Vibration Control of a Milling Machine

By

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Executive Summary

The goal of this project is to design a vibration isolator capable of limiting the displacement of a mill's cutting head to µm during operation. Three different excitations were expected, and the proposed isolator was analyzed to ensure efficacy in the expected environment. Simulink methods were implemented, and a block diagram of the system was created in order obtain the desired displacement of the cutting head.

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Introduction

Vibrations are often present in industrial applications due to unbalanced loads on motors, the motion of vehicles such as forklifts, and due to heavy machinery, such as presses. It is often desired to maintain precise control of cutting tools while in these types of environments. The excitations induced on the frame of such devices work against this, causing unwanted deflection through the excitation of the base. In order to reduce the transmissibility if these external forces, a vibration isolator is often employed which consists of a combination of a stiff member (high spring rate) and a mass member; a damper is also sometimes used, though this is not common. The addition of the new mass and new spring also changes the natural frequency. For a single degree of freedom system, the natural frequency is given by:

$$\omega_n = \sqrt{k_{eq}/m_{eq}} \tag{1}$$

Where:

- ω_n is the natural frequency (rad/s)
- k_{eq} is the equivalent spring rate of the system (N/m)
- m_{eq} is the equivalent mass of the system (kg)

In the present study, the design of a vibration isolator is discussed for use in limiting the displacement of a cutting head on a mill. Figure 1 shows the machine to be analyzed and figure 2 shows the simplified model of the device with the addition of a vibration isolator.

Figure 1. Milling Machine

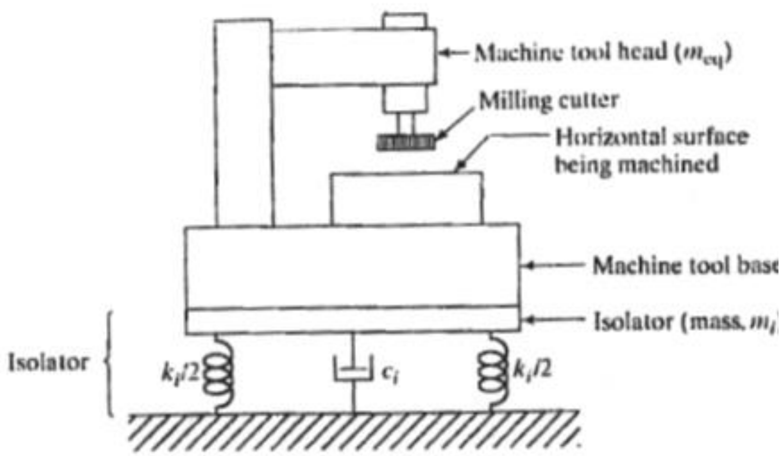

Figure 2. Simplified Model of Milling Machine

Due to the manufacturing environment, the mill is subjected to a crane (y_1), a forging press (y_2), and from an air compressor (y_3). These ground excitations are characterized by:

$$y_1(t) = Y_1 e^{-0.1\omega_1 t} \sin(\omega_1 t) \tag{2}$$
$$y_2(t) = Y_2 \sin(\omega_2 t) \tag{3}$$
$$y_3(t) = Y_3 \sin(\omega_3 t) \tag{4}$$

The amplitudes and frequencies are:

- $Y_1 = 14.82$ μm
- $Y_2 = 20.10$ μm
- $Y_3 = 20.47$ μm
- $\omega_1 = 20\pi$
- $\omega_2 = 30\pi$
- $\omega_3 = 40\pi$

Additionally, the equivalent mass, stiffness, and damping ratio of the machine tool head in vertical vibration (at the location of the cutter) are experimentally determined to be 525 kg, 460 kN/m, and 0.18 respectively. The equivalent mass of the machine tool base is 900 kg. Using this information, the mass, spring stiffness, and damping of the isolator are to be selected to limit the deflection of the tool head to 3 μm peak-to-peak due to the ground vibrations of all three sources.

Theory

To obtain the equations of motion of the system, first the internal damping of the device must be determined. The original natural frequency was calculated as 29.601 rad/s using equation 1. Next the damping ratio is defined as:

$$\zeta = \frac{c_{eq}}{2m_{eq}\omega_n} \tag{5}$$

Using this, the damping factor can is then calculated as 5594.497 N-s/m. Next, the equations of motion for both masses were found:

$$m_{eq}\ddot{y}_m + c_{eq}\dot{y}_m + k_{eq}y_m = 0 \tag{6}$$

$$m_i\ddot{y}_i + (c_{eq} + c_i)\dot{y}_i + (k_{eq} + k_i)y_i - c_{eq}\dot{y}_m - k_{eq}y_m = (\dot{y}_1 + \dot{y}_2 + \dot{y}_3)c_i + (y_1 + y_2 + y_3)k_i \tag{7}$$

This system can them be represented in Simulink using the following diagram:

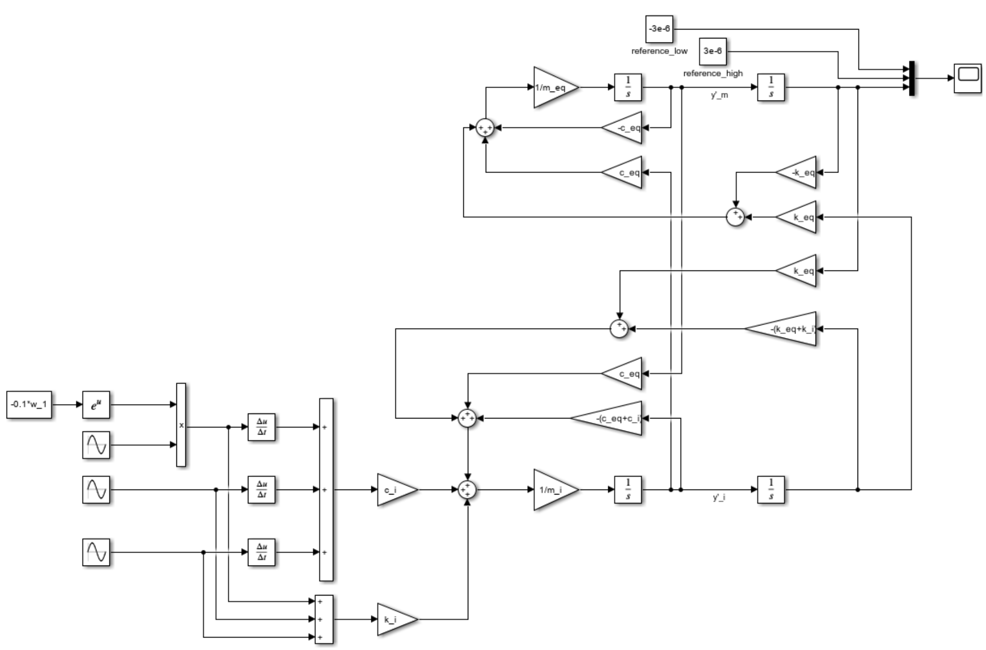

Figure 3. System Model

Results and Discussion

For an initial guess using the above system, an isolator mass of 525kg, spring stiffness of 460 kN/m, and no damping were used, which gave the following response was calculated:

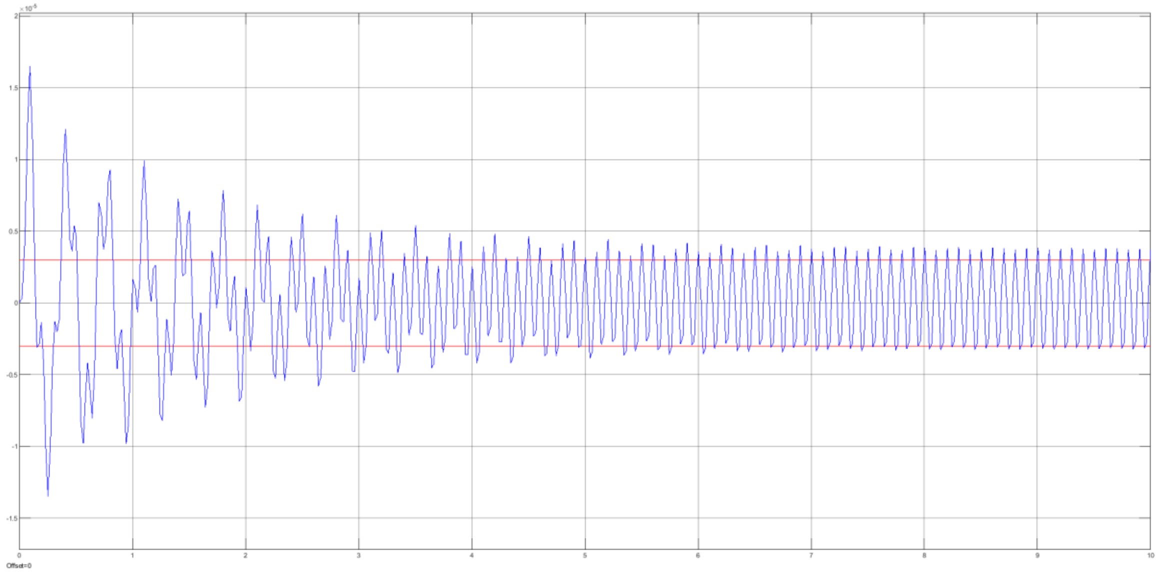

Figure 4. Trial 1 Displacement (m) vs Time

Note that the red lines represent the maximum allowable displacement of 3μm peak-to-peak. Next, the values were changed to 1050kg for the mass, 279.725 N-s/m for the damper, and 690 kN/m for the spring rate. The response of this system was found to be:

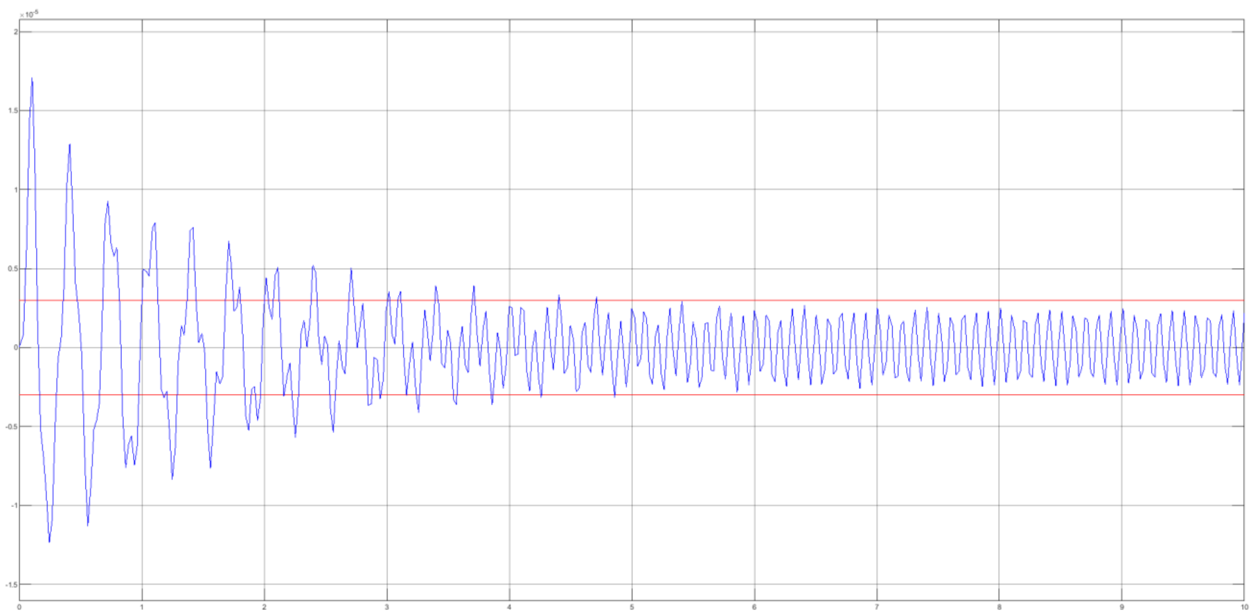

Figure 5. Trial 2 Displacement (m) vs Time

From the plot of displacement, after approximately 4 seconds the displacement is within the acceptable range. It is notable that the base of the design is significantly higher than the mass of the machine, which would likely cause the cost effectiveness of the solution to be limited. Additionally, the damping rate of the base was fairly low, which means that given a correct choice of material, the internal damping rate may make an external damper unnecessary. Finally, the spring rate of the device does not greatly exceed that of the machine, suggesting that the value used can be practically achieved.

Conclusion

Design of a vibrational isolator was discussed, with a successful selection of isolator mass, spring rate, and damping rate achieved. It is notable that the motion of the crane causes an excessive displacement that was not damped by the isolator until after 4 seconds from the initial excitation from the crane. The proposed solution parameters and their applicability to a physical prototype were considered and found to be reasonable.

Appendix A – Alternative solution methods attempted

Using the equations of motion, we calculate the transfer function of the system, using zero initial conditions:

$$m_{eq}s^2X_m(s) + c_{eq}sX_m(s) + k_{eq}X_m(s) = 0$$

$$m_is^2X_i(s) + (c_{eq} + c_i)sX_i(s) + (k_{eq} + k_i)X_i(s0 - c_{eq}sX_m(s) - k_{eq}X_m(s) = \frac{Y_1\omega_1}{(s + 0.1\omega_1)^2 + \omega_1^2} + \frac{Y_2\omega_2}{s^2 + \omega_2^2} + \frac{Y_3\omega_3}{s^2 + \omega_3^2}$$

Which can be rearranged to obtain:

$$(m_{eq}s^2 + c_{eq}s + k_{eq})X_m = 0 \tag{8}$$

$$\left(m_is^2 + (c_{eq} + c_i)s + (k_{eq} + k_i)\right)X_i(s) - (c_{eq}s + k_{eq})X_m(s) = \frac{Y_1\omega_1}{(s+0.1\omega_1)^2+\omega_1^2} + \frac{Y_2\omega_2}{s^2+\omega_2^2} + \frac{Y_3\omega_3}{s^2+\omega_3^2} \tag{9}$$

These can then be solved using Cramer's Rule[1] as:

$$X_m(s) = D_1(s)/D(s) \tag{10}$$

$$X_i(s) = D_2(s)/D(s) \tag{11}$$

Where:

$$D_1(s) = \begin{vmatrix} 0 & -\left(c_{eq}s + k_{eq}\right) \\ \frac{Y_1\omega_1}{(s + 0.1\omega_1)^2 + \omega_1^2} + \frac{Y_2\omega_2}{s^2 + \omega_2^2} + \frac{Y_3\omega_3}{s^2 + \omega_3^2} & m_is^2 + \left(c_{eq} + c_i\right)s + \left(k_{eq} + k_i\right) \end{vmatrix}$$

$$D_2(s) = \begin{vmatrix} m_{eq}s^2 + c_{eq}s + k_{eq} & 0 \\ -\left(c_{eq}s + k_{eq}\right) & \frac{Y_1\omega_1}{(s + 0.1\omega_1)^2 + \omega_1^2} + \frac{Y_2\omega_2}{s^2 + \omega_2^2} + \frac{Y_3\omega_3}{s^2 + \omega_3^2} \end{vmatrix}$$

$$D(s) = \begin{vmatrix} m_{eq}s^2 + c_{eq}s + k_{eq} & -\left(c_{eq}s + k_{eq}\right) \\ -\left(c_{eq}s + k_{eq}\right) & m_is^2 + \left(c_{eq} + c_i\right)s + \left(k_{eq} + k_i\right) \end{vmatrix}$$

Obtaining the determinants, we then get:

$$D_1(s) = \left(\frac{Y_1\omega_1}{(s + 0.1\omega_1)^2 + \omega_1^2} + \frac{Y_2\omega_2}{s^2 + \omega_2^2} + \frac{Y_3\omega_3}{s^2 + \omega_3^2}\right)\left(c_{eq}s + k_{eq}\right)$$

$$D_2(s) = \left(\frac{Y_1\omega_1}{(s + 0.1\omega_1)^2 + \omega_1^2} + \frac{Y_2\omega_2}{s^2 + \omega_2^2} + \frac{Y_3\omega_3}{s^2 + \omega_3^2}\right)\left(m_{eq}s^2 + c_{eq}s + k_{eq}\right)$$

$$D(s) = \left(m_{eq}s^2 + c_{eq}s + k_{eq}\right)\left(m_is^2 + \left(c_{eq} + c_i\right)s + \left(k_{eq} + k_i\right)\right) - \left(c_{eq}s + k_{eq}\right)^2$$

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[1] Singiresu S. Rao "Mechanical Vibrations", sixth edition, pg. 545, equation 5.56

Therefore:

$$X_m(s) = \frac{D_1(s)}{D(s)} = \frac{\left(\frac{Y_1\omega_1}{(s+0.1\omega_1)^2+\omega_1^2} + \frac{Y_2\omega_2}{s^2+\omega_2^2} + \frac{Y_3\omega_3}{s^2+\omega_3^2}\right)\left(c_{eq}s + k_{eq}\right)}{\left(m_{eq}s^2 + c_{eq}s + k_{eq}\right)\left(m_i s^2 + \left(c_{eq}+c_i\right)s + \left(k_{eq}+k_i\right)\right) - \left(c_{eq}s + k_{eq}\right)^2}$$

$$X_i(s) = \frac{D_2(s)}{D(s)} = \frac{\left(\frac{Y_1\omega_1}{(s+0.1\omega_1)^2+\omega_1^2} + \frac{Y_2\omega_2}{s^2+\omega_2^2} + \frac{Y_3\omega_3}{s^2+\omega_3^2}\right)\left(m_{eq}s^2 + c_{eq}s + k_{eq}\right)}{\left(m_{eq}s^2 + c_{eq}s + k_{eq}\right)\left(m_i s^2 + \left(c_{eq}+c_i\right)s + \left(k_{eq}+k_i\right)\right) - \left(c_{eq}s + k_{eq}\right)^2}$$

Re-expressing these equations in matrix form:

$$[m]\ddot{x}(t) + [c]\dot{x} + [k]x(t) = \vec{f}(t) \tag{12}$$

we get the equation:

$$\begin{bmatrix} m_{eq} & 0 \\ 0 & m_i \end{bmatrix}\begin{bmatrix} \ddot{y}_m \\ \ddot{y}_i \end{bmatrix} + \begin{bmatrix} c_{eq} & 0 \\ -c_{eq} & c_{eq}+c_i \end{bmatrix}\begin{bmatrix} \dot{y}_m \\ \dot{y}_i \end{bmatrix} + \begin{bmatrix} k_{eq} & 0 \\ -k_{eq} & k_{eq}+k_i \end{bmatrix}\begin{bmatrix} y_m \\ y_i \end{bmatrix} = \begin{bmatrix} 0 \\ y_1+y_2+y_3 \end{bmatrix} \tag{13}$$

In order to calculate the necessary values of the isolator, the method of principle coordinate systems can be used.