International Institute of Information Technology Hyderabad

Modern Complexity Theory (CS1.405)

Assignment 3
Deadline: November 7, 2025 (Friday), 17:00 PM
Venue for Hard-copy Submission:
CSTAR, A3-110, Vindhya Block, IIIT Hyderabad
Total Marks: 100

NOTE: It is strongly recommended that no student is allowed to copy from others.
No assignment will be taken after the deadline.
Write the following while submitting ONLY HARDCOPY:

Modern Complexity Theory (CS1.405)
Assignment 3
Name:
Roll No.:

Questions

1. Let
$$BOTH_{\text{NFA}} = \{\langle M_1, M_2\rangle \mid M_1 \text{ and } M_2 \text{ are NFAs where } L(M_1) \cap L(M_2) \neq \emptyset\}.$$
Show that $BOTH_{\text{NFA}}$ is NL-complete. **[10]**

2. Consider a game played on a partially ordered set (poset) $(P, \leq)$. Two players take turns. In each turn, a player chooses an element $x \in P$. That element x and all elements $y \in P$ such that $x \leq y$ (i.e., all elements "greater" than or equal to x) are removed from the set. The player who is left with no element to choose (i.e., P is empty at the start of their turn) loses.
Let *POSET-GAME* be the language defined as:
$$\textit{POSET-GAME} = \{P \mid P \text{ is a poset for which the first player has a winning strategy}\}.$$
Show that, *POSET-GAME* is PSPACE-complete. **[10]**

3. Let $MULT = \{a\#b\#c \mid a, b, c \text{ are binary natural numbers and } a \times b = c\}$. Show that, $MULT \in L$. **[10]**

4. Consider a deterministic Turing machine M, an input string x, and a nonnegative integer k (given in unary encoding, i.e., 1^k). Does M accept the input x without ever using more than k cells of its tape during its computation?
$$\textit{BOUNDED-TAPE-ACCEPTANCE} = \{\langle M, x, 1^k\rangle \mid M \text{ accepts } x \text{ using at most } k \text{ tape cells}\}.$$
Prove that, *BOUNDED-TAPE-ACCEPTANCE* is PSPACE-complete. **[10]**

5. Given a finite directed graph $G = (V, E)$, and a distinguished start vertex $s \in V$.
   - Two players, Player I and Player II, take turns moving a token that initially sits on vertex s.
   - On a player's turn, if the token is on vertex v, the player must choose an outgoing edge $(v, w) \in E$ that has not been used before, and move the token to vertex w.
   - Once an edge is used, it cannot be used again by either player.
   - If a player cannot make a move on their turn (no unused outgoing edges remain), that player loses.

   Does Player I (the first player) have a winning strategy starting from vertex s?
$$\textit{GENERALIZED-GEOGRAPHY} = \{\langle G, s\rangle \mid \text{Player I has a winning strategy in the game on } G \text{ starting at } s\}.$$
Prove that *GENERALIZED-GEOGRAPHY* is PSPACE-complete. **[10]**

6. Given $n \times n$ Boolean matrix A, indices $i, j \in \{1, \ldots, n\}$, and an integer k (given in unary, 1^k).Decide whether $(A^k)_{ij} = 1$ under Boolean matrix multiplication. Prove that, it is NL-complete. **[10]**

   Hint: This is equivalent to asking whether there exists a path of length k from vertex i to vertex j in the directed graph represented by A.

7. Show that, 2SAT is in NL. **[10]**

8. Consider the language,

$$EQ_{NFA} = \{\langle N, N' \rangle \mid N, N' \text{ are NFAs with the same alphabet and } L(N) = L(N')\}$$

   Show that $EQ_{NFA} \in PSPACE$. **[10]**

9. A *ladder* is a sequence of strings $s_1, s_2, \ldots, s_k$, wherein every string differs from the preceding one in exactly one character. For example the following is a ladder of English words, starting with "head" and ending with "free":

   head, hear, near, fear, bear, beer, deer, deed, feed, feet, fret, free.

   Let $LADDER_{\text{DFA}} = \{\langle M, s, t \rangle \mid M \text{ is a DFA and } L(M) \text{ contains a ladder of strings, starting with } s \text{ and ending with } t\}$. Show that $LADDER_{\text{DFA}}$ is in PSPACE. **[10]**

10. The cat-and-mouse game is played by two players, "Cat" and "Mouse," on an arbitrary undirected graph. At a given point each player occupies a node of the graph. The players take turns moving to a node adjacent to the one that they currently occupy. A special node of the graph is called "Hole." Cat wins if the two players ever occupy the same node. Mouse wins if it reaches the Hole before the preceding happens. The game is a draw if a situation repeats (i.e., the two players simultaneously occupy positions that they simultaneously occupied previously and it is the same player's turn to move).

    $\mathit{HAPPY\text{-}CAT} = \{(G, c, m, h) \mid G, c, m, h$, are respectively a graph, and positions of the Cat, Mouse, and Hole, such that Cat has a winning strategy if Cat moves first$\}$.

    Show that $\mathit{HAPPY\text{-}CAT}$ is in PSPACE. **[10]**

    Hint: The solution is not complicated and doesn't depend on subtle details in the way the game is defined. Consider the entire game tree. It is exponentially big, but you can search it in polynomial time.

All the best!!!