CL Logbook

15/10/2021

MIMO $\quad n_t \times n_r$

$\underline{2 \times 2}$

$$\Rightarrow \quad y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\vec{y} = \sqrt{\frac{\rho}{n_T}}\, H \vec{x} + \vec{n}$$

$$H = \begin{bmatrix} \text{randn} + i \cdot \text{randn} & \text{randn} + i \cdot \text{randn} \\ \text{randn} + i \cdot \text{randn} & \text{randn} + i \cdot \text{randn} \end{bmatrix}$$

2×2

Gaussian

Unit variance

$$n = \begin{bmatrix} \text{randn} + i \cdot \text{randn} \\ \text{randn} + i \cdot \text{randn} \end{bmatrix}$$

2×1

Gaussian

ex 1 Capacity

$$C_H = \log_2 \left| I + \frac{\rho}{n_T} H H^H \right|$$

Conjugate transpose

$$\begin{bmatrix} h_1 & h_3 \\ h_2 & h_4 \end{bmatrix} \rightarrow \begin{bmatrix} h_1^* & h_2^* \\ h_3^* & h_4^* \end{bmatrix}$$

in Matlab : H'

$\rho = 0 - 30\,dB$

take 31 samples : $SNR_dB = [0, 1, 2, \dots, 30]$

$$SNR = 10^{\frac{SNR_dB}{10}} = [1, \dots, 1000]$$

16/10/2021

To get Cergodic → take 10000 random H → each H get a C_H (10000 C_H)

Cergodic ← mean

First try to write code in Matlab and plot a graph for $n_r = n_t = 1, 2, 3, 4$ to compare with the provided graph

Let C_H = zeros(10000, p)

each row stores a C_H

→ samples (1, 2, ..., 31)

0 dB 1 dB 30 dB

I = eye(nr(j)) Build the identical matrix

j = [1, 2, 3, 4]

$n_r=1$ $n_r=2$ $n_r=4$

10000 loops

H(m, n) = (rand n + 1i * rand n)

$n_r = n_t = 2$

0~30 dB

C_H(l, k) = $\log_2$(det(I + SNR(k)/n_t(j) * (H * H'))))

10000 loops k = 1 : p

C_ergodic(j, :) = mean(C_H)

Inorder to plot the figure without warning,

take abs (C_ergodic)

its imaginary part is about $1.0e^{-16}$ degree, ignorable

Then plot

the result is

1.0e-16 *

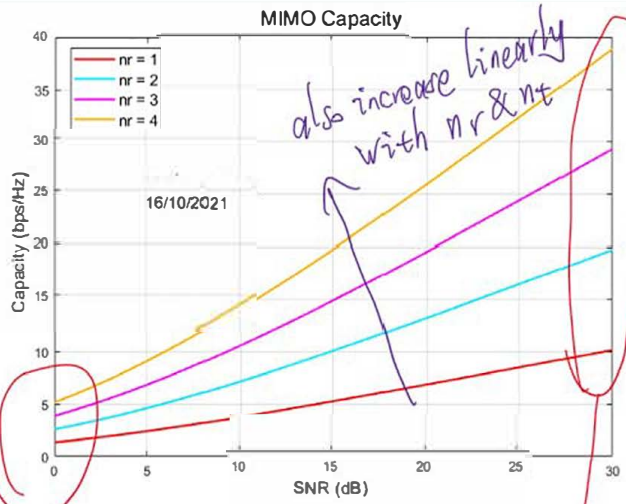
列 1 至 9

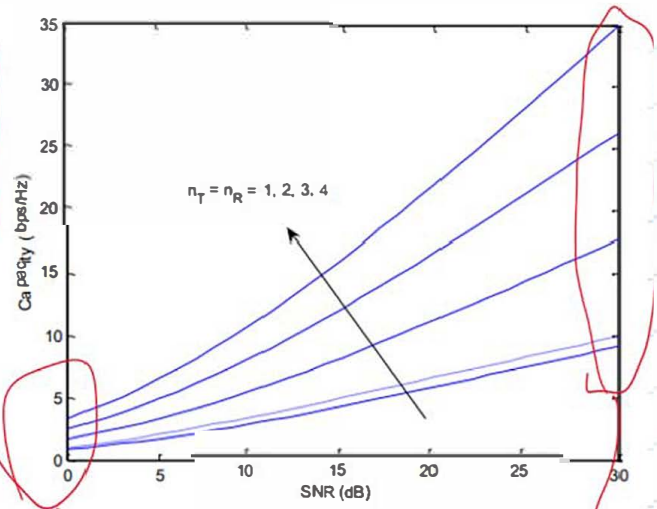
0.0000 + 0.0000i	0.0000 + 0.0000i	0.0000 + 0.0000i	0.0000 + 0.0000i	0.0000 + 0.0000i
0.0000 + 0.0069i	0.0000 - 0.0040i	0.0000 - 0.0057i	0.0000 + 0.0028i	0.0000 + 0.0096i
0.0000 - 0.0011i	0.0000 + 0.0053i	0.0000 + 0.0079i	0.0000 - 0.0096i	0.0000 + 0.0084i
0.0000 + 0.0022i	0.0000 + 0.0042i	0.0000 + 0.0059i	0.0000 - 0.0045i	0.0000 - 0.0002i

列 10 至 18

0.0000 + 0.0000i	0.0000 + 0.0000i	0.0000 + 0.0000i	0.0000 + 0.0000i	0.0000 + 0.0000i
0.0000 - 0.0000i	0.0000 + 0.0117i	0.0000 - 0.0257i	0.0000 - 0.0009i	0.0000 + 0.0115i
0.0000 - 0.0079i	0.0000 - 0.0326i	0.0000 + 0.0045i	0.0000 - 0.0015i	0.0000 - 0.0557i
0.0000 + 0.0182i	0.0000 + 0.0129i	0.0000 + 0.0048i	0.0000 + 0.0547i	0.0000 - 0.0195i

列 19 至 27





These two figures are about the same.

But, there is a problem

It is clear that, my result is a bit larger than the provided one.

Why?

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I forget to normalize H, it does not has unit variance

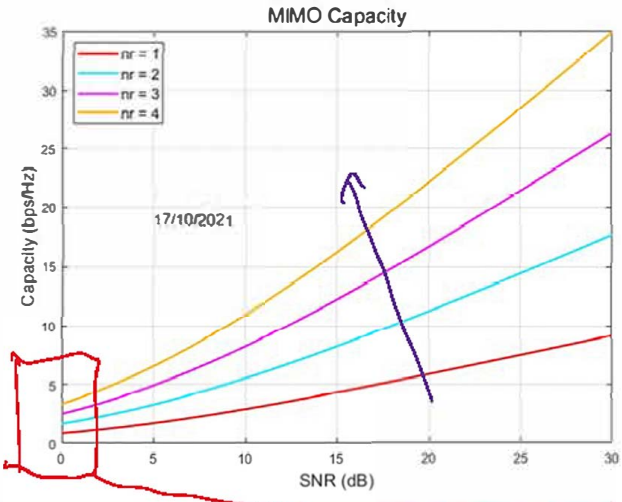
$$\cancel{H = \begin{bmatrix} randn + i\cdot randn & randn + i\cdot randn \\ randn + i\cdot randn & randn + i\cdot randn \end{bmatrix}}$$

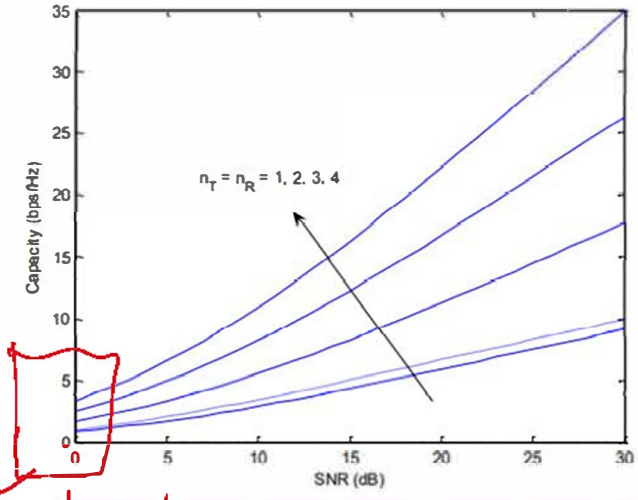
$$\Downarrow$$

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} \sim & \sim \\ \sim & \sim \end{bmatrix}$$

No ise should also has unit variance in ex 2 & 3 !!!

∴ The right figure is:





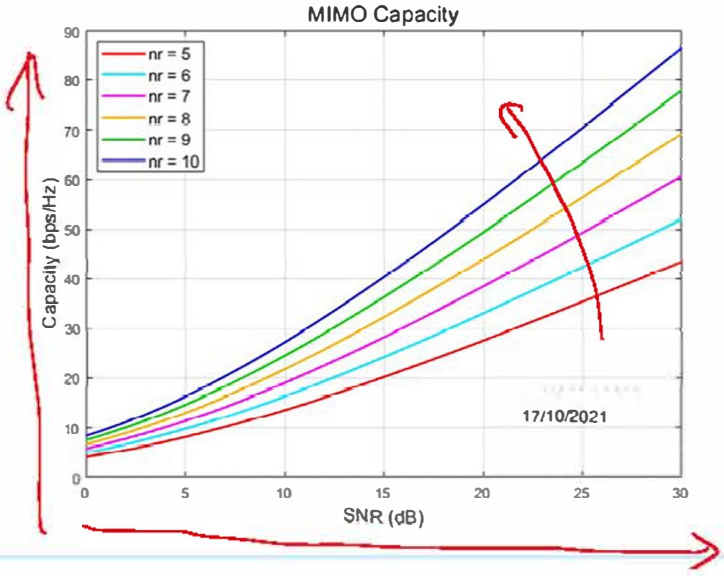
They are the same.

good, the same

∴ Let $n_r = n_t = 5, 6, 7, 8, 9, 10$

$j = [1, 2, 3, 4, 5, 6]$

Result:



Capacity increase linearly with nr & nt & SNR

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ex 2 Transmission and Reception

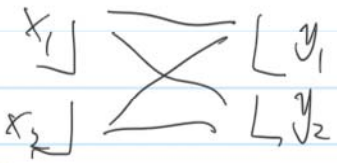
$$\vec{y} = \sqrt{\frac{P}{n_T}}\, H \vec{x} + \vec{n}$$

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} \text{randn} + i \cdot \text{randn} & \text{randn} + i \cdot \text{randn} \\ \text{randn} + i \cdot \text{randn} & \text{randn} + i \cdot \text{randn} \end{bmatrix}$$

2x2
Gaussian
unit variance

$$n = \frac{1}{\sqrt{2}} \begin{bmatrix} \text{randn} + i \cdot \text{randn} \\ \text{randn} + i \cdot \text{randn} \end{bmatrix}$$

2x1
Gaussian



$$y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

After getting y, detection

① ML

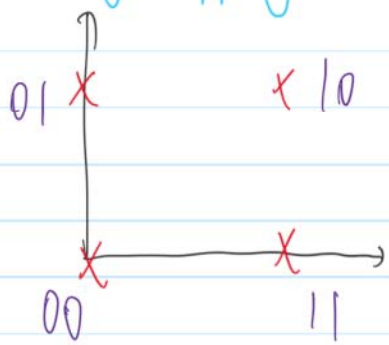
$$\hat{x} = \arg\min_{x \in C} \left\| y - \sqrt{\frac{P}{n_T}}\, H x \right\|^2$$

QPSK Modulation
with Gray mapping

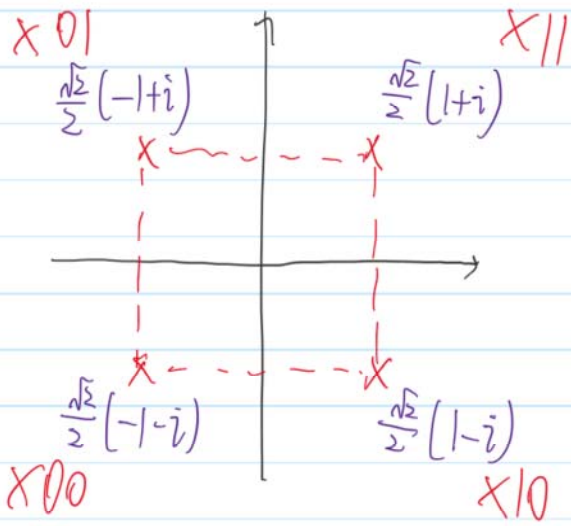
e.g: a set of bits <u>00011011</u>

00
01
10
11

Gray mapping



QPSK $\Longrightarrow$



$\because$ 2x2 MIMO, each time transmit 2 symbols

$$\therefore \quad x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} \pm 1 \pm i \\ \pm 1 \pm i \end{bmatrix}$$

$2 \times 2 \times 2 \times 2 = 16$ combinations

00	00	00	00	01	01	01	01	10	10	10	10	11	11	11	11
00	01	10	11	00	01	10	11	00	01	10	11	00	01	10	11

$\therefore$ After getting y

Find $y - \sqrt{\frac{\rho}{n_T}} H x \Rightarrow$ 16 values find the minimum one

$\Downarrow$

The corresponding x is regarded as the original two symbols

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Write code in Matlab

$x_00 = \frac{1}{\sqrt{2}}(-1-1i)$ $\quad$ $x_10 = \frac{1}{\sqrt{2}}(1-1i)$

$x_01 = \frac{1}{\sqrt{2}}(-1+1i)$ $\quad$ $x_11 = \frac{1}{\sqrt{2}}(1+1i)$

$$x_gray = \underbrace{\begin{bmatrix} x_00 & x_00 & \cdots & x_11 \\ x_00 & x_01 & \cdots & x_11 \end{bmatrix}}_{16 \text{ columns}}$$

$$C = \frac{1}{\sqrt{2}}\begin{bmatrix} \pm 1 \pm 1i \\ \pm 1 \pm 1i \end{bmatrix} \rightarrow \frac{1{,}000{,}000}{4} \text{ columns}$$

It is suggested to use 10^6 bits

Each column of C contain 2 symbols // 4 bits

$$C_all = \frac{1}{\sqrt{2}}\left(\text{randsrc}\left(n, \frac{b}{4}\right) + 1i * \text{randsrc}\left(n, \frac{b}{4}\right)\right)$$

for L=1: Loop 300 loops → plot figure use average of (300 SER)

$$H = \frac{1}{\sqrt{2}} * \left(\text{randn}(n,n) + 1i * \text{rand}(n,n)\right)$$

Generate a H, carries 10^6 bits

for k=1: p : Because the program is slow, let p=8 to tak 8 samples

for $o = 1 : \frac{b}{4}$ transmission for each row of C_all

$A = \sqrt{\frac{\rho}{n}}$ $\quad$ $y = AHc + n$

Then calculate norm $(y - AH\,x_gray)$ → one of 16 columns

find the min(16 values)

the received two symbol is the corresponding column of x_gray

check: if symbol $1 \neq c(1)$

error = error + 1

if symbol2 ≠ C(2)

error = error + 1

$$\therefore BER_ML(L, k) = \frac{error}{b/2} \times \frac{1}{2}$$

symbol errors

symbols

symbol error rate

Bit Error Rate

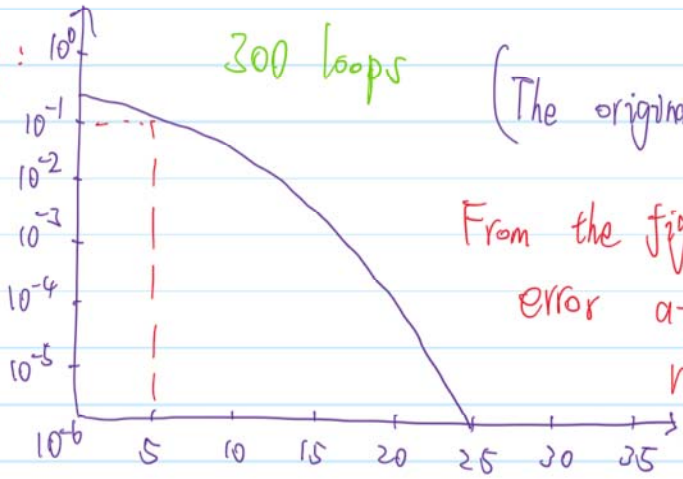
BER_ML_mean = mean(BER_ML, 1)

mean of rows

Run

20/10/2021

Result:



300 loops

(The original figure is deleted)

From the figure, only about 1 bit error at 25dB, while more than 10% of bits are wrong when SNR < about 5dB

Then add ZF & MMSE to the program

R_ZF = (AH)\y $\quad (A = \sqrt{\frac{\rho}{n_t}})$

$$R_MMSE = \left(A^2(H'H) + I\right) \backslash (AH'y)$$

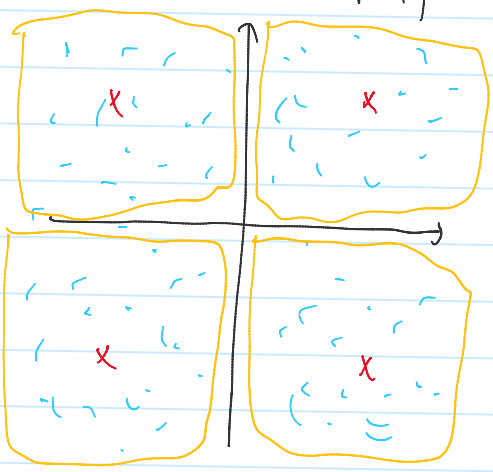
Here $R_ZF = \begin{bmatrix} a + bi \\ c + di \end{bmatrix}$ compare with $c\begin{bmatrix} e + fi \\ g + hi \end{bmatrix}$

If $\text{sign}(a) \neq \text{sign}(e)$ or $\text{sign}(b) \neq \text{sign}(f)$

error = error + 1

If $\text{sign}(c) \neq \text{sign}(g)$ or $\text{sign}(d) \neq \text{sign}(h)$

error = error + 1



The received symbol locates at the 1st/2nd/3rd/4th quadrant

⇓

consider it as the corresponding X_gray

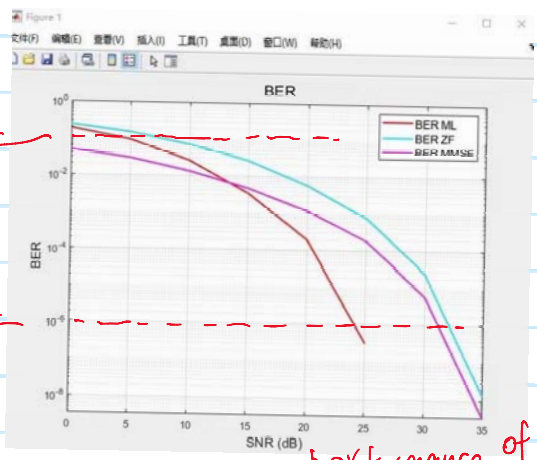
⇓

If it does not equals to the original symbol

error = error + 1

It is the same for R_MMSE

Result



10% wrong bits

only 1 bit is wrong

Only runs a few loops so there are some problems at the beginning

But the trend is right

performance of ML > ZF > MMSE

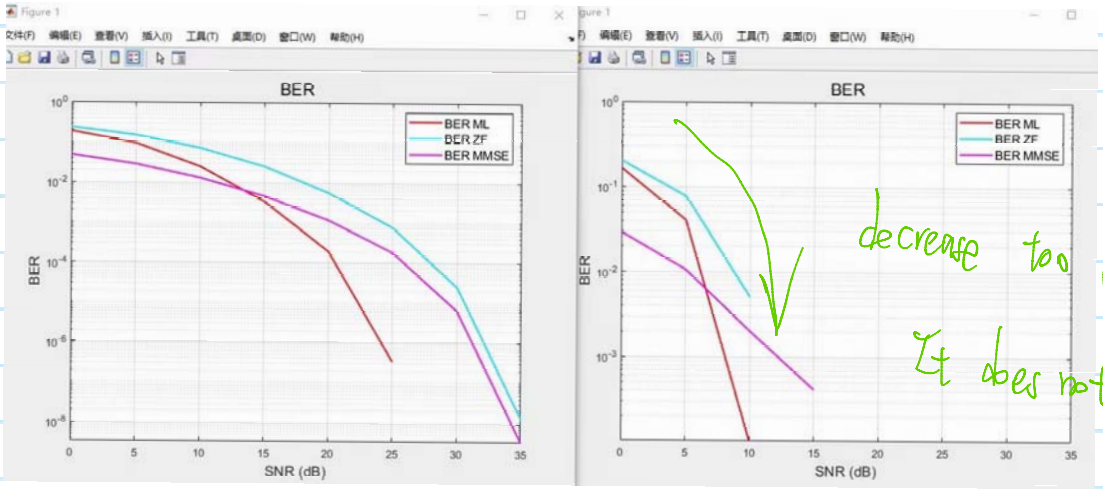
But I find a problem, will the noise change with SNR?

Because signal power is fixed, noise power should change to keep SNR

Hence I tried to change the power of noise

$$n = \frac{1}{\sqrt{2}}\begin{bmatrix} x \\ x \end{bmatrix} \Rightarrow \sqrt{\frac{1}{2SNR}}\begin{bmatrix} x \\ x \end{bmatrix}$$

Result for a few loops (everything is the same except noise power)



decrease too fast

It does not make sense

Wrong!

After checking, I find that

this noise is the noise in wireless channel, it is irrelevant to SNR

this SNR is the SNR at the receive antenna

ex3 Space-Time Coding

Since ex3 can be implemented by changing ex2 a bit, I decided to put them together

21/10/2021

$$X = \begin{bmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \end{bmatrix} \qquad N = \begin{bmatrix} a+bi & e+fi \\ c+di & g+hi \end{bmatrix}$$

$$y = \begin{bmatrix} X & X \\ X & X \end{bmatrix}$$ others are the same

After getting y

Calculate $\left(\text{norm}(y - A\,H\,x, \text{'fro'})\right)^2$

frobenius norm

e.g. $x_gray = \begin{bmatrix} x_{01} \\ x_{11} \end{bmatrix}$, here $X = \begin{bmatrix} x_{01} & -x_{11}^* \\ x_{11} & x_{01}^* \end{bmatrix}$

find the minimum

and find the corresponding x_gray

check: if symbol 1 $\neq$ C(1)

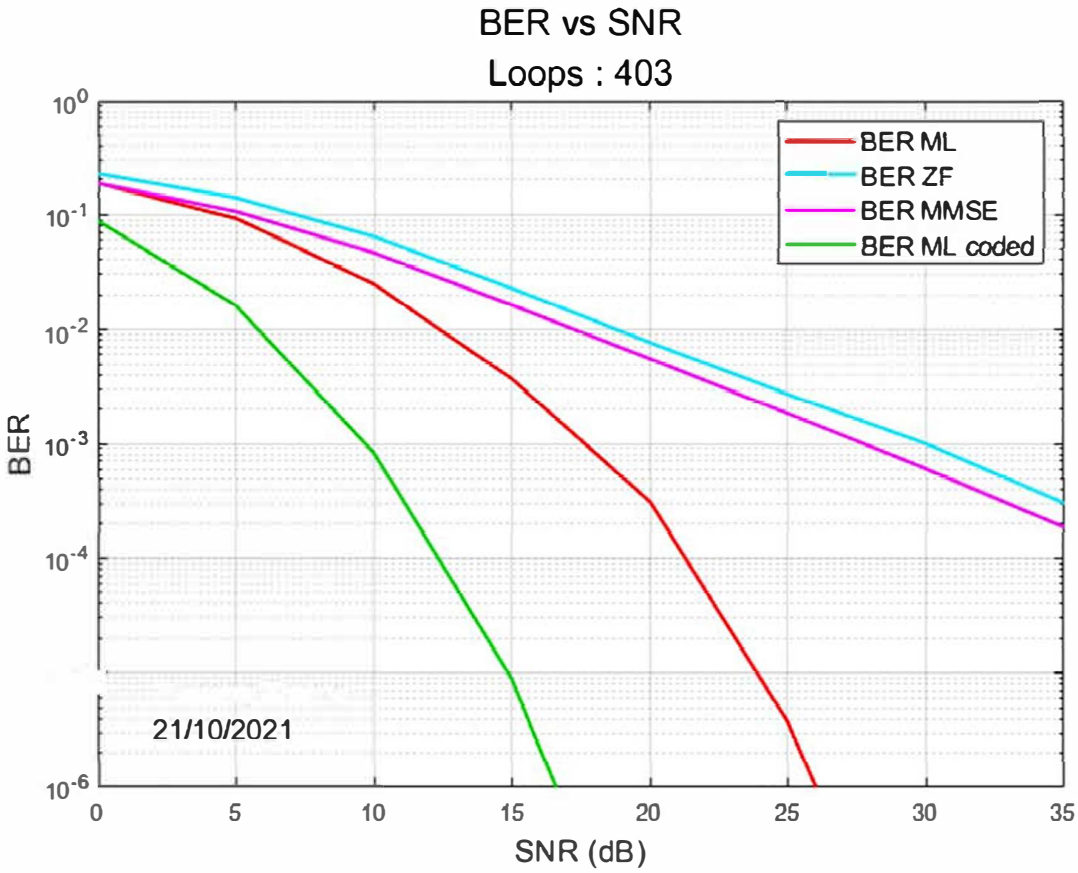
error = error + 1

if symbol 2 $\neq$ C(2)

error = error +1

The calculate the BER

Final Result



Good result.

1. BER give by ZF is constantly the largest one.
2. At the beginning, ML and MMSE show similar performance. But as SNR ↑, ML performs much better.
3. MMSE constantly lower than ZF, but the gap between them is not very large.

4. ML almost do not have error after SNR > 26 dB
MMSE and ZF still show a relatively large
BER at 35 dB

5. If the transmit symbols are coded, the performance
is significantly better than uncoded symbols.