MSc Laboratory Experiment

Multi-Input Multi-Output (MIMO) Communication

Academic supervisor: Dr. Cong Ling

Aims:

The main aim of this experiment is to give the students the concept of multi-input multi-output (MIMO) communication. Moreover, it will give them the idea of channel capacity, fading channel, QAM modulation, MIMO detection, space-time coding and computer simulation of MIMO communication. All the experiments will be performed using MATLAB.

Introduction:

MIMO is one of the key enabling technologies of broadband wireless. It makes use of multiple antennas in communication. Multi-antenna is not new; it has been used in diversity reception for decades. Traditionally, fading in wireless channels was regarded as a nuisance, which was combated by means of diversity, namely, receiving many copies of a signal from different antennas. In contrast, fading is now seen as a beneficial factor in MIMO communication. In MIMO communication, antennas are usually far spaced so as to achieve independent fading. It in fact takes advantage of independence naturally provided by fading to create multiple spatial channels in parallel. Hence, higher channel capacity is obtained over the same bandwidth, thereby higher spectral efficiency.

MIMO Setup:

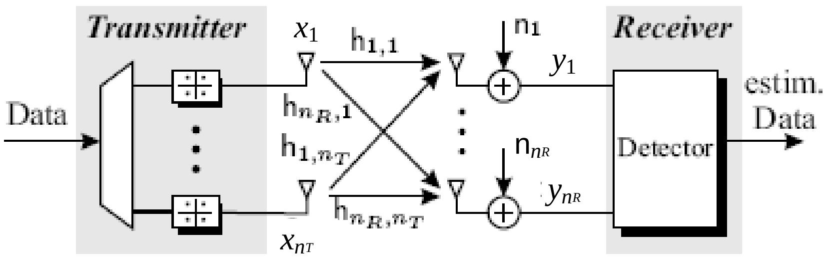

Fig. 1. Block diagram of a MIMO system.

Fig. 1 shows an n_R-by-n_T MIMO system. The n_R-dimensional received signal **y** can be written as

$$\mathbf{y} = \sqrt{\frac{\rho}{n_T}}\mathbf{H}\mathbf{x} + \mathbf{n} \tag{1}$$

where **x** the n_T-dimensional complex transmitted vector, **H** is the n_R-by-n_T complex Gaussian channel matrix with i.i.d. entries, and **n** is the n_R-dimensional i.i.d. complex Gaussian noise vector. Each entry of **H** or **n** has unit variance. The transmitted signal **x** satisfies the unit power constraint so that ρ represents the average signal to noise ration (SNR) at each receive antenna. Precise knowledge about **H** is assumed at the receiver, which can be acquired by sending pilots through the channel.

Capacity:

Now we assume $n_R = n_T = n$. Conditioned on a particular channel realization **H**, the instantaneous MIMO capacity is given by

$$C_{\mathbf{H}} = \log_2 \left| \mathbf{I} + \frac{\rho}{n_T} \mathbf{H}\mathbf{H}^H \right| \qquad \text{(bps/Hz)} \tag{2}$$

where | . | denotes matrix determinant.

Assume the channel exhibits sufficiently fast fading so that channel matrices **H** are statistically independent over the time. For such ergodic channels, the Shannon capacity is equal to the statistical average

$$C_{\text{ergodic}} = E_{\mathbf{H}}[C_{\mathbf{H}}]. \tag{3}$$

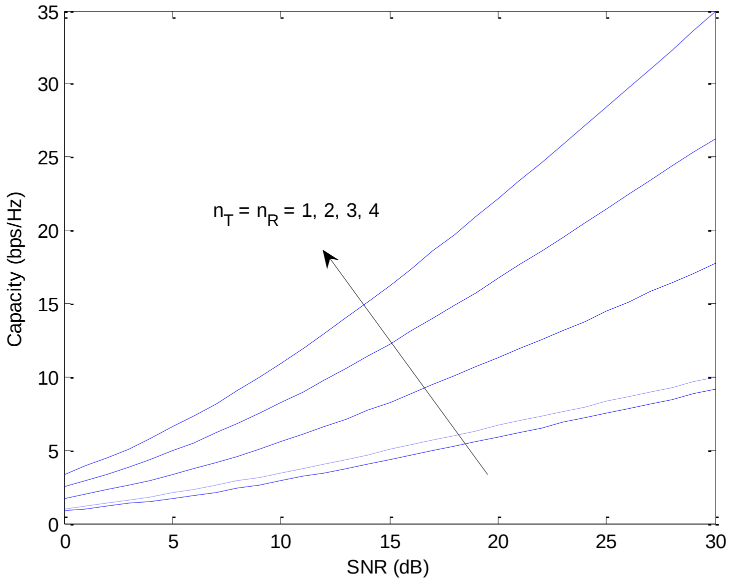

Fig. 2. MIMO capacity. Solid: MIMO fading channel; dotted: AWGN channel.

Fig. 2 contrasts the MIMO capacity to the capacity $\log_2(1+\rho)$ of a single-antenna complex AWGN channel. It is clear that the MIMO capacity can easily surpass that of a Gaussian channel. The MIMO capacity actually increases linearly with the antenna number. Hence fading is indeed a beneficial factor. This is in contrast to the common intuition that fading impairs the performance. In fact, the fading penalty is insignificant even for the single antenna case, as shown by the bottom two curves in Fig. 2.

Exercise 1: It is difficult to derive the closed-form expression for MIMO capacity (3). To generate a graph like Fig. 2, you can use Monte Carlo simulation, i.e., you generate a large number (e.g., 10000) of random matrices **H**, and take the average of $C_{\mathbf{H}}$. Plot a graph of MIMO capacity for $n_R = n_T = 5, 6, \ldots, 10$, for SNR from 0 to 30 dB.

Note: In communications, SNR is usually measured in decibel (dB). One has the formula $\rho(\text{dB}) = 10\log_{10}\rho$.

Transmission and Reception:

The incoming date are split into n substreams and simultaneously transmitted through n antennas, as shown in Fig. 1. The receiver has to detection the transmitted data. There are many ways to detection. Here we study a few of them.

The standard maximum-likelihood (ML) detector is given by

$$\hat{\mathbf{x}} = \arg\min_{\mathbf{x}\in C} \left\| \mathbf{y} - \sqrt{\frac{\rho}{n_T}}\mathbf{H}\,\mathbf{x} \right\|^2 \tag{4}$$

where C denotes the transmitter constellation and ||.|| denotes the Euclidean norm. However, it is slow for a large number of antennas.

Some suboptimal decoding strategies have been used in MIMO detection. The most obvious strategy is zero-forcing (ZF)

$$\hat{\mathbf{x}} = Q\left\{ \left(\sqrt{\frac{\rho}{n_T}}\mathbf{H} \right)^{-1} \mathbf{y} \right\} \tag{5}$$

where $Q\{.\}$ denotes the quantization rule. A well known drawback of ZF is the effect of noise amplification, since the channel matrix **H** might be ill-conditioned.

The performance of ZF can be improved by taking into account the knowledge of noise variance, which gives rise to minimum mean-square error (MMSE) detection

$$\hat{\mathbf{x}} = Q\left\{ \left(\frac{\rho}{n_T}\mathbf{H}^H\mathbf{H} + \mathbf{I} \right)^{-1} \sqrt{\frac{\rho}{n_T}}\mathbf{H}^H\mathbf{y} \right\}. \tag{6}$$

Fig. 3 shows the bit error rate of a 4-by-4 MIMO system with 64 QAM modulation with different detectors. Gray mapping is used for QAM. It is a rule mapping bits to QAM symbols and has the feature that neighboring symbols only differ in one bit. This is to minimize the bit error rate when a symbol is in error.

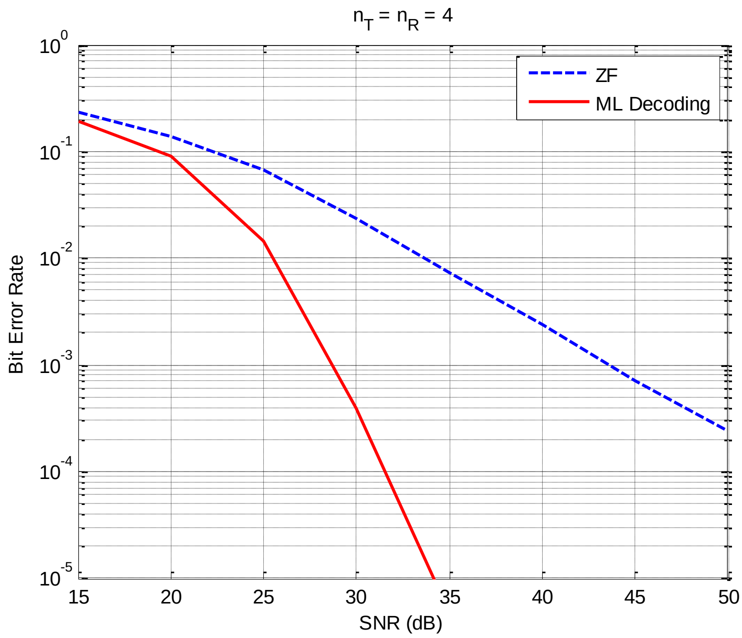

Fig. 3. Bit error rate of a 4-by-4 MIMO system with 64 QAM modulation

Exercise 2: Simulate a 2-by-2 MIMO system with QPSK modulation with Gray mapping, and produce the performance curves of ML, ZF and MMSE detection like Fig. 3, for SNR from 0 to 35 dB.

Space-Time Coding:

In the previous section, data are uncoded. Space-time codes can be used to improve the performance of MIMO. There are many types of space-time codes. Here we restrict to the famous Alamouti code for 2 antennas. Its code matrix is given by

$$\mathbf{X} = \begin{bmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \end{bmatrix}$$

where x_1 and x_2 are information symbols and * denotes complex conjugation. That is, in the first time slot x_1 and x_2 are respectively sent through Antenna 1 and Antenna 2, while in the second time slot $-x_2^*$ and x_1^* are respectively sent through Antenna 1 and Antenna 2. It can be seen that **X** is a unitary matrix.

In this case, the received signal is also a 2×2 matrix

$$\mathbf{Y} = \sqrt{\frac{\rho}{n_T}}\mathbf{H}\mathbf{X} + \mathbf{N} \tag{7}$$

The ML detection rule reads

$$\hat{\mathbf{x}} = \arg \min_{(x_1, x_2) \in C} \left\| \mathbf{Y} - \sqrt{\frac{\rho}{n_T}}\mathbf{H}\mathbf{X} \right\|_F^2 \tag{8}$$

where the Frobenius norm of a matrix is defined as $\|\mathbf{A}\|_F^2 = \sum_{i,j} a_{i,j}^2$.

Exercise 3: Simulate the Alamouti code with QPSK modulation and Gray mapping, and produce the performance curve of ML detection for SNR from 0 to 35 dB. Compare it with Exercise 2.

Remark 1: The ML detector (8) requires exhaustive search. In fact, it can be simplified to symbol-by-symbol detection, thanks to orthogonality of code matrix **X**. If you are interested, please refer to [1, Chap. 5] for details.

Remark 2: For Exercises 2 and 3, it is recommended to generate about 300 random matrices **H**, each carrying 1,000,000 bits.

References:

[1] Paulraj A., Nabar R. and Gore D., *Introduction to Space-Time Wireless Communications*, Cambridge Univ. Press, May 2003.
[2] Biglieri, E., *MIMO Wireless Communications*, Cambridge University Press, 2007.