Advanced Comm. Theory Class Tutorial 2

November 8, 2021

1 Aims

1. To generate m-sequences given primitive polynomials;
2. To generate gold-sequences using two m-sequences with arbitrary delays;
3. To examine/plot the auto-correlation and cross-correlation properties of both m-sequence and gold-sequence.

Note: these functions will be used in the Coursework Part A.

2 Task A: m-sequence

Consider a primitive polynomial of degree m of the following form:

$$c_m D^m + c_{m-1} D^{m-1} + \cdots + c_1 D^1 + c_0, \qquad c_0 = 1, \tag{1}$$

1. Write a MATLAB function to produce the m-sequence based on the above primitive polynomial. That is,
$$\underline{\alpha} = \texttt{fMSeqGen}(\underline{c}), \tag{2}$$
where $\underline{c} = [c_m, c_{m-1}, \ldots, c_0]^T \in \mathcal{R}^{(m+1)\times 1}$ is a column vector containing polynomial coefficients and $\underline{\alpha} \in \mathcal{R}^{N_c \times 1}$ is the resultant m-sequence of +1s and -1s and $N_c = 2^m - 1$.

   Note that:

   - the function should be written in a way that is capable of producing m-sequence of any permissible length, i.e. any value of m;
   - the initial states of the shift register should be set to all ones.

2. For the polynomial $D^3 + D + 1$, the vector $\underline{c} = [1, 0, 1, 1]^T$, produce the corresponding m-sequence.
3. Plot the auto-correlation of the m-sequence in 2).

3 Task B: Gold-sequence

Consider two different primitive polynomials of the same degree m and coefficients $\underline{c}_1, \underline{c}_2 \in \mathcal{R}^{(m+1)\times 1}$, the corresponding two m-sequences are denoted by $\underline{\alpha}_1$ and $\underline{\alpha}_2$, respectively.

1. Write a MATLAB function to produce the gold-sequence based on the two m-sequences. That is,
$$\underline{b} = \texttt{fGoldSeqGen}(\underline{\alpha}_1, \underline{\alpha}_2, k), \tag{3}$$
where k is delay (k-bits) added to the 2nd m-sequence and $\underline{b} \in \mathcal{R}^{N_c \times 1}$ is the resultant gold-sequence of 1s and -1s. Please note $\underline{\alpha}_1$ and $\underline{\alpha}_2$ used for producing gold sequence should be sequences of 0s and 1s for XOR operation, namely, the output sequence of the shift register before transforming to a sequence of ± 1s.

2. Consider the following two polynomials

   (a) $D^5 + D^2 + 1$;
   (b) $D^5 + D^4 + D^3 + D^2 + 1$,

   construct matrix $\mathbf{b}$ containing all possible gold-sequences (including two m-sequences alone) as

   $$\mathbf{b} = [\underline{b}_1, \underline{b}_2, \ldots, \underline{b}_{N_c}, \underline{\alpha}_1, \underline{\alpha}_2] \in \mathcal{R}^{N_c \times (N_c+2)}, \tag{4}$$

   where $\underline{b}_k, \forall k = 1, 2, \ldots, N_c$ denotes the gold-sequence obtained by adding k-bits delay to the 2nd m-sequence. Identify all the **"balanced"** gold-sequences in matrix $\mathbf{b}$.

3. Plot the auto-correlation for the balanced gold-sequence with the smallest delay.

4. Plot the cross-correlation for the two balanced gold-sequences with the two smallest delays.

4 Submission

Submission via OneNote Class Exercise. No later than Sunday 21st November.

5 Marking

Each tutorial submission will be marked as "Pass" or "Fail": Pass = 1 mark; Fail = 0 mark.