AM Logbook

24/11/2021

Objects:

$$\boxed{\text{Detect}} \xrightarrow{\hat{M}} \boxed{\text{Estimate}} \rightarrow \boxed{\text{reception}}$$

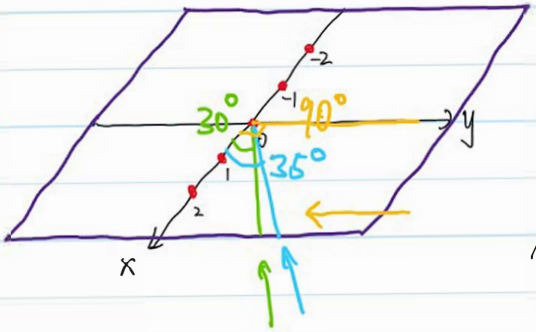
① estimate M

② provide complete information about directions of emitting sources

③ investigate the use of an array sensors for detecting and rejecting in-band interferences

④ estimate relative powers / cross-correlations / etc.

Model: linear array 5 sensors [-2 0 0; -1 0 0; 0 0 0; 1 0 0; 2 0 0]



directions: [30, 0; 35, 0; 90, 0]

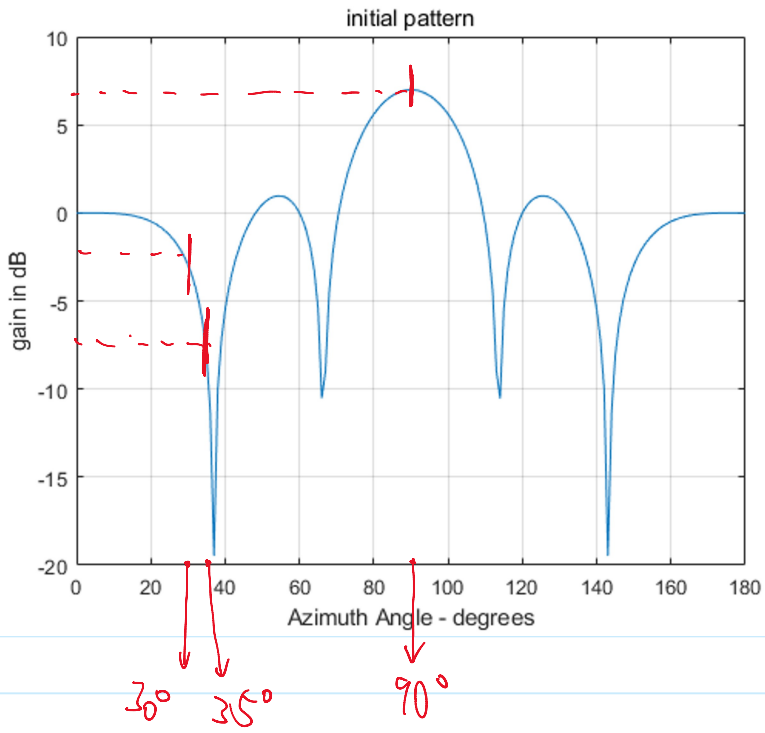
Using a multi-cross-correlator

estimate R_{xx} (second order statistics / Covariance matrix of the received signal)

- detection problem
- estimation problem ← address
- interference suppression problem

Task:

① Use the provided function pattern,



90° has the largest gain

30° & 35° has relatively small gain

② Source position vector / manifold vector,

S =

0.6661 - 0.7458i	0.4210 - 0.9071i	1.0000 + 0.0000i	→ (-2,0)
-0.9127 + 0.4086i	-0.8429 + 0.5381i	1.0000 + 0.0000i	→ (-1,0)
1.0000 + 0.0000i	1.0000 + 0.0000i	1.0000 + 0.0000i	→ (0,0)
-0.9127 - 0.4086i	-0.8429 - 0.5381i	1.0000 - 0.0000i	→ (1,0)
0.6661 + 0.7458i	0.4210 + 0.9071i	1.0000 - 0.0000i	→ (2,0)
30°	35°	90°	

S is a complex matrix, with 5 rows → 5 sensors, 3 columns → 3 sources

uncorrelated equal powers

The covariance matrix for 3 sources R_{mm}, since

$$\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix} = R_{mm}$$

∴ Covariance matrix for the signal input:

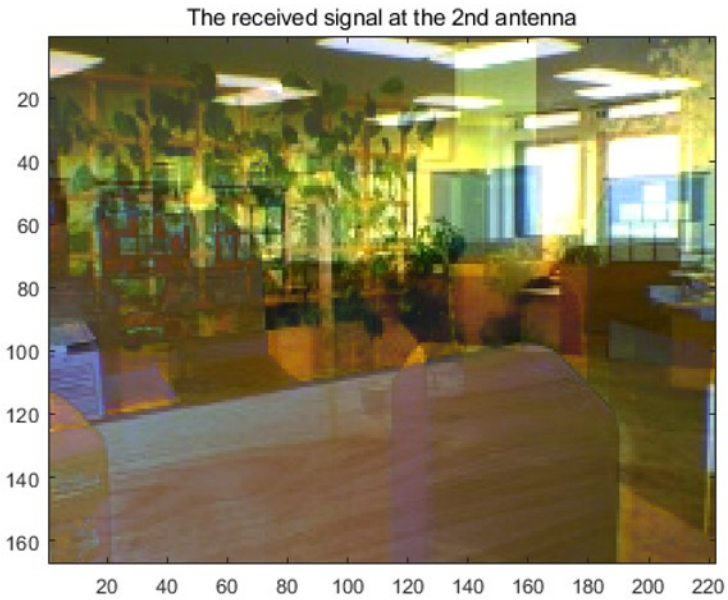
Rxx_theoretical = $R_{xx,\,theoretical} = S \cdot R_{mm} \cdot S^H + \sigma^2 I_5$

3.0001 + 0.0000i	-0.7556 + 0.9467i	2.0871 - 1.6529i	0.8300 + 1.9440i	0.2419 - 1.7573i
-0.7556 - 0.9467i	3.0001 + 0.0000i	-0.7556 + 0.9467i	2.0871 - 1.6529i	0.8300 + 1.9440i
2.0871 + 1.6529i	-0.7556 - 0.9467i	3.0001 + 0.0000i	-0.7556 + 0.9467i	2.0871 - 1.6529i
0.8300 - 1.9440i	2.0871 + 1.6529i	-0.7556 - 0.9467i	3.0001 + 0.0000i	-0.7556 + 0.9467i
0.2419 + 1.7573i	0.8300 - 1.9440i	2.0871 + 1.6529i	-0.7556 - 0.9467i	3.0001 + 0.0000i

The first term of $R_{xx} = S \cdot R_{mm} \cdot S^H = SS^H$

③

```
Rxx_au =

  1.0e+05 *

   1.1997 + 0.0000i  -0.5181 - 0.1012i   1.0504 - 0.8297i   0.0147 + 0.5545i   0.2695 - 1.1453i
  -0.5181 + 0.1012i   0.2914 + 0.0000i  -0.3767 + 0.4751i  -0.0138 - 0.2595i   0.0147 + 0.5545i
   1.0504 + 0.8297i  -0.3767 - 0.4751i   1.5086 + 0.0000i  -0.3767 + 0.4751i   1.0504 - 0.8297i
   0.0147 - 0.5545i  -0.0138 + 0.2595i  -0.3767 - 0.4751i   0.2914 + 0.0000i  -0.5181 - 0.1012i
   0.2695 + 1.1453i   0.0147 - 0.5545i   1.0504 + 0.8297i  -0.5181 + 0.1012i   1.1997 + 0.0000i

Rxx_im =

  1.0e+04 *

   6.4023 + 0.0000i  -1.4333 - 0.9032i   5.9094 - 3.8583i   0.8760 + 2.2303i   2.3832 - 5.4652i
  -1.4333 + 0.9032i   1.4890 + 0.0000i  -0.6549 + 2.2141i   0.1619 - 0.7647i   0.8760 + 2.2303i
   5.9094 + 3.8583i  -0.6549 - 2.2141i   8.0591 + 0.0000i  -0.6549 + 2.2141i   5.9094 - 3.8583i
   0.8760 - 2.2303i   0.1619 + 0.7647i  -0.6549 - 2.2141i   1.4890 + 0.0000i  -1.4333 - 0.9032i
   2.3832 + 5.4652i   0.8760 - 2.2303i   5.9094 + 3.8583i  -1.4333 + 0.9032i   6.4023 + 0.0000i
```



④ Set direction / Rmm / S / sigma2 to be unknown.

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⑤ Only have Rxx, theoretical

R_{xx}

```
Rxx_theoretical =

   3.0001 + 0.0000i  -0.7556 + 0.9467i   2.0871 - 1.6529i   0.8300 + 1.9440i   0.2419 - 1.7573i
  -0.7556 - 0.9467i   3.0001 + 0.0000i  -0.7556 + 0.9467i   2.0871 - 1.6529i   0.8300 + 1.9440i
   2.0871 + 1.6529i  -0.7556 - 0.9467i   3.0001 + 0.0000i  -0.7556 + 0.9467i   2.0871 - 1.6529i
   0.8300 - 1.9440i   2.0871 + 1.6529i  -0.7556 - 0.9467i   3.0001 + 0.0000i  -0.7556 + 0.9467i
   0.2419 + 1.7573i   0.8300 - 1.9440i   2.0871 + 1.6529i  -0.7556 - 0.9467i   3.0001 + 0.0000i
```

$eig(R_{xx})$

0.0001
0.0001 } power of noise

0.0953
4.9681
9.9369 } 3 sources

The eigendecomposition of the matrix R_{xx} provides the power of noise and the number of sources.

$eig(R_{xx_re})$

```
1.0e+05 *

   0.0000
   0.0000
   0.0017
   0.1072
   4.3818
```

$eig(R_{xx_im})$

```
1.0e+05 *

  -0.0000
   0.0000
   0.0019
   0.2039
   2.1784
```

The two elements that have small figures show the power of noise, while the remaining elements have large figures reflect the number of sources

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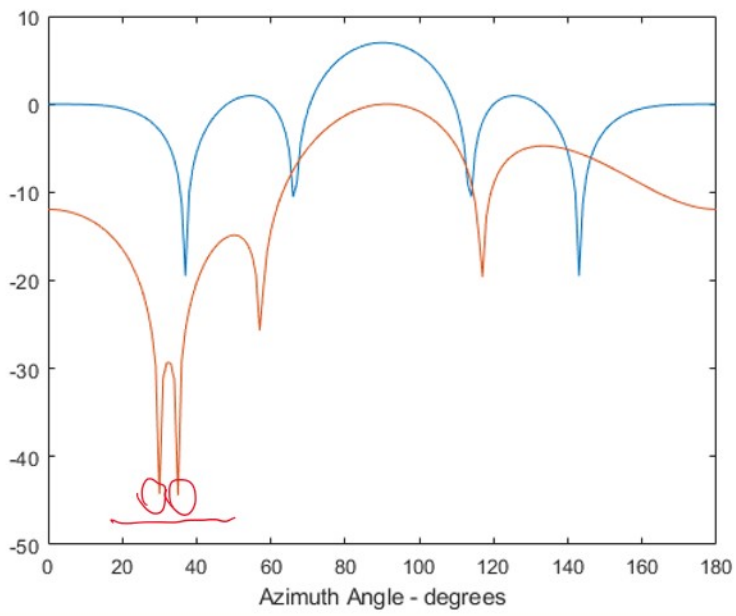
(b)



From the figure, it is clear that at about 30° & 35°, the gain is extremely low.

⑦ The covariance matrix R_{xx}:

3.1000 + 0.0000i	-0.7556 + 0.9467i	2.0871 - 1.6529i	0.8300 + 1.9440i	0.2419 - 1.7573i
-0.7556 - 0.9467i	3.1000 + 0.0000i	-0.7556 + 0.9467i	2.0871 - 1.6529i	0.8300 + 1.9440i
2.0871 + 1.6529i	-0.7556 - 0.9467i	3.1000 + 0.0000i	-0.7556 + 0.9467i	2.0871 - 1.6529i
0.8300 - 1.9440i	2.0871 + 1.6529i	-0.7556 - 0.9467i	3.1000 + 0.0000i	-0.7556 + 0.9467i
0.2419 + 1.7573i	0.8300 - 1.9440i	2.0871 + 1.6529i	-0.7556 - 0.9467i	3.1000 + 0.0000i

Compare it with the previous one:

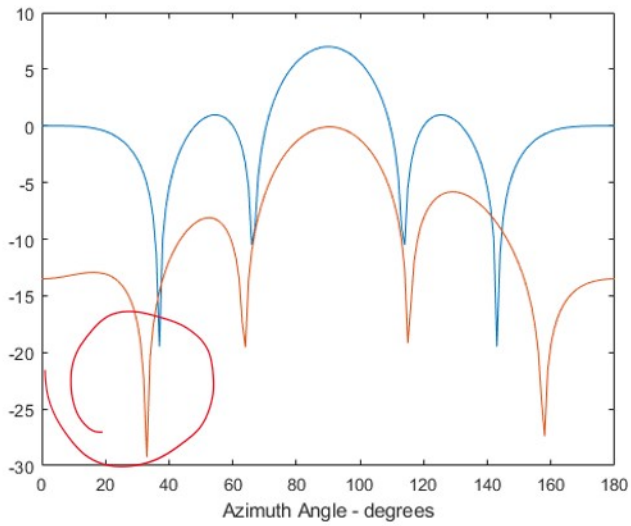
From these, the change of noise is reflected

Rxx_theoretical =

3.0001 + 0.0000i	-0.7556 + 0.9467i	2.0871 - 1.6529i	0.8300 + 1.9440i	0.2419 - 1.7573i
-0.7556 - 0.9467i	3.0001 + 0.0000i	-0.7556 + 0.9467i	2.0871 - 1.6529i	0.8300 + 1.9440i
2.0871 + 1.6529i	-0.7556 - 0.9467i	3.0001 + 0.0000i	-0.7556 + 0.9467i	2.0871 - 1.6529i
0.8300 - 1.9440i	2.0871 + 1.6529i	-0.7556 - 0.9467i	3.0001 + 0.0000i	-0.7556 + 0.9467i
0.2419 + 1.7573i	0.8300 - 1.9440i	2.0871 + 1.6529i	-0.7556 - 0.9467i	3.0001 + 0.0000i

0.1000
0.1000
0.1952
5.0680
10.0368

As the increase of noise, it's hard to determine the number of sources, since they looks like noise.



⑧ The increases of noise power make it hard to realise estimation.

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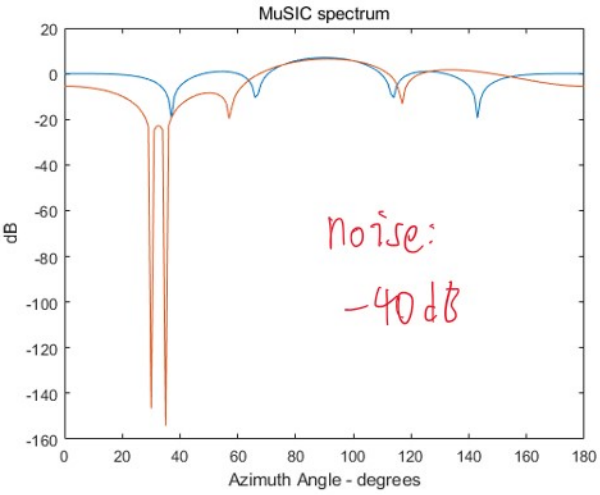
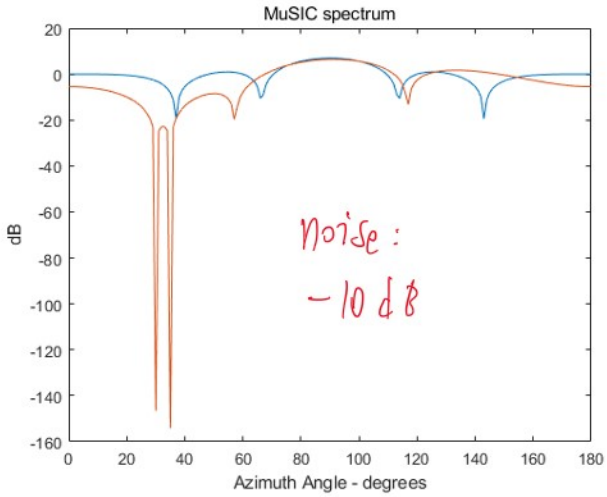
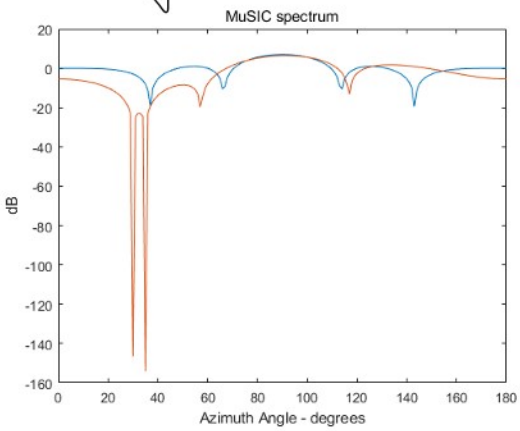
⑨ By using the MuSIC Algorithm, the result is shown



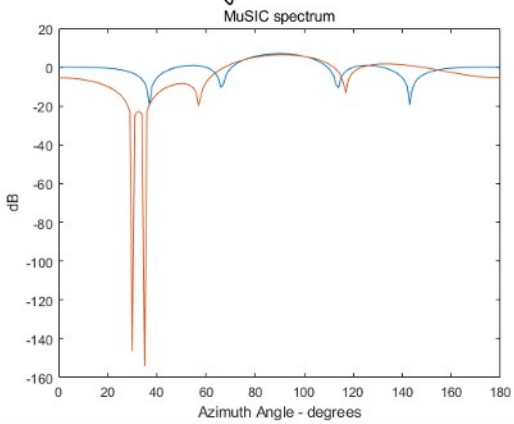


(10)

Using Rxx_qu



Using Rxx_im



They looked the same.

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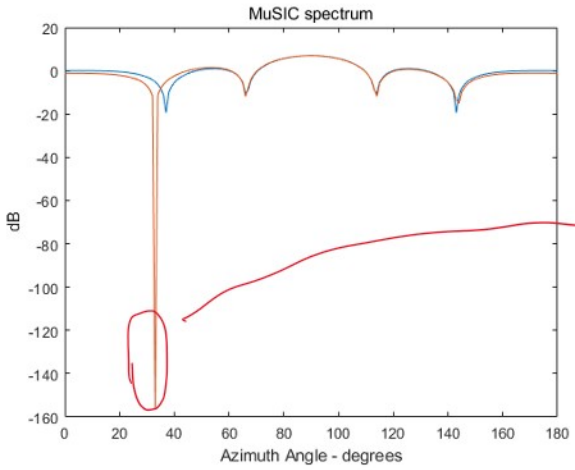
(11)

```
19.8142 + 0.0000i
 4.9704 - 0.0000i
 0.0001 + 0.0000i
 0.0001 - 0.0000i
 0.0001 - 0.0000i
```

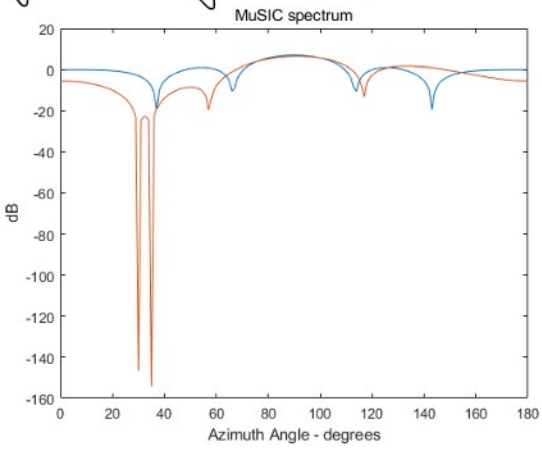
From this, we know that there are 2 sources, But actually there are 3 sources!



Because two sources (30° & 35°) are fully correlated, we can no longer detect them.

1/12/2021

By realizing the spatial smoothing technique, the result is:



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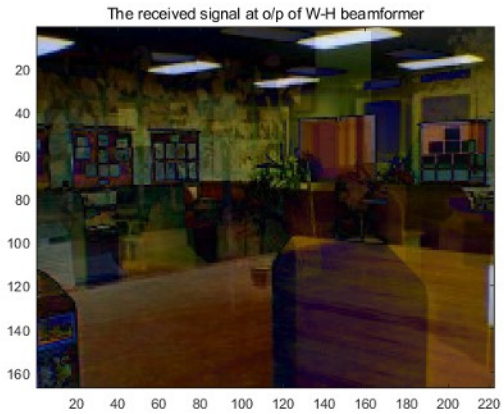
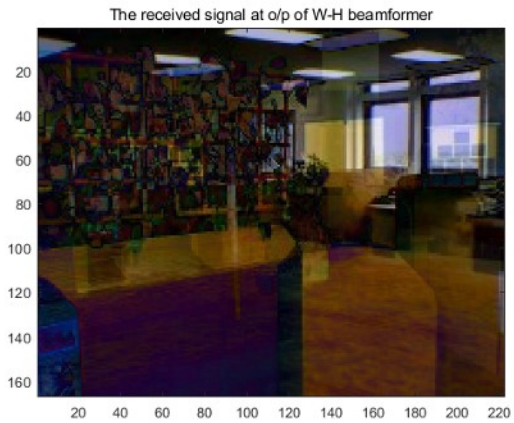
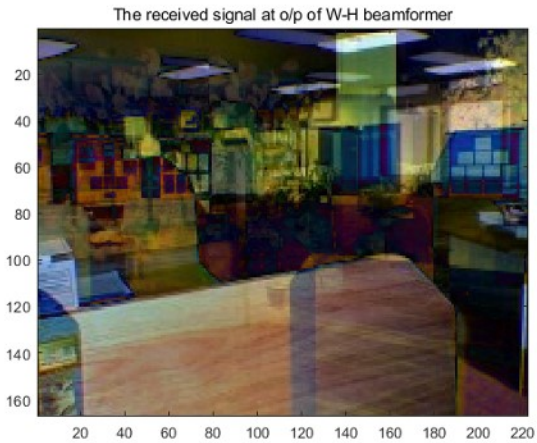
12

1) Comparing with the previous noise-like sound, it can be found that there are 3 pieces of songs. However, they are still not very clear, mixed with other 2 songs.

2)

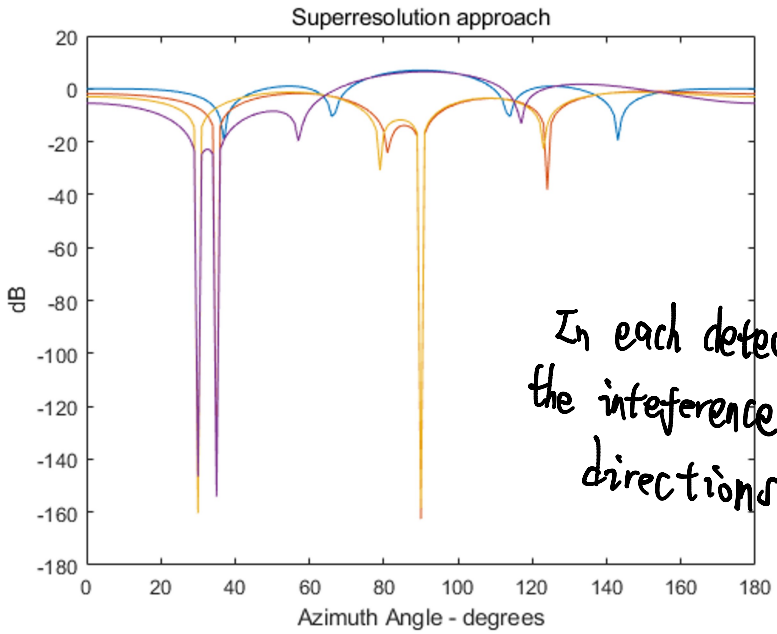






It can be discovered that there are 3 figures, but they are not very clear.

3)



For R_{xx} - theoretical the result is shown in the figure

In each detection, it eliminate the interference in the other 2 directions.

Results are clearly shown

4) The result of W-H beamformer is not very good.

With the superresolution approach, from 3), 3 figures are recovered

quite well. It is beause from the first figure in 3), the receiver correctly figure out which image is transmitted in which direction (distinguish 3 sources).

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13

Try 1

6.4859 + 0.0000i	-1.6336 + 2.0466i	4.5120 - 3.5734i	1.7944 + 4.2027i	0.5229 - 3.7991i
-1.6336 - 2.0466i	6.4859 + 0.0000i	-1.6336 + 2.0466i	4.5120 - 3.5734i	1.7944 + 4.2027i
4.5120 + 3.5734i	-1.6336 - 2.0466i	6.4859 + 0.0000i	-1.6336 + 2.0466i	4.5120 - 3.5734i
1.7944 - 4.2027i	4.5120 + 3.5734i	-1.6336 - 2.0466i	6.4859 + 0.0000i	-1.6336 + 2.0466i
0.5229 + 3.7991i	1.7944 - 4.2027i	4.5120 + 3.5734i	-1.6336 - 2.0466i	6.4859 + 0.0000i

0.0002
0.0002
0.2059
10.7405
21.4825

3 sources

Try 2

5.6654 + 0.0000i	-1.4269 + 1.7877i	3.9413 - 3.1214i	1.5674 + 3.6710i	0.4567 - 3.3185i
-1.4269 - 1.7877i	5.6654 + 0.0000i	-1.4269 + 1.7877i	3.9413 - 3.1214i	1.5674 + 3.6710i
3.9413 + 3.1214i	-1.4269 - 1.7877i	5.6654 + 0.0000i	-1.4269 + 1.7877i	3.9413 - 3.1214i
1.5674 - 3.6710i	3.9413 + 3.1214i	-1.4269 - 1.7877i	5.6654 + 0.0000i	-1.4269 + 1.7877i
0.4567 + 3.3185i	1.5674 - 3.6710i	3.9413 + 3.1214i	-1.4269 - 1.7877i	5.6654 + 0.0000i

0.0002
0.0002
0.1799
9.3818
18.7649

3 sources

Try 3

6.2208 + 0.0000i	-1.5668 + 1.9629i	4.3277 - 3.4274i	1.7210 + 4.0309i	0.5015 - 3.6439i
-1.5668 - 1.9629i	6.2208 + 0.0000i	-1.5668 + 1.9629i	4.3277 - 3.4274i	1.7210 + 4.0309i
4.3277 + 3.4274i	-1.5668 - 1.9629i	6.2208 + 0.0000i	-1.5668 + 1.9629i	4.3277 - 3.4274i
1.7210 - 4.0309i	4.3277 + 3.4274i	-1.5668 - 1.9629i	6.2208 + 0.0000i	-1.5668 + 1.9629i
0.5015 + 3.6439i	1.7210 - 4.0309i	4.3277 + 3.4274i	-1.5668 - 1.9629i	6.2208 + 0.0000i

0.0002
0.0002
0.1975
10.3016
20.6046

3 sources

Although I think there are 3 sources, the differences between the last 2 elements and the third element is quiet large.

If the noise is quite large, for example, 10^{-1} :

6.6337 + 0.0000i	-1.6170 + 2.0258i	4.4662 - 3.5371i	1.7761 + 4.1600i	0.5176 - 3.7605i
-1.6170 - 2.0258i	6.6337 + 0.0000i	-1.6170 + 2.0258i	4.4662 - 3.5371i	1.7761 + 4.1600i
4.4662 + 3.5371i	-1.6170 - 2.0258i	6.6337 + 0.0000i	-1.6170 + 2.0258i	4.4662 - 3.5371i
1.7761 - 4.1600i	4.4662 + 3.5371i	-1.6170 - 2.0258i	6.6337 + 0.0000i	-1.6170 + 2.0258i
0.5176 + 3.7605i	1.7761 - 4.1600i	4.4662 + 3.5371i	-1.6170 - 2.0258i	6.6337 + 0.0000i

0.2140
0.2140
0.4176
10.8452
21.4779

I cannot be sure 0.4176 means a source, since it is close to the noise term.

$$\text{AIC}: \quad \min_k \Big(\underbrace{-2\ln\big(\max_k LF^{(k)}\big) + 2k}_{\text{AIC}(k)} \Big)$$

$$\text{MDL}: \quad \min_k \Big(\underbrace{-\ln\big(\max_k LF^{(k)}\big) + \tfrac{1}{2}k\ln L}_{\text{MDL}(k)} \Big)$$

$$LF^{(k)} = -L\ln\left(\det\left(R_{xx}^{(k)}\right)\right) - \mathrm{Tr}\left(R_{xx}^{(k)}\right)^{-1}\hat{R}_{xx}$$

$$= \ln\left(\frac{\prod_{l=k+1}^{N} d_l^{1/(N-k)}}{\frac{1}{N-k}\sum_{l=k+1}^{N} d_l}\right)^{(N-k)L}$$

$$\therefore \text{AIC}(k) = -2\ln\left(\frac{\prod_{l=k+1}^{N} d_l^{1/N-k}\cdot(N-k)}{\sum_{l=k+1}^{N} d_l}\right) + 2k(2N-k)$$

$$\text{MDL}(k) = -\ln\left(\frac{\prod_{l=k+1}^{N} d_l^{1/N-k}\cdot(N-k)}{\sum_{l=k+1}^{N} d_l}\right) + \tfrac{1}{2}k(2N-k)\ln L$$

Write these two expression in Matlab, and find the value of k that minimise them. (More clear vector format is in lecture slide 4A, P38)

The result:

```
AIC_sourceNo =

     3


MDL_sourceNo =

     3
```

noise: 10^{-4}

```
AIC_sourceNo =

     3


MDL_sourceNo =

     3
```

noise: 10^{-1}

∴ They both give the correct answer.

The result of AIC(k) and MDL(k) for different k

k	AIC (1.0e+04 *)	MDL (1.0e+03 *)
0	1.1183	5.5914
1	1.0074	5.0352
2	0.5246	2.6201
3	0.0042	0.0169
4	0.0048	0.0193

5/12/2021

(14) I tried RLS algorithm in Matlab, but have no idea to combine it with this lab.