International Institute of Information Technology Hyderabad

Modern Complexity Theory (CS1.405)

Assignment 2
Deadline: October 10, 2025 (Friday), 17:00 PM
Venue for Hard-copy Submission:
CSTAR, A3-110, Vindhya Block, IIIT Hyderabad
Total Marks: 100

NOTE: It is strongly recommended that no student is allowed to copy from others.
No assignment will be taken after the deadline.
Write the following while submitting ONLY HARDCOPY:

Modern Complexity Theory (CS1.405)
Assignment 2
Name:
Roll No.:

Questions

1. Prove that if a unary language is NP-complete then $P = NP$. **[10]**

2. Imagine you and your group of friends are on a covert mission. You are the brain of the entire operation. One of the areas that your team has to cross, is scattered with mines. You, being the brain of the operation, have to ensure that your team moves swiftly without setting it off and alerting the enemy (also, avoid the death of anyone in the operation). So essentially, you become the *minesweeper*, like the game. You decide to mathematically model the area and stumble across the result that $MINESWEEPER \in NP - Complete$. Your superior doubts your capabilities and decides to replace you- but you are adamant and decide to hand him the proof to prove your capabilities. Show the mathematical model and the main result of $MINESWEEPER \in NP - complete$ you would show your superior. **[10]**

3. Consider the language $L_{NP} = \{\langle V, x, 1^n, 1^t \rangle : \exists u \in \{0,1\}^n$ such that V accepts $\langle x, u \rangle$ within t steps$\}$, where V is an encoding of a deterministic Turing machine. Prove that, L_{NP} is $NP - complete$. **[10]**

4. In a directed graph, the *indegree* of a node is the number of incoming edges and the *outdegree* is the number of outgoing edges. Show that the following problem is $NP - complete$. Given an undirected graph G and a designated subset C of G's nodes, is it possible to convert G to a directed graph by assigning directions to each of its edges so that every node in C has indegree 0 or outdegree 0, and every other node in G has indegree at least 1? Prove that it is $NP - complete$. **[10]**

5. Let SUMEXP $= \{\langle a, b, c, p \rangle \mid a, b, c, p$ are positive binary integers and $a^b + b^a \equiv c \pmod{p}\}$. Show that SUMEXP $\in P$. **[10]**

6. *MONOCHROME-ROW-COVER:* You are given an $m \times n$ board. Each cell of the board may contain a red stone, a blue stone, or be empty. You are allowed to delete (remove) stones from the board if you wish. Your goal is to decide whether it is possible, after some deletions, to make the board satisfy the following two conditions:

   Monochromatic rows: In the final board, every row that still contains stones must contain stones of only one color (all red or all blue). Rows are also allowed to become completely empty.

   Column coverage: In the final board, every column must contain at least one stone (so no column can be left empty).

   Formally, the decision problem is:

   Instance: An $m \times n$ board with red/blue stones and empty cells. Question: Can we delete some stones so that each row is monochromatic (or empty) and each column has at least one stone?

   Prove that, *MONOCHROME-ROW-COVER* is $NP - complete$. **[10]**

7. You are given a box and a collection of cards as indicated in the following figure 1. Because of the pegs in the box and the notches in the cards, each card will fit in the box in either of two ways. Each card contains two columns of holes, some of which may not be punched out. The puzzle is solved by placing all

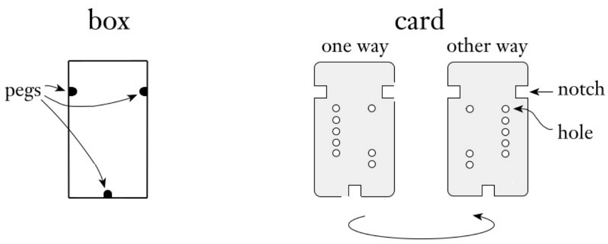

Figure 1

the cards in the box so as to completely cover the bottom of the box (i.e., every hole position is blocked by at least one card that has no hole there).

Let PUZZLE $= \{\langle c_1, \ldots, c_k\rangle \mid$ each c_i represents a card and this collection of cards has a solution$\}$. Show that, PUZZLE is $NP-complete$. **[10]**

8. A *coloring* of a graph is an assignment of colors to its nodes so that no two adjacent nodes are assigned the same color. Let

$$3COLOR = \{\langle G\rangle \mid G \text{ is colorable with 3 colors}\}.$$

Show that, *3COLOR* is $NP-complete$. (Hint: Use the following three subgraphs in figure 2.) **[10]**

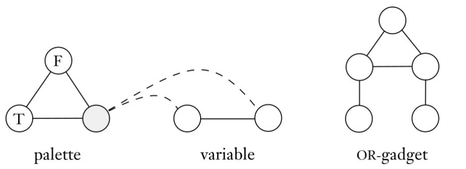

Figure 2

9. A subset of the nodes of a graph G is a *dominating set* if every other node of G is adjacent to some node in the subset. Let

$$\text{DOMINATING-SET} = \{\langle G, k\rangle \mid G \text{ has a dominating set with } k \text{ nodes}\}.$$

Show that it is $NP-complete$ by giving a reduction from VERTEX-COVER. **[10]**

10. Two Boolean formulas are said to be *equivalent* if they have the same set of variables and are true on the same set of assignments to those variables (i.e., they describe the same Boolean function). A Boolean formula is *minimal* if no shorter Boolean formula is equivalent to it. Let MIN-FORMULA be the collection of minimal Boolean formulas. Show that if $P = NP$, then MIN-FORMULA $\in$ P. **[10]**

All the best!!!