BC Logbook

9/11/2021

frequency selective channel

SISO Single user

point-to-point communication + AWGN

III. A.

input : band-limited to W

baseband equivalent : limited to $\frac{W}{2}$

$$x_b(t) = \sum_n x[n] \operatorname{sinc}(Wt - n)$$

$$x[n] = x_b\left(\frac{n}{W}\right)$$

$$\operatorname{sinc}(t) = \frac{\sin(\pi t)}{\pi t}$$

$$y_b(t) = \sum_i a_i^b(t)\, x_b(t - \tau_i(t)) + w(t)$$

complex channel response

$$a_i^b(t) = \underset{\text{attenuation factor}}{a_i(t)}\, e^{-j 2\pi \overset{\text{carrier frequency}}{f_c} \underset{\text{delay}}{\tau_i(t)}}$$

Ex 1

1) $$y_b(t) = \sum_i a_i^b(t) \cdot \sum_n x[n] \operatorname{sinc}\left[W(t - \tau_i(t)) - n\right] + w(t)$$

2) Let $t = \frac{m}{W}$

$$y[m] = y_b\left(\frac{m}{W}\right) = \sum_i a_i^b\left(\frac{m}{W}\right) \sum_n x[n] \operatorname{sinc}\left(W\left(\frac{m}{W} - \tau_i\left(\frac{m}{W}\right)\right) - n\right) + w\left(\frac{m}{W}\right)$$

$$= \sum_n x[n] \sum_i a_i^b(m/W) \operatorname{sinc}\left(m - n - W\tau_i(m/W)\right) + w[m]$$

3) let $l \triangleq m - n$

$$y[m] = \sum_l x[m-l] \sum_i a_i^b(m/W) \operatorname{sinc}\left(l - W \cdot \tau_i(m/W)\right) + w[m]$$

Let $h_l[m] = \sum_i a_i^b(m/W) \operatorname{sinc}\left(l - \tau_i(m/W) \cdot W\right)$

$$y[m] = \sum_l h_l[m]\, x[m-l] + w[m]$$

lth complex channel filter tap at time m

If a_i^b & τ_i of the different paths are time-invariant

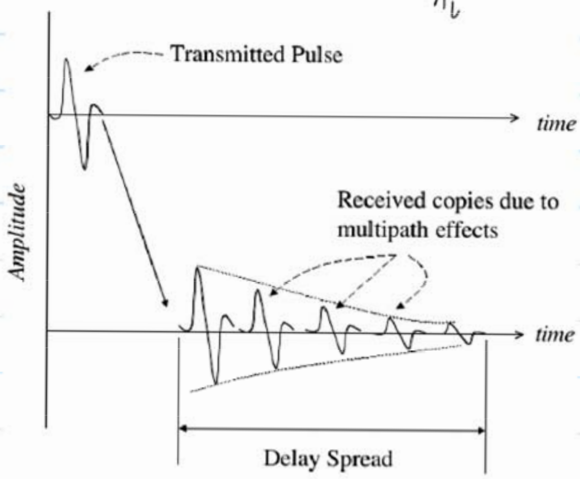
$$h_l[m] = \sum_i a_i^b \operatorname{sinc}(l - \tau_i W) = h_l$$

$$h_l[m] = \sum_i a_i^b \, \mathrm{sinc}(l - \tau_i W) = h_l$$

B. Delay spread : $T_d \triangleq \max_{i,j} |\tau_i(t) - \tau_j(t)|$

Coherence bandwidth $W_c \triangleq \frac{1}{2T_d}$

- $W < W_c$: flat fading (different frequency components see the same channel)
- $W > W_c$: frequency-selective fading (different frequency components see different channels)



C. Assume $h_l[m] = h_l$

$$y[m] = \sum_{l=0}^{L-1} h_l x[m-l] + w[m]$$

L nonzero taps

$\downarrow$ ith OFDM block

Average power constraint P

$$y_n[i] = \underbrace{h_n[i]}_{\text{DFT of the channel}} d_n[i] + w_n[i]$$

frequency selection channel $\rightarrow$ N_c independent sub-carriers

$n = 0, \dots, N_c - 1$

$w_n[i]$: Complex circularly symmetric Gaussian $\mathcal{CN}(0, N_0)$

$$E[\|d[i]\|^2] \le P$$

$d[i] = [d_0[i], \dots, d_{N_c-1}[i]]$

$w[i] = [w_0[i], \dots, w_{N_c-1}[i]]$

$y[i] = [y_0[i], \dots, y_{N_c-1}[i]]$

DFT

P_n : power of the nth subchannel

$\sum_n P_n = P$

maximum rate of reliable comm : $\sum_{n=0}^{N_c-1} \log_2\left(1 + \frac{P_n |h_n|^2}{N_0}\right)$ bit/OFDM symbol

$$\therefore \max_{P_0, \dots, P_{N_c-1}} \sum_{n=0}^{N_c-1} \log\left(1 + \frac{P_n |h_n|^2}{N_0}\right) \quad \text{s.t.} \begin{cases} \sum_0^{N_c-1} P_n \le P \\ P_n \ge 0, \; n = 0, \dots, N_c - 1 \end{cases}$$

standard $\Downarrow$ form

IV.

$$\min_{P_0, \dots, P_{N_c-1}} - \sum_{n=0}^{N_c-1} \log\left(1 + \frac{P_n |h_n|^2}{N_0}\right) \quad \text{s.t.} \begin{cases} -P_n \le 0 & n = 0, \dots, N_c - 1 \\ \sum_{n=0}^{N_c-1} P_n - P \le 0 \end{cases}$$

Ex 2

1) $$L(\lambda_1, \lambda_2, P_0, \ldots, P_{N_c-1}) = -\sum_{n=0}^{N_c-1} \log\left(1+\frac{P_n|h_n|^2}{N_0}\right) + \underbrace{\underbrace{\lambda}_{\geq 0}\underbrace{\left(\sum_{n=0}^{N_c-1} P_n - P\right)}_{\leq 0}}_{=0} + \cancel{\sum_{n=0}^{N_c-1} \lambda_n(-P_n)}$$

$$\frac{\partial L}{\partial P_n} = -\frac{\frac{|h_n|^2}{N_0}}{1+\frac{P_n|h_n|^2}{N_0}} + \lambda$$

$$= -\frac{1}{\frac{N_0}{|h_n|^2}+P_n} + \lambda = 0$$

$$P_n = \frac{1}{\lambda} - \frac{N_0}{|h_n|^2}$$

if $P_n < 0$, let $P_n = 0$

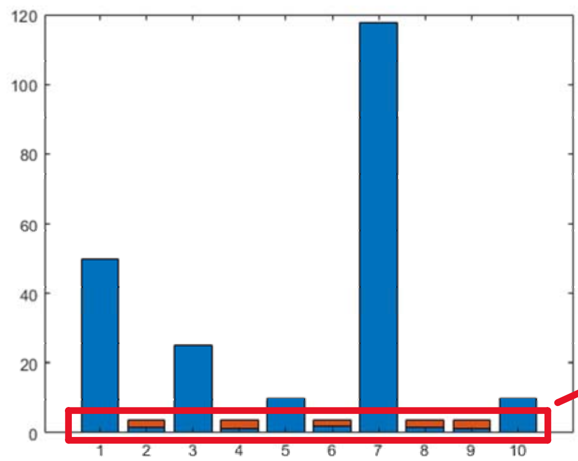
2)

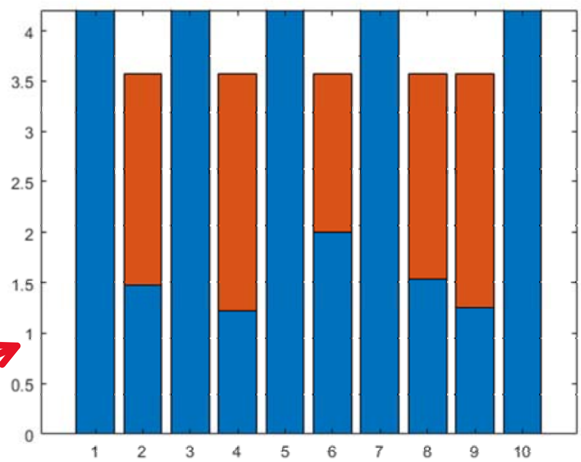
```
lambda = 0.28;
Nc = 10;
N0 = 1;
h = [0.1+0.1i 0.2+0.8i 0.01+0.2i 0.1+0.9i 0.3+0.1i 0.1+0.7i 0.09+0.02i 0.1+0.8i 0.4+0.8i 0.1+0.3i];
Pn = zeros(1,Nc);

for i = 1:Nc
    Pn(i) = 1/lambda - N0/(abs(h(i)))^2;
    if Pn(i)<0
        Pn(i) = 0;
    end
end

bb = [N0./(abs(h)).^2;Pn];
bar([bb'], 'stacked')
```

3) The result is :





bad channels

Pn =

0 2.1008 0 2.3519 0 1.5714 0 2.0330 2.3214 0

4) Channel 1, 3, 5, 7, 10 are bad channels, due to their bad channel conditions, no power will be allocated to them.

As for channel 2, 4, 6, 8, 9, they are relatively good channels, hence some power is allocated to them.

This result is only the optimal solution for $\lambda = 0.28$, not the optimal solution for the transmission. Actually, iteration is required.

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V. For the output $y_n = h_n x_n + W_n \qquad n = 0, \dots, N_c - 1$

Transmitter has fixed power P, hence

$$\sum_0^{N_c-1} P_n \leq P$$

The total power delivered at the receiver is required to be more than a given threshold P_d, hence

$$\sum_0^{N_c-1} E[|y_n|^2] \geq P_d$$

$$\Downarrow$$

$$\sum_0^{N_c-1} |h_n|^2 P_n + N_0 \geq P_d$$

The target is to maxmize the information rate:

$$\therefore \max_{P_0, \dots, P_{N_c-1}} \sum_{n=0}^{N_c-1} \log\left(1 + \frac{P_n |h_n|^2}{N_0}\right) \quad \text{s.t.} \begin{cases} \Sigma_0^{N_c-1} P_n \leq P \\ P_n \geq 0, \ n = 0, \dots, N_c - 1 \\ \Sigma_0^{N_c-1} |h_n|^2 P_n + N_0 \geq P_d \end{cases}$$

standard form $\Downarrow$

$$\min_{P_0, \dots, P_{N_c-1}} -\sum_{n=0}^{N_c-1} \log\left(1 + \frac{P_n |h_n|^2}{N_0}\right) \quad \text{s.t.} \begin{cases} -P_n \leq 0 \quad n = 0, \dots, N_c - 1 \\ \sum_{n=0}^{N_c-1} P_n - P \leq 0 \\ P_d - N_0 - \Sigma_0^{N_c-1} |h_n|^2 P_n \leq 0 \end{cases}$$

1) $$L(\lambda_1, \lambda_2, P_0, \dots, P_{N_c-1}) = -\sum_{n=0}^{N_c-1} \log\left(1 + \frac{P_n |h_n|^2}{N_0}\right) + \lambda\left(\sum_{n=0}^{N_c-1} P_n - P\right) + \mu\left(P_d - N_0 - \Sigma_0^{N_c-1} |h_n|^2 P_n\right)$$

2) $$\frac{\partial L}{\partial P_n} = -\frac{\frac{|h_n|^2}{N_0}}{1+\frac{P_n|h_n|^2}{N_0}} + \lambda - \mu|h_n|^2$$

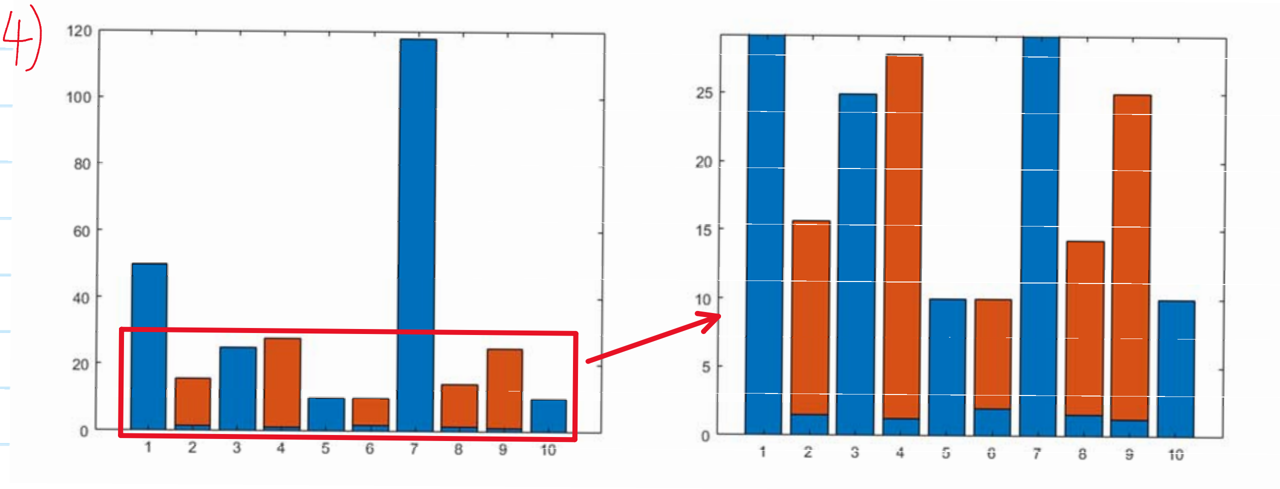
$$= -\frac{1}{\frac{N_0}{|h_n|^2}+P_n} + \lambda - \mu|h_n|^2$$

$$\Rightarrow \quad P_n = \frac{1}{\lambda-\mu|h_n|^2} - \frac{N_0}{|h_n|^2}$$

if $P_n < 0$, let $P_n = 0$

3)
```
lambda = 0.2;
mu = 0.2;
Nc = 10;
N0 = 1;
h = [0.1+0.1i 0.2+0.8i 0.01+0.2i 0.1+0.9i 0.3+0.1i 0.1+0.7i 0.09+0.02i 0.1+0.8i 0.4+0.8i 0.1+0.3i];
Pn = zeros(1,Nc);

for i = 1:Nc
    Pn(i) = 1/(lambda-mu*(abs(h(i)))^2) - N0/(abs(h(i)))^2;
    if Pn(i)<0
        Pn(i) = 0;
    end
end
Pn
bb = [N0./(abs(h)).^2;Pn];
bar([bb'], 'stacked')
```

4)


Similarly, channel 1, 3, 5, 7, 10 are bad channels, no power is allocated to them.

As for channel 2, 4, 6, 8, 9, more power is allocated to them comparing with the result in Ex 2. This is not only caused by the change of λ, but also a new energy requirement at the receiver. In order to satisfy this constraint, more power should be allocated to these good channels.

According to these, it is clear that P is correlated with λ, P_d is correlated with μ, and definitely, P should be larger than a value determined by P_d, h_n, W_n, otherwise the received energy requiremend cannot be satisrfied.