# EC5.406 – Signal Detection and Estimation Theory – Quiz 2

Date: 28th October, 2025
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Maximum marks: 20
Exam duration: 45 minutes

1. [10 marks]
A random signal $x[n]$ is known to follow one of the two hypothesis,

$$\mathrm{H}_0 : x[n] \sim \mathcal{N}(0, \sigma^2),$$
$$\mathrm{H}_1 : x[n] \sim \mathcal{N}(A, \sigma^2),$$

where $A > 0$ and $\sigma^2$ are some known constants. A Neyman-Pearson detector is designed with a false alarm probability of $\alpha$ when $N$ observations are available.

It was later realized that under hypothesis $\mathrm{H}_1$, the distribution of $x[n]$ is in fact $\mathcal{N}(\beta A, \sigma^2)$ where $\beta > 1$.

(a) If for the same $P_{FA} = \alpha$, the probability of detection has increased by 10%, find $\beta$.

(b) To maintain the same $P_{FA}$ and probability of detection, it is decided to reduce the number of observations $N$. Find $\beta$ such that only half of the original observations, $N/2$, are required.

2. [10 marks]

(a) In Bayesian hypothesis testing, derive the detector which minimizes the probability of error.

(b) For the binary signal detection problem, derive the detector structure for minimum probability of error detection when the two hypotheses are equally likely. Assume that the noise is additive white Gaussian.