EC5.406 - Signal Detection and Estimation Theory
Final Exam

Date: 27th November, 2025
Instructor: Santosh Nannuru

Maximum marks: 60
Exam duration: 180 minutes

Instructions:

a) There are a total of 7 questions for 60 marks.

b) A single page (A4, two sided), self made, handwritten cheat sheet is allowed. It must be submitted along with the answer book.

b) Clearly show the steps used to arrive at the solutions.

Q1. [10 marks]
We are investigating a zero-mean random process $w[n]$ whose variance could increase with time. Under hypothesis H_0, the process is a white Gaussian noise process with variance of σ_0^2 for the complete observation duration of N samples. Under hypothesis H_1, the process is a white Gaussian noise process with variance of σ_0^2 for the first $n_0 < N$ samples, after which the variance increases to $\sigma_0^2 + \Delta$ for the rest of the observation and $\Delta > 0$.

(a) [5] For known σ_0^2 and unknown Δ, show that the UMP exists for this problem.

(b) [2] Find the expression for the ROC curve. Plot rough sketches of this ROC for two distinct values of Δ, for example $\Delta_2 > \Delta_1$.

(c) [3] For $N = 3$ and $n_0 = 2$, find and sketch the decision regions.

Q2. [10 marks]
We wish to check whether the noise process $x[n] \in \mathbb{R}$ follows a Gaussian or Laplacian distribution. The two hypotheses are as follows,

$$H_0 : p(x[n]; H_0) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{x^2[n]}{2\sigma^2}\right), n = 0, 1, \ldots, N-1$$
$$H_1 : p(x[n]; H_1) = \frac{1}{\lambda} \exp\left(-\frac{2}{\lambda}|x[n]|\right), n = 0, 1, \ldots, N-1$$

where σ^2 and λ are unknown positive parameters.

(a) [4] Find the MLE estimates of the unknown parameters.

(b) [3] Find the GLRT detector and express the test statistic in simplified form.

(c) [3] For $N = 2$, find and sketch the decision regions.

Q3. [10 marks]
Consider the problem of constant signal ($A > 0$) detection in the presence of Gaussian noise from $N = 2$ samples,

$$H_0 : x[n] = w[n], n = 0, 1$$
$$H_1 : x[n] = A + w[n], n = 0, 1 .$$

The noise $w[n]$ has zero-mean but variance of $\alpha\sigma^2$ and $\beta\sigma^2$ for $n = 0$ and $n = 1$ respectively. The Neyman-Pearson detector is known to be of the form $4x[0] + 3x[1] > \gamma$ to decide in favor of the hypothesis H_1.

(a) [7] If $\alpha + \beta = 10$, find α and β.

(b) [3] Find expression for the ROC curve.

Q4. [10 marks]
The parameter θ is known to have uniform prior distribution $\theta \sim U[0, \alpha]$. The observed data $x[n]$ are i.i.d. with the conditional distribution of $x[n] \sim U[0, \theta], n = 0, 1, \ldots, N-1$.

(a) [4] For $N = 1$, find and plot the exact posterior distribution of θ.

(b) [3] Find the MAP estimate of θ from N observations of $x[n]$.

(c) [3] Find the MMSE estimate of θ from N observations of $x[n]$.

Q5. [10 marks]
Under hypothesis H_0, the random variable x is Gaussian distributed with mean and variance μ_0 and σ_0^2 respectively. Alternately, under H_1 it is Gaussian with mean and variance μ_1 and σ_1^2 respectively. All parameters are known and additionally it is given that $\mu_1 > \mu_0$ as well as $\sigma_1^2 > \sigma_0^2$.

(a) [5] Find the likelihood ratio test detector and the simplified test statistic.

(b) [5] If $\mu_1 = \mu_0$, identify the decision regions. Derive the expression for ROC curve and comment on their behavior as σ_1^2 increases for a fixed σ_0^2.

Q6. [4 marks]
Answer the following giving equations,

(a) [2] State the Cramér-Rao lower bound theorem.

(b) [2] State the asymptotic property of the maximum likelihood estimator.

Q7. **[6 marks]**
Let x and y be two independent Gaussian random variables with distributions $\mathcal{N}(0, \sigma^2)$ and $\mathcal{N}(1, \sigma^2)$ respectively. Two observations w_0 and w_1 are available based on which we want to distinguish between the following two hypothesis,

$$\begin{aligned} H_0 : w_0 &= x + y \\ w_1 &= x \\ H_1 : w_0 &= x + y \\ w_1 &= y \, . \end{aligned}$$

(a) [3] Find the Neyman-Pearson detector for this hypothesis testing problem.
(b) [3] Derive the simplified test statistic and find its probability distribution under each of the two hypothesis.

Note that w_0 and w_1 are not independent random variables.