FILTERING EXAMPLE

$$P(X_{t+1} \mid e_{1:t+1}) = \alpha P(e_{t+1} \mid X_{t+1}) \sum_{x_t} P(X_{t+1} \mid x_t) P(x_t \mid e_{1:t})$$

PROBLEM On day 1, umbrella appears $\Rightarrow u_1 = \text{true}$
Let's calc. prob. dist. for Rain on day 1
$P(R_1 \mid u_1)$

* $t+1 = 1 \Rightarrow t = 0$ to fit into our eq.

$$P(R_1 \mid u_1) = \alpha \underbrace{P(u_1 \mid R_1)}_{\text{evidence model CPT}} \sum_{r_0} P(R_1 \mid r_0) P(r_0)$$

$$= \alpha \langle 0.9, 0.2 \rangle \sum_{r_0} P(R_1 \mid r_0) P(r_0)$$

sum out

$$= \alpha \langle 0.9, 0.2 \rangle \big(\underbrace{P(R_1 \mid r_0) P(r_0) + P(R_1 \mid \neg r_0) P(\neg r_0)}_{\text{transition models}} \big)$$

$$= \alpha \langle 0.9, 0.2 \rangle \big(\langle 0.7, 0.3 \rangle \underbrace{P(r_0)}_{\text{prior prob}} + \langle 0.3, 0.7 \rangle P(\neg r_0) \big)$$

$$= \alpha \langle 0.9, 0.2 \rangle \big(\langle 0.7, 0.3 \rangle 0.5 + \langle 0.3, 0.7 \rangle 0.5 \big)$$

$$= \alpha \langle 0.9, 0.2 \rangle \big(\langle 0.35, 0.15 \rangle + \langle 0.15, 0.35 \rangle \big)$$

$$= \alpha \langle 0.9, 0.2 \rangle \langle 0.5, 0.5 \rangle$$

$$= \alpha \langle 0.45, 0.1 \rangle$$

need to normalize to make a true prob.

$$= \left\langle \frac{0.45}{0.55}, \frac{0.1}{0.55} \right\rangle$$

$$= \langle 0.8181, 0.1819 \rangle$$

PROBLEM On day 2, umbrella appears, so $u_2 = \text{true}$

lets calc. $P(R_2 | u_1, u_2)$

☆ $t+1 = 2 \Rightarrow t = 1$ to fit into our eq.

$$P(R_2|u_1,u_2) = \alpha P(u_2|R_2) \sum_{r_1} P(R_2|r_1) P(r_1|u_1)$$

(evidence model)

$$= \alpha \langle 0.9, 0.2 \rangle \sum_{r_1} P(R_2|r_1) P(r_1|u_1)$$

(sum out; transition model)

$$= \alpha \langle 0.9, 0.2 \rangle \left(P(R_2|r_1) P(r_1|u_1) + P(R_2|\neg r_1) P(\neg r_1|u_1 \right.$$

$$= \alpha \langle 0.9, 0.2 \rangle \left(\langle 0.7, 0.3 \rangle P(r_1|u_1) + \langle 0.3, 0.7 \rangle P(\neg r_1|u_1 \right.$$

our last filter answer!

$$= \alpha \langle 0.9, 0.2 \rangle (\langle 0.7, 0.3 \rangle 0.8181 + \langle 0.3, 0.7 \rangle 0.1818)$$

$$= \alpha \langle 0.9, 0.2 \rangle (\langle 0.5727, 0.2454 \rangle + \langle 0.05454, 0.1273 \rangle)$$

$$= \alpha \langle 0.9, 0.2 \rangle (\langle 0.6272, 0.3727 \rangle)$$

$$= \alpha \langle 0.5645, 0.0745 \rangle \quad \text{sum} = 0.6389$$

$$= \langle 0.88, 0.117 \rangle$$

INTUITION

Prob of Rain from day 1 to day 2 increases b/c rain persists

SMOOTHING

PROBLEM: Smoothed estimate for rain at $t=1$, given umbrella on days 1 and 2

$k=1 \quad t=2$

~~$P(R_1|u_1,u_2) = \alpha P(R_1|u_1) P(u_2|R_1)$~~

$$P(X_k|e_{1:t}) = \alpha P(X_k|e_{1:k}) P(e_{k+1:t}|X_k)$$

$$P(R_1|u_1,u_2) = \alpha P(R_1|u_1) P(u_2|R_1)$$

filtering! $\langle 0.818, 0.182 \rangle$

$$P(e_{k+1:t}|X_k) = \sum_{X_{k+1}} P(e_{k+1}|X_{k+1}) P(e_{k+2:t}|X_{k+1}) P(X_{k+1}|X_k)$$

$$P(u_2|R_1) = \sum_{r_2} P(u_2|r_2) P(\ |r_2) P(r_2|R_1)$$

$$= P(u_2|r_2) P(r_2|R_1) + P(u_2|\neg r_2) P(\neg r_2|R_1)$$
$$= 0.9\langle 0.7, 0.3\rangle + 0.2\langle 0.3, 0.7\rangle$$
$$= \langle 0.63, 0.27\rangle + \langle 0.06, 0.14\rangle$$
$$= \langle 0.69, 0.41\rangle$$

$$P(R_1|u_1,u_2) = \alpha \langle 0.818, 0.182\rangle \langle 0.69, 0.41\rangle$$
$$= \langle 0.883, 0.117\rangle$$

INTUITION

Smoothed estimate is higher b/c ~~rain~~ umbrella on 2 makes it likely to rain on 2, thus also rain on 1