International Institute of Information Technology Hyderabad

Modern Complexity Theory (CS1.405)

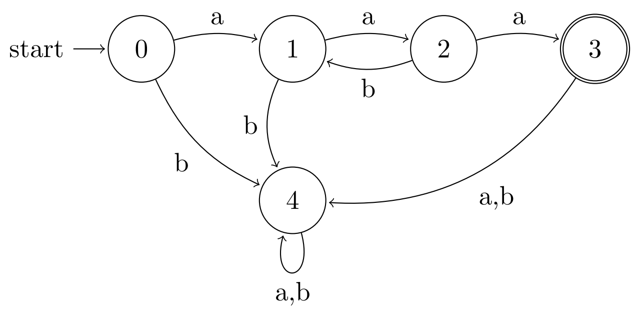
Assignment 1
Deadline: August 30, 2025 (Saturday), 17:00 PM
Venue for Hard-copy Submission:
CSTAR, A3-110, Vindhya Block, IIIT Hyderabad
Total Marks: 100

NOTE: It is strongly recommended that no student is allowed to copy from others.
No assignment will be taken after the deadline.
Write the following while submitting ONLY HARDCOPY:

Modern Complexity Theory (CS1.405)
Assignment 1
Name:
Roll No.:

Questions

1. (a) Determine the language accepted by the below DFA.



   (b) Let

$$ALL_{NFA} = \left\{ \langle N \rangle \;\middle|\; N \text{ is an NFA over alphabet } \Sigma \text{ and } L(N) = \Sigma^* \right\}.$$

   That is, ALL_{NFA} is the set of NFAs that accept all possible strings over Σ. Prove whether ALL_{NFA} is decidable.

[(5+5)=10]

2. (a) Explain: If L is accepted by an NFA with ϵ transition, then show that L is accepted by an NFA without ϵ transition.

   (b) Construct a DFA equivalent to the following NFA.
   $M = (\{p, q, r, s\}, \{0, 1\}, \delta, p, \{q, s\})$ where δ is defined in the following table:

	0	1
p	$\{q, s\}$	$\{q\}$
q	$\{r\}$	$\{q, r\}$
r	$\{s\}$	$\{p\}$
s	$-$	$\{p\}$

[(5+5)=10]

3. Let L_1 be strings with an even number of a's, and L_2 be strings with an odd number of b's over $\Sigma = \{a, b\}$. Construct a DFA for $L_1 \cap L_2$. **[10]**

4. Prove or disprove that the regular languages are closed under concatenation, union, complement and Kleene star. **[(2.5+2.5+2.5+2.5)=10]**

5. The symmetric difference between two sets A and B is defined as

$$(A - B) \cup (B - A).$$

If L_1 and L_2 be two regular languages (over some alphabet Σ), then show that their symmetric difference $(L_1 - L_2) \cup (L_2 - L_1)$ is also regular. **[10]**

6. Suppose that a MTM has four tapes and the tapes contents are shown below:

$$\begin{aligned} &\text{tape 1:} \quad 0\,1^{\downarrow}\,0\,1\,0\,\sqcup\,\ldots \\ &\text{tape 2:} \quad a\,a\,a^{\downarrow}\,\sqcup\,\ldots \\ &\text{tape 3:} \quad b^{\downarrow}\,a\,\sqcup\,\ldots \\ &\text{tape 4:} \quad x\,y\,z^{\downarrow}\,\sqcup\,\ldots \end{aligned}$$

where $\downarrow$ is the current tape position for a tape. Design a single-tape Turing machine, S for the above MTM. **[10]**

7. Construct a Turing Machine that accepts the language

$$L = \{\, 1^{n^2} 2^n 3^n \mid n \geq 1 \,\}.$$

Give the transition diagram for the Turing Machine obtained and also show the moves made by the Turing Machine for the string

111111222333

(where the number of 1's is the square of the number of 2's and 3's). . **[10]**

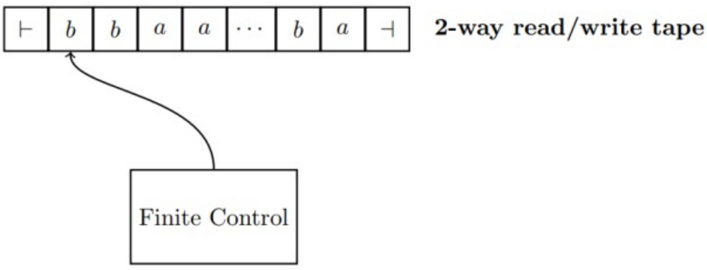
8. Let

$$A = \left\{ \langle R \rangle \;\middle|\; \begin{array}{l} R \text{ is a regular expression over } \{0,1\}, L(R) \text{ contains at least one string } w \\ \text{with 11 as a substring and no 000 as a substring.} \end{array} \right\}.$$

Show that A is decidable. **[10]**

9. Design an NTM to decide $L = \{w \mid w = w^R\}$, where w^R is the reverse of w. Show how a DTM would need to simulate all possible choices, and estimate the time difference. **[10]**

10. A linear bounded automaton (LBA) is exactly like a 1-tape Turing Machine, except that the input string $x \in \Sigma^*$ is enclosed in left and right endmarkers $\vdash$ and $\dashv$ which may not be overwritten. The machine is constrained never to move left of $\vdash$ or right of $\dashv$. It is allowed to read/write between these markers.



   (a) Give a rigorous formal definition of deterministic linearly bounded automata, including definitions of configurations and acceptance.

   (b) Let $\mathcal{M}$ be an LBA with state set Q of size k and tape alphabet Γ of size m. How many possible configurations are there on input x with $|x| = n$?

[(5+5)=10]

All the best!!!