EECS 336 Fall 2015
Homework Problem 7.2

Algorithm

1. Find the maximum flow f from s to t in $G = (V, E)$.
2. Derive residual graph G_f after f.
3. Let $S = \{v \in V; v \text{ is reachable from } s \text{ in } G_f\}$
4. Let $T = \{(u, w) \in E; u \in S, w \notin S\}$
5. If $d \leq |T|$, return any d edges in T. Else if $d > |T|$, return T and any other $d - |T|$ edges.

Correctness

T contain the edges that cross the minimum (s, t) cut $(S, V - S)$. Therefore, capacity $c(S, V - S) = |f|$ according to Max Flow Min Cut duality. Removing any d edges in T would drop the capacity of $(S, V - S)$ to $|f| - d$, which is still a minimum (s, t) cut. Since all edges have unit capacities, d is the maximum reduction flow that can be made to the damage. Therefore, the algorithm gives correct solution.

Runtime Analysis

Using Ford-Fulkerson with unit edge capacities, with maximum value f of $|V|$, finding f takes $O(|V|\,|E|)$. Finding G_f takes $O(|E|)$. Finding S takes $O(|V|)$. Finding T takes $O(|E|)$. Therefore, algorithm runs in $O(|V|\,|E|)$.