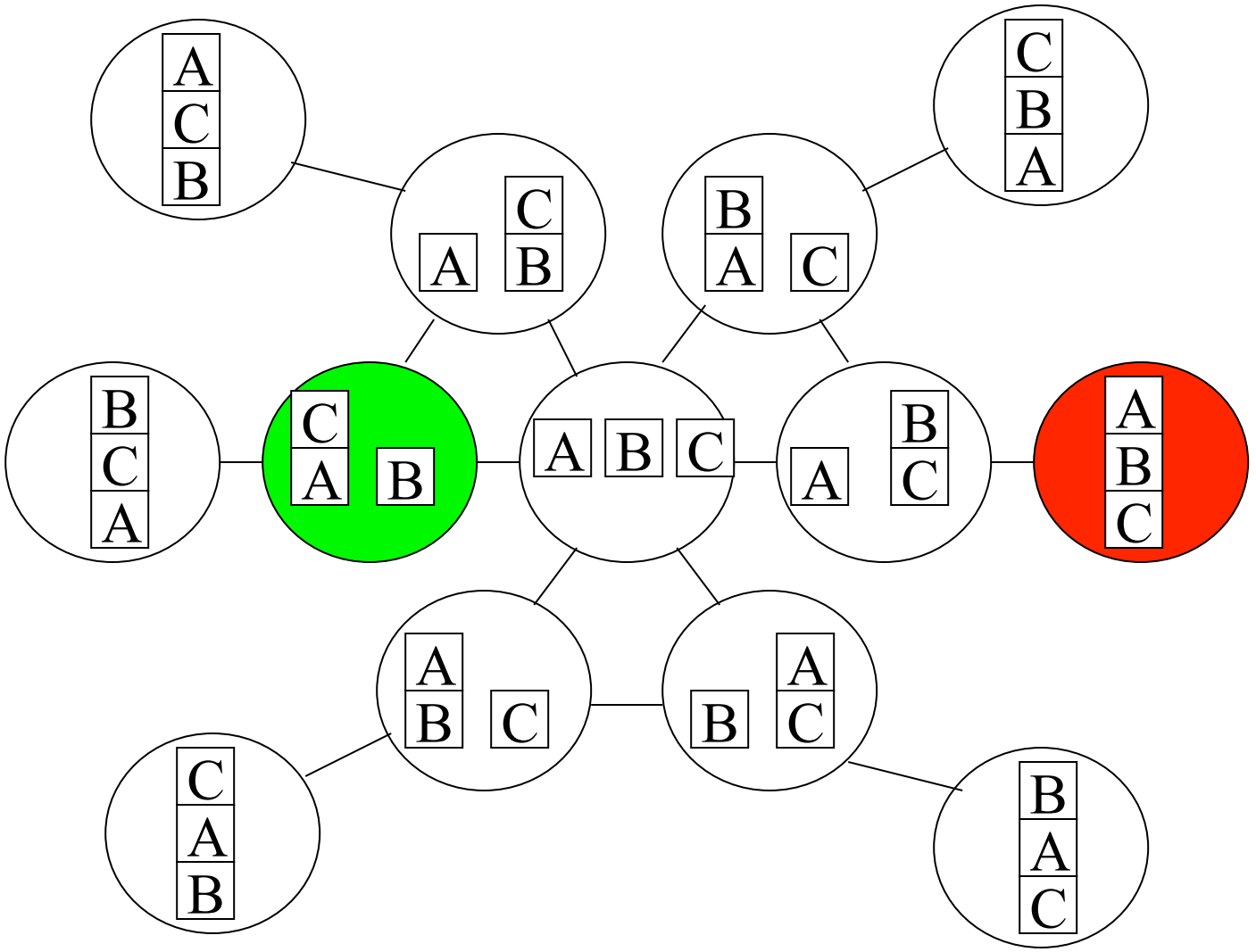
Using A* in Planning



EECS 348: Adversarial Search

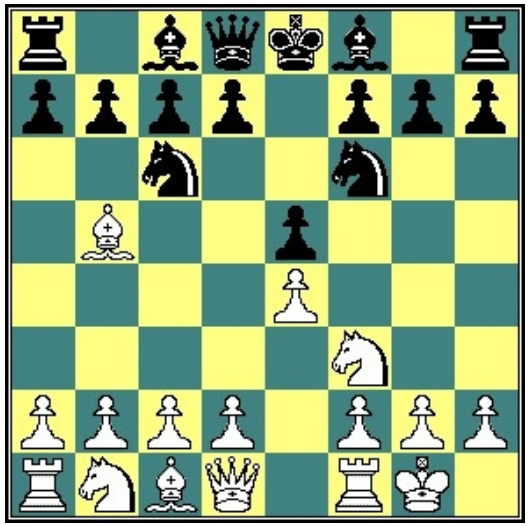
Strategic thinking $\stackrel{?}{=}$ intelligence

Two-player games have been a key focus of AI as long as computers have been around…

Humans and computers have different relative strengths in these games:

humans

good at evaluating the strength of a board for a player



computers

good at looking ahead in the game to find winning combinations of moves

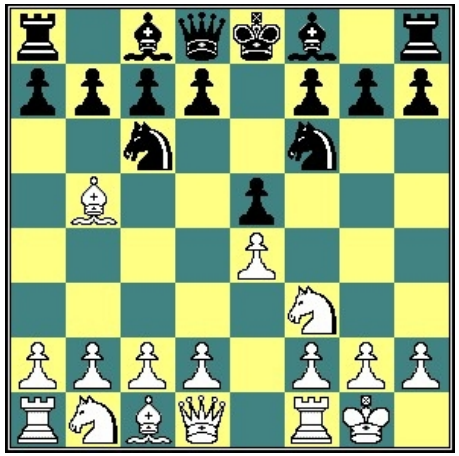
How humans play games…

An experiment (by deGroot) was performed in which chess positions were shown to novice and expert players…

- experts could reconstruct these perfectly
- novice players did far worse…

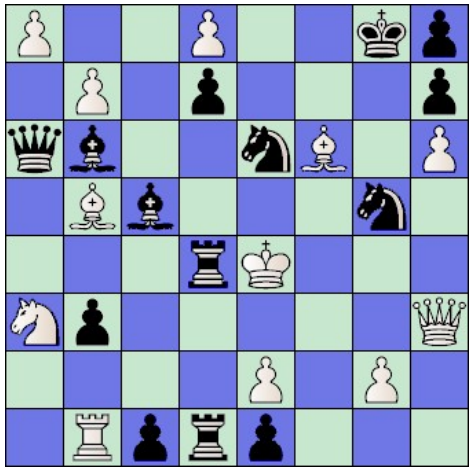
How humans play games…

An experiment (by deGroot) was performed in which chess positions were shown to novice and expert players…



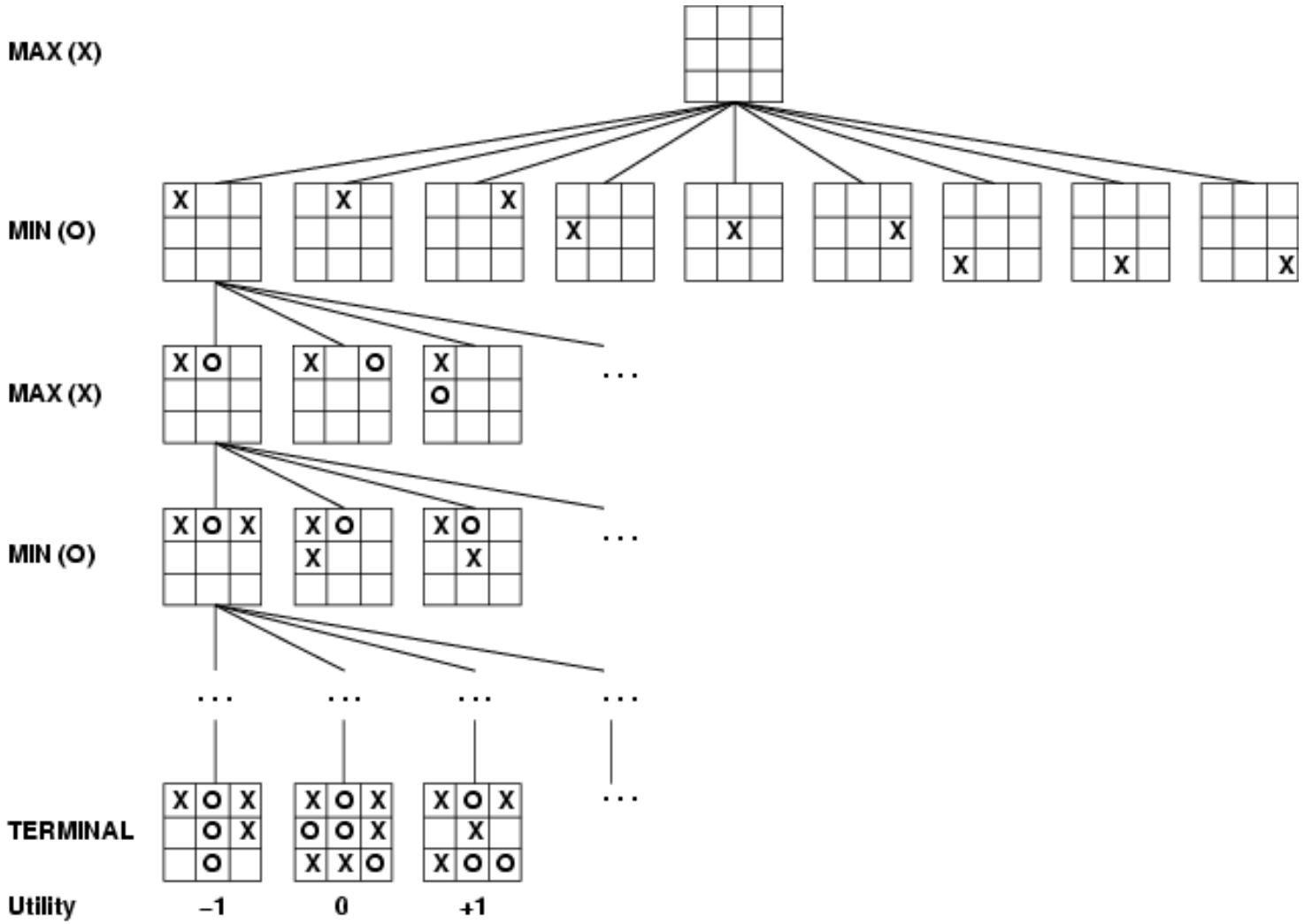
- experts could reconstruct these perfectly
- novice players did far worse…

<u>Random</u> chess positions (not legal ones) were then shown to the two groups



- experts and novices did just as badly at reconstructing them!

How computers play games…

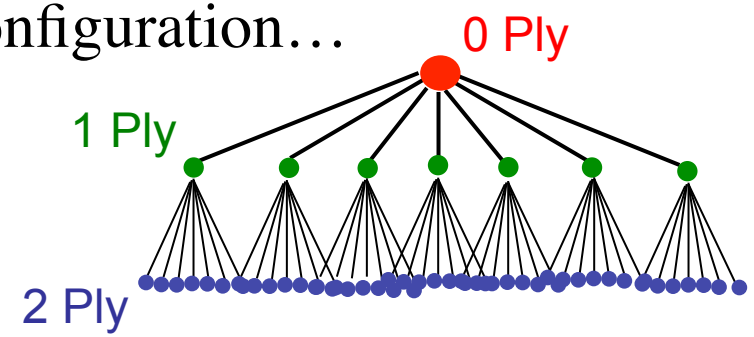


Games' Branching Factors

• On average, there are fewer than 40 possible moves that a chess player can make from any board configuration…

18 Ply!!

Hydra at home in the United Arab Emirates…



Branching Factor Estimates
for different two-player games

Tic-tac-toe	4
Connect Four	7
Checkers	10
Othello	30
Chess	40
Go	300

Optimal Strategy

- An Optimal Strategy is one that is as least as good as any other, no matter what the opponent does
  - If there's a way to force the win, it will
  - Will only lose if there's no other option

Minimax: An Optimal Strategy

```
function MINIMAX-DECISION(state) returns an action
    v ← MAX-VALUE(state)
    return the action in SUCCESSORS(state) with value v

function MAX-VALUE(state) returns a utility value
    if TERMINAL-TEST(state) then return UTILITY(state)
    v ← −∞
    for a, s in SUCCESSORS(state) do
        v ← MAX(v, MIN-VALUE(s))
    return v

function MIN-VALUE(state) returns a utility value
    if TERMINAL-TEST(state) then return UTILITY(state)
    v ← ∞
    for a, s in SUCCESSORS(state) do
        v ← MIN(v, MAX-VALUE(s))
    return v
```

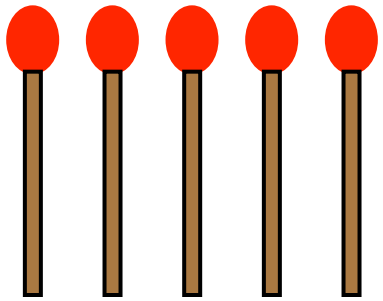
Minimax Algorithm: An Optimal Strategy

Choose the best move based on the resulting states' MINIMAX-VALUE…

MINIMAX-VALUE(n) =

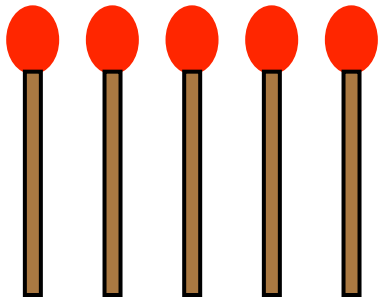
if n is a terminal state
then Utility(n)
else if MAX's turn
the MAXIMUM MINIMAX-VALUE
of all possible successors to n
else if MIN's turn
the MINIMUM MINIMAX-VALUE
of all possible successors to n

Baby Nim

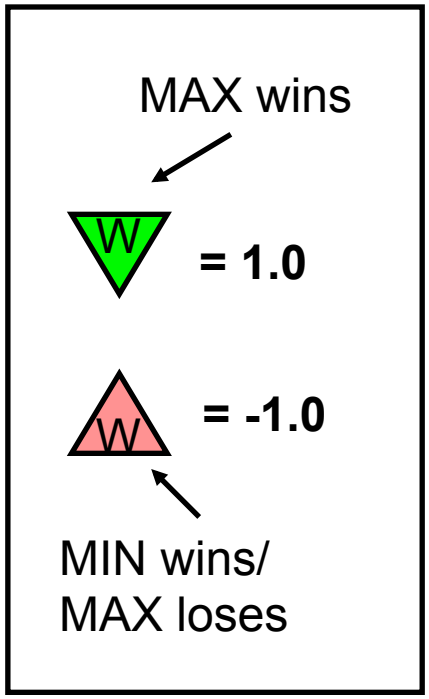


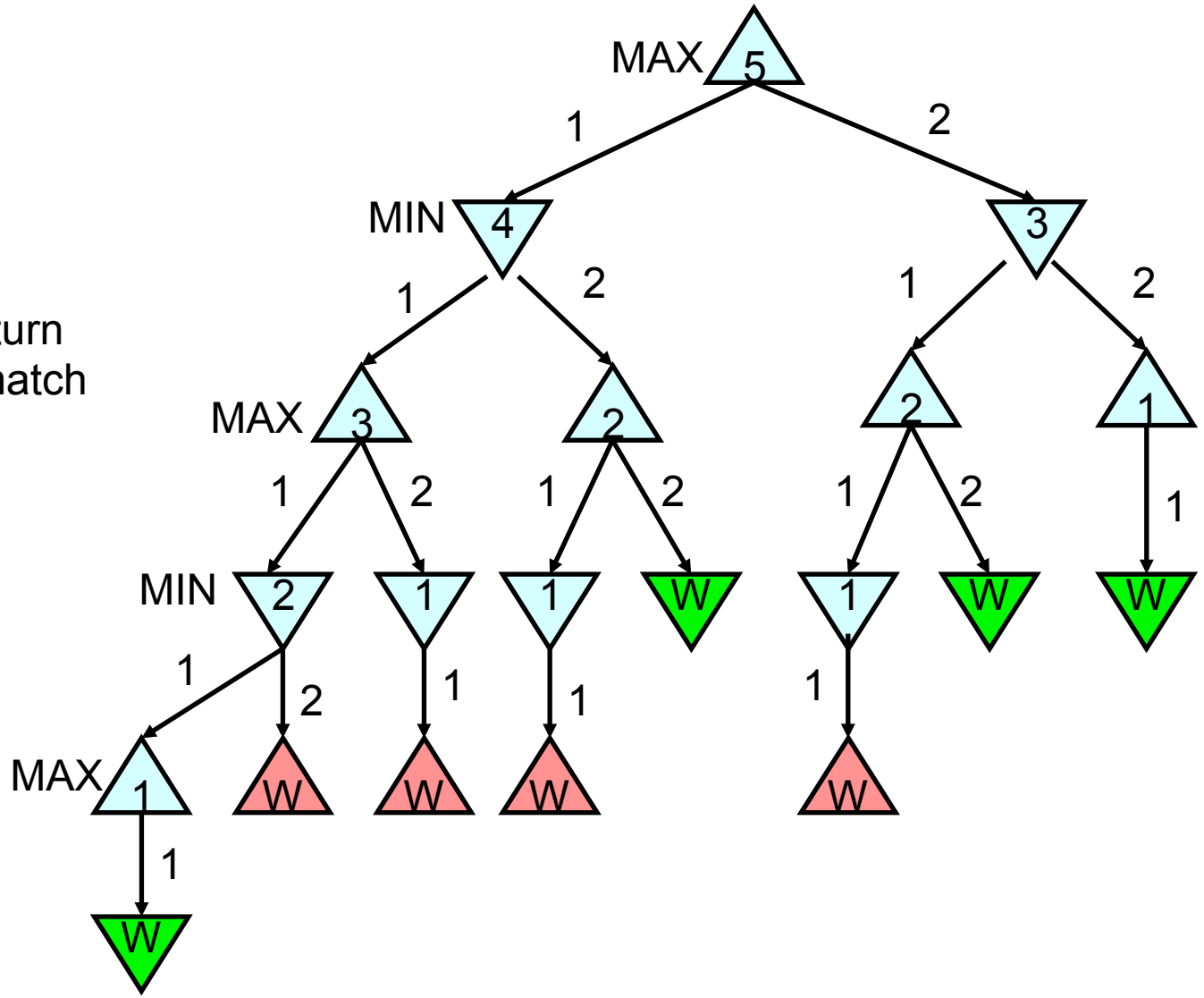
Take 1 or 2 at each turn
Goal: take the last match

Baby Nim

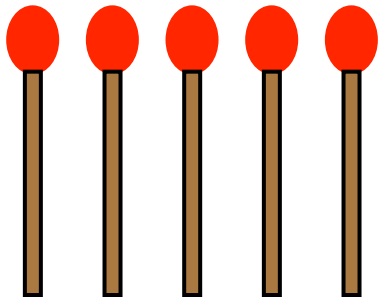


Take 1 or 2 at each turn
Goal: take the last match

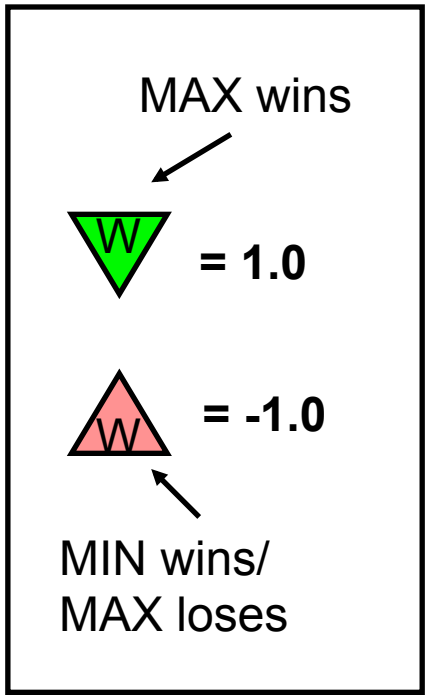


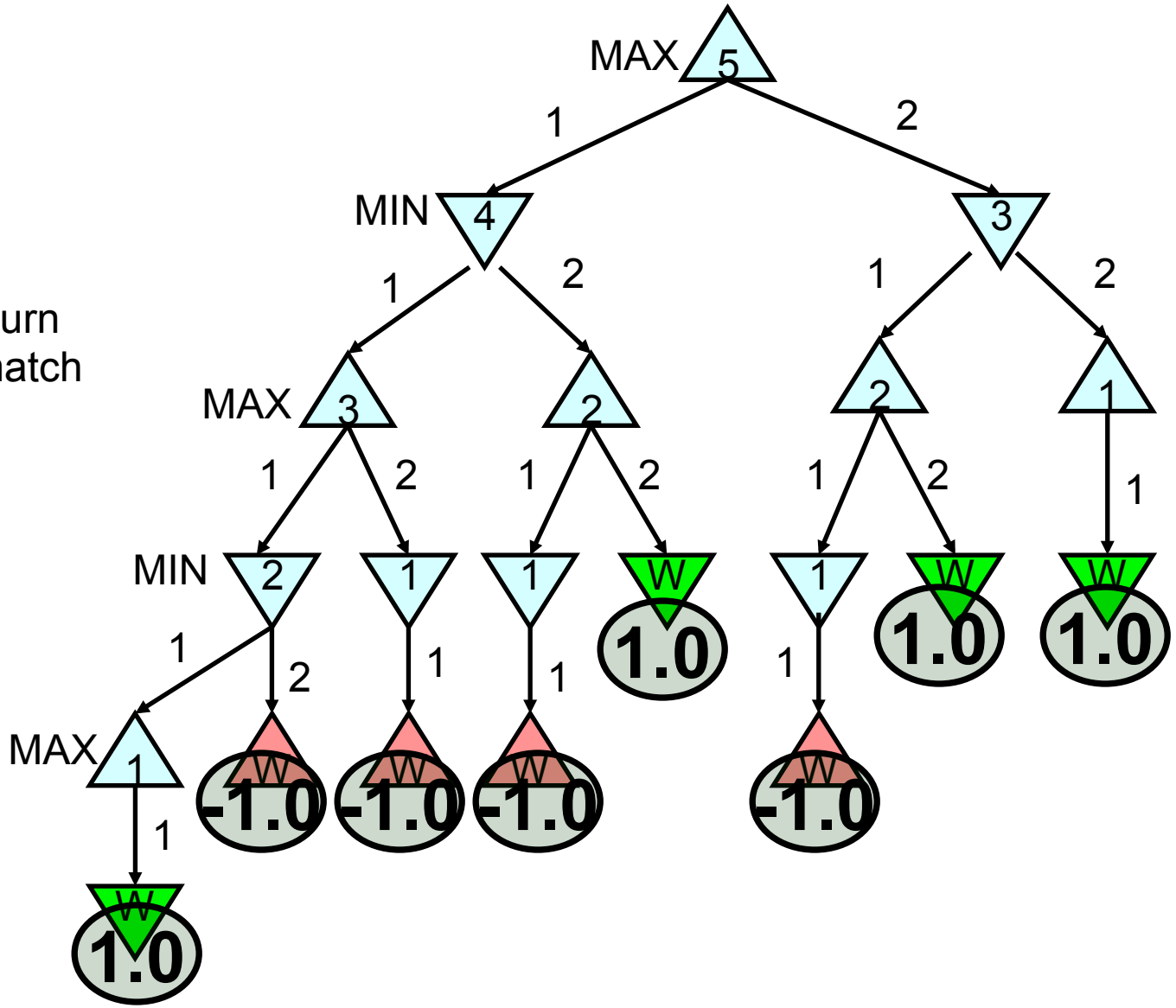


Baby Nim

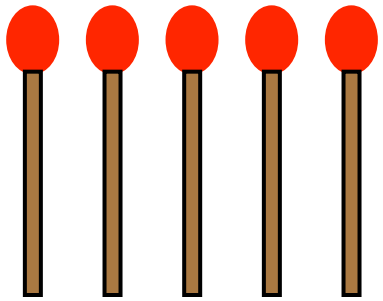


Take 1 or 2 at each turn
Goal: take the last match

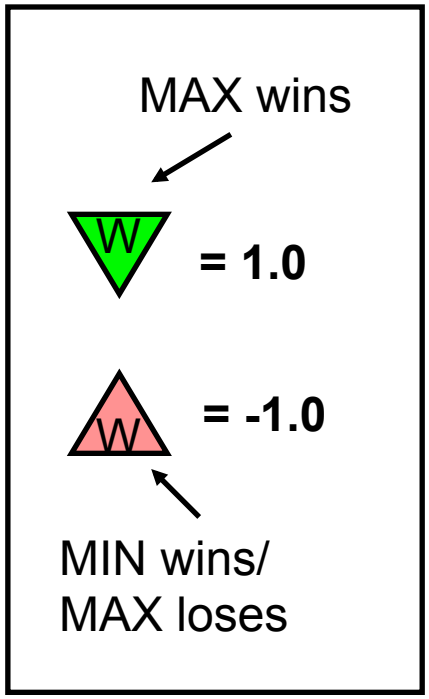


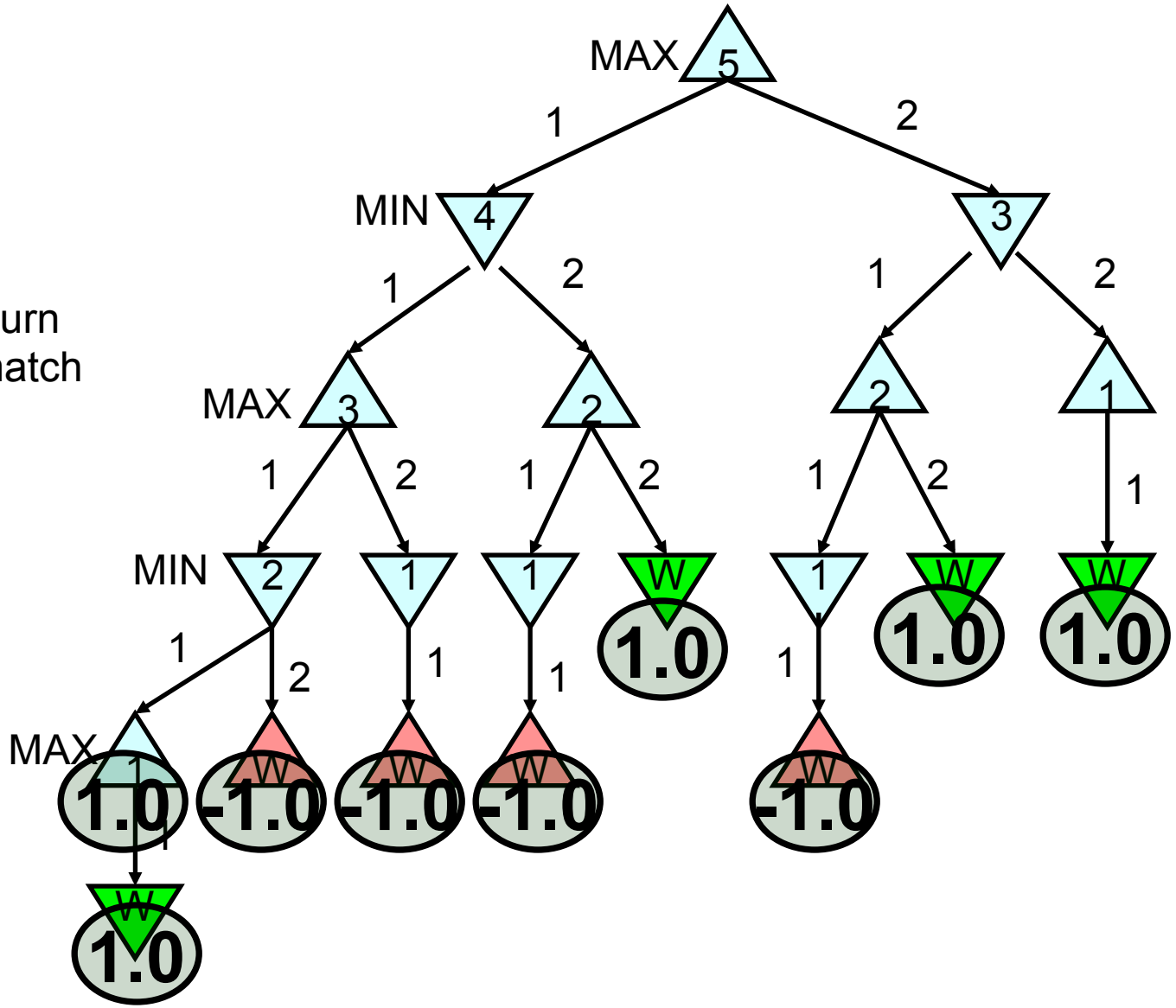


Baby Nim

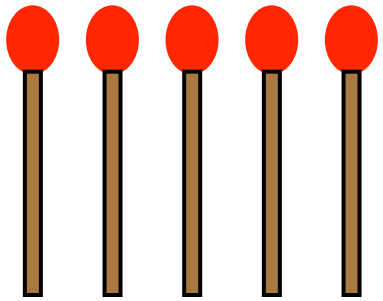


Take 1 or 2 at each turn
Goal: take the last match

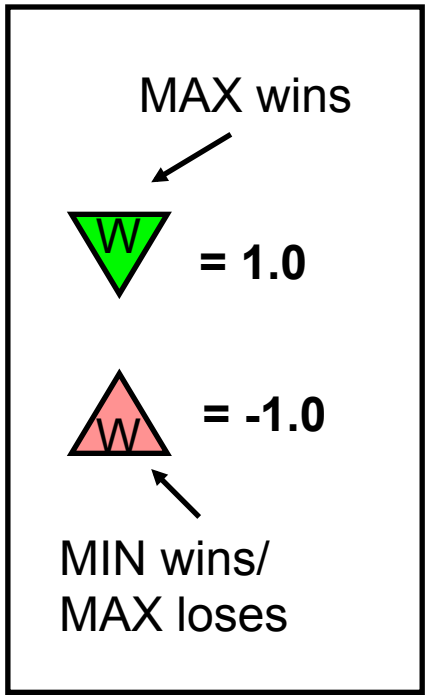


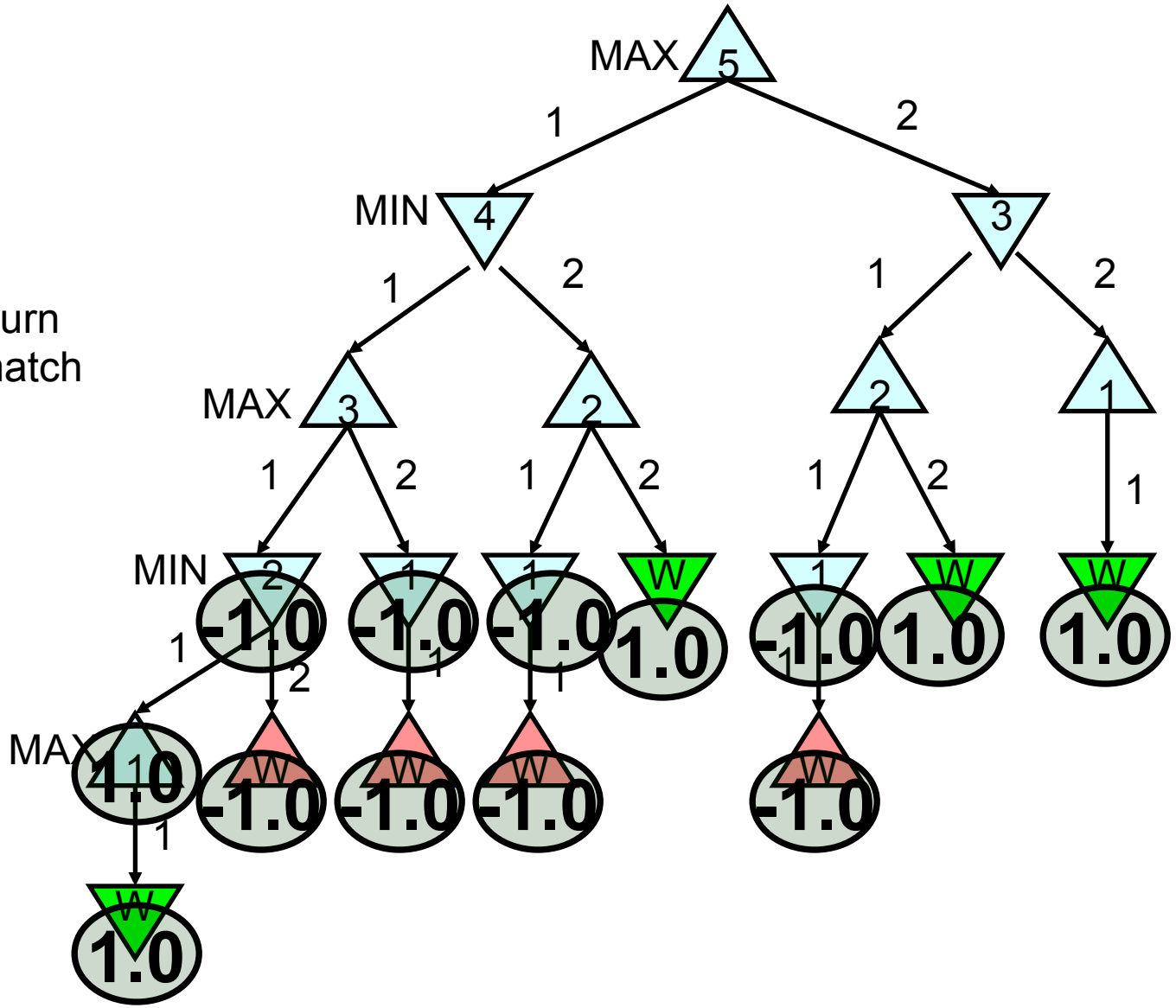


Baby Nim

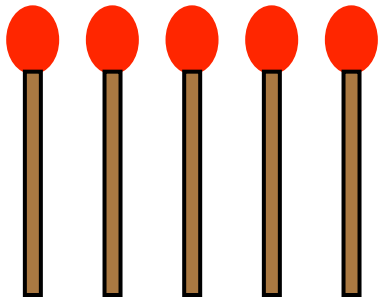


Take 1 or 2 at each turn
Goal: take the last match

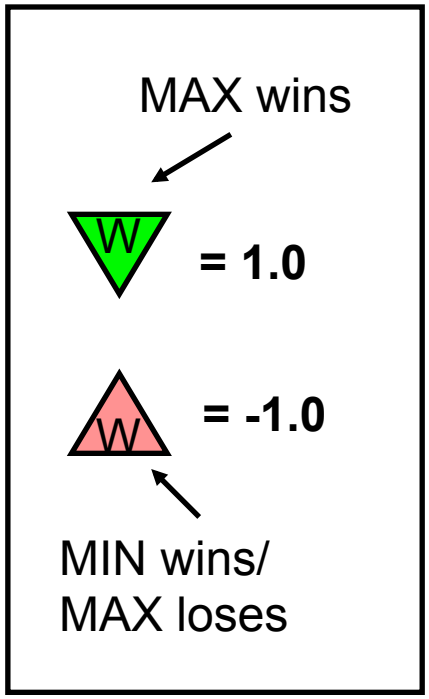


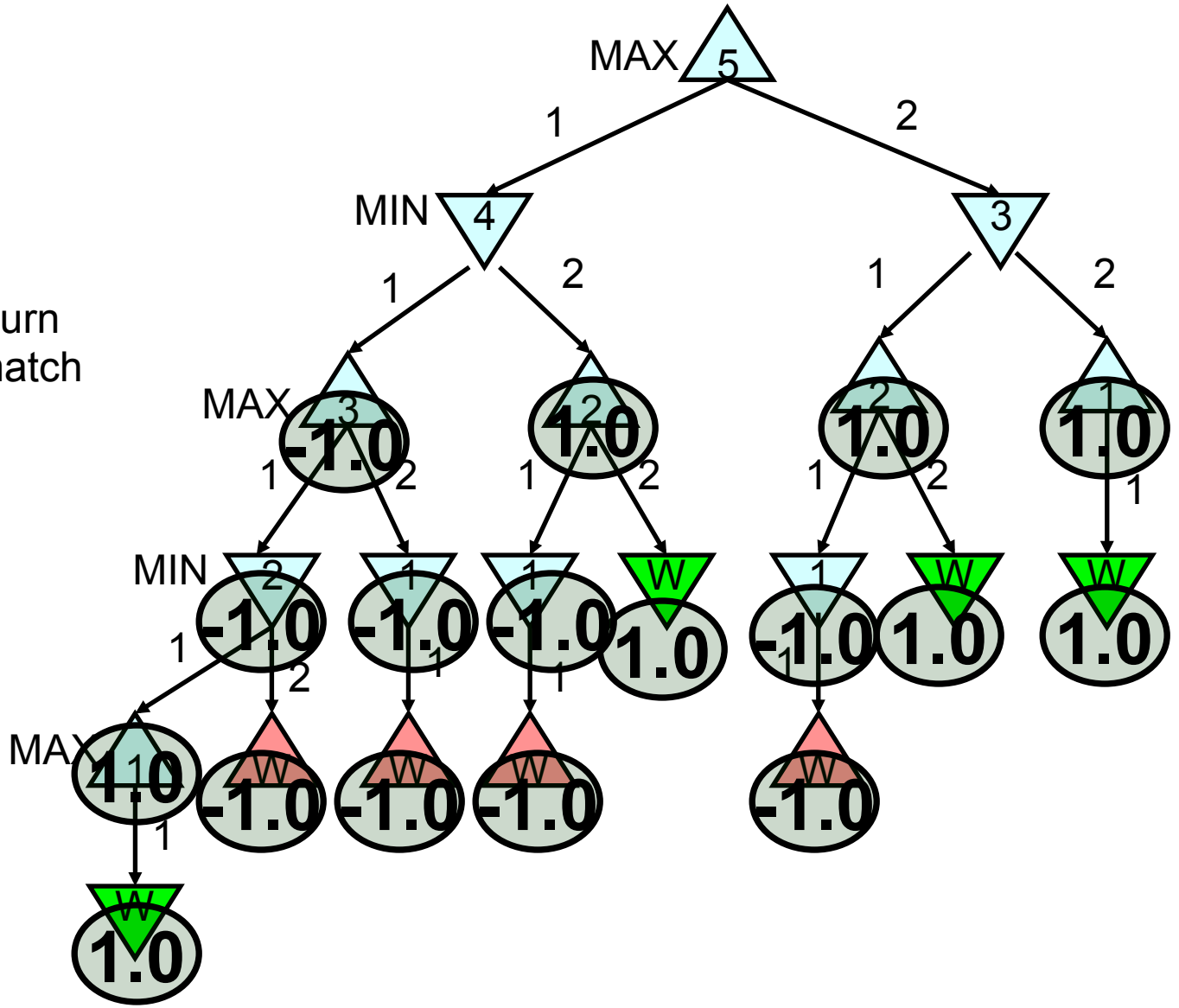


Baby Nim

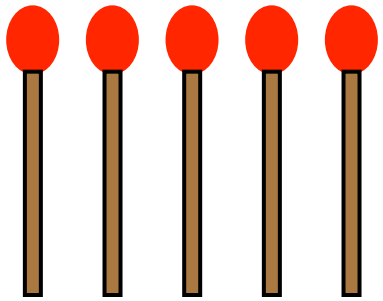


Take 1 or 2 at each turn
Goal: take the last match

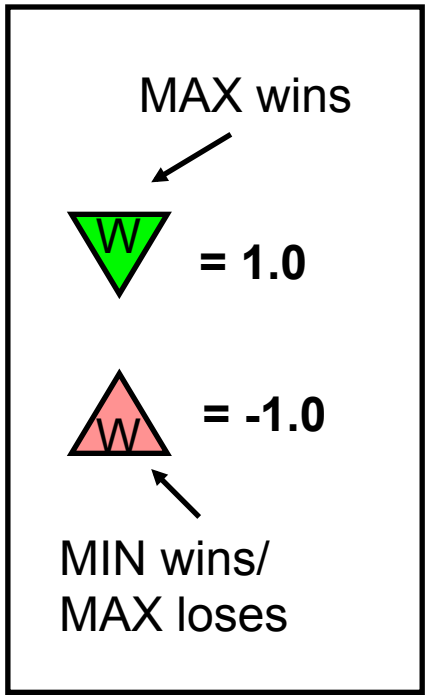


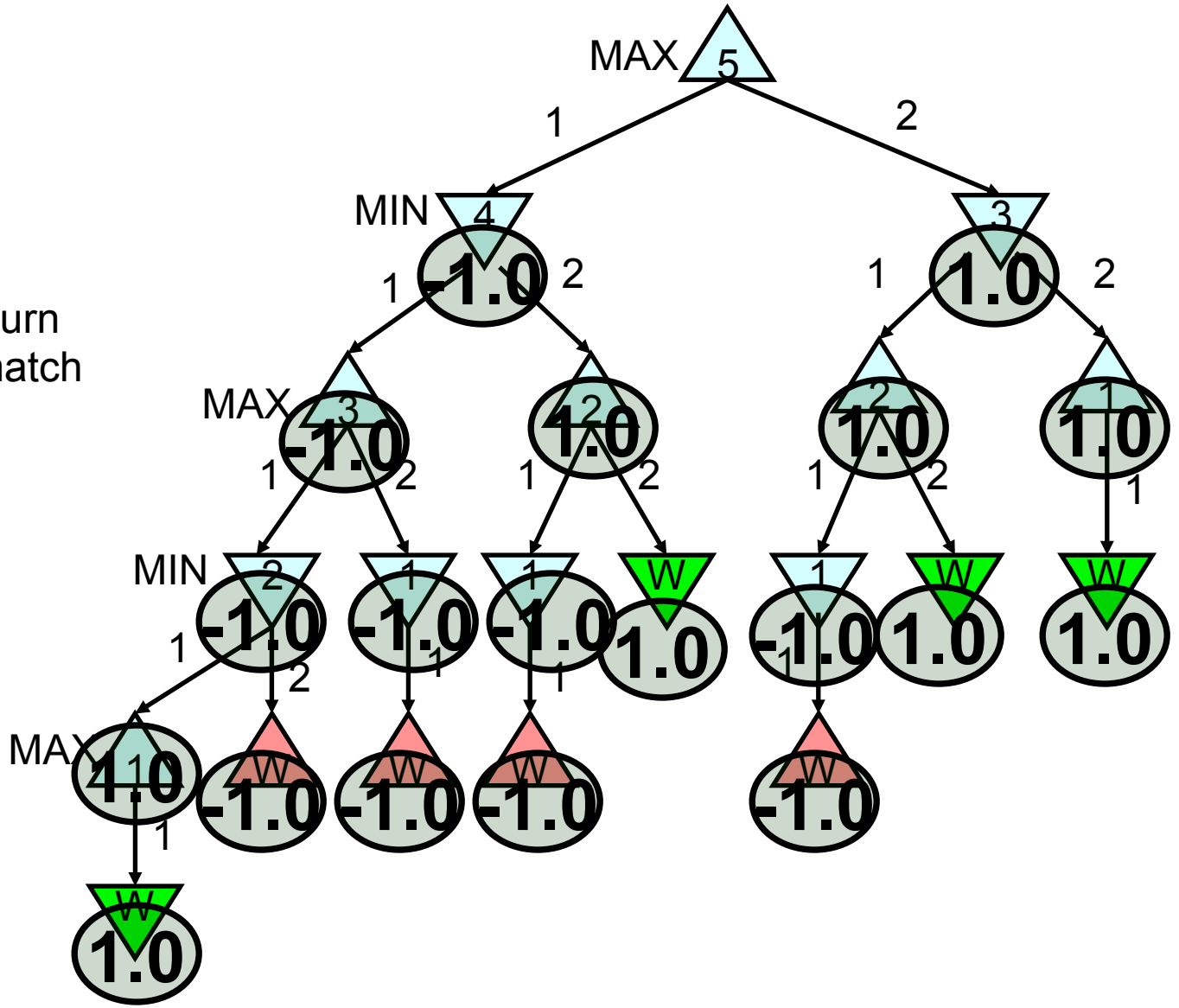


Baby Nim

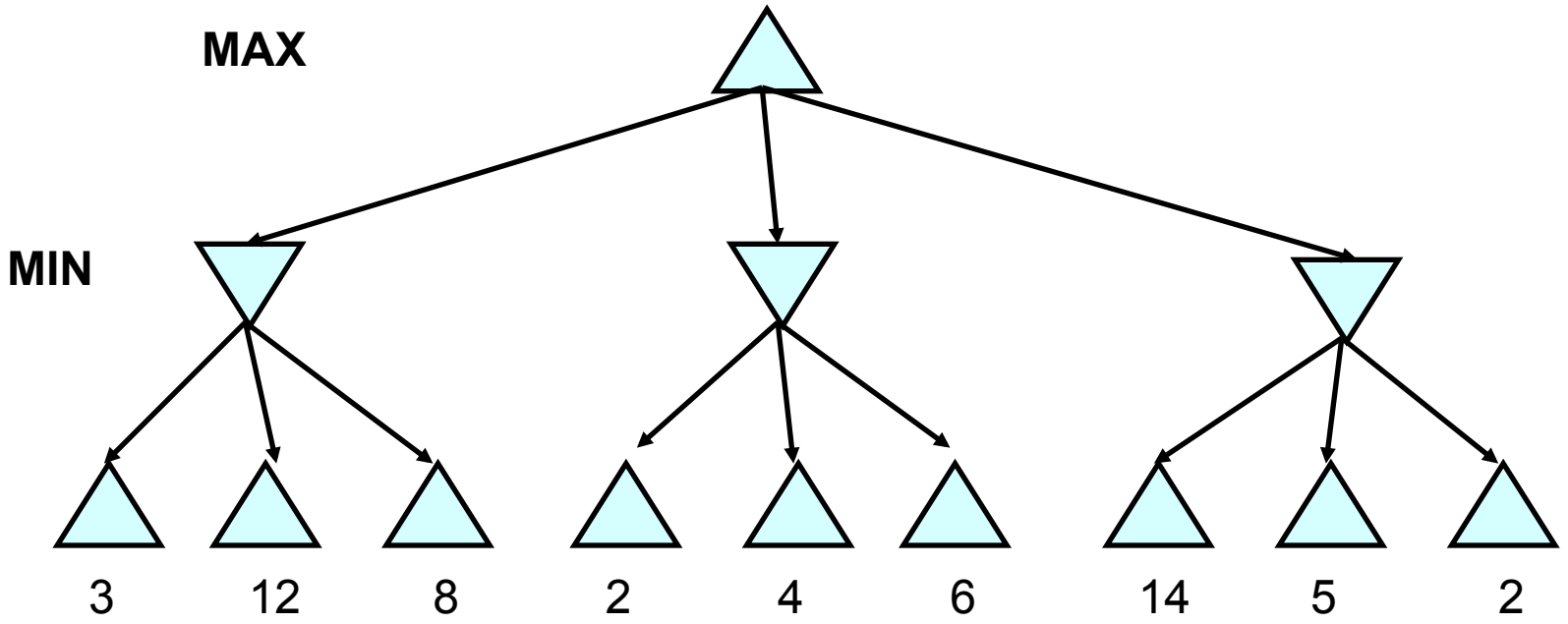


Take 1 or 2 at each turn
Goal: take the last match





MINIMAX example 2

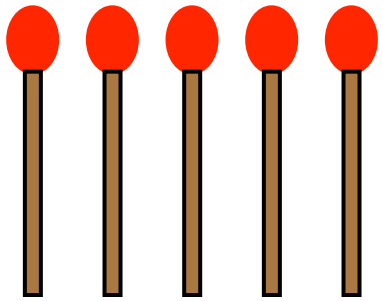


Properties of minimax

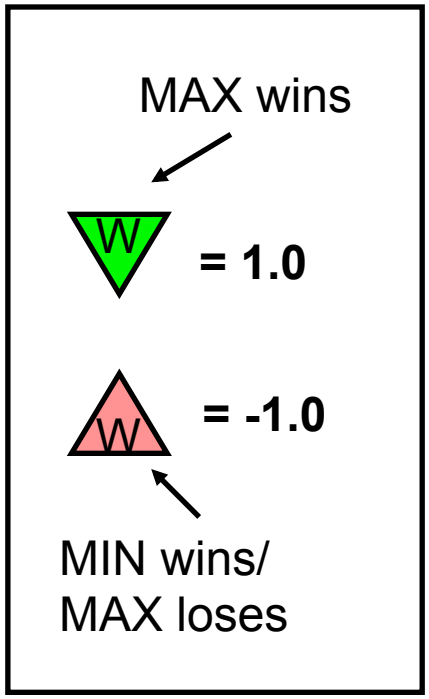
- For chess, b ≈ 35, d ≈100 for "reasonable" games
  → exact solution completely infeasible

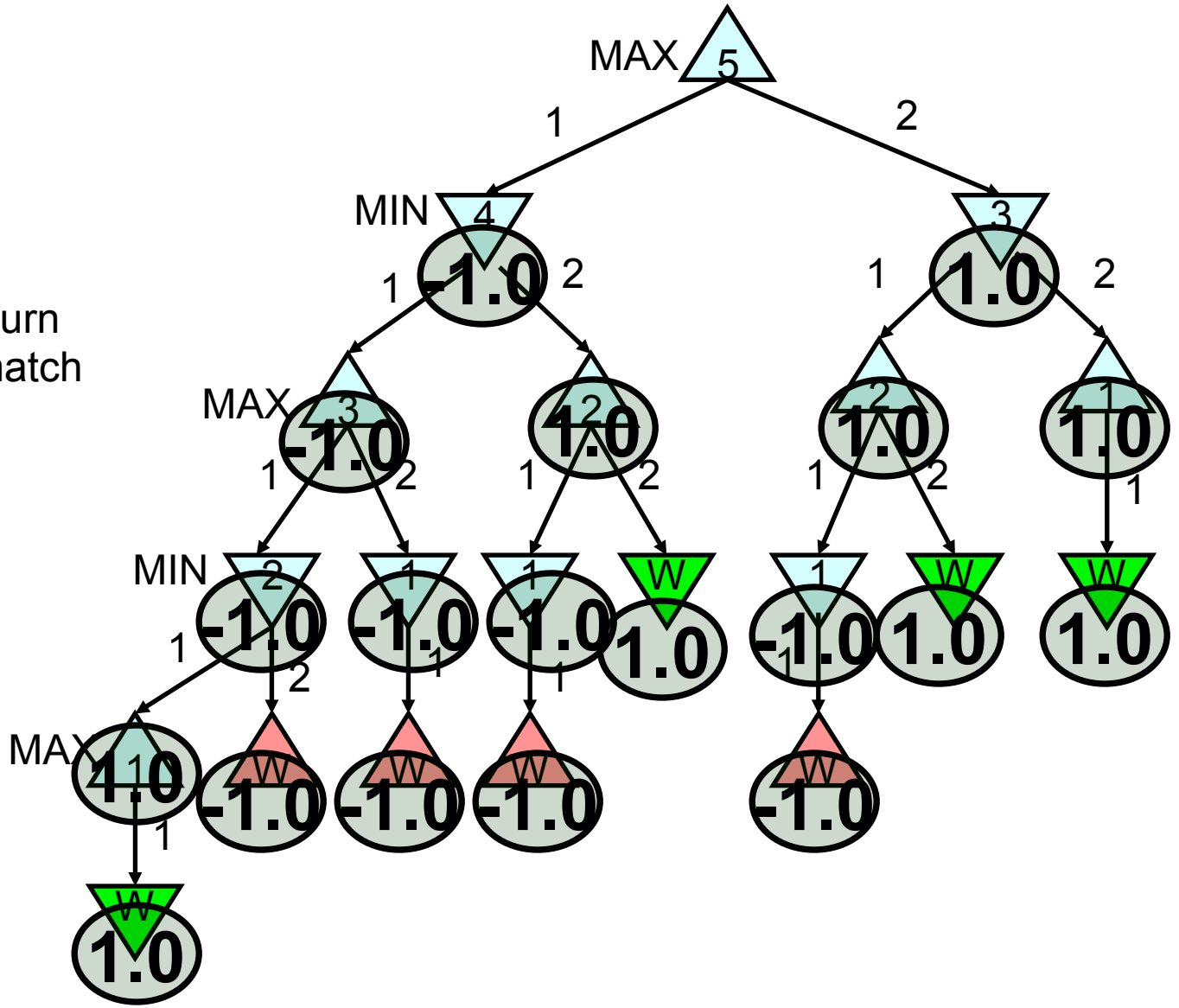
- Is minimax reasonable for
  - Mancala?
    - B?
    - D?
  - Tic Tac Toe?
    - B?
    - D?

Baby Nim



Take 1 or 2 at each turn
Goal: take the last match





Alpha-Beta Pruning

Pruning

eliminate parts of the tree from consideration

Alpha-Beta pruning

prunes away branches that can't possibly influence the final decision

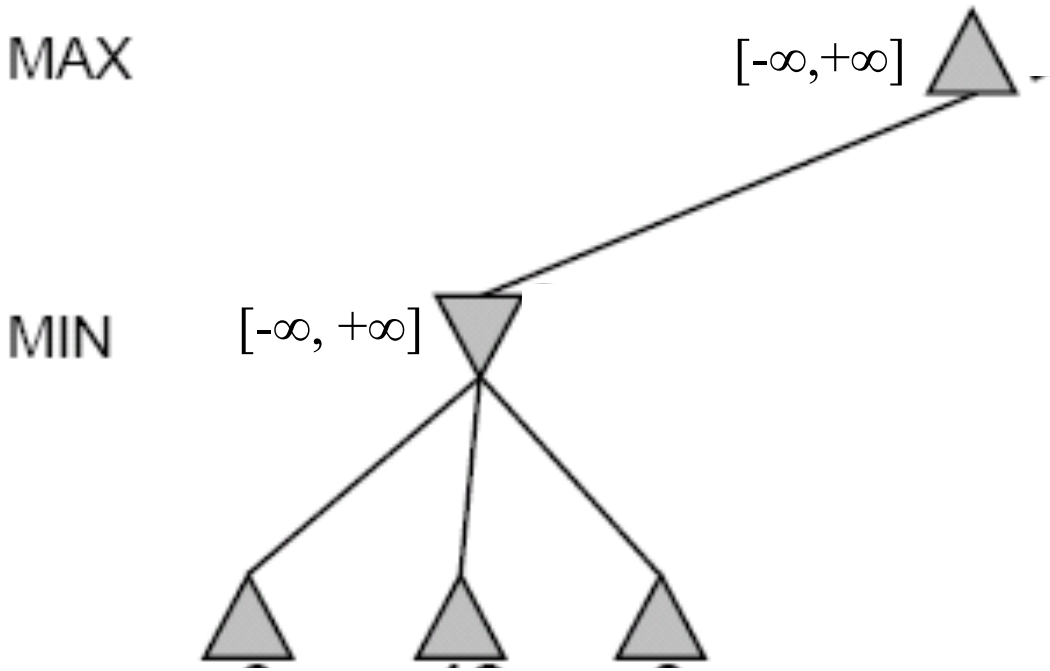
Consider a node *n*

If a player has a better choice *m* (at a parent or further up), then *n* will never be reached

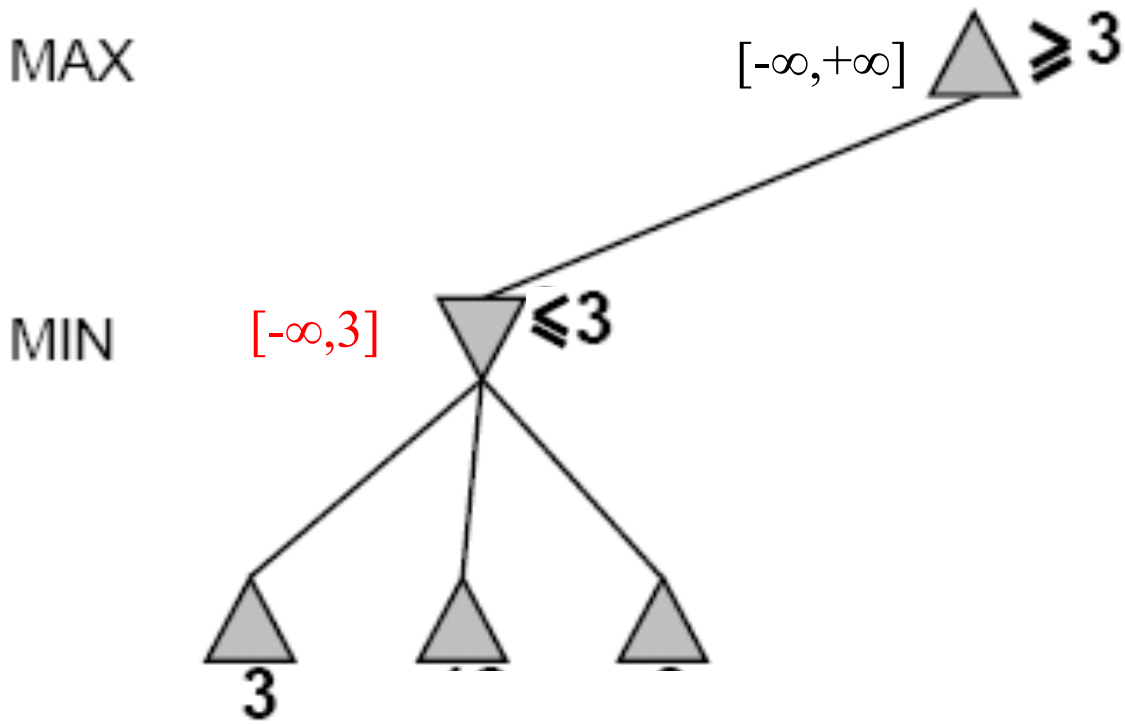
So, once we know enough about *n* by looking at some successors, then we can prune it.

Alpha-Beta Example

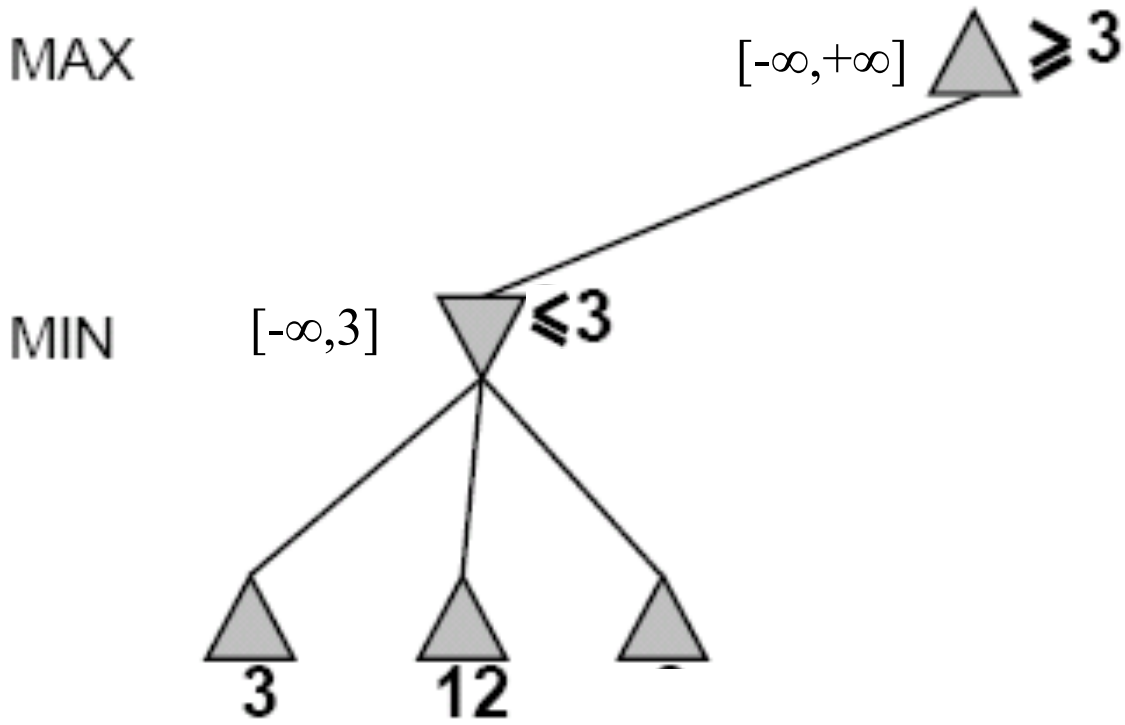
Do DF-search until first leaf



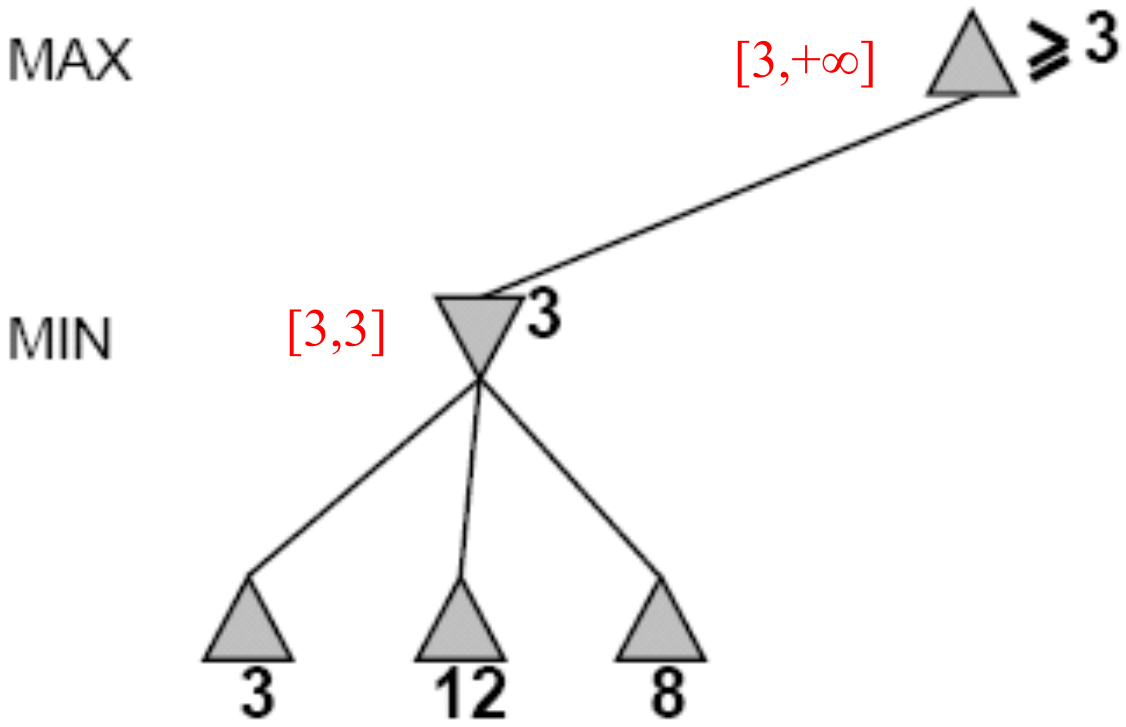
Alpha-Beta Example (continued)



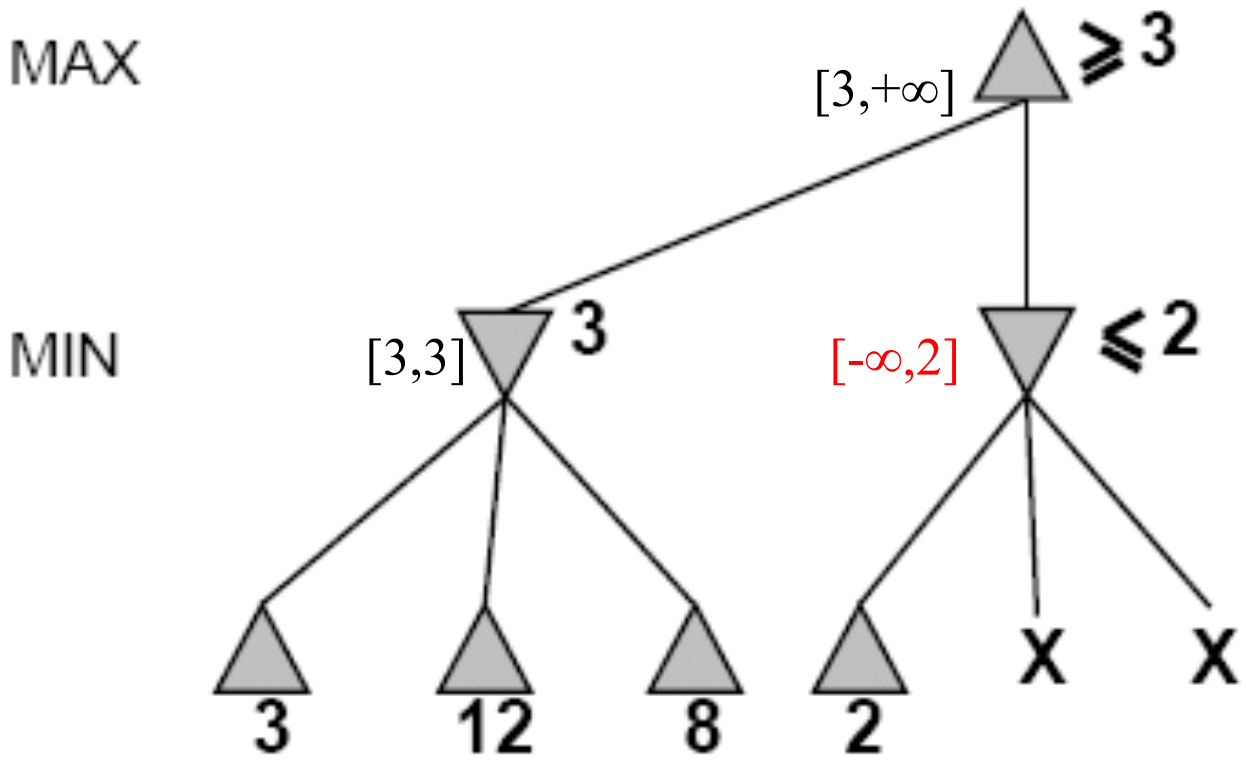
Alpha-Beta Example (continued)



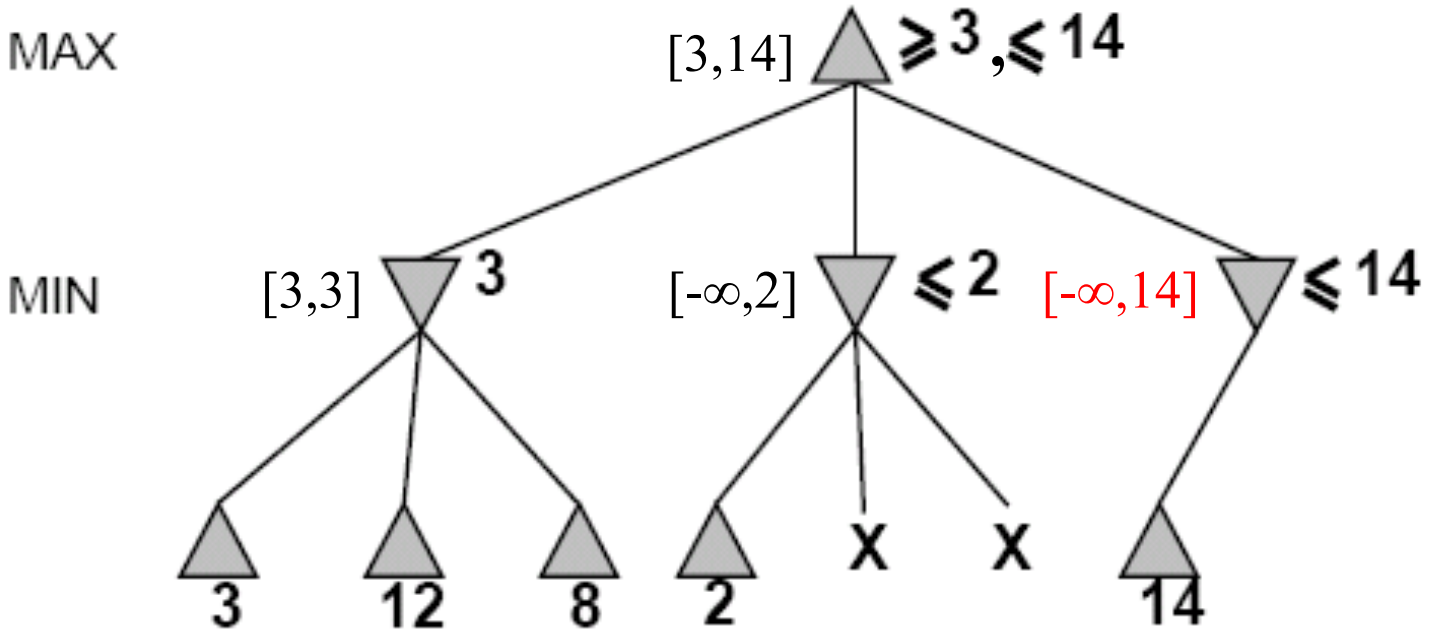
Alpha-Beta Example (continued)



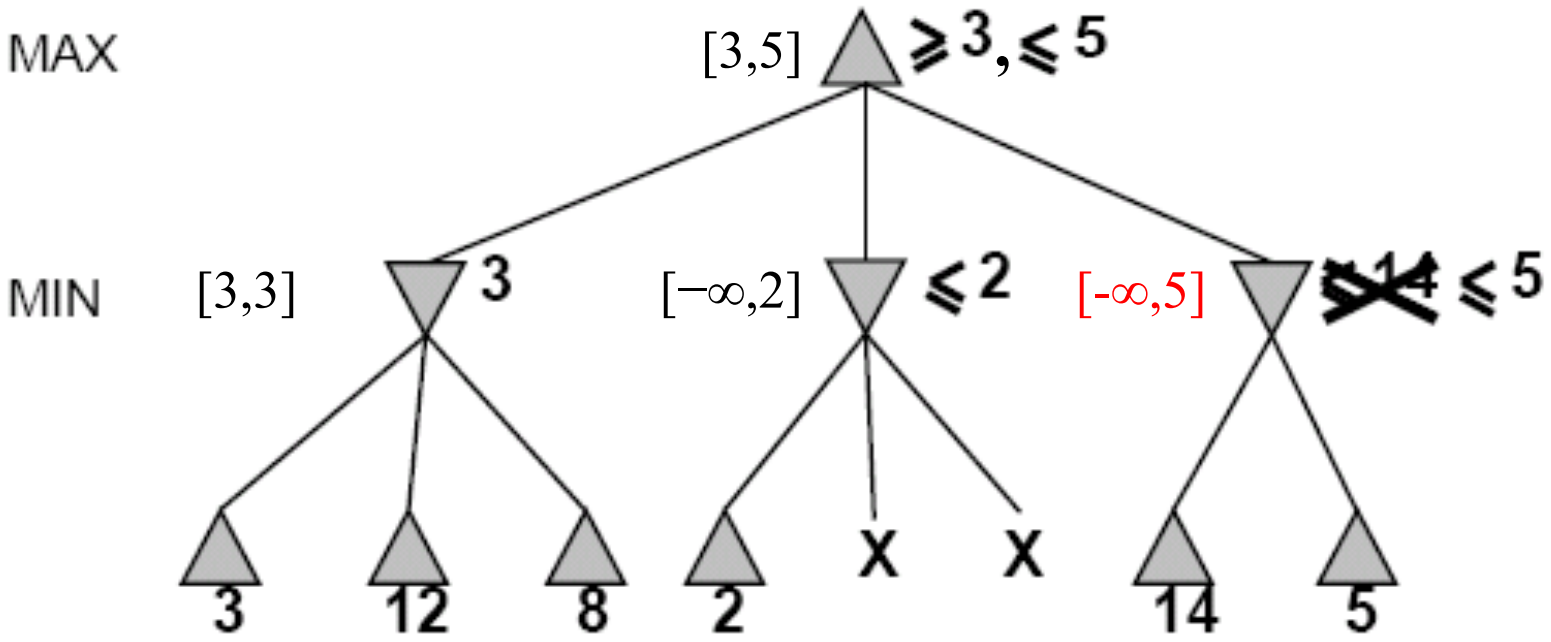
Alpha-Beta Example (continued)



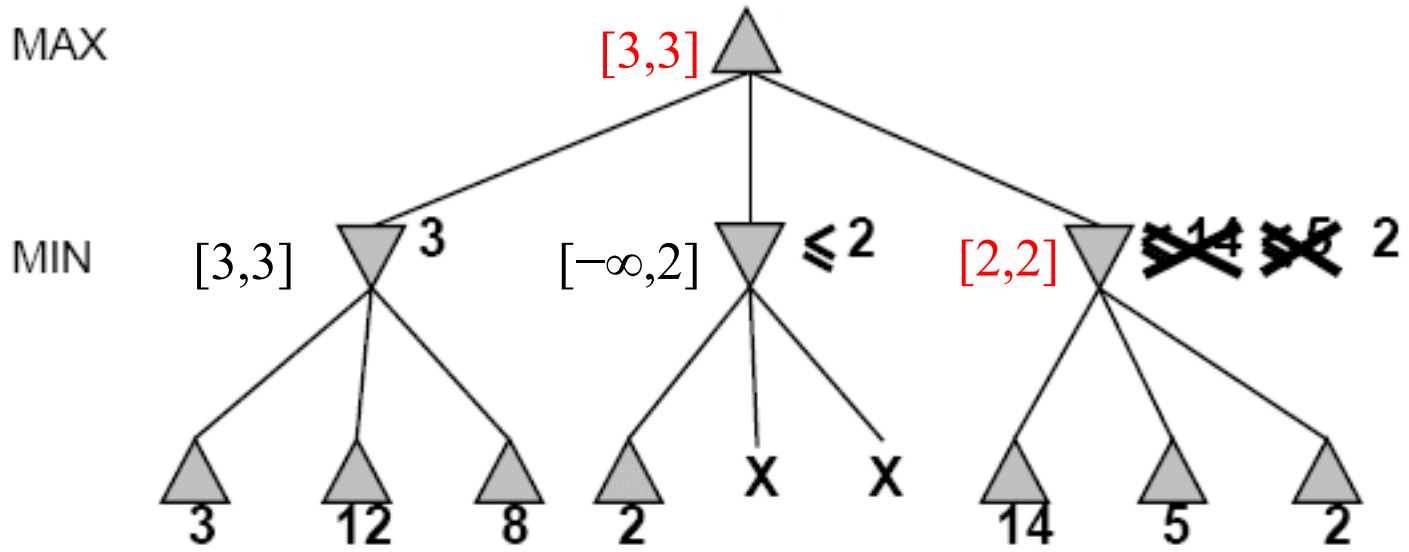
Alpha-Beta Example (continued)



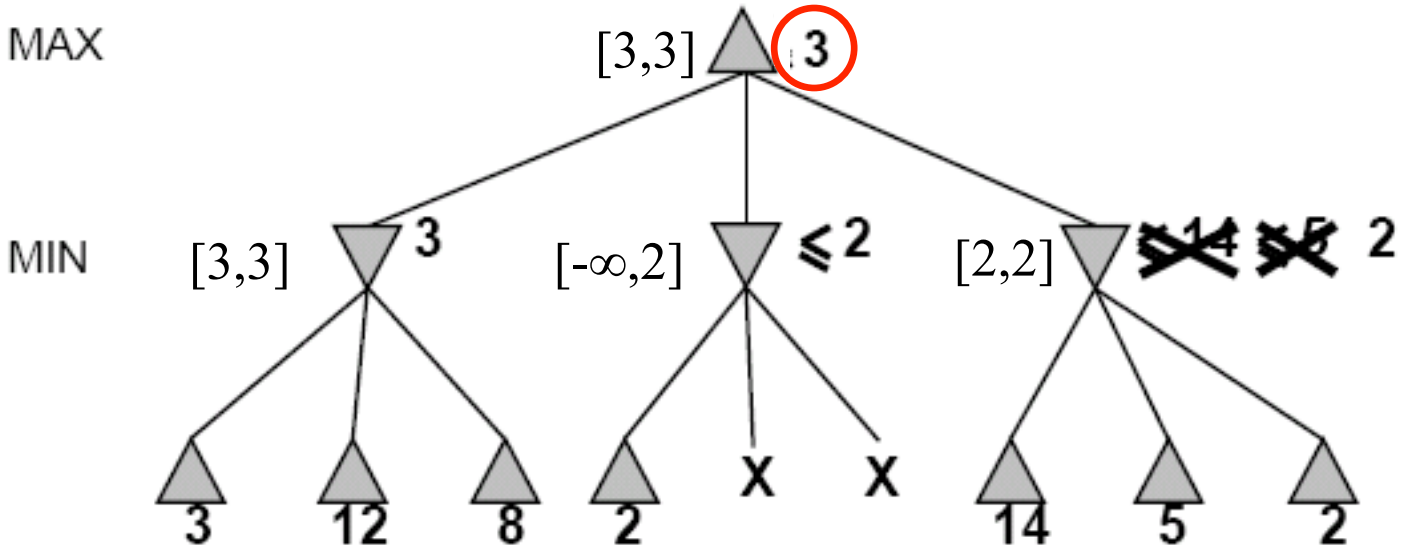
Alpha-Beta Example (continued)



Alpha-Beta Example (continued)



Alpha-Beta Example (continued)

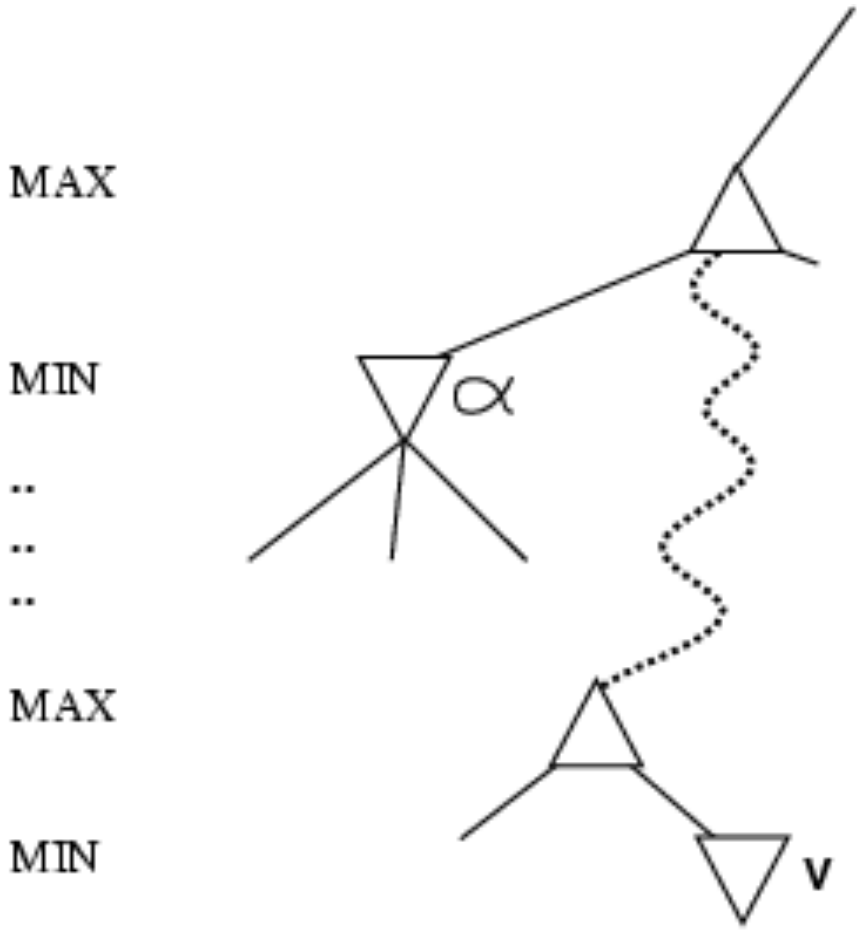


Properties of α-β

- Pruning does not affect final result
- However, effectiveness of pruning affected by…?
- What impact can it have on running time?

Why is it called α-β?

- α is the value of the best (i.e., highest-value) choice found so far at any choice point along the path for *max*
- If *v* is worse than α, *max* will avoid it
  → prune that branch
- Define β similarly for *min*



```
function ALPHA-BETA-SEARCH(state) returns an action
  inputs: state, current state in game

  v ← MAX-VALUE(state, −∞, +∞)
  return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  inputs: state, current state in game
          α, the value of the best alternative for MAX along the path to state
          β, the value of the best alternative for MIN along the path to state

  if TERMINAL-TEST(state) then return UTILITY(state)
  v ← −∞
  for a, s in SUCCESSORS(state) do
    v ← MAX(v, MIN-VALUE(s, α, β))
    if v ≥ β then return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  inputs: state, current state in game
          α, the value of the best alternative for MAX along the path to state
          β, the value of the best alternative for MIN along the path to state

  if TERMINAL-TEST(state) then return UTILITY(state)
  v ← +∞
  for a, s in SUCCESSORS(state) do
    v ← MIN(v, MAX-VALUE(s, α, β))
    if v ≤ α then return v
    β ← MIN(β, v)
  return v
```

Problems with AB Pruning?

Resource limits

Suppose we have 100 secs, and can explore 10^4 nodes/sec
→ can explore 10^6 nodes per move

Standard approach (Shannon, 1950):

- evaluation function
  = estimated desirability of position

- cutoff test:
  e.g., depth limit

Cutting off search

- Change:
  - if TERMINAL-TEST(state) then return UTILITY(state)
- into
  - if CUTOFF-TEST(state,depth) then return EVAL(state)

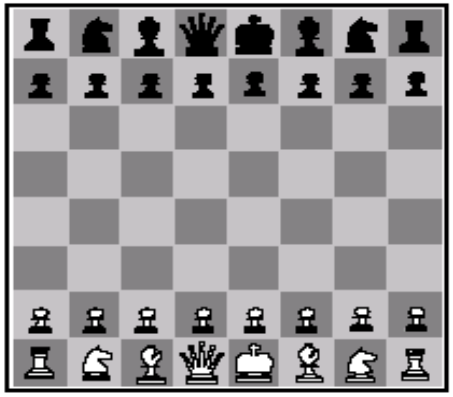
- Introduces a fixed-depth limit
  - Is selected so that the amount of time will not exceed what the rules of the game allow.
- When cuttoff occurs, the evaluation is performed.

Heuristic EVAL

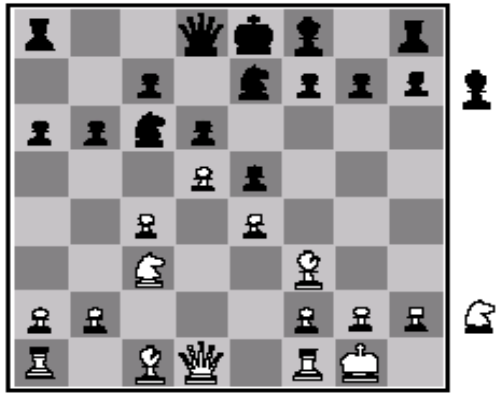
- Idea: produce an estimate of the expected utility of the game from a given position.
- Performance depends on quality of EVAL.
- Requirements:
  - EVAL should order terminal-nodes in the same way as UTILITY.
  - Computation may not take too long.
  - For non-terminal states the EVAL should be strongly correlated with the actual chance of winning.

Simple Mancala Heuristic: Goodness of board = # stones in my Mancala minus the number of stones in my opponents.

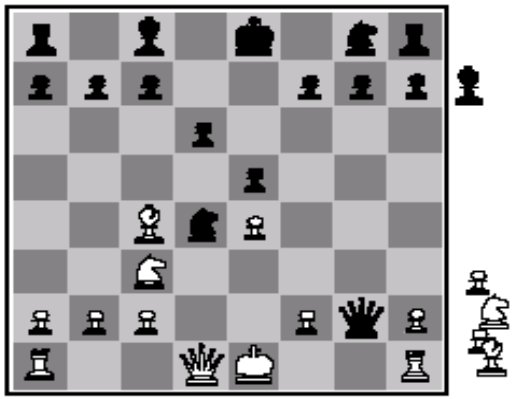
Heuristic EVAL example



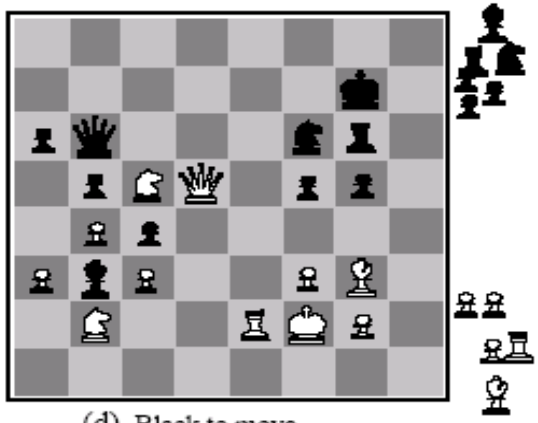
(a) White to move
Fairly even



(b) Black to move
White slightly better



(c) White to move
Black winning

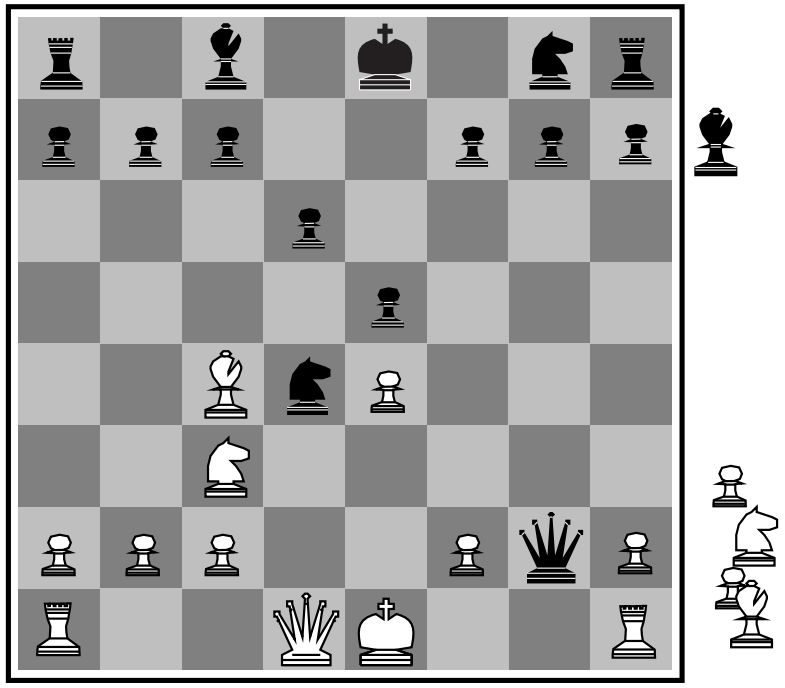


(d) Black to move
White about to lose

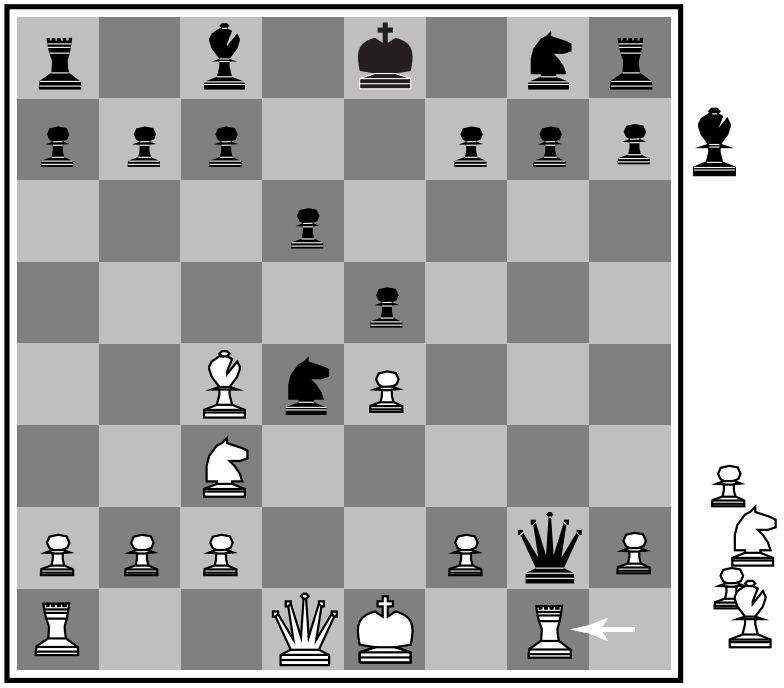
$$Eval(s) = w_1 f_1(s) + w_2 f_2(s) + \ldots + w_n f_n(s)$$

Heuristic difficulties

Simple heuristic - weighing the pieces by material value



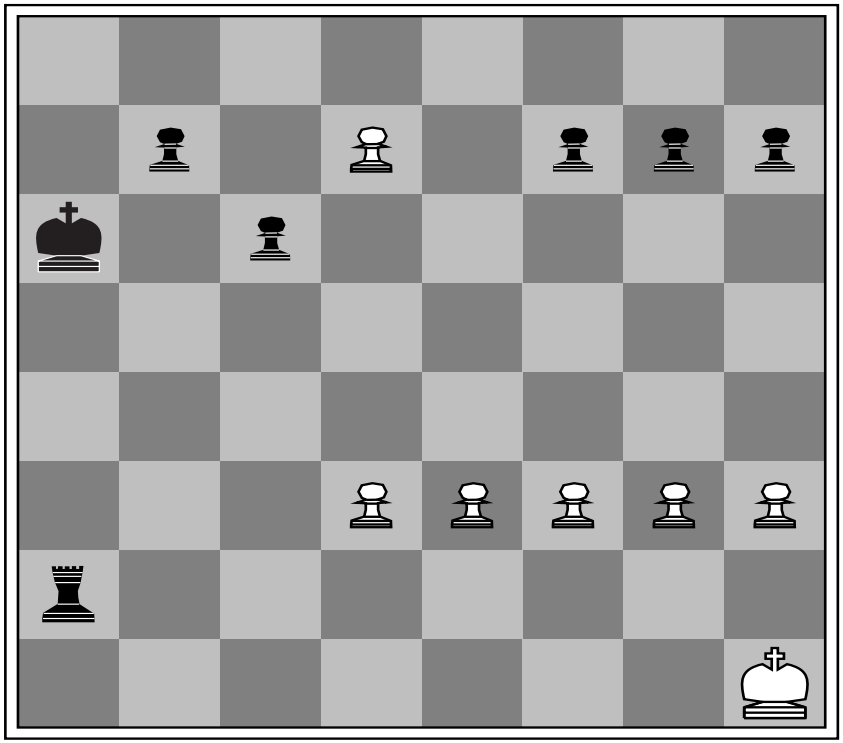
(a) White to move



(b) White to move

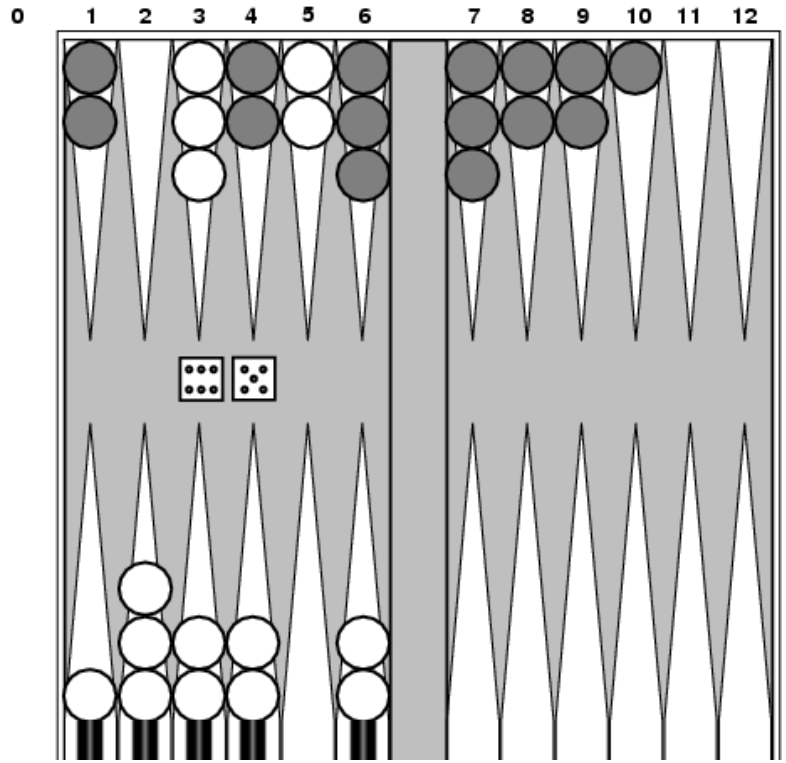
Horizon effect

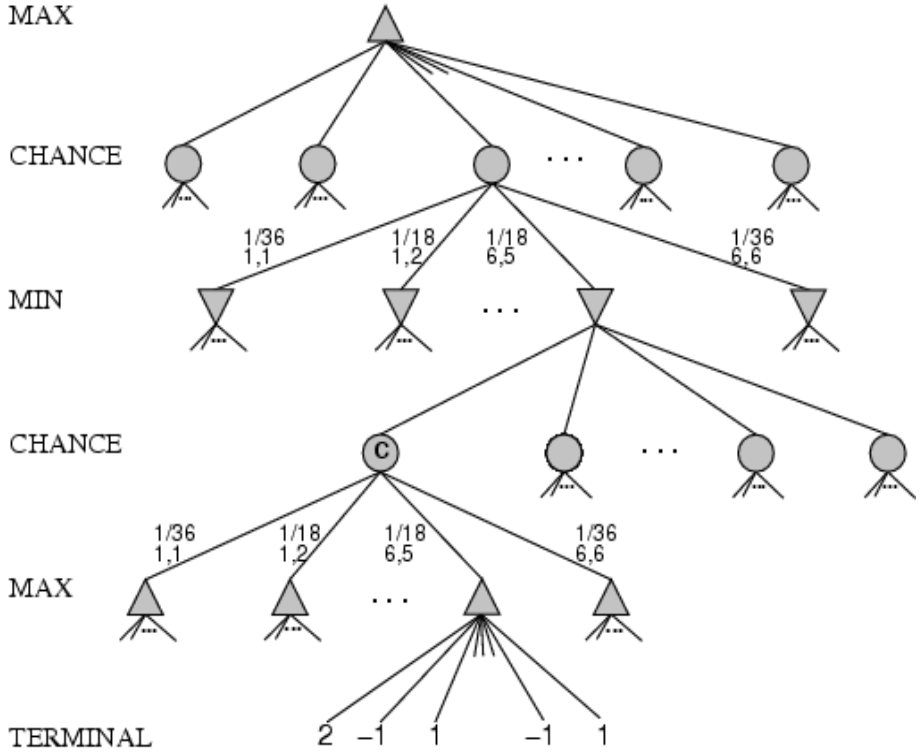
Fixed depth search thinks it can avoid the queening move



Black to move

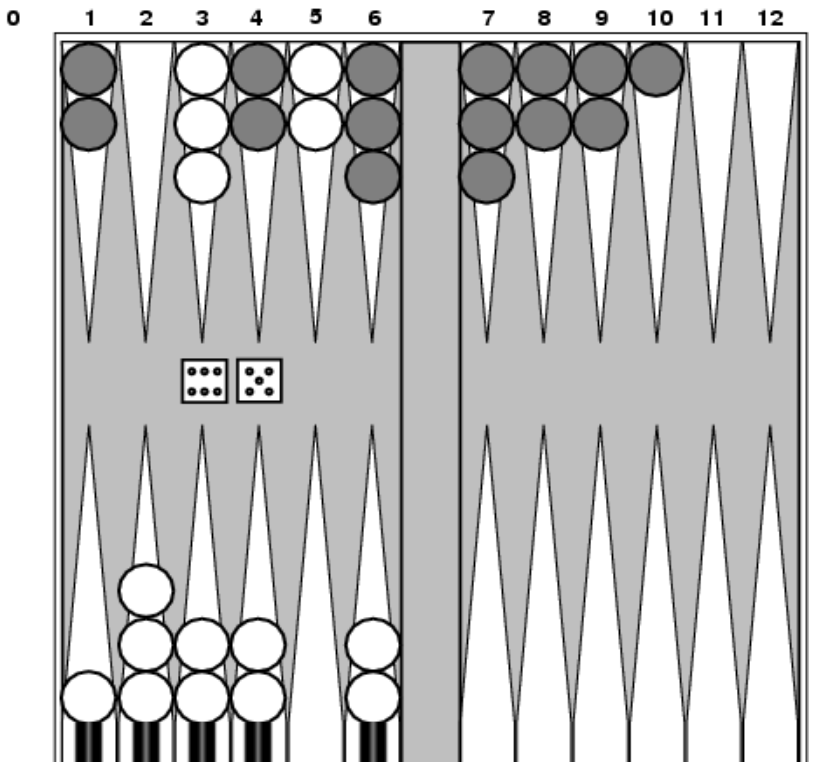
Games that include chance

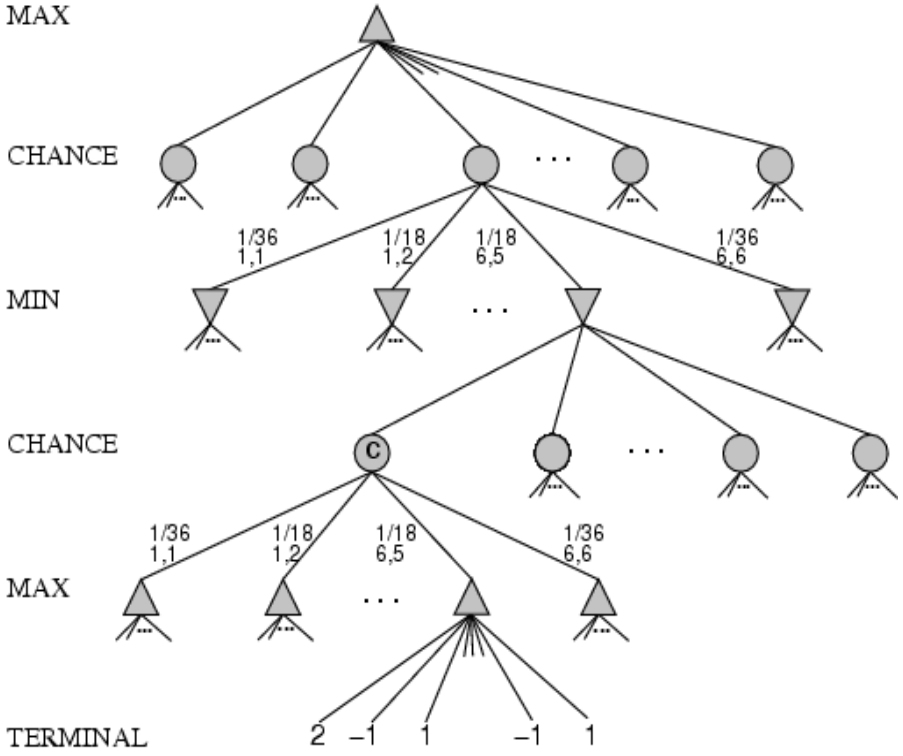




- Whites turn, After rolling a 5 and a 6
- Possible moves (5-10,5-11), (5-11,19-24),(5-10,10-16) and (5-11,11-16)

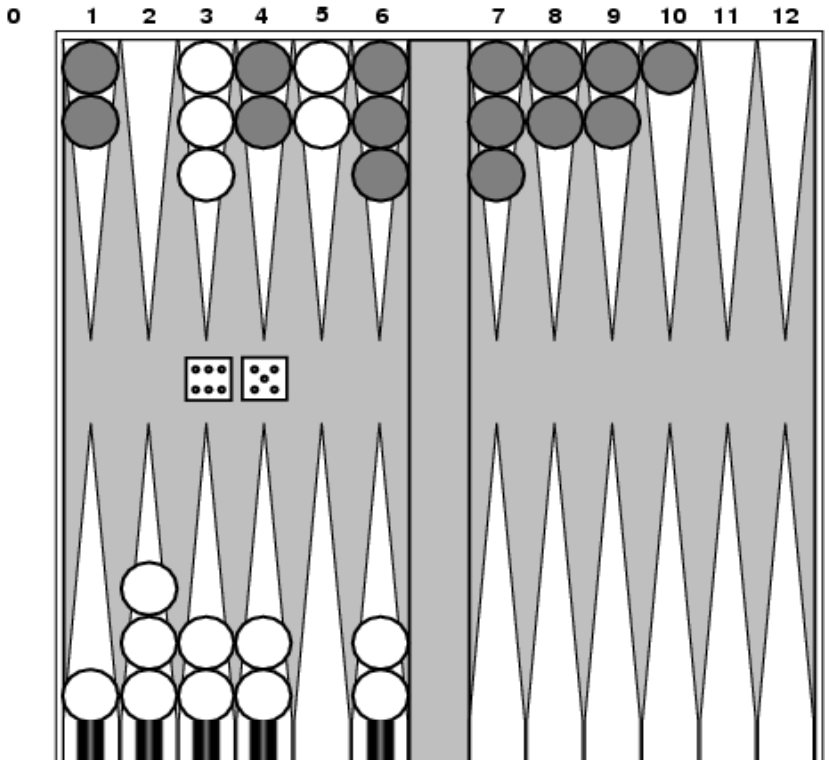
Games that include chance

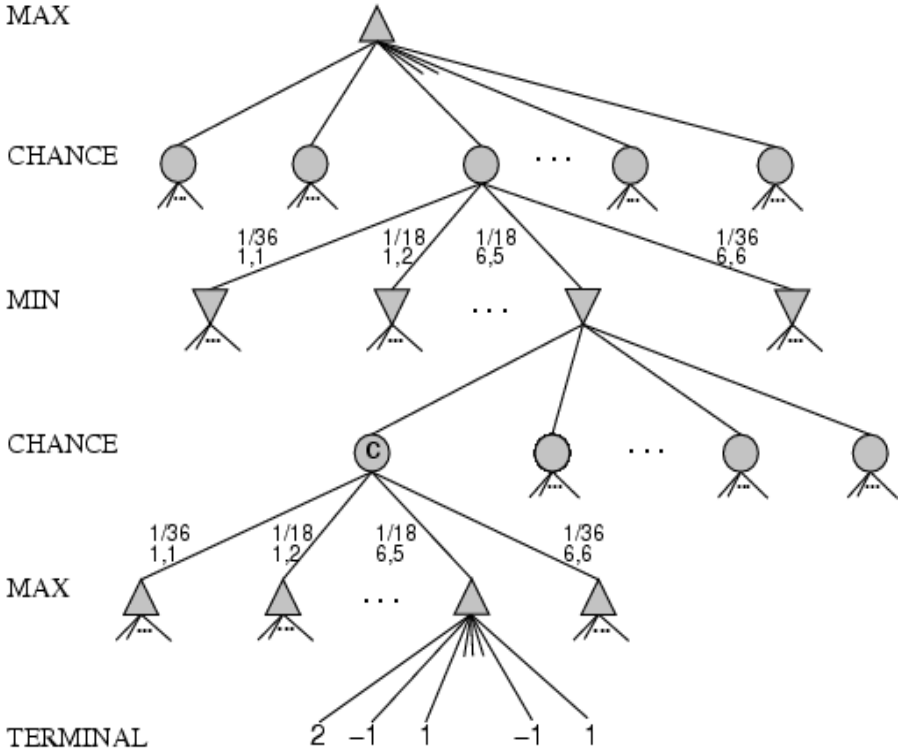




- Possible moves (5-10,5-11), (5-11,19-24),(5-10,10-16) and (5-11,11-16)
- [1,1], [6,6] chance 1/36, all other chance 1/18

Games that include chance





- [1,1], [6,6] chance 1/36, all other chance 1/18
- Can not calculate definite minimax value, only *expected* value

Expecti minimax value

EXPECTI-MINIMAX-VALUE(n)=

UTILITY(n) If n is a terminal

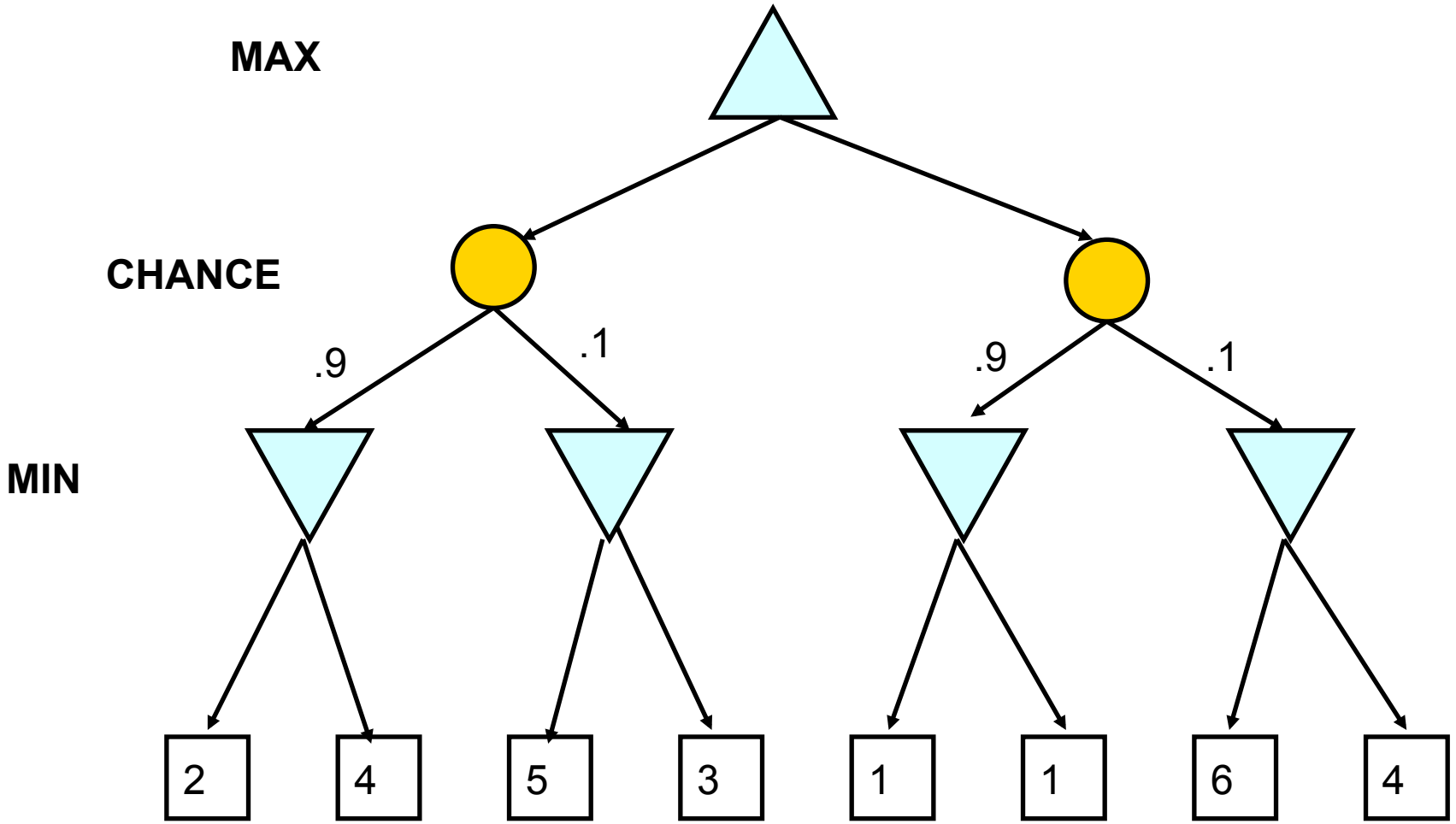
$\max_{s \in successors(n)}$ MINIMAX-VALUE(s) If n is a max node

$\min_{s \in successors(n)}$ MINIMAX-VALUE(s) If n is a min node

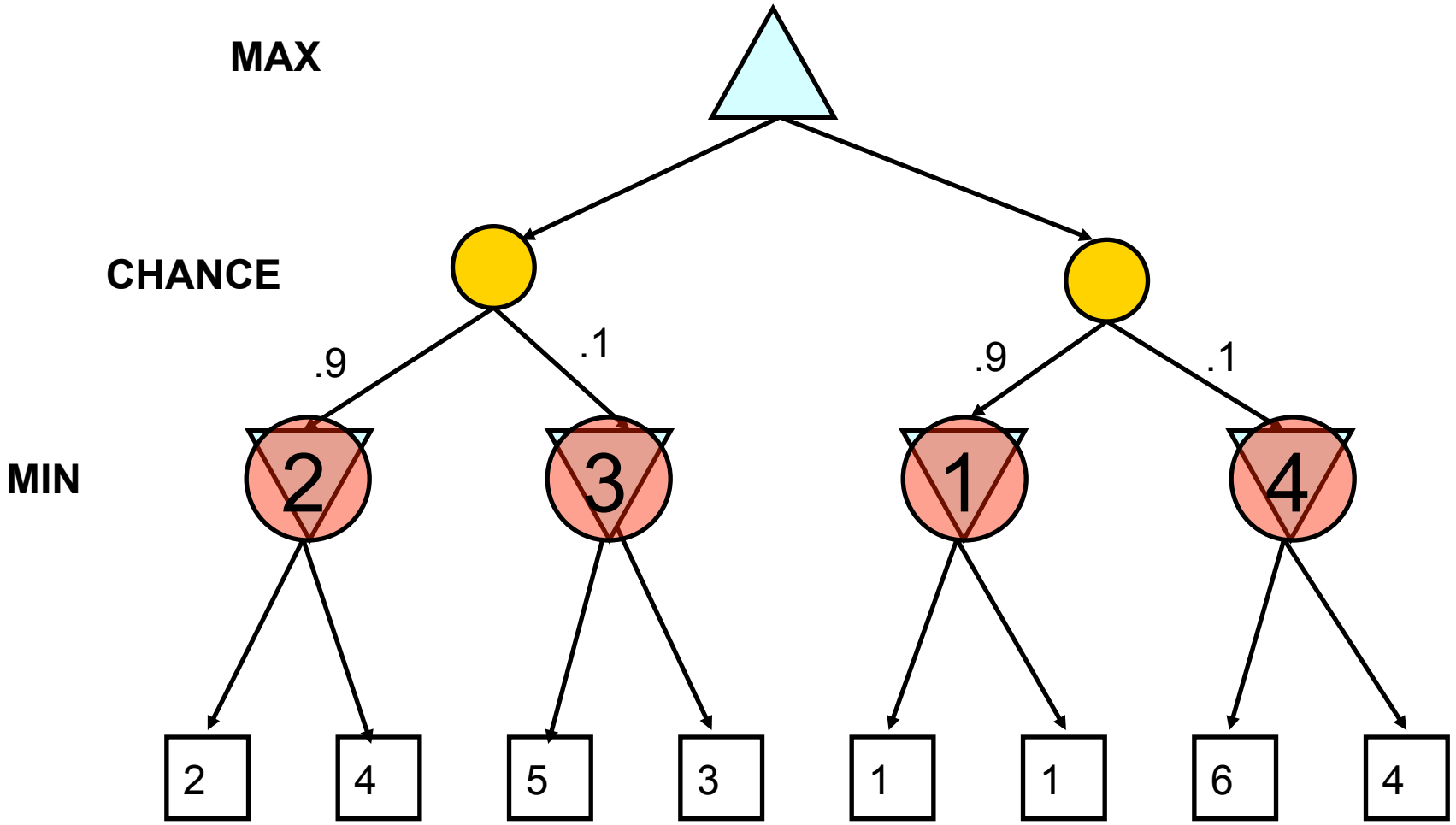
$\sum_{s \in successors(n)} P(s)$. EXPECTIMINIMAX(s) If n is a chance node

These equations can be backed-up recursively all the way to the root of the game tree.

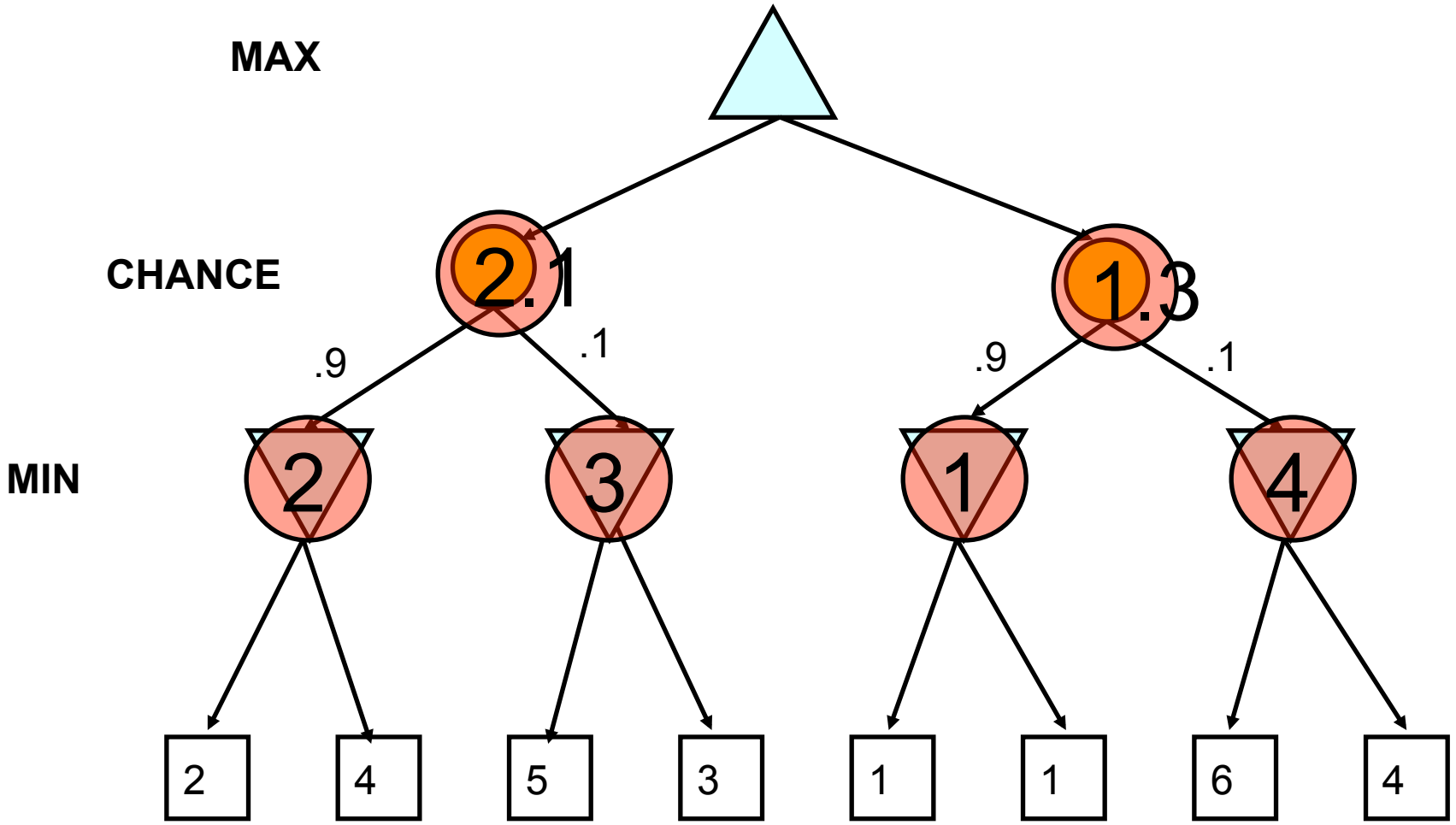
EXPECTEDMINIMAX example



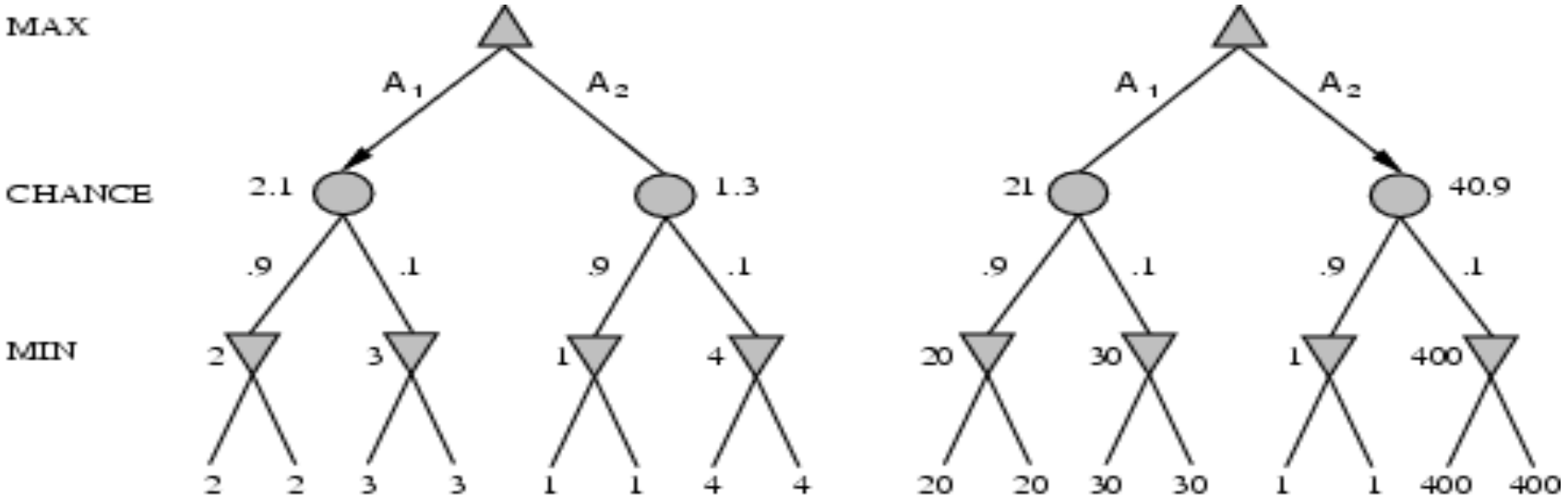
EXPECTIMINIMAX example

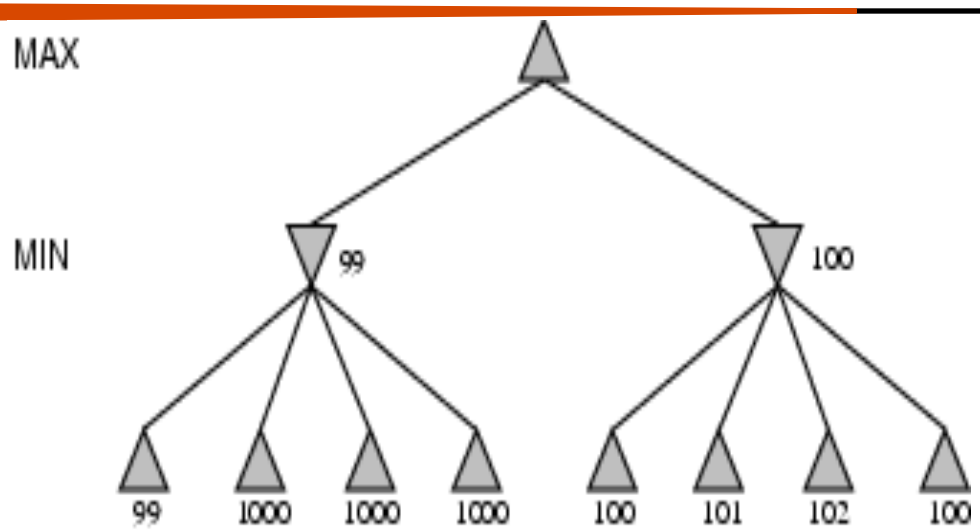


EXPECTIMINIMAX example



Position evaluation with chance nodes





- What will minimax do here?
- Is that OK?
- What might you do instead?

Summary

- Games are fun
- They illustrate several important points about AI
  - Perfection is unattainable -> approximation
  - Uncertainty constrains the assignment of values to states & broadens the state space