 \times 10^{-5}$, and $\Omega_A = 0$, $\Omega_K^0 = 0$. [8]

At transition,

$$S_F = \rho_M$$

~~$$S_F = \rho_F S_R$$~~

~~$$\rho_F \propto \frac{1}{a^4} \quad \rho_M \propto \frac{1}{a^3}$$~~

~~$$\frac{\rho_M}{(1+z)^3} = \frac{\rho_F}{(1+z)^4}$$~~

$$\rho_F \propto \frac{1}{a^4}$$

$$S_F = \frac{S_R}{a^4}$$

$$\frac{\rho_M}{(1+z)^3} = \frac{\rho_F}{(1+z)^4}$$

$$\rho_M = \frac{\rho_M}{a^3}$$

$$S_F \propto (1+z)^4 = S_M (1+z)^2$$

$$1+z = \frac{\rho_M}{\rho_F}$$

$$= \frac{0.3}{3 \times 10^{-5}}$$

$$= 3 \times 10^4$$

$$1+z = 10000$$

$$z = 999$$