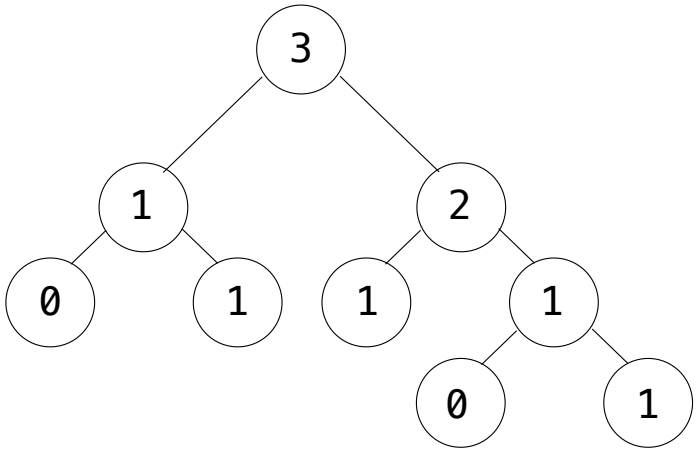
Trees

Announcements

Trees

Tree Abstraction



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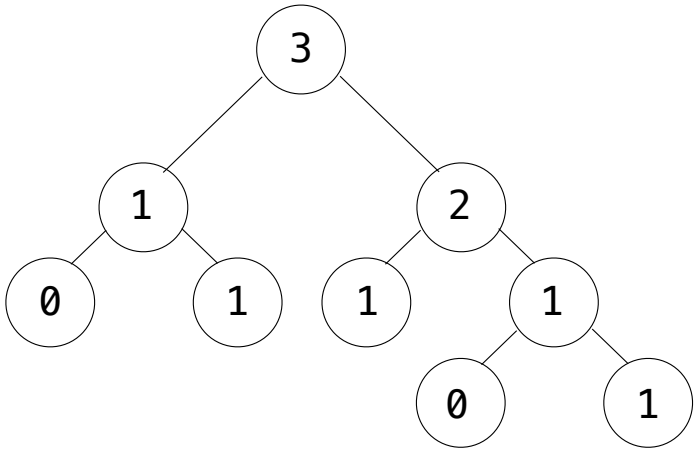

Recursive description (wooden trees):

Relative description (family trees):

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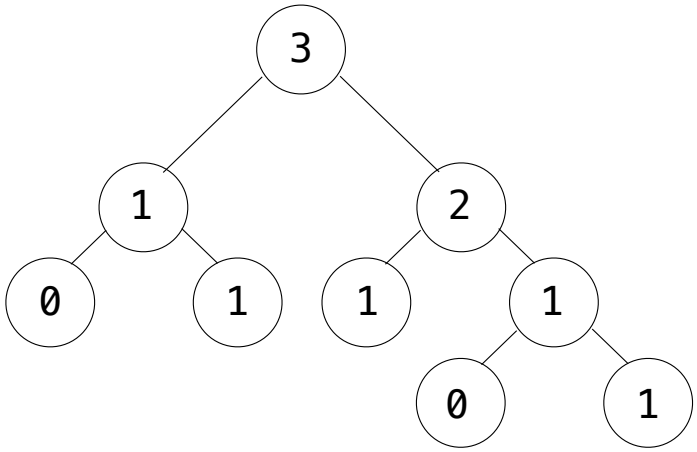

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A **tree** has a **root label** and a list of **branches**

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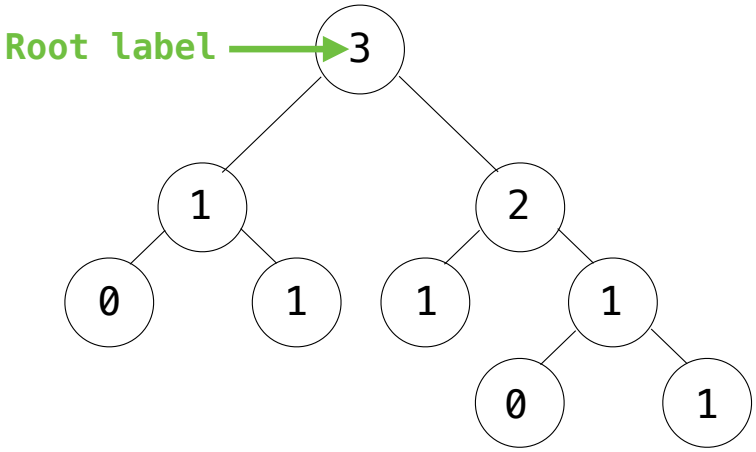

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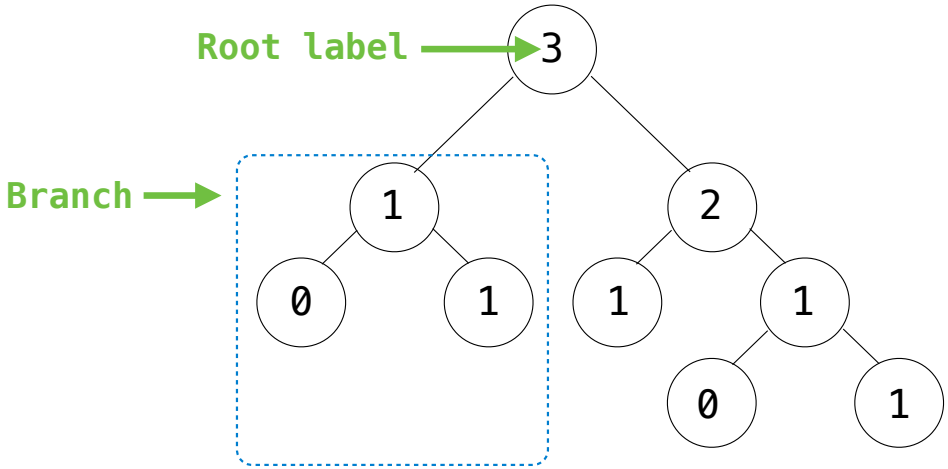

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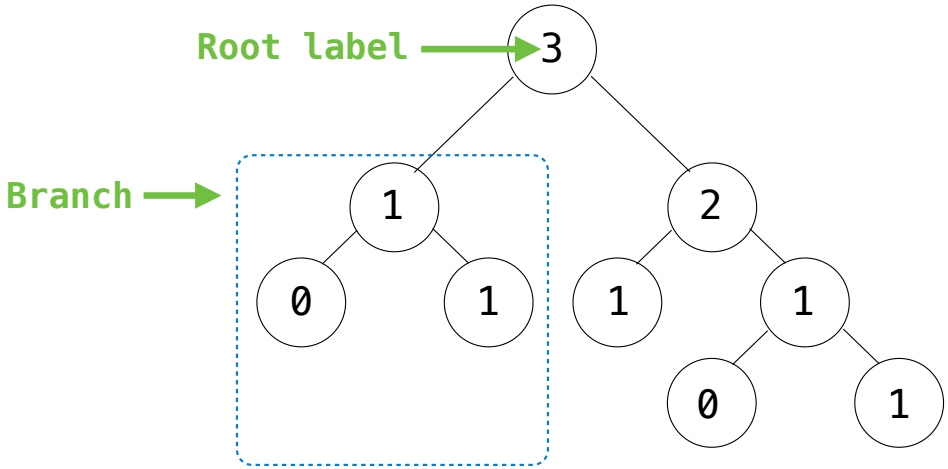

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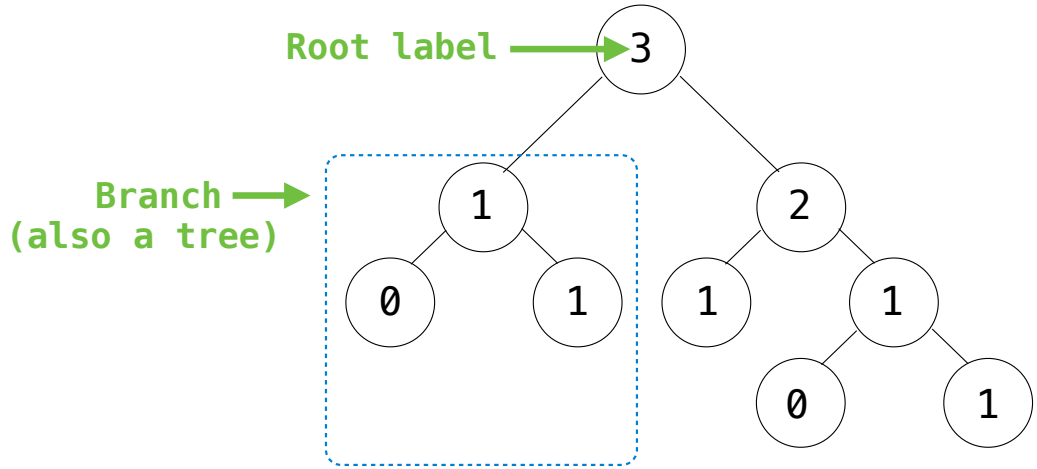

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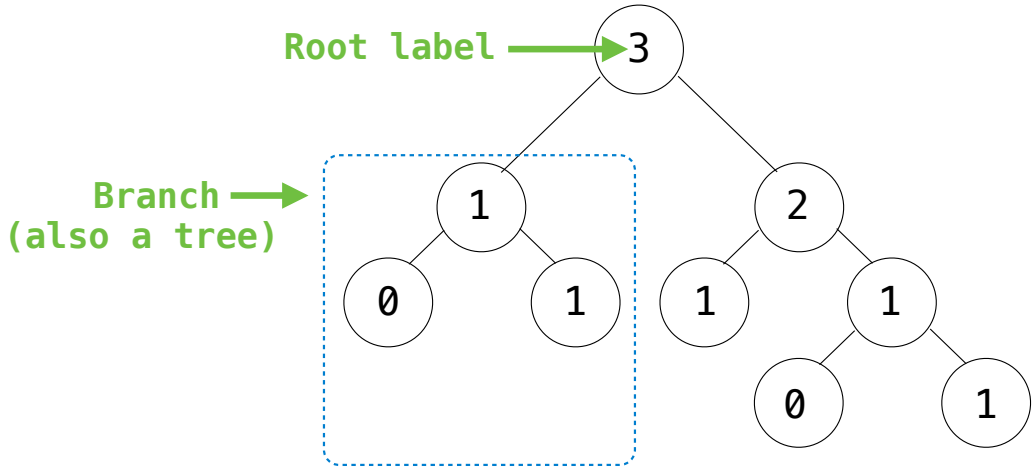

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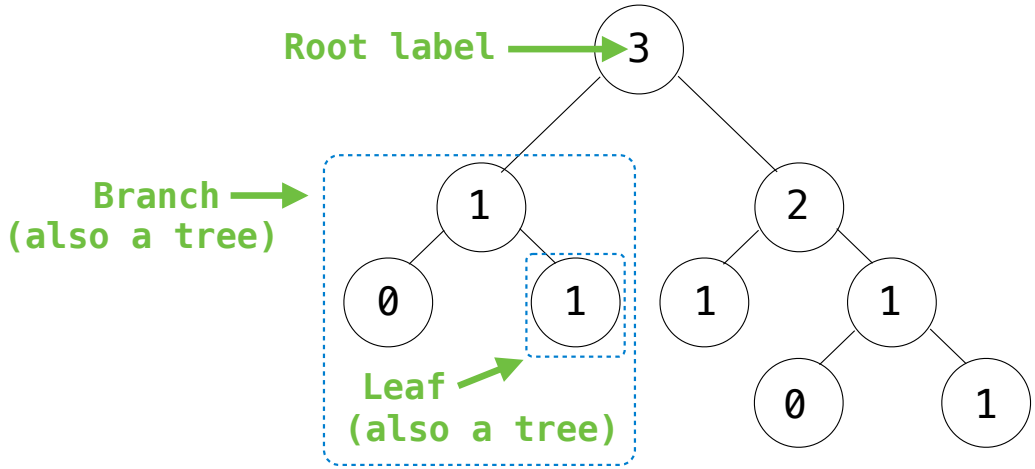

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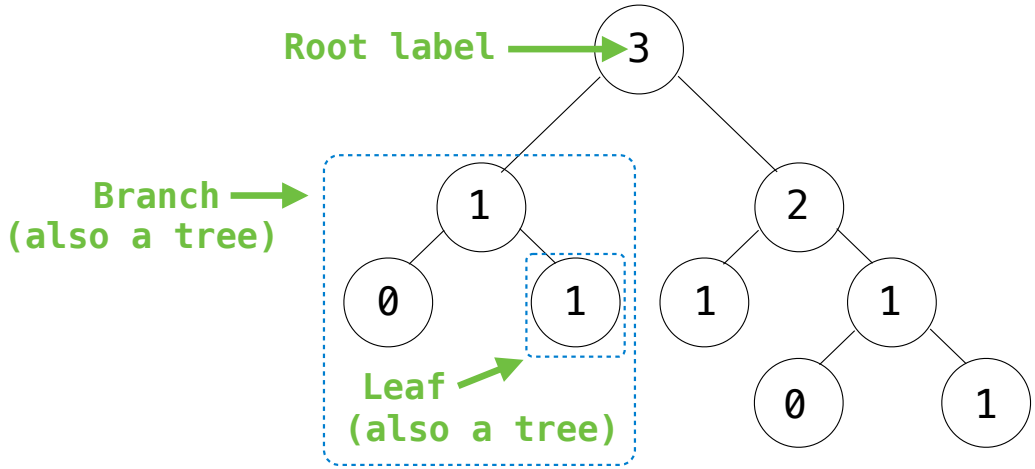

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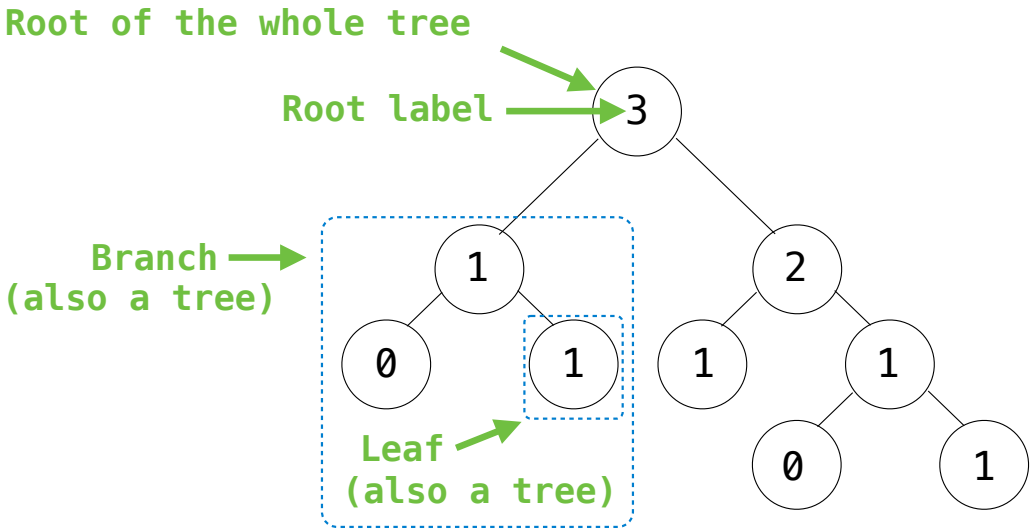

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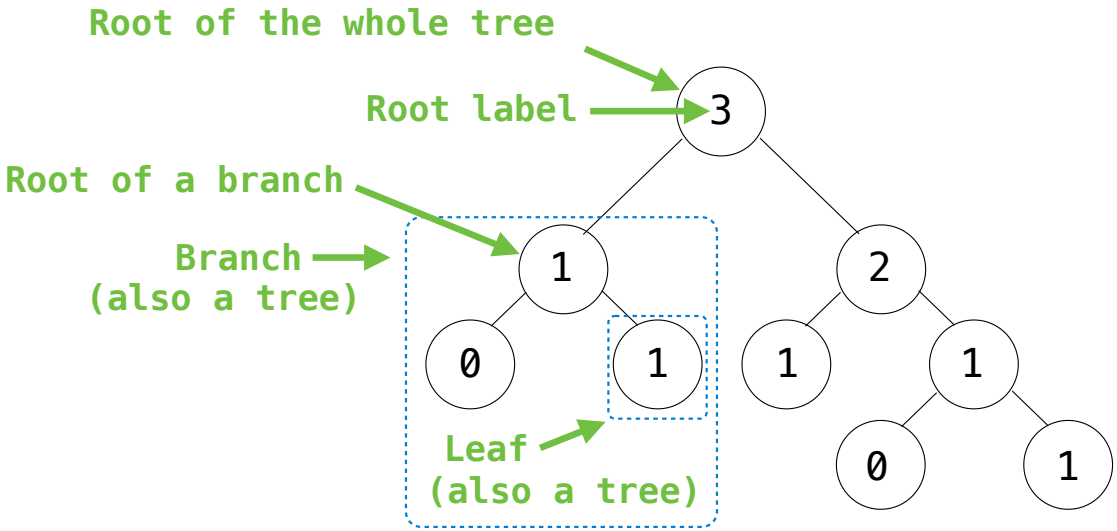

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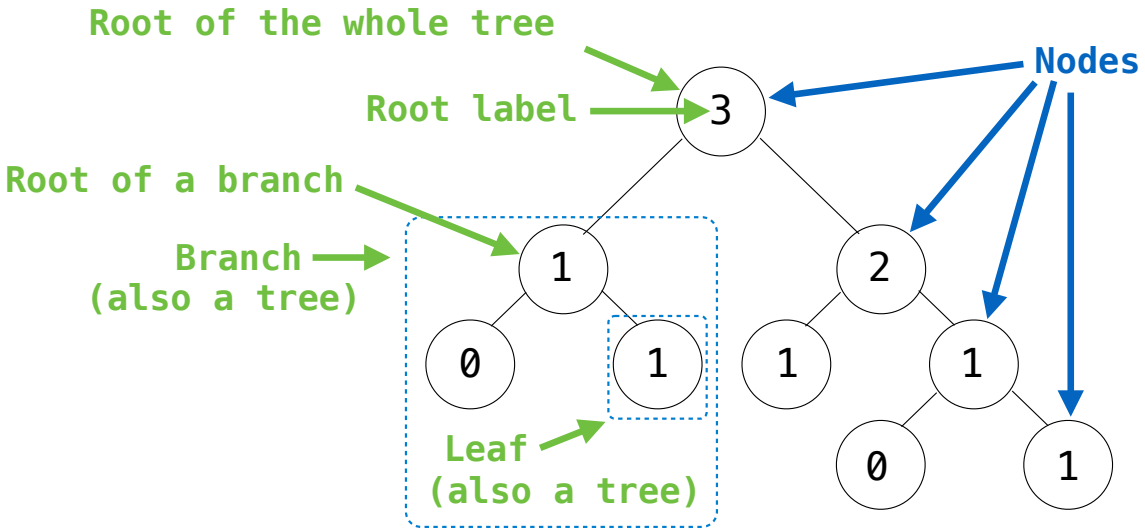

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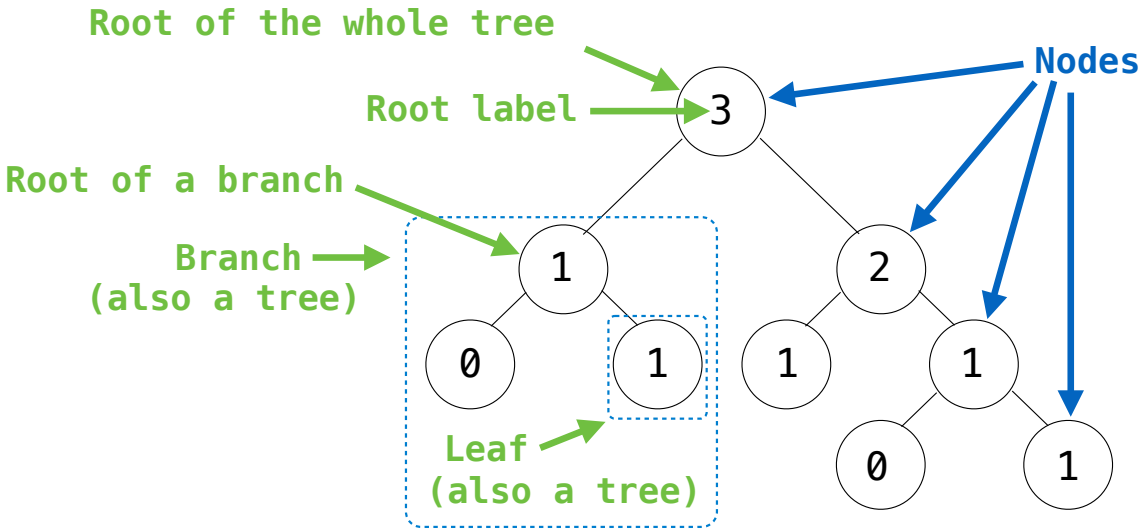

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Each **node** has a **label** that can be any value

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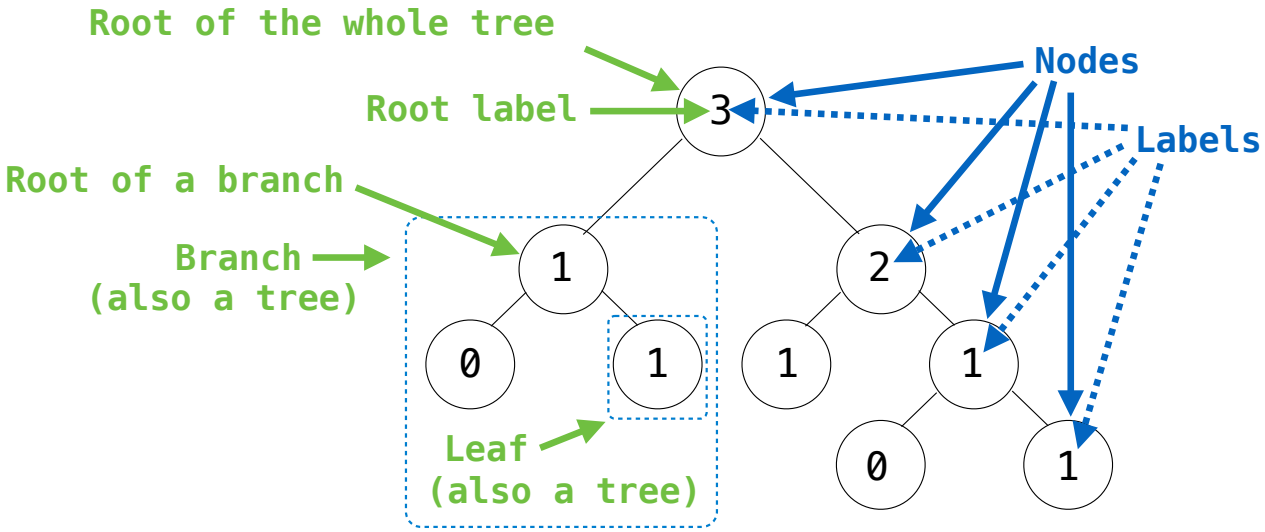

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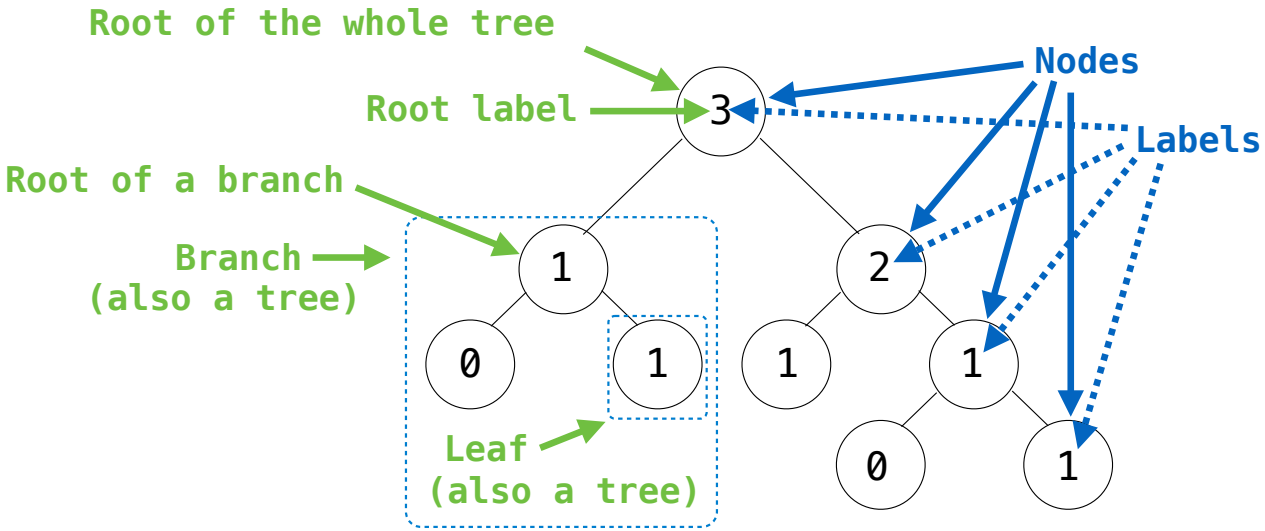

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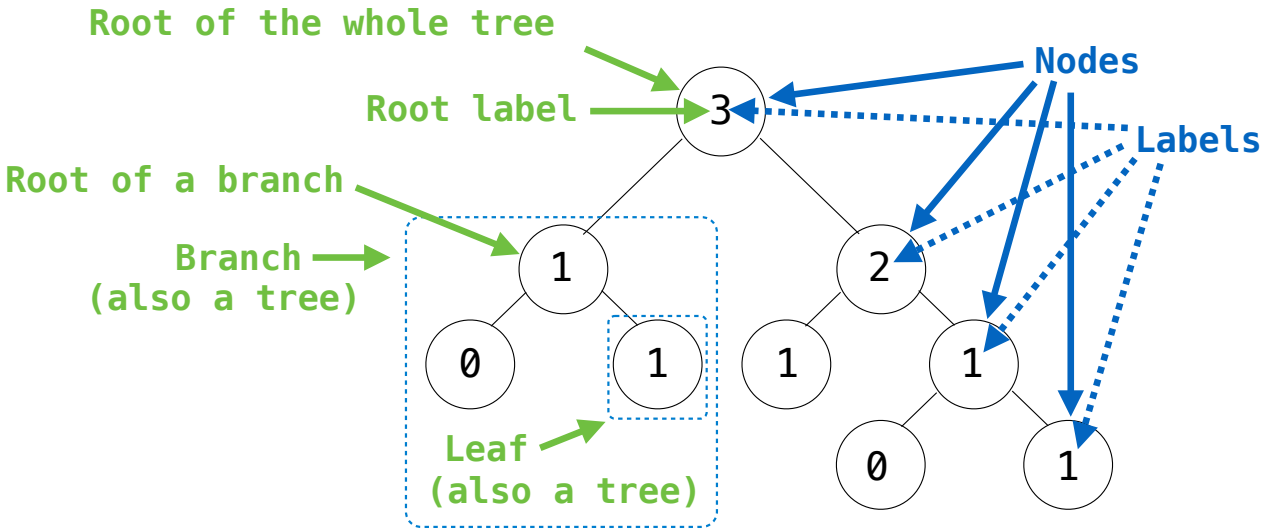

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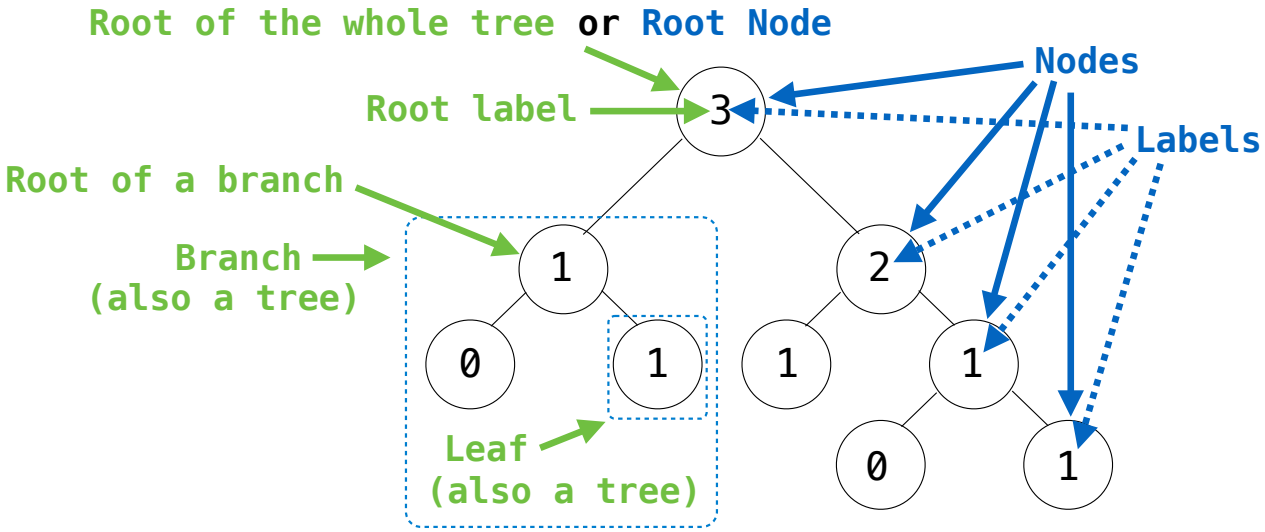

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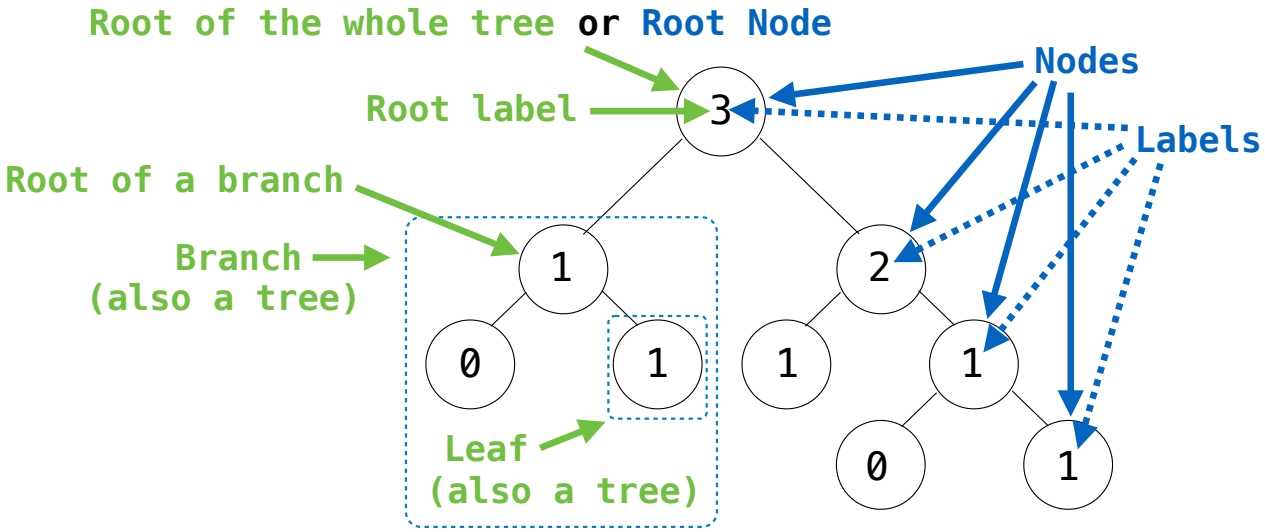

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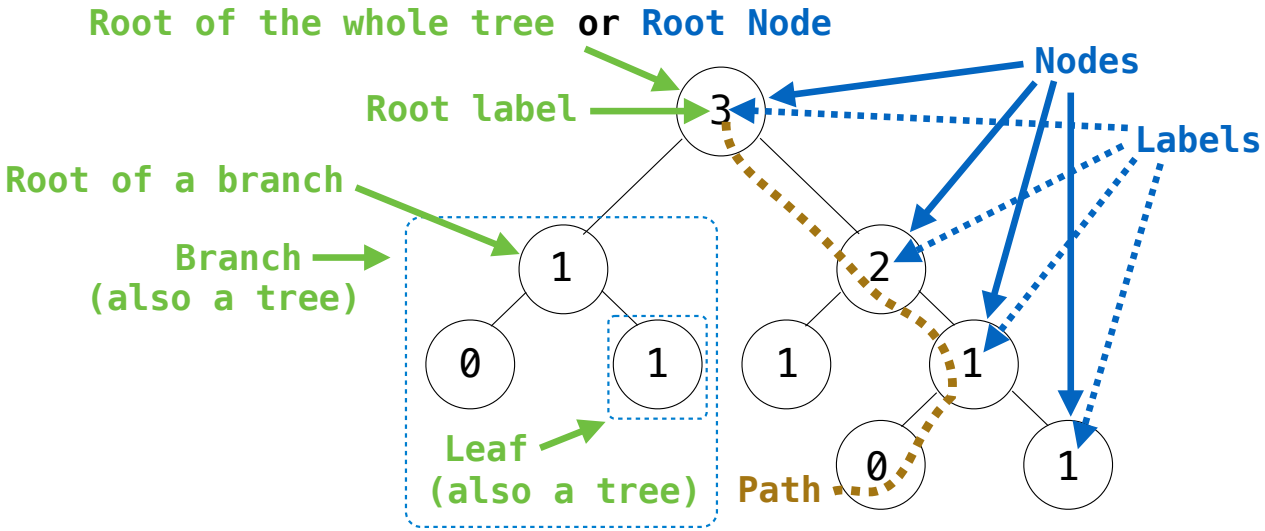

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- Each branch is a tree

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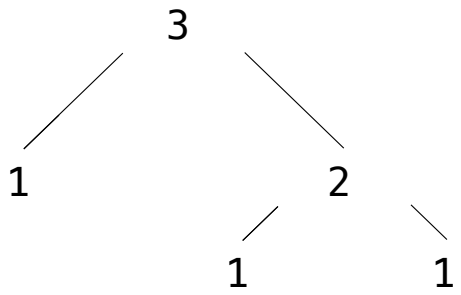

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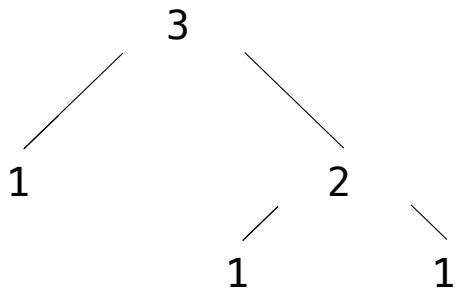


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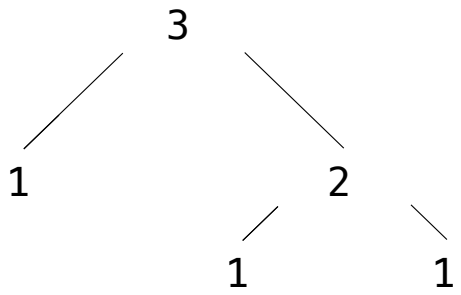


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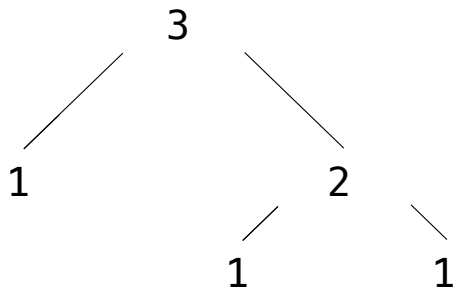


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def tree(label, branches=[]):
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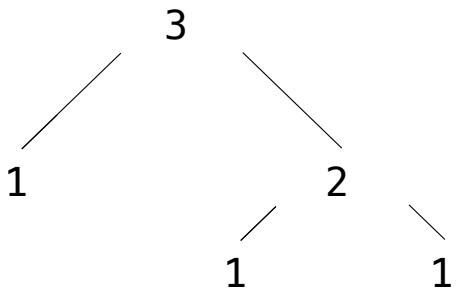


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def label(tree):
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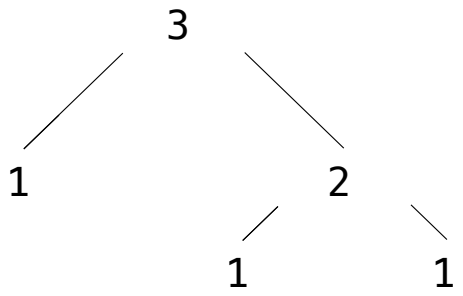


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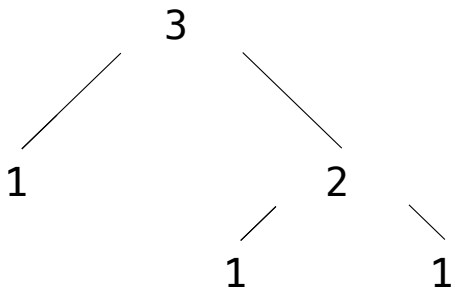


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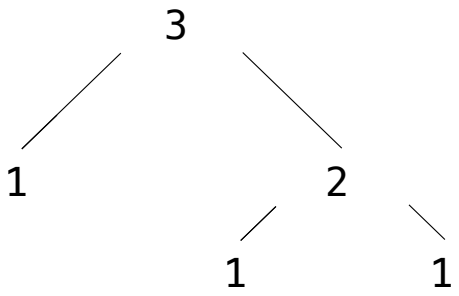


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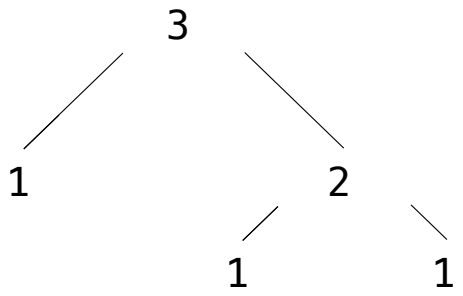


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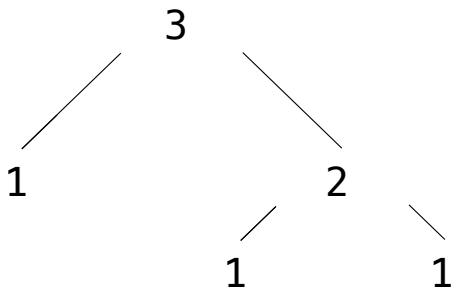


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Creates a list from a sequence of branches

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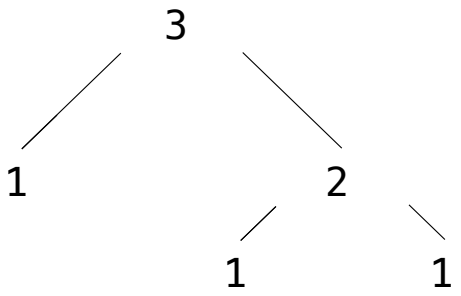


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Verifies the tree definition

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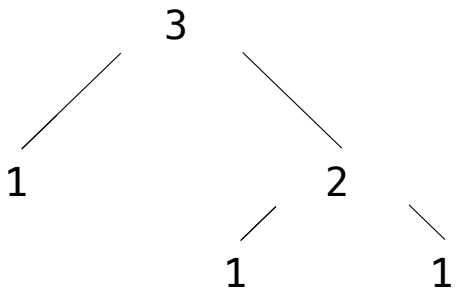


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def is_tree(tree):
    if type(tree) != list or len(tree) < 1:
        return False
    for branch in branches(tree):
        if not is_tree(branch):
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Verifies the tree definition

Creates a list from a sequence of branches

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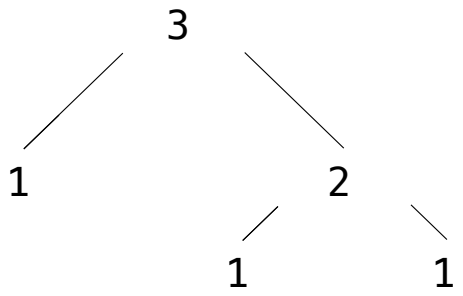


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Verifies the tree definition

Creates a list from a sequence of branches

Verifies that tree is bound to a list

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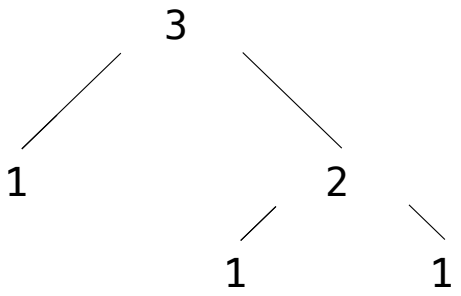


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Implementing the Tree Abstraction

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def tree(label, branches=[]):
    for branch in branches:
        assert is_tree(branch)
    return [label] + list(branches)

def label(tree):
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def branches(tree):
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def is_tree(tree):
    if type(tree) != list or len(tree) < 1:
        return False
    for branch in branches(tree):
        if not is_tree(branch):
            return False
    return True
```

Verifies the tree definition

Creates a list from a sequence of branches

Verifies that tree is bound to a list

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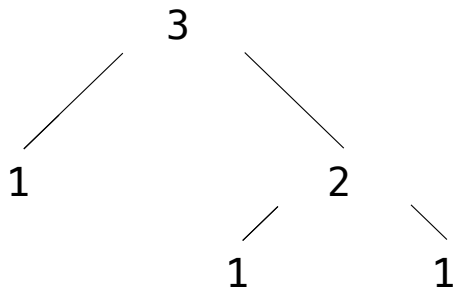


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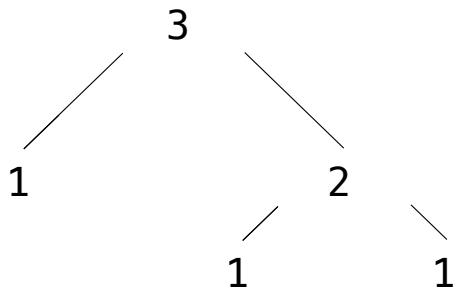


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(Demo)

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(Demo)

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(Demo)

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    else:
        return sum(List of leaf labels for each branch, [])
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Example: Printing Trees

(Demo)

Example: Summing Paths

(Demo)

Example: Counting Paths

Count Paths that have a Total Label Sum

```
def count_paths(t, total):
    """Return the number of paths from the root to any node in tree t
    for which the labels along the path sum to total.

    >>> t = tree(3, [tree(-1), tree(1, [tree(2, [tree(1)]), tree(3)]), tree(1, [tree(-1)])])
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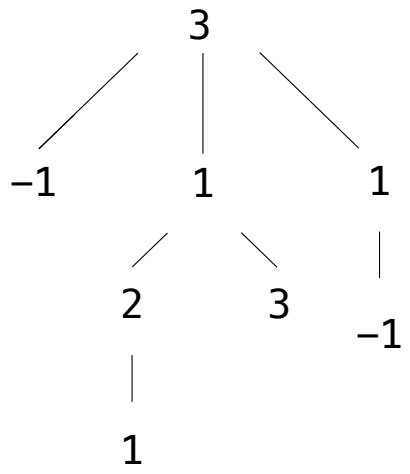

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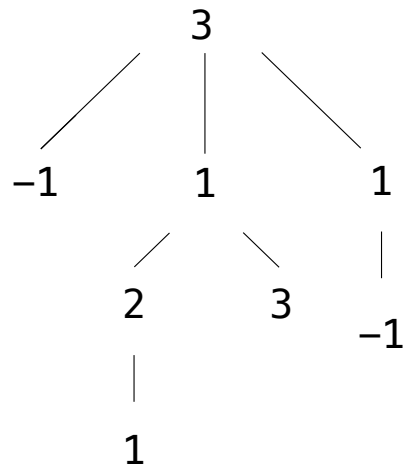

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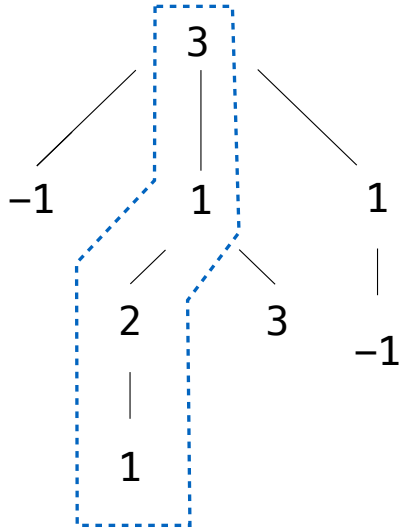

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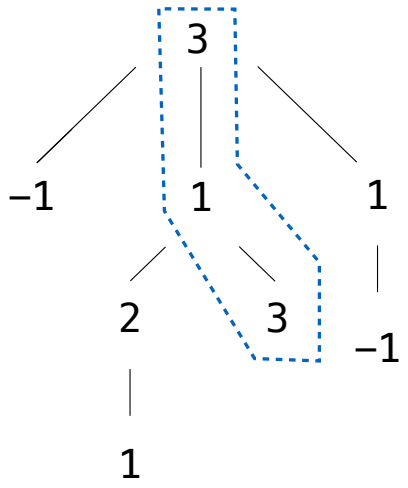

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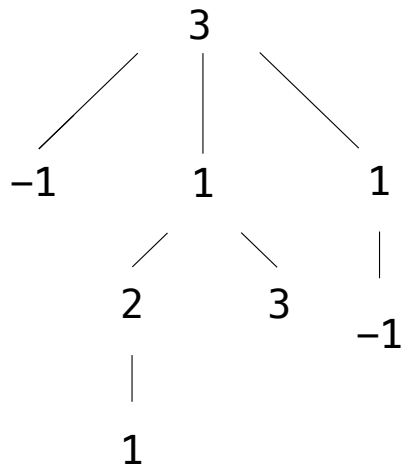

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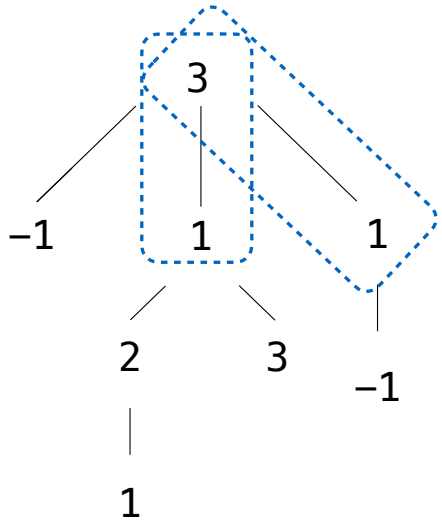

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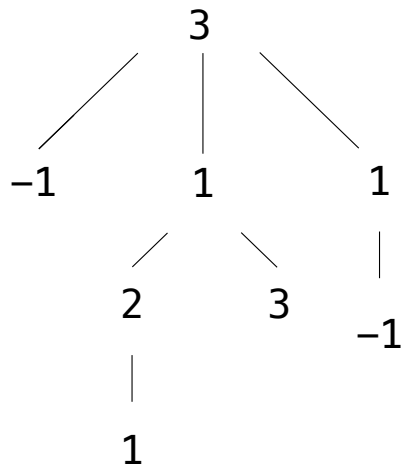

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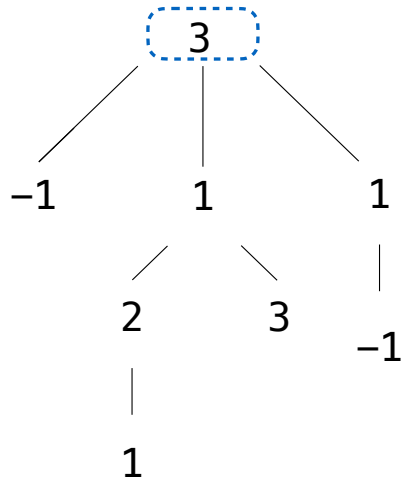

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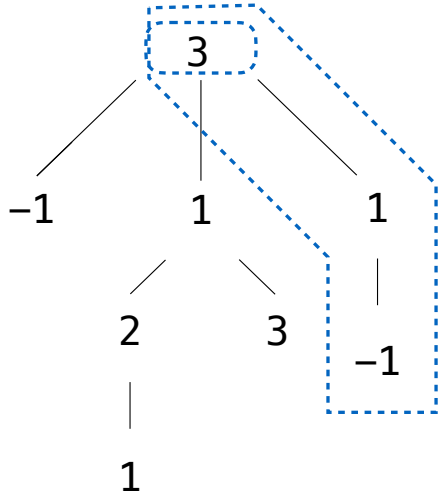

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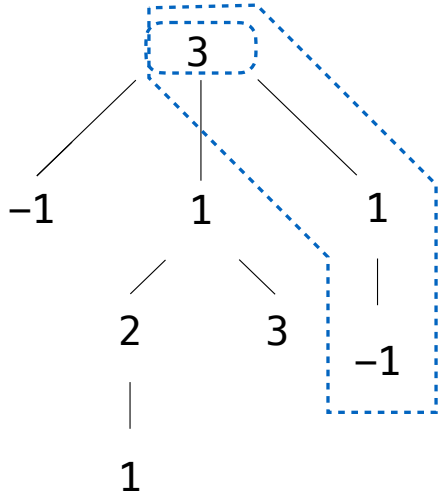
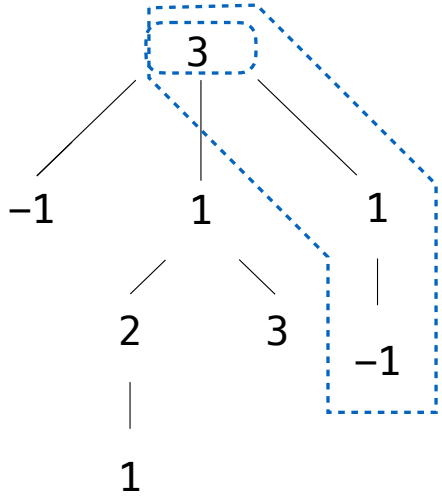
    >>> count_paths(t, 3)
    2
    >>> count_paths(t, 4)
    2
    >>> count_paths(t, 5)
    0
    >>> count_paths(t, 6)
    1
    >>> count_paths(t, 7)
    2
    """
    if label(t) == total:
        found = _____________
    else:
        _____________________
    return found + _________([_____________________________________ for b in branches(t)])
```



Count Paths that have a Total Label Sum

```
def count_paths(t, total):
    """Return the number of paths from the root to any node in tree t
    for which the labels along the path sum to total.

    >>> t = tree(3, [tree(-1), tree(1, [tree(2, [tree(1)]), tree(3)]), tree(1, [tree(-1)])])
    >>> count_paths(t, 3)
    2
    >>> count_paths(t, 4)
    2
    >>> count_paths(t, 5)
    0
    >>> count_paths(t, 6)
    1
    >>> count_paths(t, 7)
    2
    """
    if label(t) == total:
        found = 1
    else:
        _____________________
    return found + _________([_____________________________________ for b in branches(t)])
```



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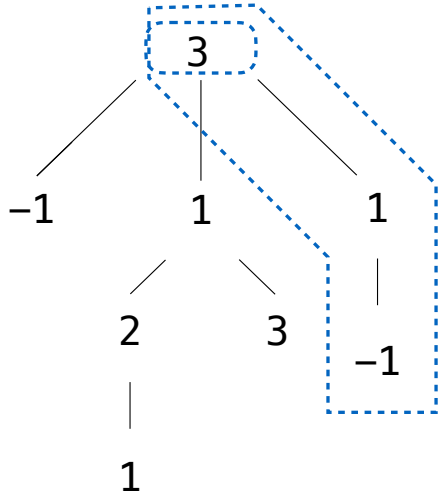
    2
    >>> count_paths(t, 4)
    2
    >>> count_paths(t, 5)
    0
    >>> count_paths(t, 6)
    1
    >>> count_paths(t, 7)
    2
    """
    if label(t) == total:
        found = ____1____
    else:
        found = 0
    return found + _____([_____ for b in branches(t)])
```



Count Paths that have a Total Label Sum

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    >>> count_paths(t, 3)
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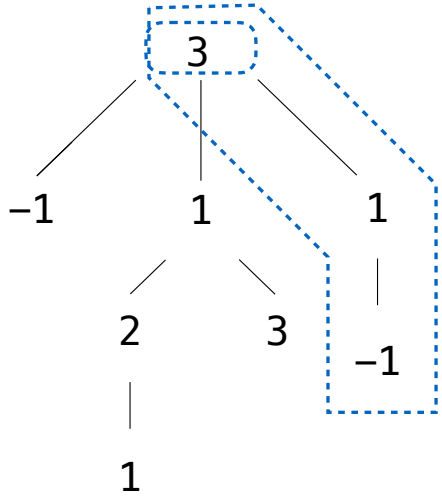
    2
    >>> count_paths(t, 5)
    0
    >>> count_paths(t, 6)
    1
    >>> count_paths(t, 7)
    2
    """
    if label(t) == total:
        found = 1
    else:
        found = 0
    return found + sum([_____ for b in branches(t)])
```



Count Paths that have a Total Label Sum

```
def count_paths(t, total):
    """Return the number of paths from the root to any node in tree t
    for which the labels along the path sum to total.

    >>> t = tree(3, [tree(-1), tree(1, [tree(2, [tree(1)]), tree(3)]), tree(1, [tree(-1)])])
    >>> count_paths(t, 3)
    2
    >>> count_paths(t, 4)
    2
    >>> count_paths(t, 5)
    0
    >>> count_paths(t, 6)
    1
    >>> count_paths(t, 7)
    2
    """
    if label(t) == total:
        found = 1
    else:
        found = 0
    return found + sum([count_paths(b, total - label(t)) for b in branches(t)])
```