Environments

Announcements

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(Demo)

Names can be Bound to Functional Arguments

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1  def apply_twice(f, x):
2      return f(f(x))
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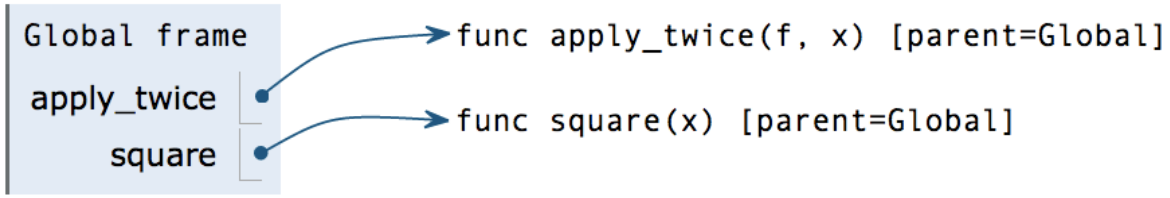

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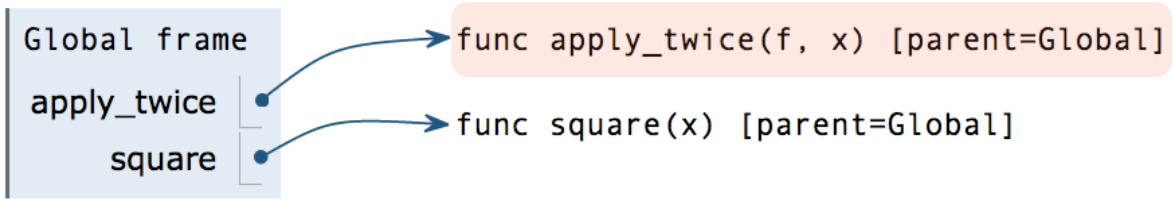

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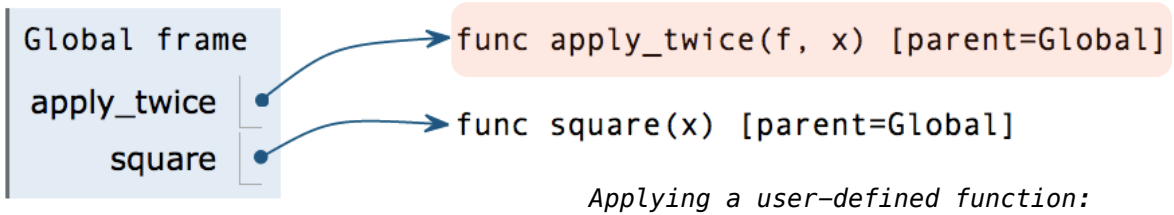

Applying a user-defined function:

- Create a new frame
- Bind formal parameters (f & x) to arguments
- Execute the body: return f(f(x))

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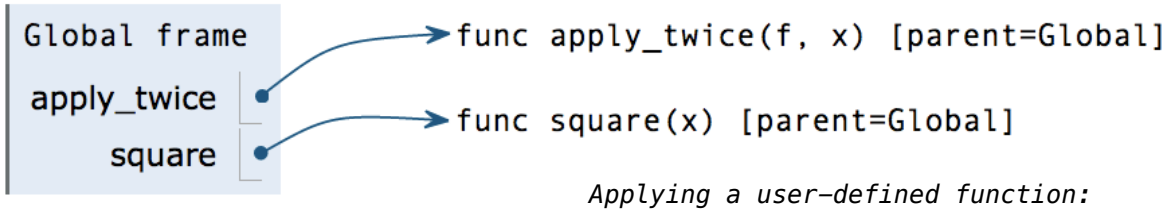

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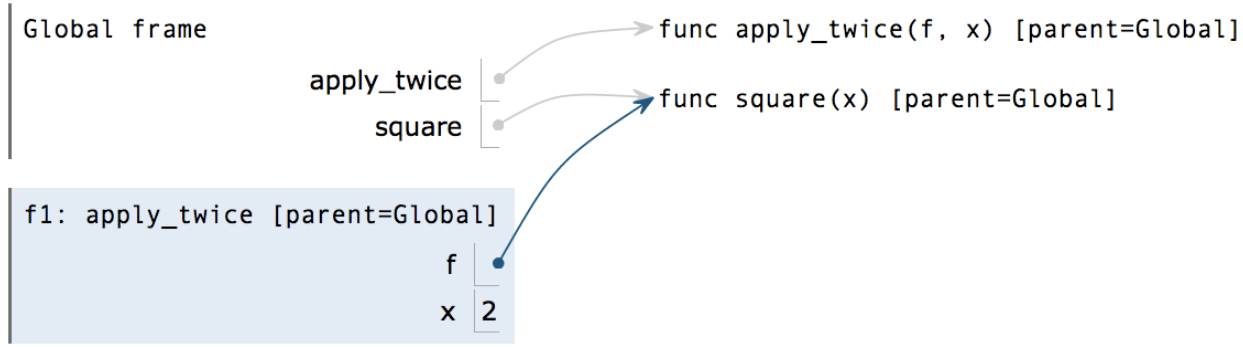

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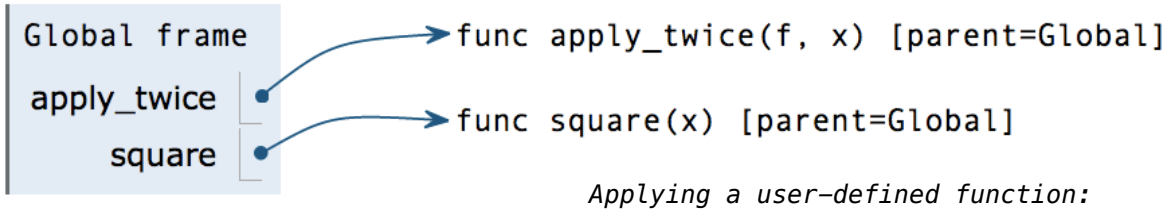

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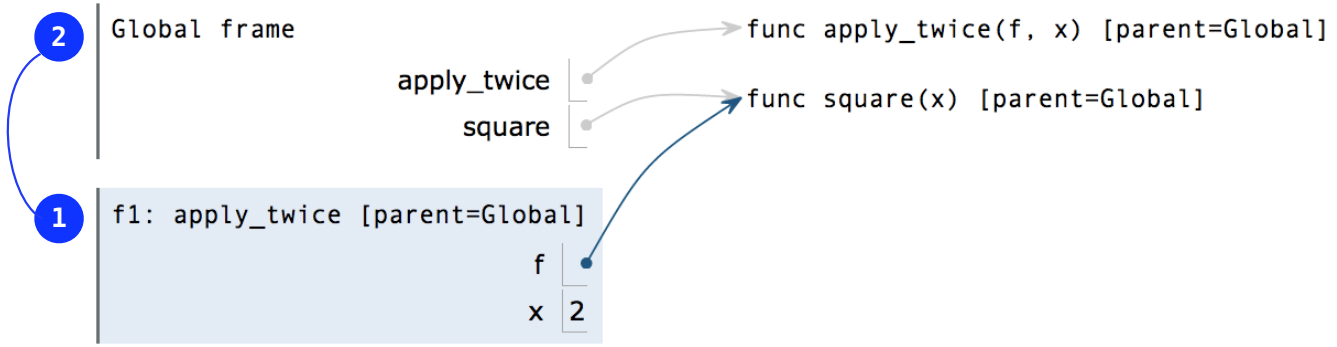

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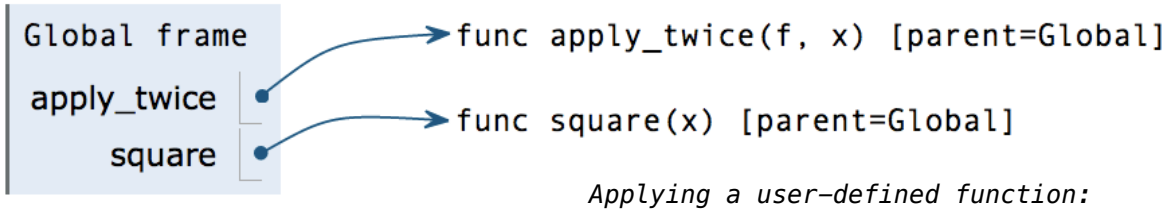

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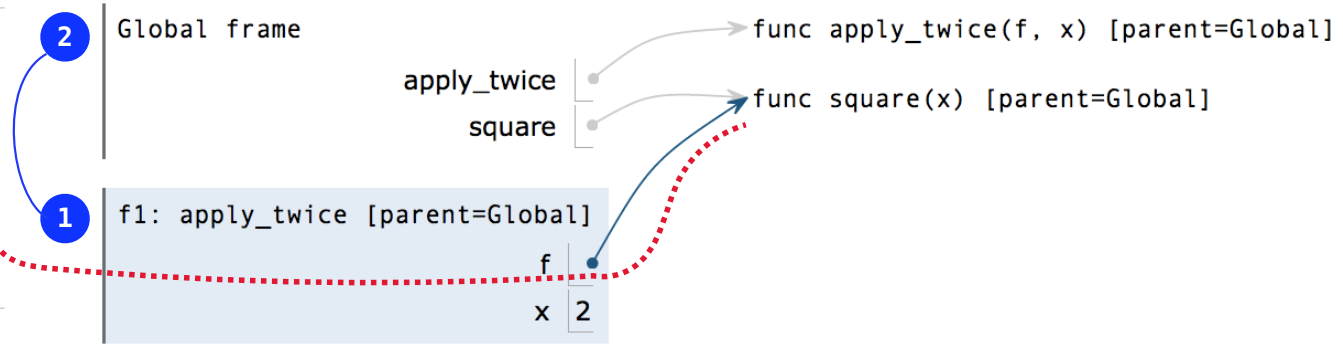

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Environments for Nested Definitions

(Demo)

Environment Diagrams for Nested Def Statements

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1  def make_adder(n):
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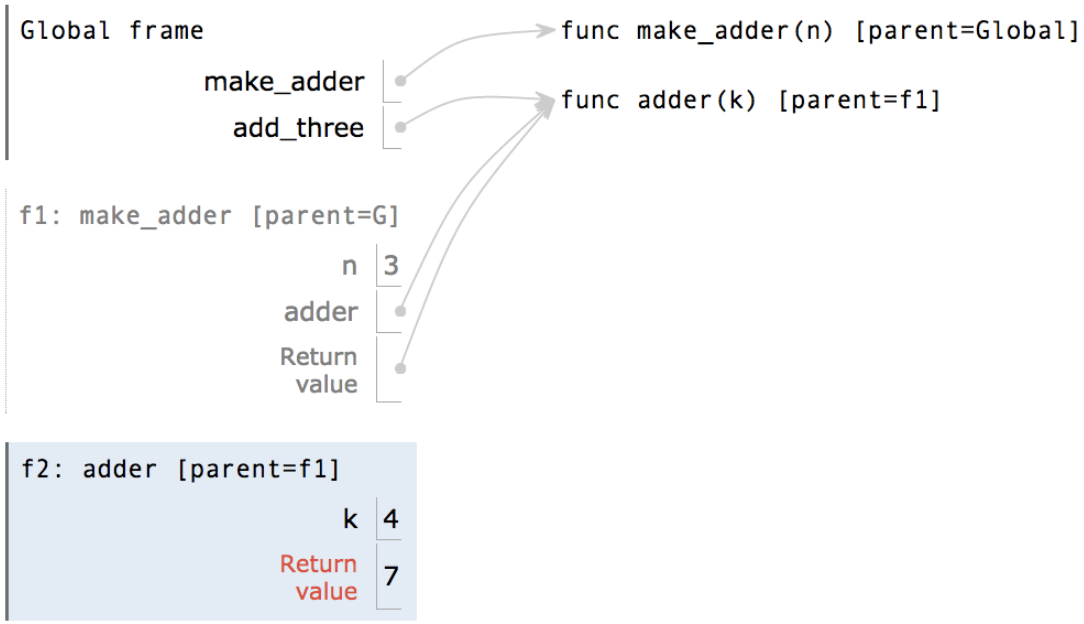

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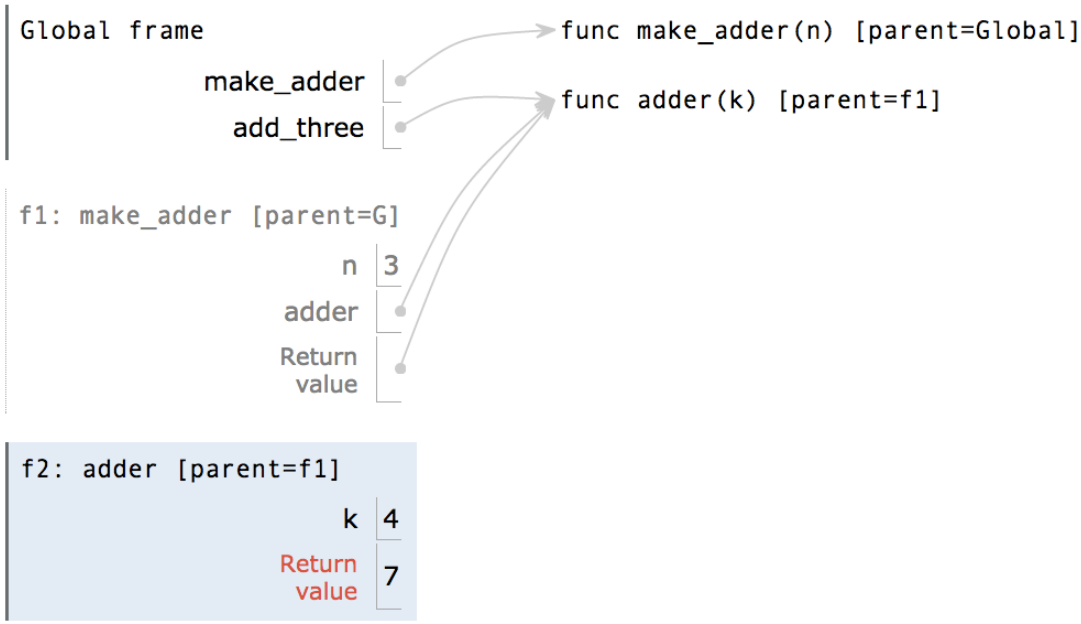

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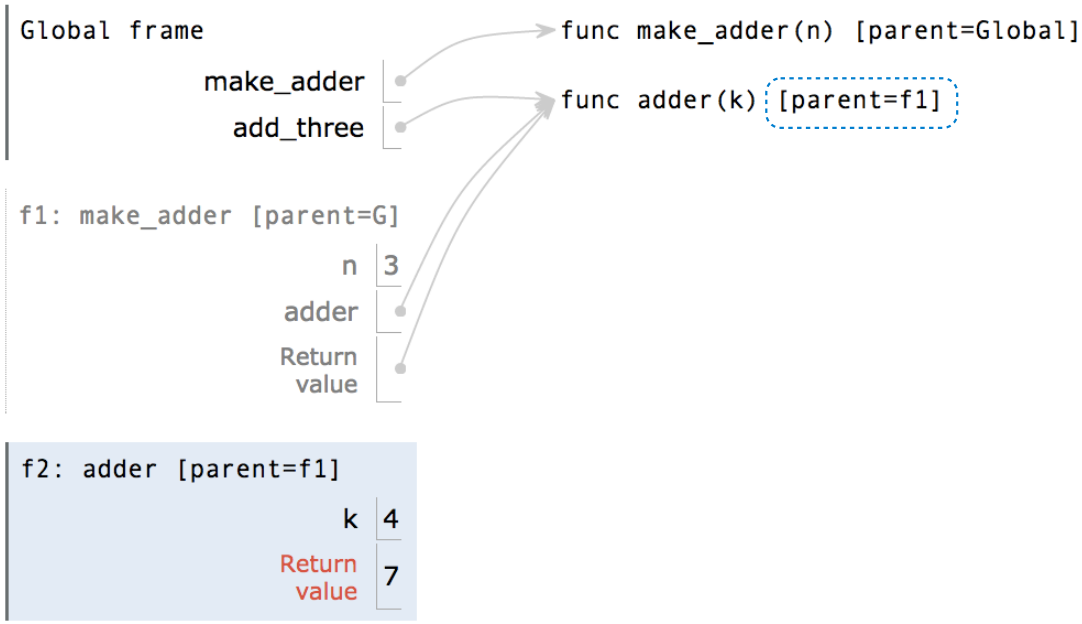

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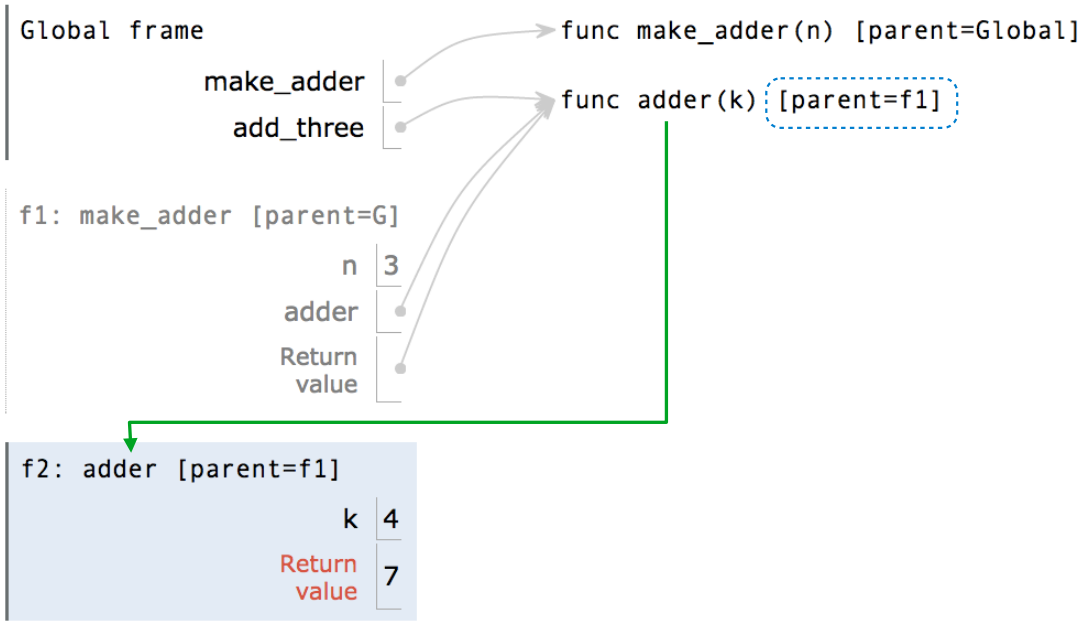

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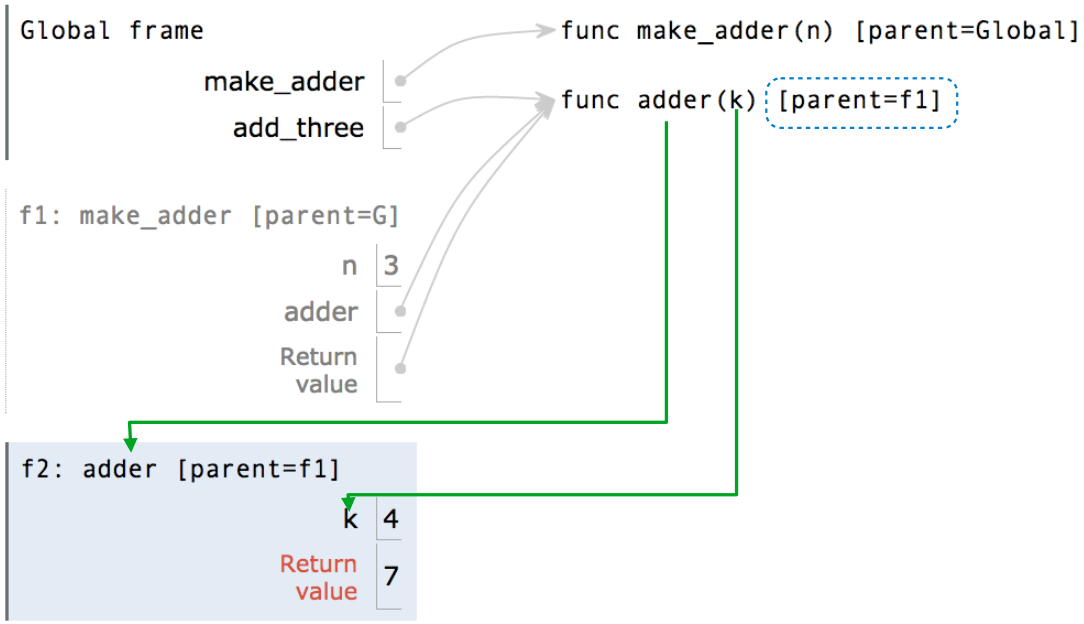

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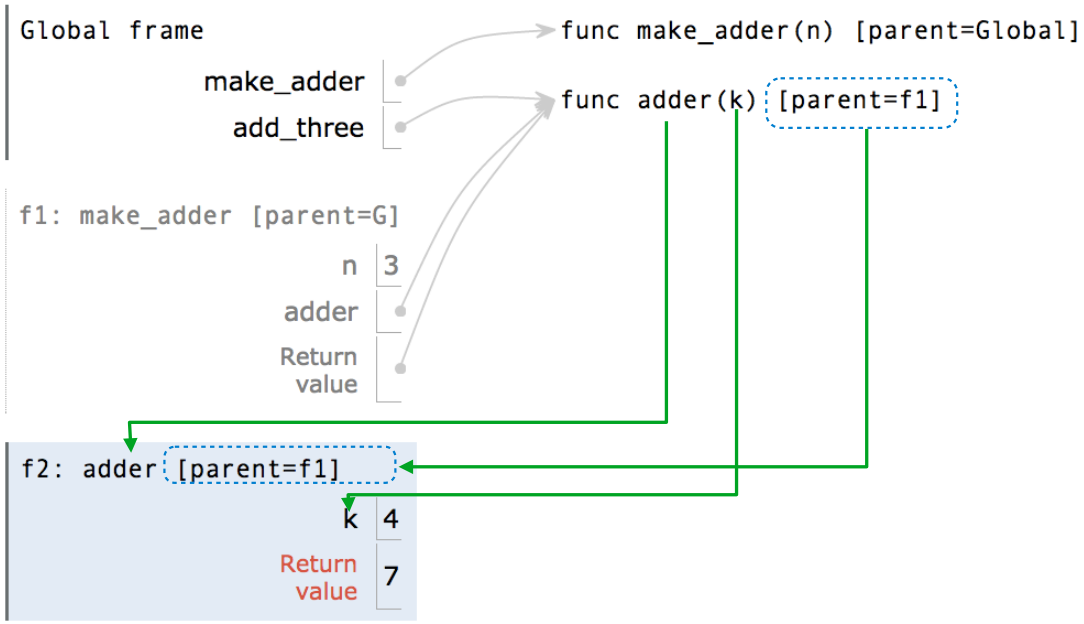

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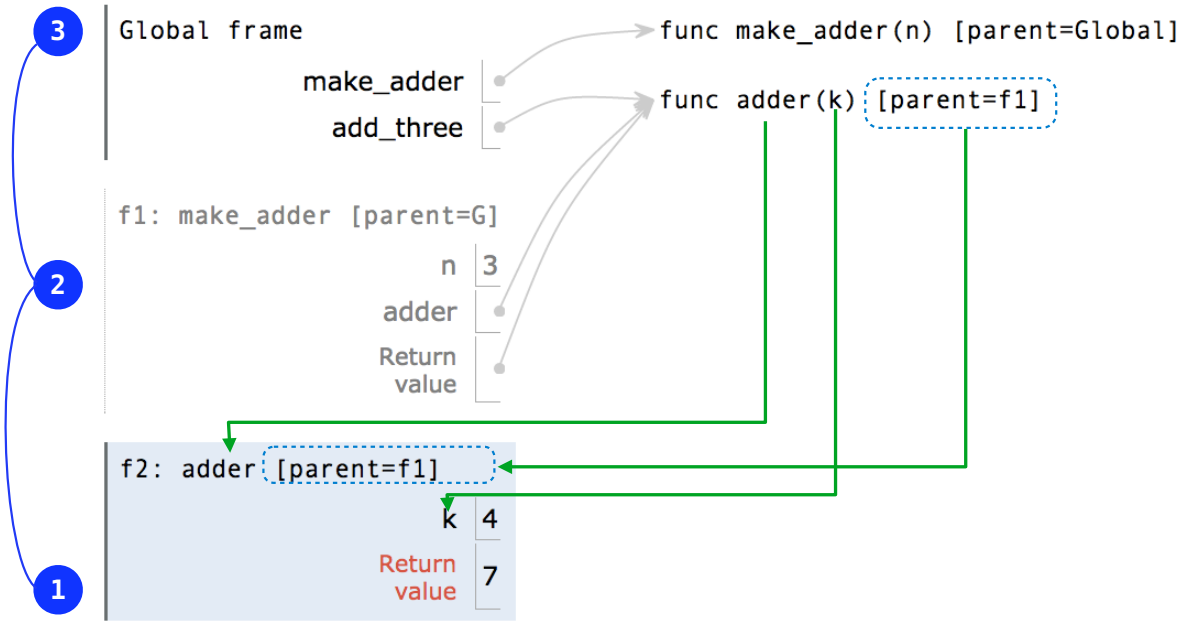

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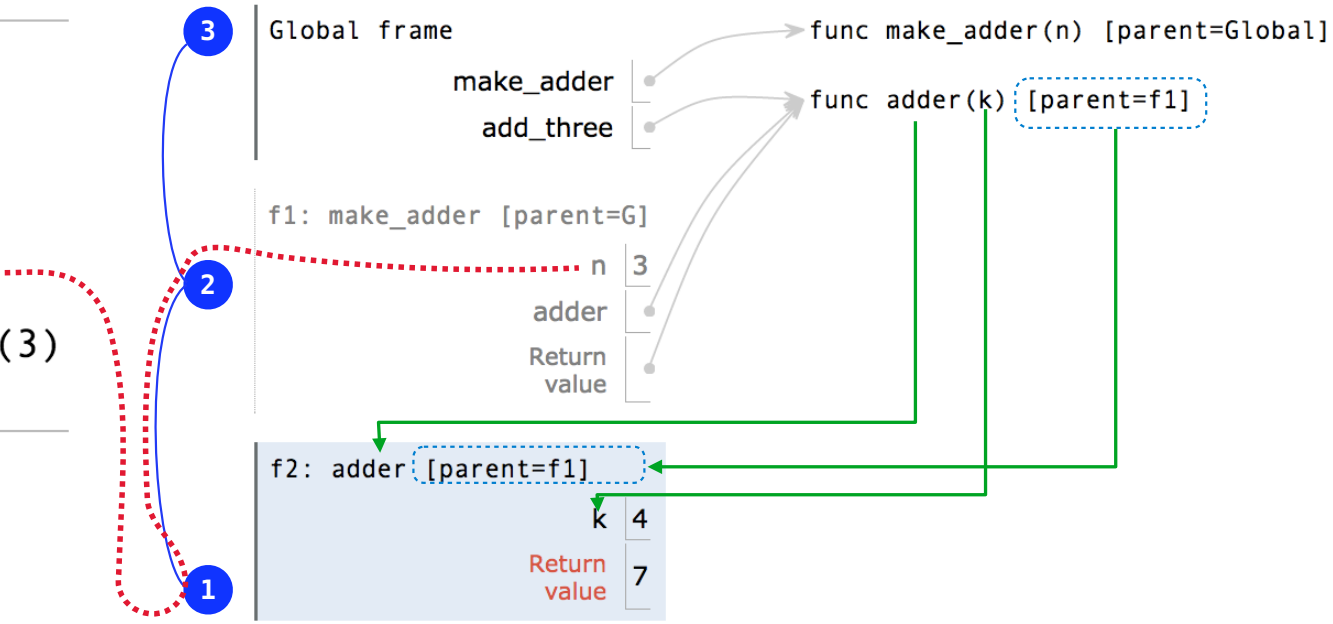

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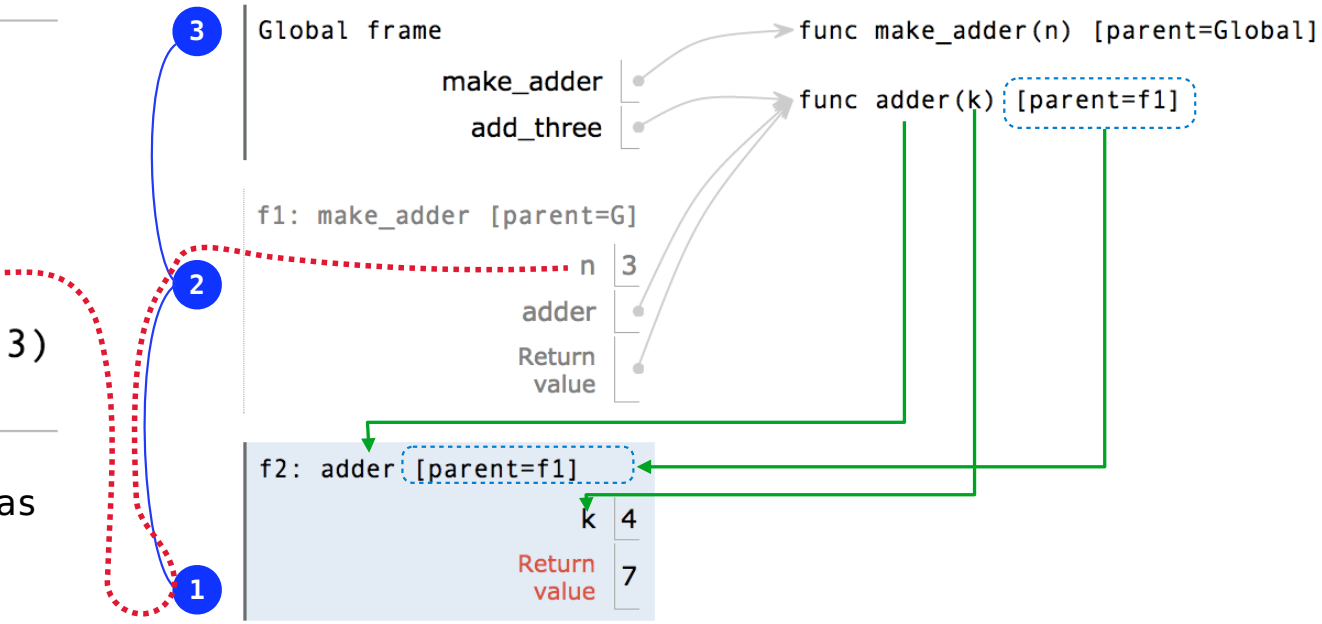

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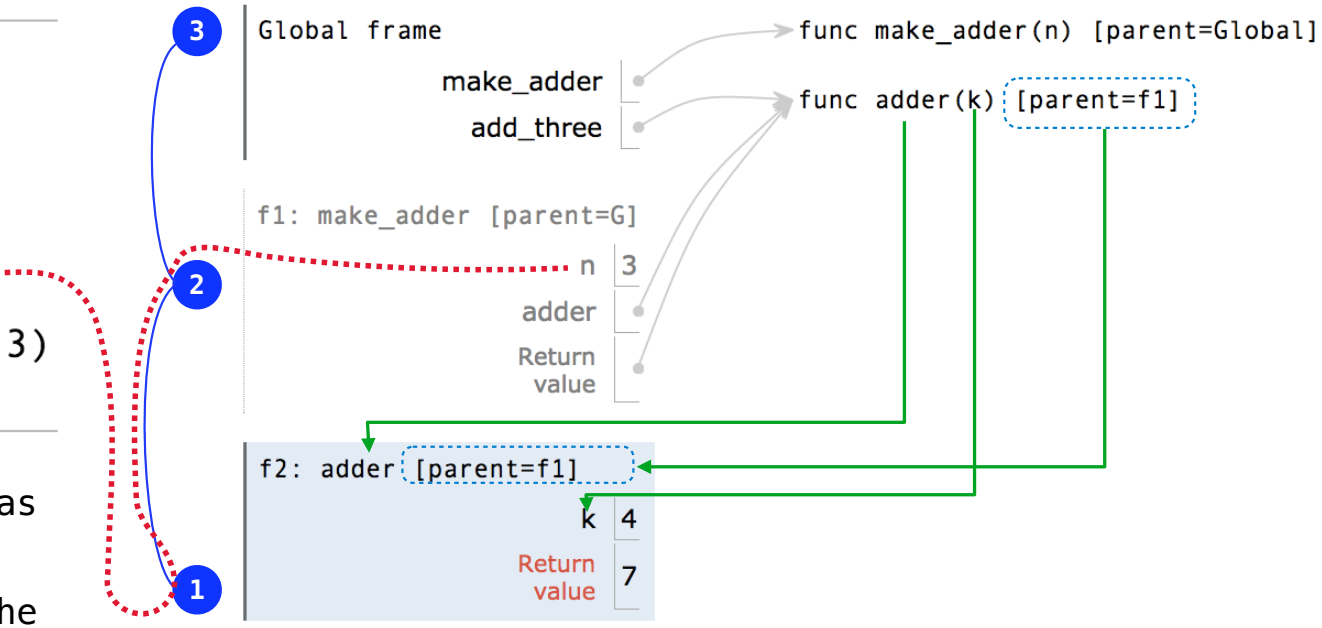

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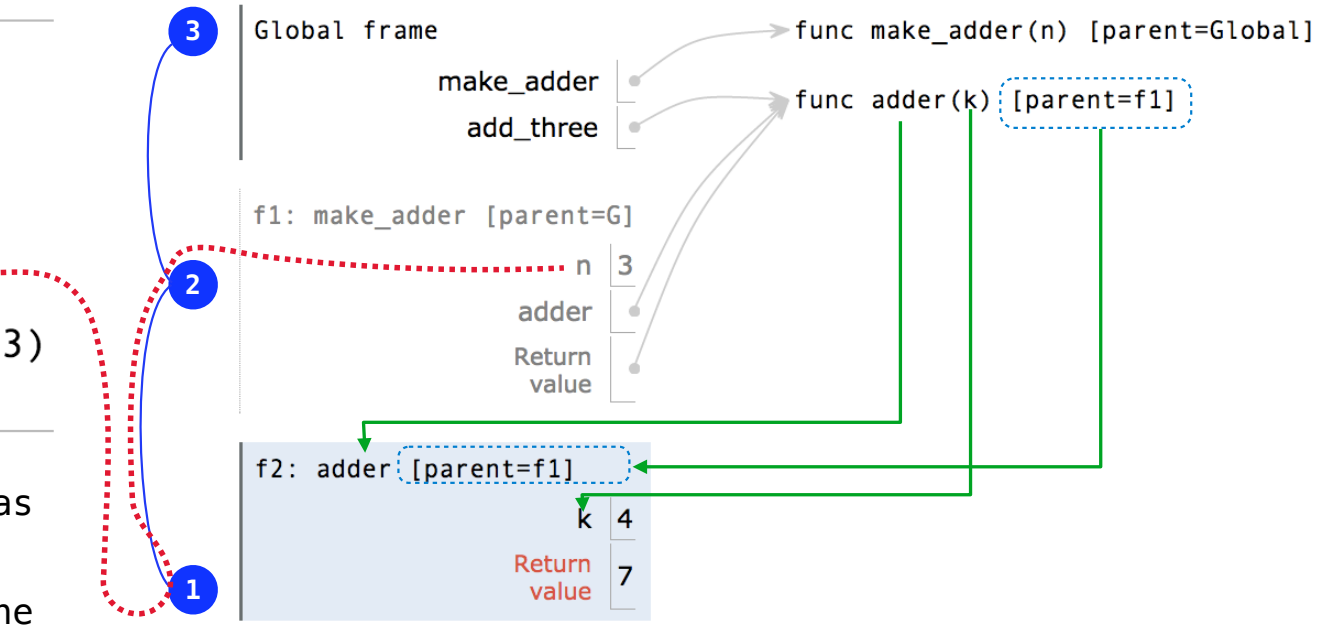

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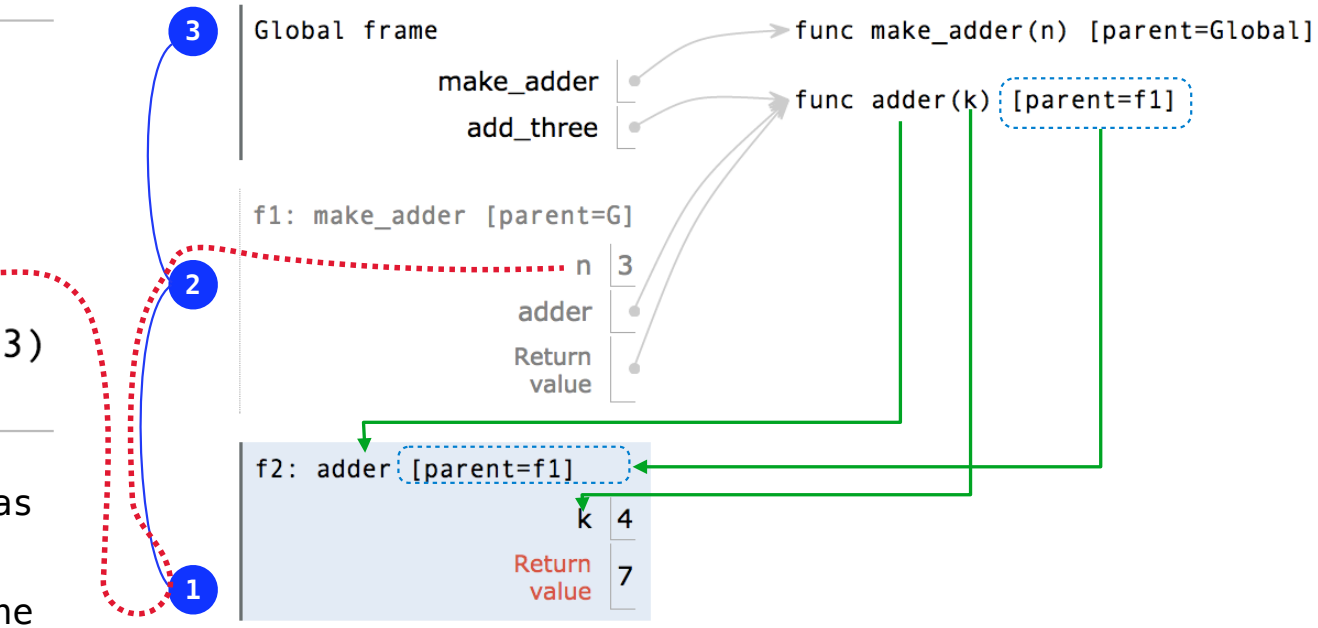

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Its parent is the current frame.

How to Draw an Environment Diagram

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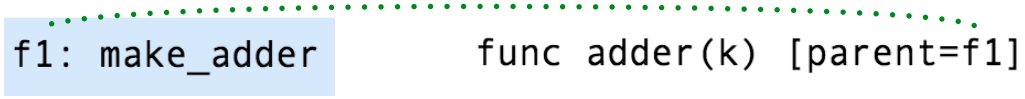

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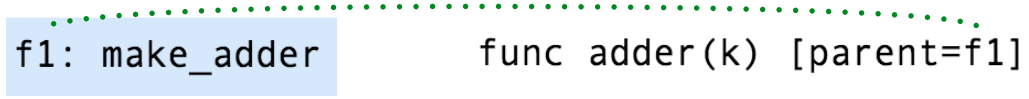

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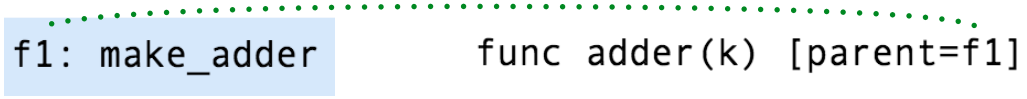

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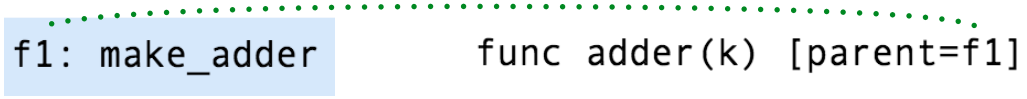

Bind `<name>` to the function value in the current frame

When a function is called:

1. Add a local frame, titled with the `<name>` of the function being called.

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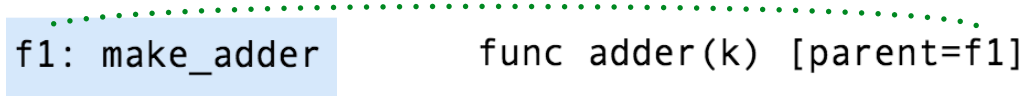

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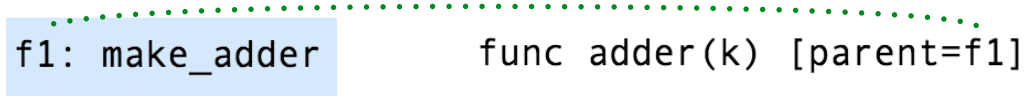

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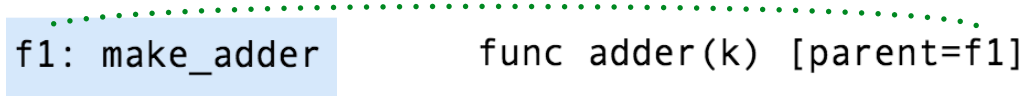

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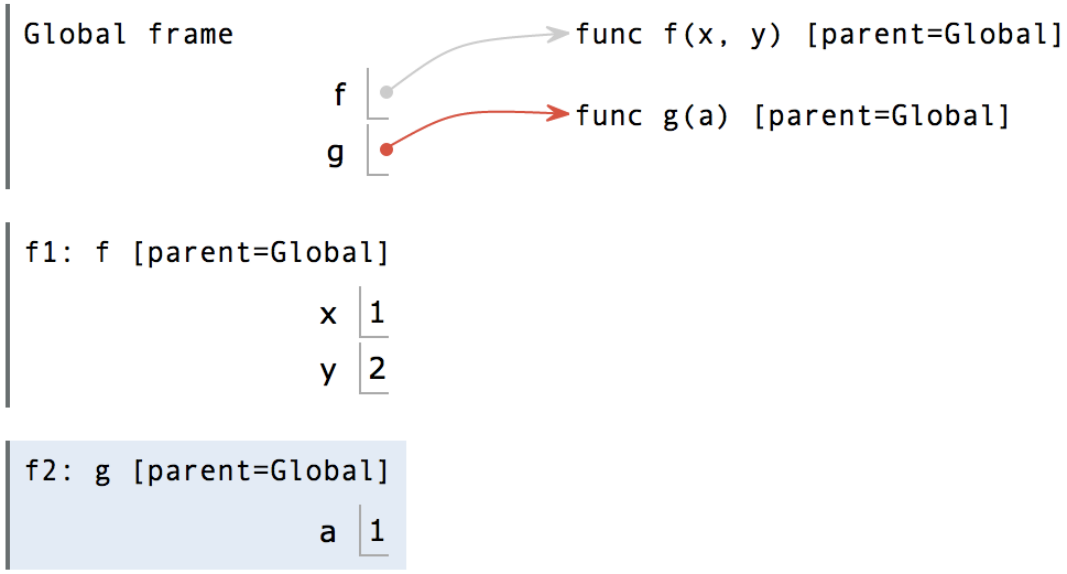
2. Copy the parent of the function to the local frame: [parent=<label>]
3. Bind the <formal parameters> to the arguments in the local frame.
4. Execute the body of the function in the environment that starts with the local frame.

Local Names

(Demo)

Local Names are not Visible to Other (Non-Nested) Functions

```
1  def f(x, y):
2      return g(x)
3
4  def g(a):
5      return a + y
6
7  result = f(1, 2)
```



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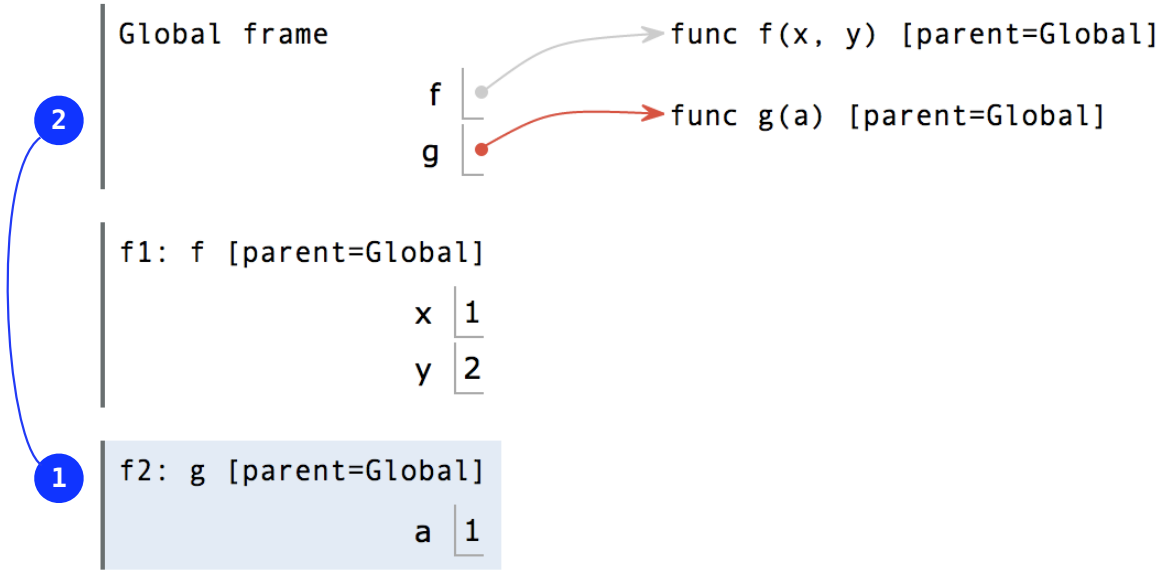

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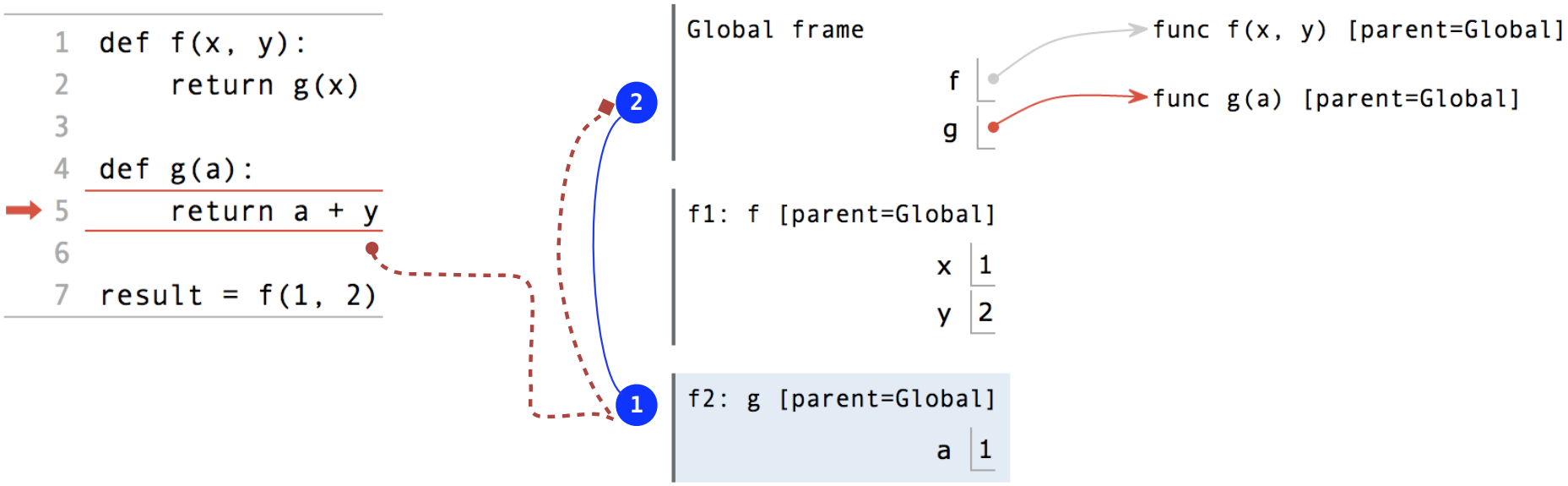

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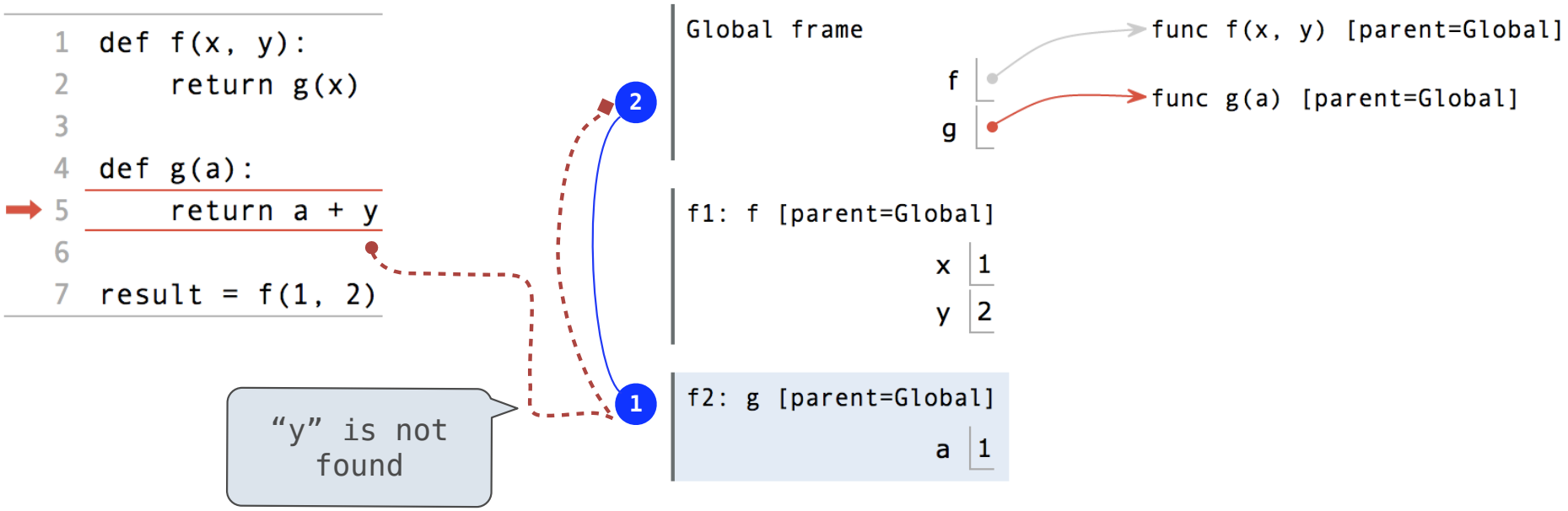

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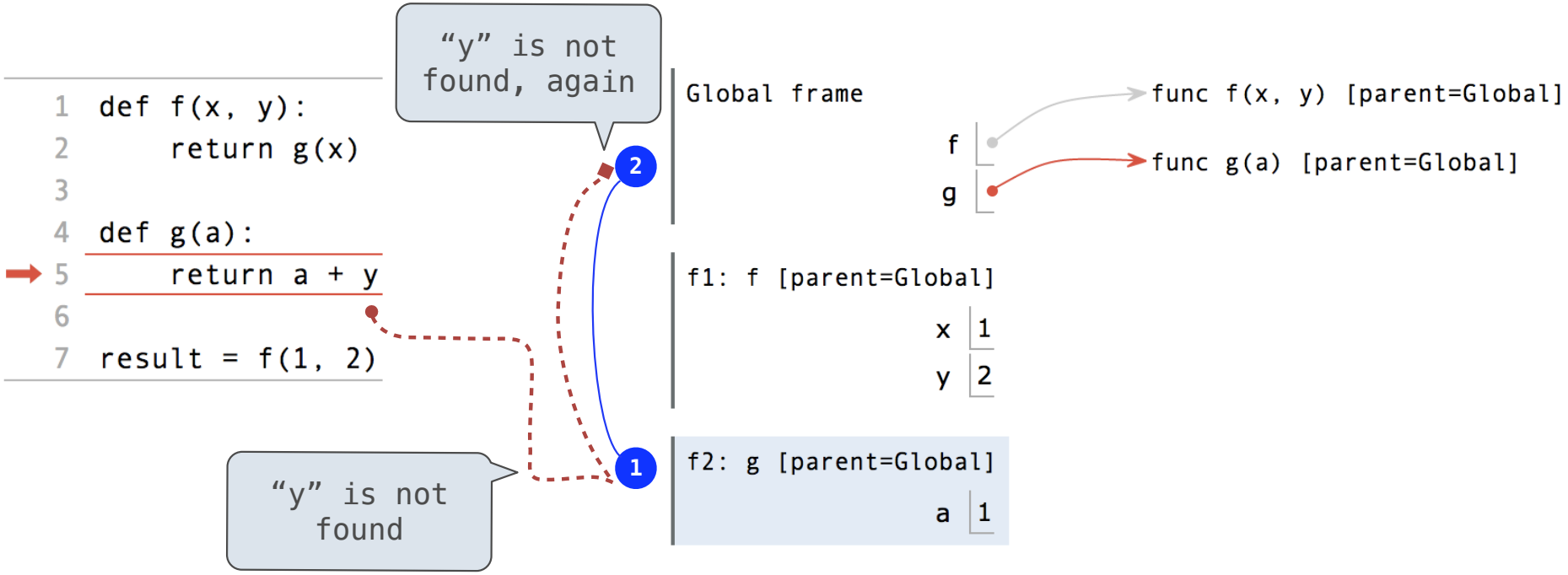

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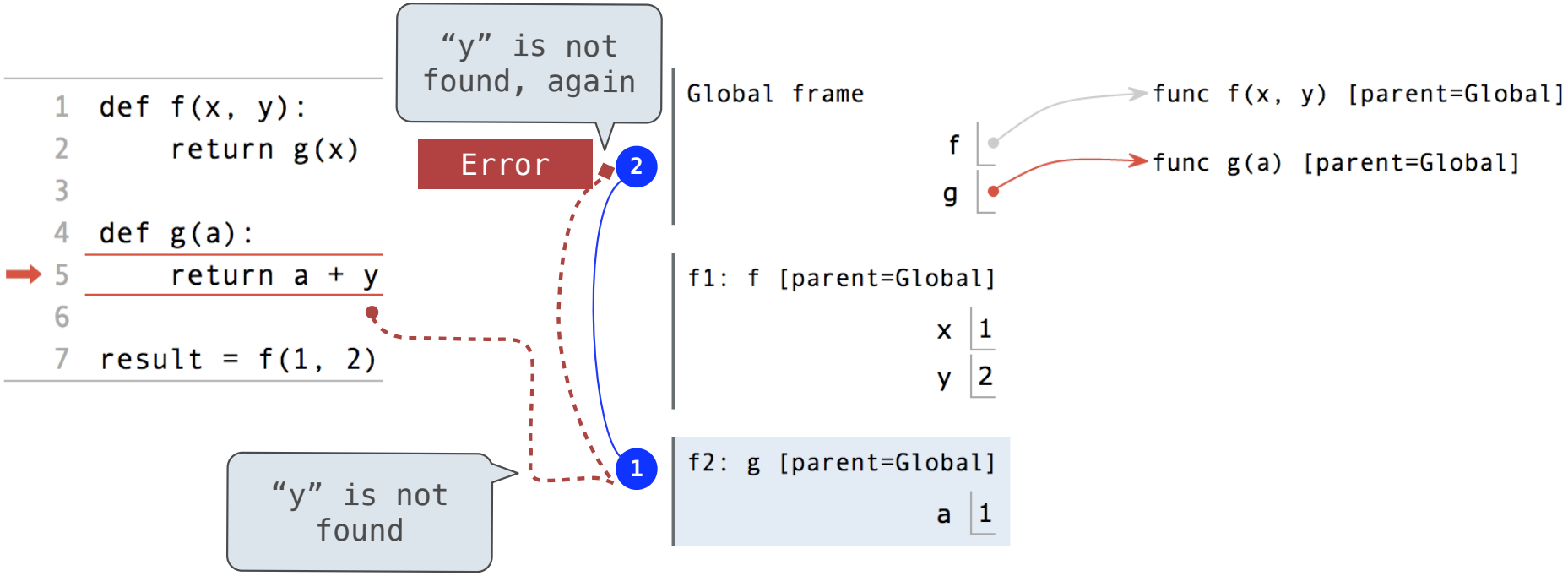

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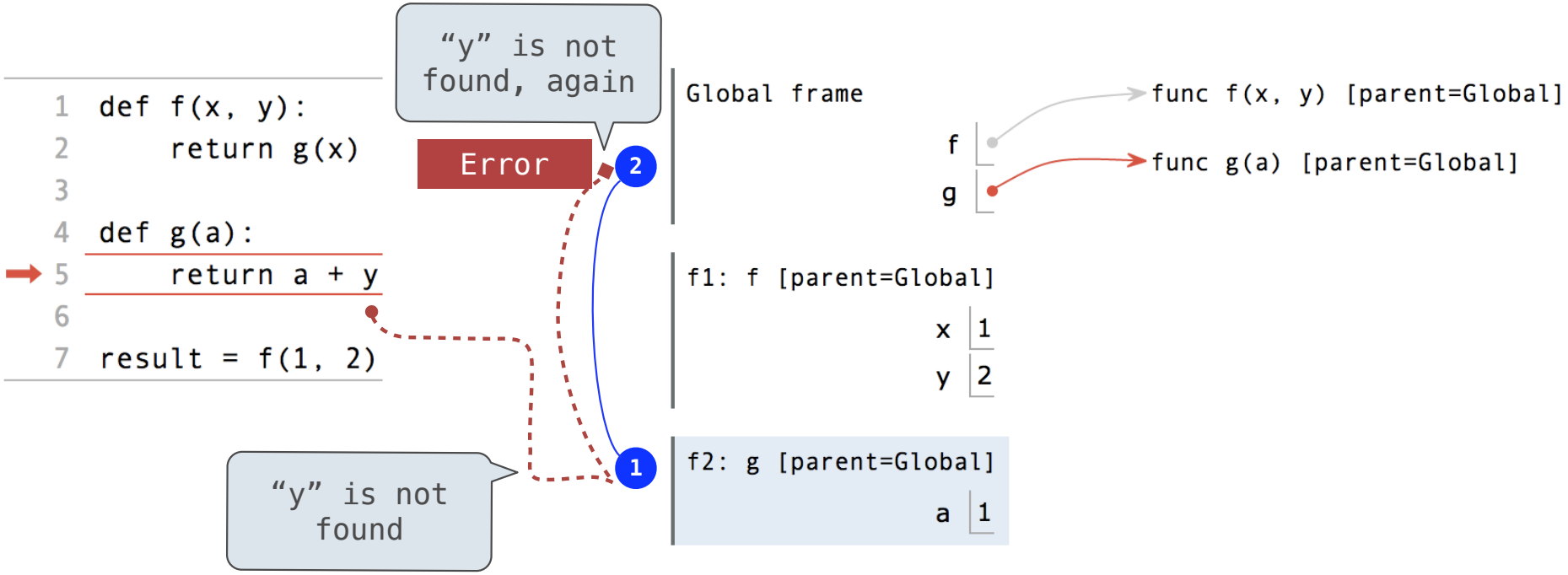

- An environment is a sequence of frames.

http://pythontutor.com/composingprograms.html#code=def%20f%28x,
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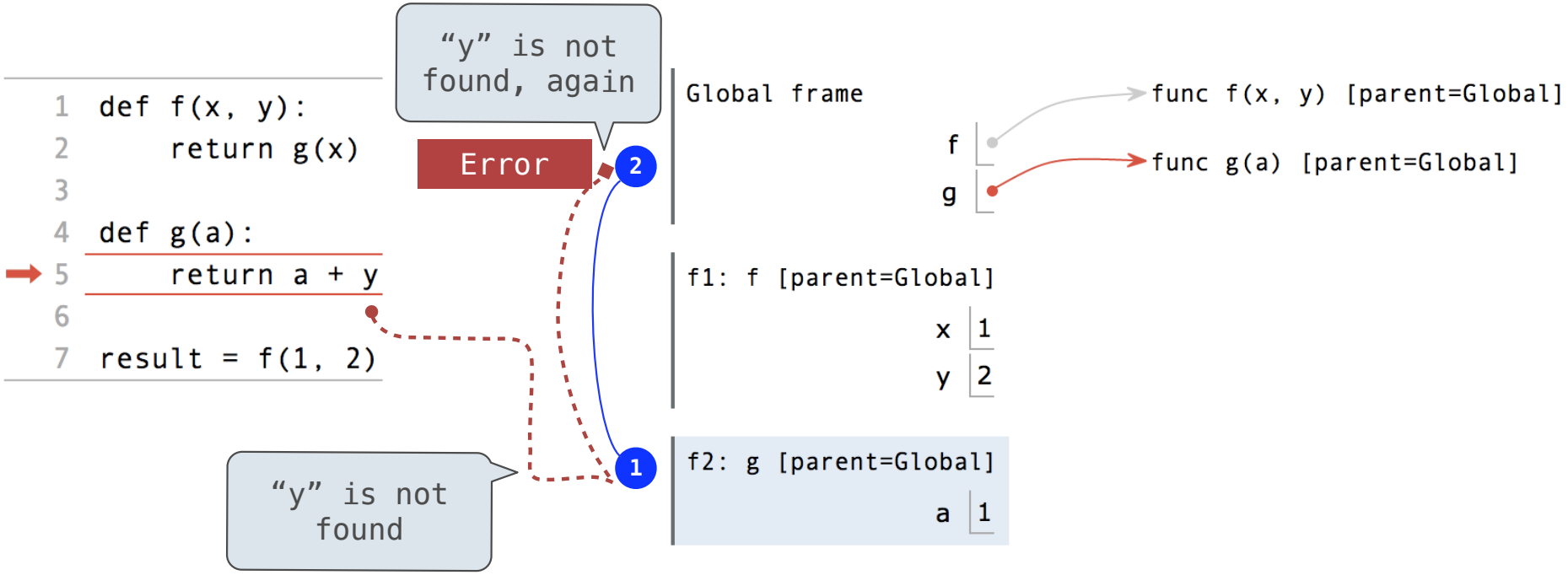

- An environment is a sequence of frames.
- The environment created by calling a top-level function (no def within def) consists of one local frame, followed by the global frame.

http://pythontutor.com/composingprograms.html#code=def%20f%28x,
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Function Composition

(Demo)

The Environment Diagram for Function Composition

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2      return x * x
3
4  def make_adder(n):
5      def adder(k):
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7      return adder
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9  def compose1(f, g):
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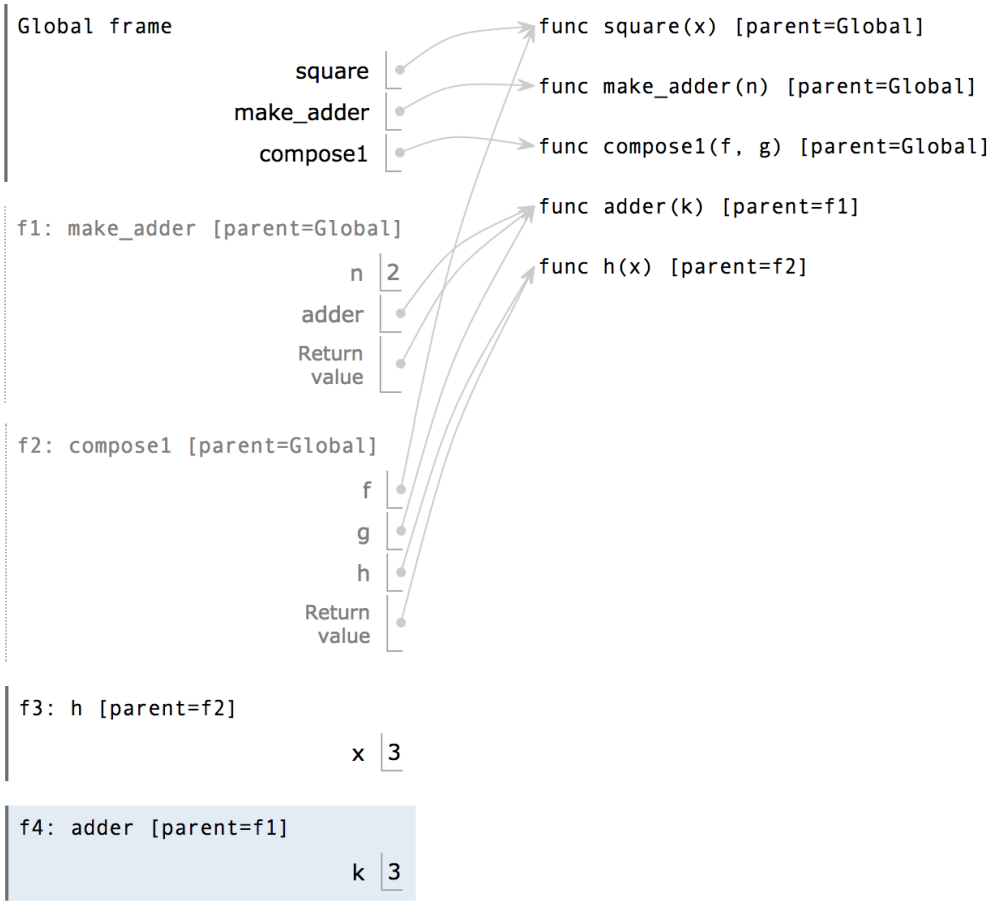

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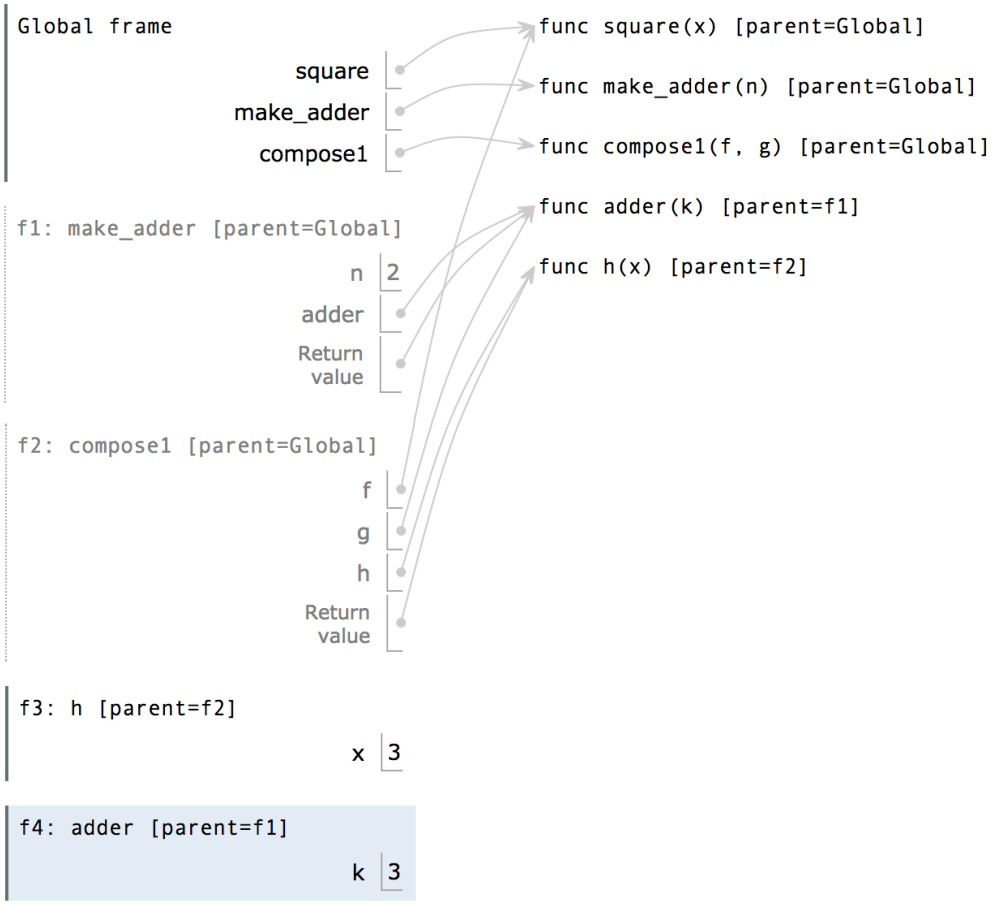

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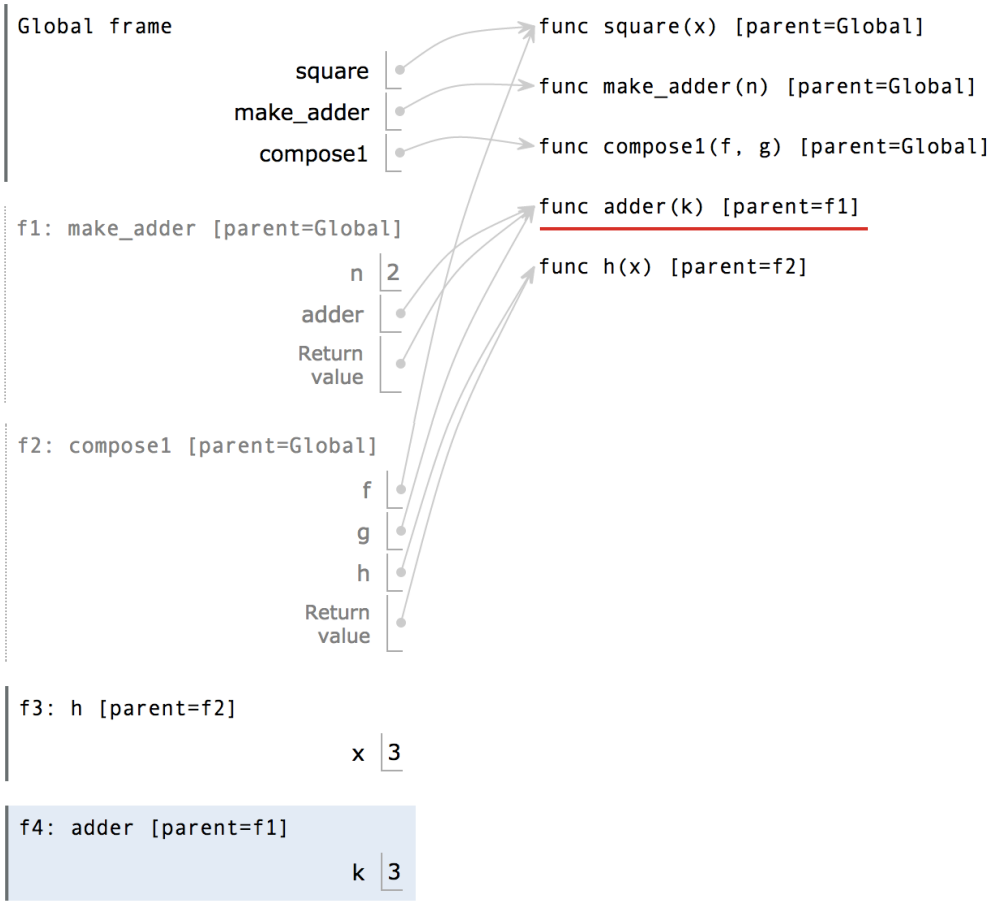

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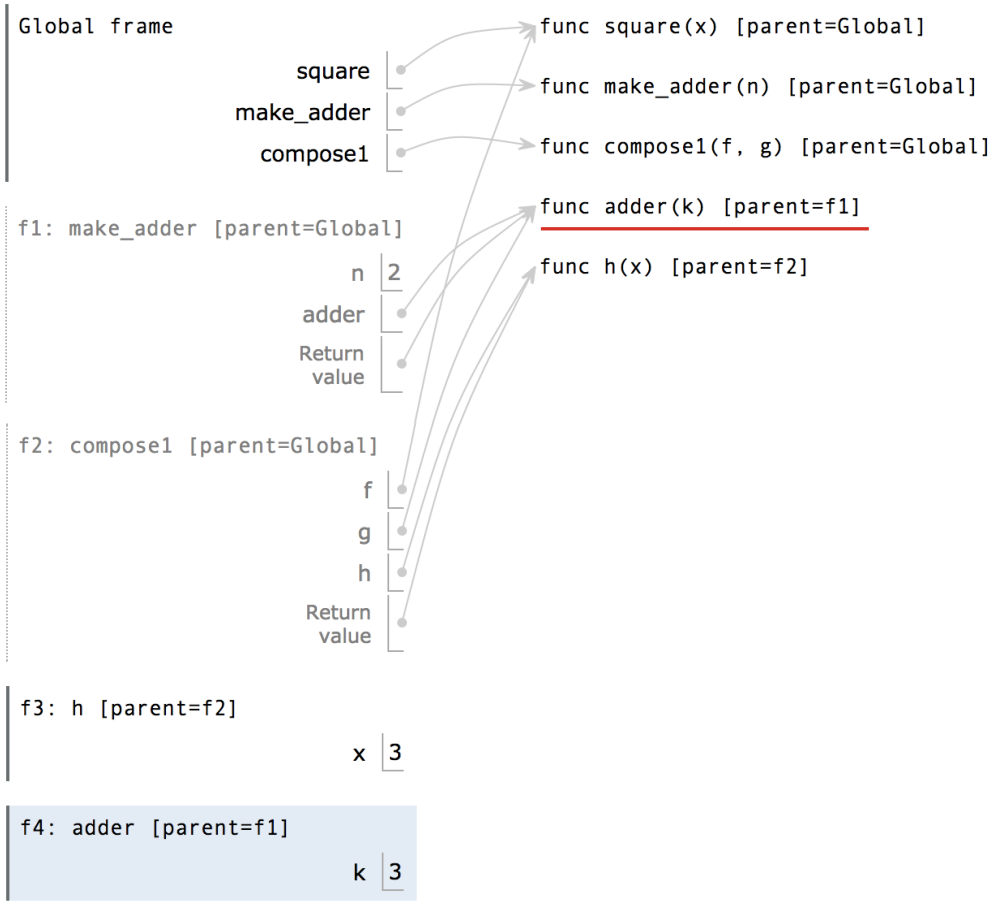

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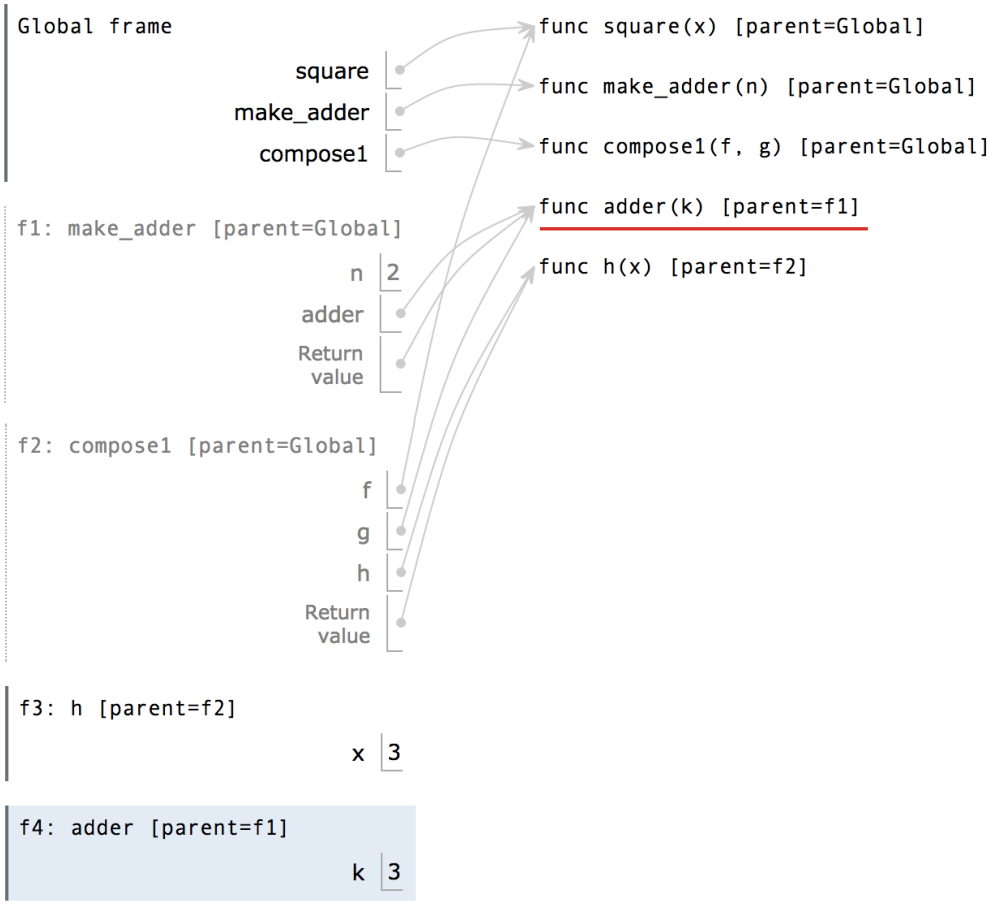
Return value of make_adder is an argument to compose1



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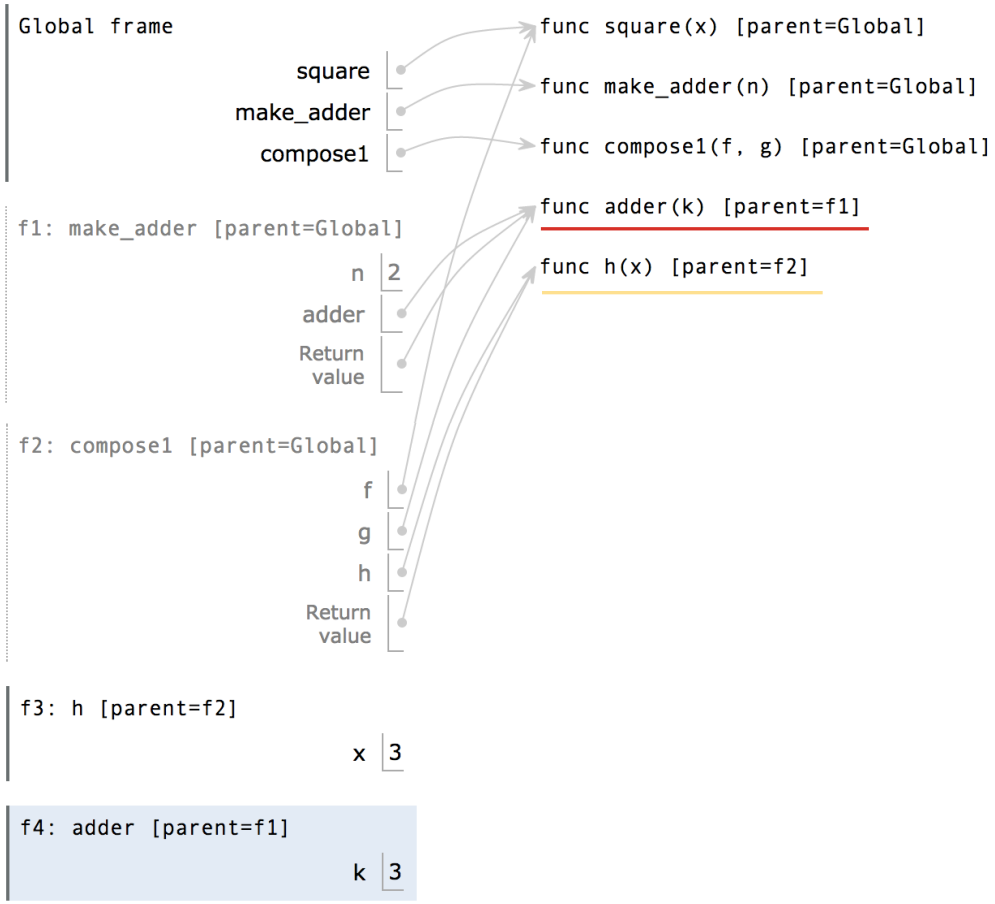

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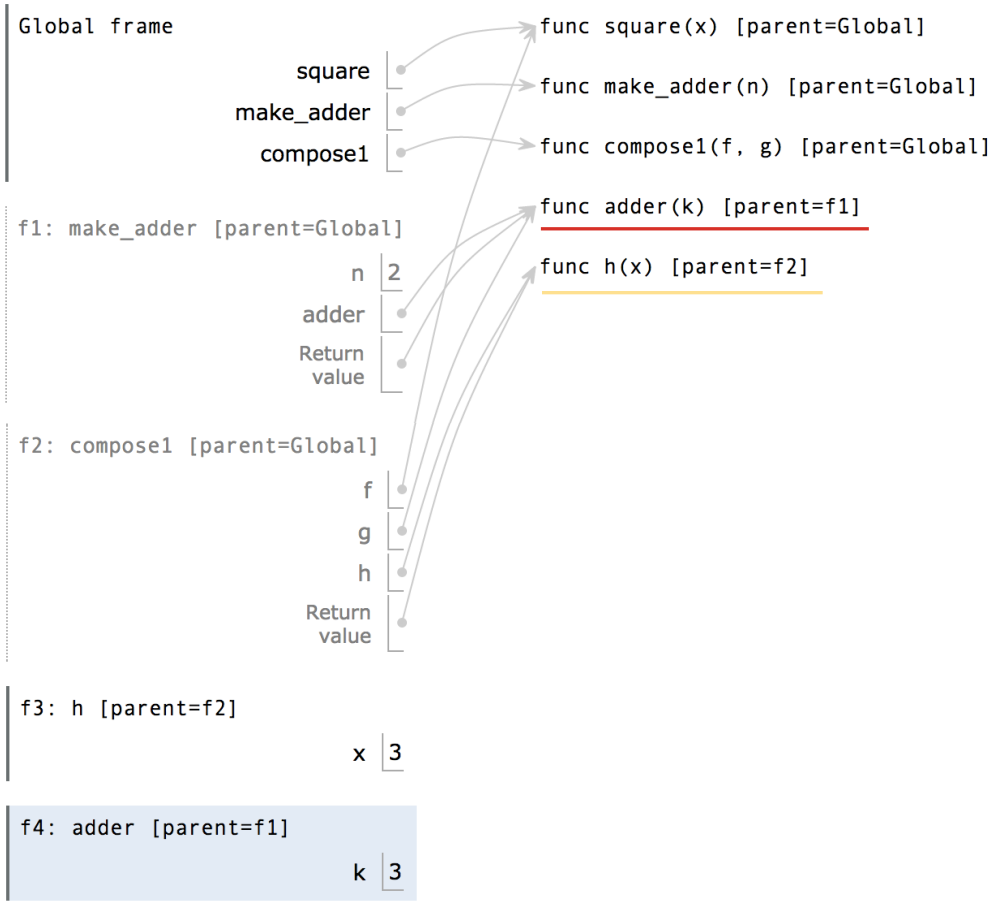

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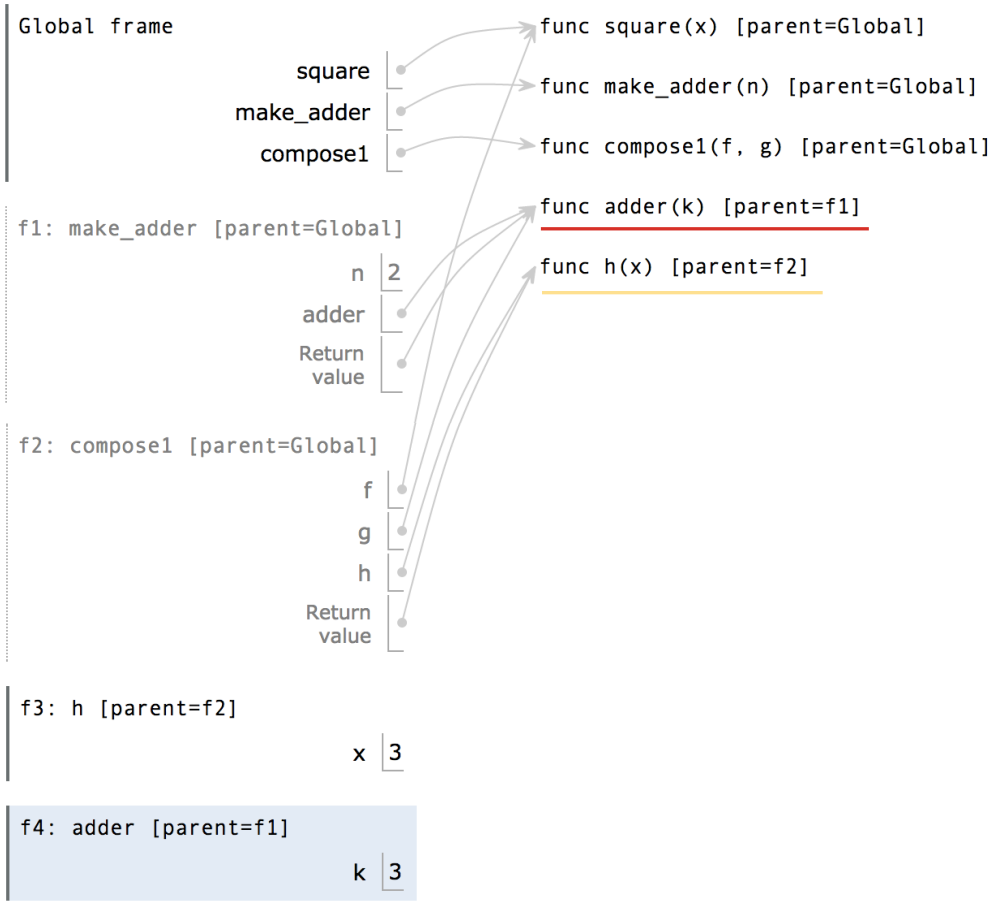
Return value of make_adder is an argument to compose1



http://pythontutor.com/composingprograms.html#code=def%20square%28x%29%3A%0A%20%20%20%20return%20x%20*%20x%0A%0Adef%20make_adder%28n%29%3A%0A%20%20%20%20def%20adder%28k%29%3A%0A%20%20%20%20%20%20%20%20return%20k%20%2B%20n%0A%20%20%20%20return%20adder%0A%20%20%20%20%0Adef%20compose1%28f,%20g%29%3A%0A%20%20%20%20def%20h%28x%29%3A%0A%20%20%20%20%20%20%20%20return%20f%28g%28x%29%29%0A%20%20%20%20return%20h%0A%20%20%20%20%0Acompose1%28square,%20make_adder%282%29%29%283%29&cumulative=true&curInstr=0&mode=display&origin=composingprograms.js&py=3&rawInputLstJSON=%5B%5D

The Environment Diagram for Function Composition

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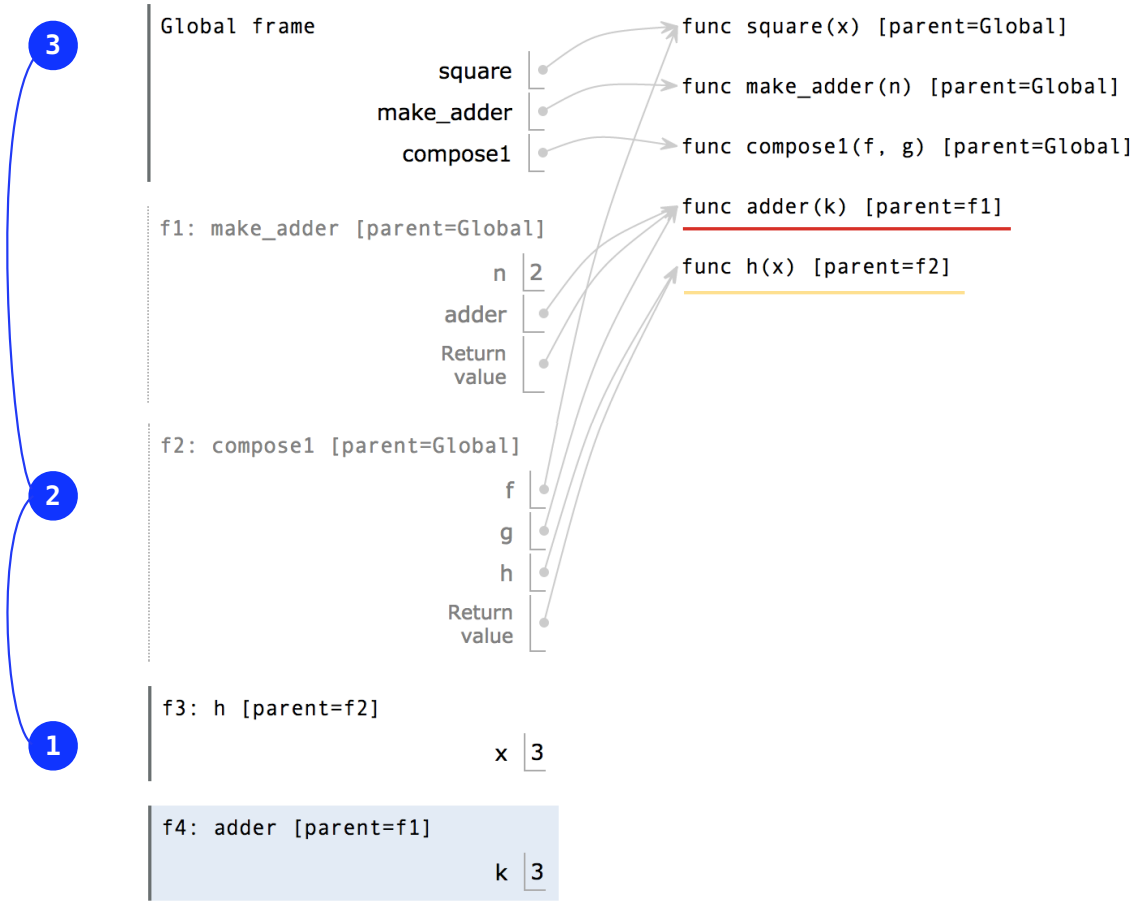

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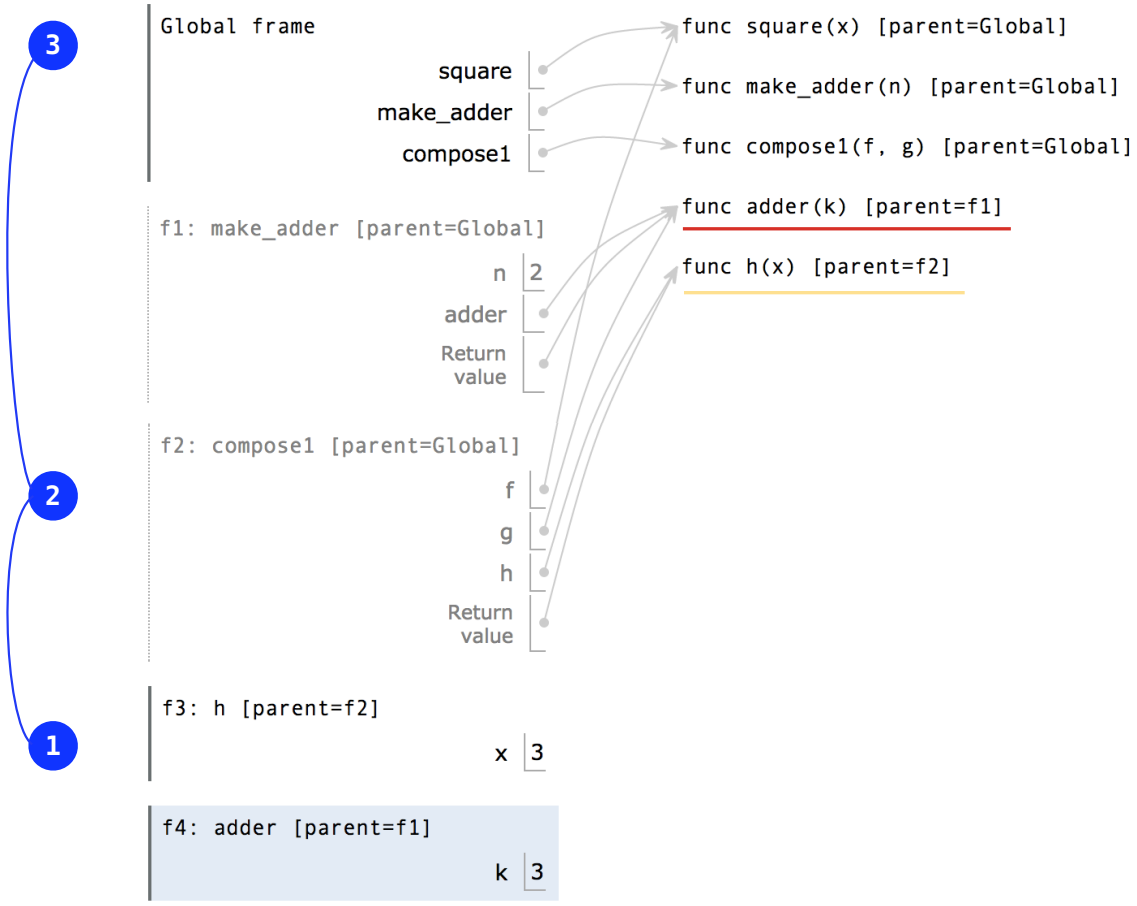

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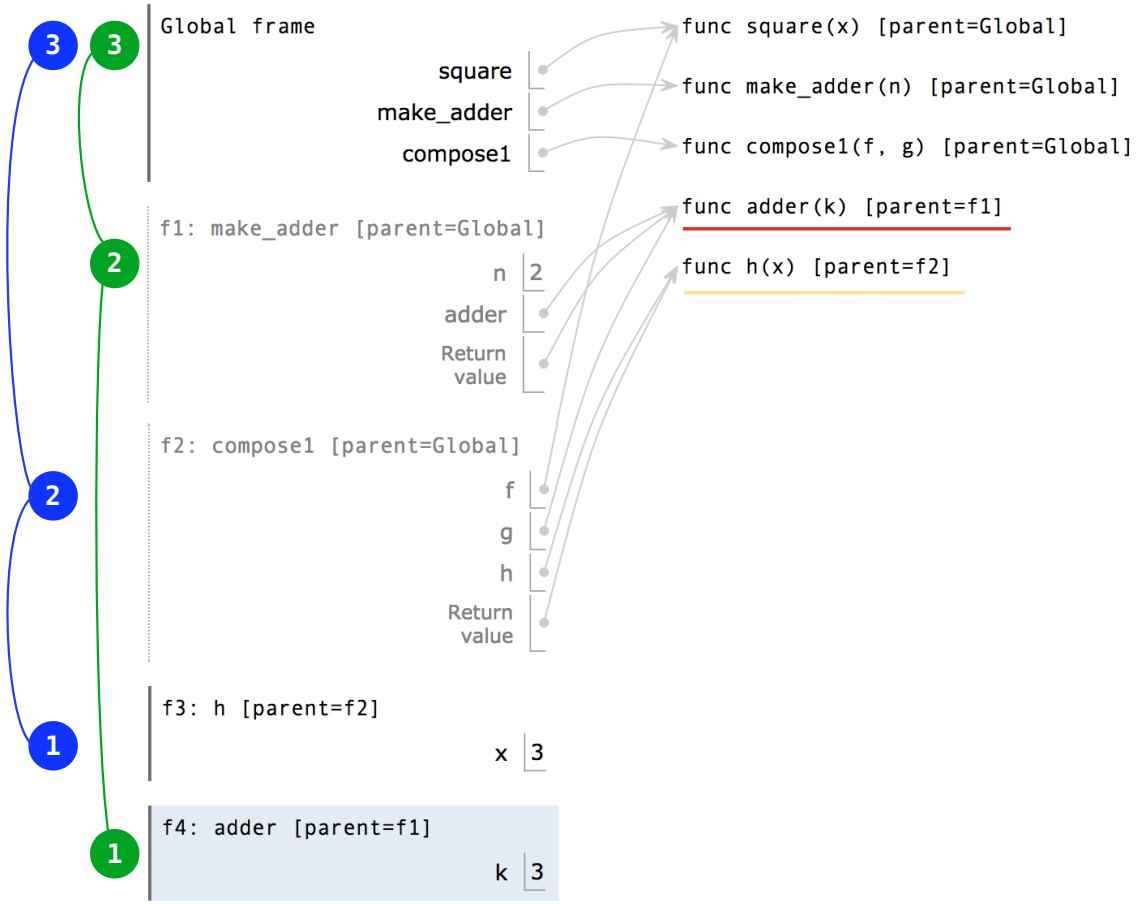

Lambda Expressions

(Demo)

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>>> x = 10
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A function

with formal parameter x

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Lambda expressions in Python cannot contain statements at all!

Lambda Expressions Versus Def Statements

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VS

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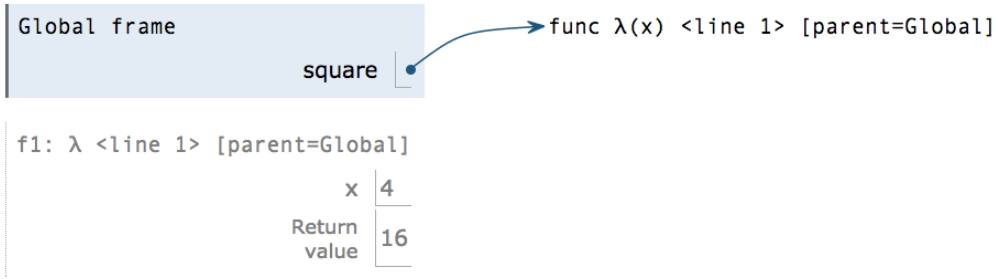

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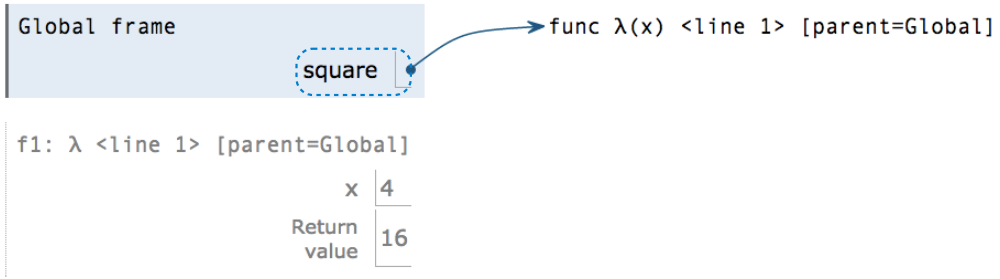

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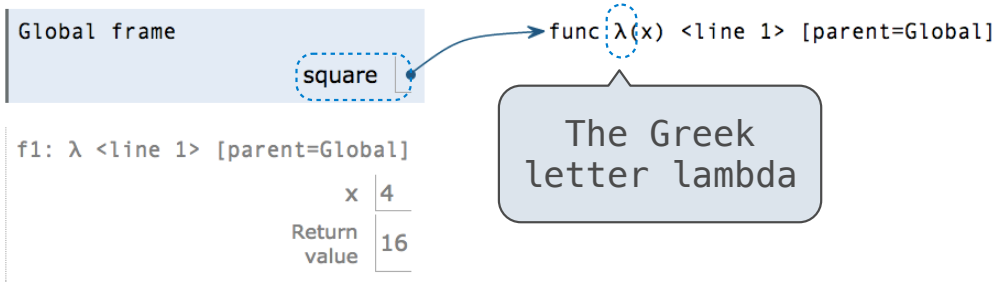

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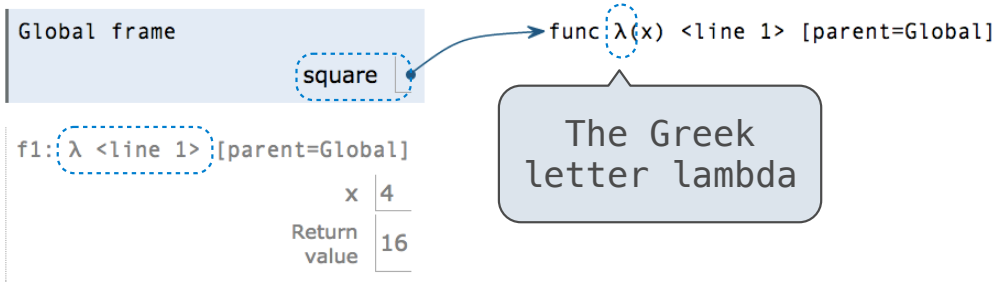

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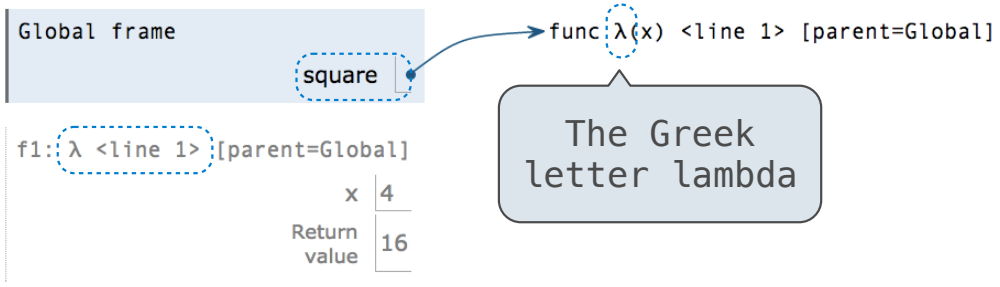

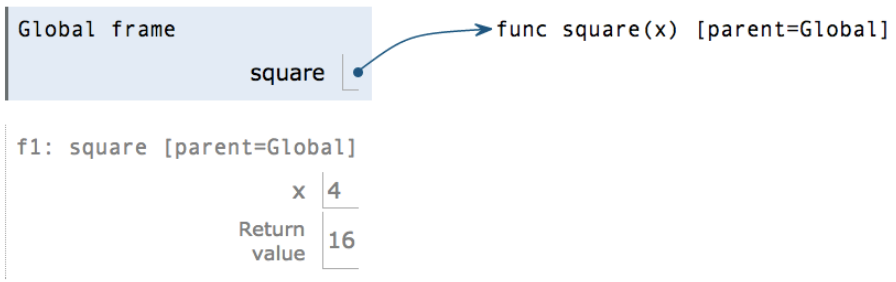

Currying

Function Currying

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    return lambda k: n + k
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(Demo)

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(Demo)

Curry: Transform a multi-argument function into a single-argument, higher-order function