(CS1.406) Advanced Algorithms (Spring 2026) Time: 45 minutes

Quiz 1

Question 1 **[8 marks]**

A set $X = \{1, 2, 3, 4\}$ is partitioned into subsets by flipping an independent fair coin for each element: if HEADS, the element goes to subset A, otherwise to subset B.

(a) What is the probability that $A = \{1, 3\}$ and $B = \{2, 4\}$? [3 marks]

(b) What is the probability that $|A| = |B| = 2$? [5 marks]

Question 2 **[10 marks]**

A random variable Y takes only non-negative integer values with the following distribution:

y	0	2	4	8
$P(Y = y)$	0.4	0.3	0.2	0.1

(a) Calculate $\mathbb{E}[Y]$. [3 marks]

(b) Use Markov's inequality to find an upper bound on $P(Y \geqslant 6)$. [3 marks]

(c) Calculate the actual value of $P(Y \geqslant 6)$ and compare it with your bound from part (b). Is the Markov bound tight in this case? [4 marks]

Question 3 **[12 marks]**

Consider flipping a fair coin n times independently.

(a) What is the probability that exactly k flips result in HEADS? [3 marks]

(b) If we generate a random subset $S \subseteq [n]$ by including element i if the i-th flip is HEADS, what is the expected size of S? [3 marks]

(c) If two such subsets S_1 and S_2 are generated independently, what is the probability that $S_1 \cap S_2 = \emptyset$? [6 marks]

Question 4 **[10 marks]**

A multiset $S = \{2, 2, 5, 7, 7, 7\}$ can be represented by the polynomial:

$$p_S(x) = x^2 + x^2 + x^5 + x^7 + x^7 + x^7$$

(a) Write the polynomial representation for multiset $T = \{1, 3, 3, 5, 5, 5\}$. [2 marks]

(b) Explain how you would use polynomial identity testing to verify if $S = T$. [4 marks]

(c) What is an advantage of this randomized approach over sorting? [4 marks]