CS 348:
First Order Logic/
First Order Predicate Calculus
and Knowledge Representation

Last Time: Propositional Logic

alarm ^ nighttime => burglar
stars => nighttime
nighttime => dark
dark => nighttime
burglar => crime
crime ^ dark => unsafe
alarm => noise
noise ^ nighttime => annoyed-neighbors
alarm
dark

Prove that this neighborhood is unsafe the above KB of facts

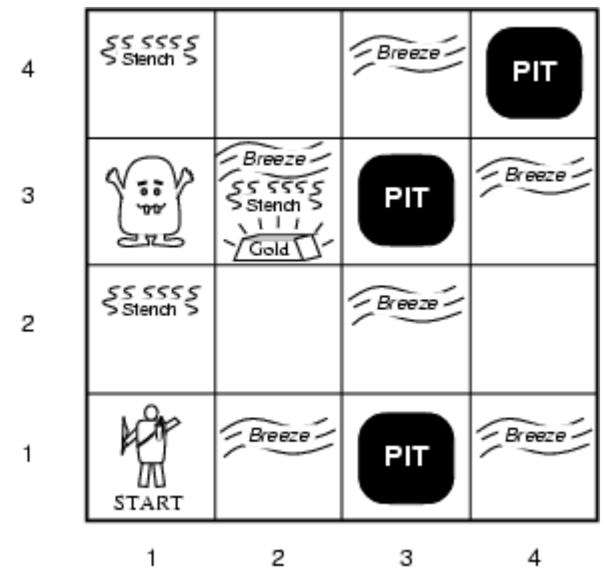
Problems with Propositional Logic

Impossible to make general assertions

"Pits cause breezes in adjacent squares"

$B2,1 \Leftrightarrow (P1,1 \vee P2,2 \vee P3,1)$

$P3,1 \Leftrightarrow (B2,1 \wedge B3,2 \wedge B4,1)$



Pros and cons of propositional logic

- ☺ Propositional logic is declarative
- ☺ Propositional logic allows partial/disjunctive/negated information
  - (unlike most data structures and databases)
- ☺ Propositional logic is compositional:
  - meaning of $B_{1,1} \wedge P_{1,2}$ is derived from meaning of $B_{1,1}$ and of $P_{1,2}$
- ☺ Meaning in propositional logic is context-independent
  - (unlike natural language, where meaning depends on context)

- ☹ Propositional logic has very limited expressive power
  - (unlike natural language)
  - E.g., cannot say "pits cause breezes in adjacent squares“
    - except by writing one sentence for each square

- Propositional Logic
  - Is simple
  - Illustrates important points:
    - Model, satisfiability, inference
  - Is restrictive: world is a set of facts
  - Lacks expressiveness (world contains FACTS)
- First-Order Logic
  - More symbols (objects, properties, relations)
  - More connectives (quantifiers)

First-order logic

- Whereas propositional logic assumes the world contains facts,
- first-order logic (like natural language) assumes the world contains
  - Objects: people, houses, numbers, colors, baseball games, wars, …
  - Relations: red, round, prime, brother of, bigger than, part of, comes between, …
  - Functions: father of, best friend, one more than, plus, …

Propositional Logic vs. FOL/FOPC

- Propositional Logic
  - The world consists of propositions (sentences) which can be true or false.
- Predicate Calculus (First Order Logic)
  - The world consists of objects, functions and relations between the objects.

Syntax of FOL: Basic elements

• Constants	KingJohn, 2, NUS,...
• Predicates	Brother, >,...
• Functions	Sqrt, LeftLegOf,...
• Variables	x, y, a, b,...
• Connectives	$\neg$, $\Rightarrow$, $\wedge$, $\vee$, $\Leftrightarrow$
• Equality	=
• Quantifiers	$\forall$, $\exists$

Atomic sentences

Atomic sentence = *predicate* ($term_1$,...,$term_n$)
or $term_1 = term_2$

Term = *function* ($term_1$,...,$term_n$)
or *constant* or *variable*

- E.g., *Brother(KingJohn,RichardTheLionheart) > (Length(LeftLegOf(Richard)), Length(LeftLegOf(KingJohn)))*

Complex sentences

- Complex sentences are made from atomic sentences using connectives

$$\neg S,\ S_1 \land S_2,\ S_1 \lor S_2,\ S_1 \Rightarrow S_2,\ S_1 \Leftrightarrow S_2,$$

E.g. *Sibling(KingJohn,Richard)* $\Rightarrow$ *Sibling(Richard,KingJohn)*

$>(1,2) \lor \leq (1,2)$

$>(1,2) \land \neg >(1,2)$

Universal quantification

- $\forall$<*variables*> <*sentence*>

Everyone at NU is smart:
$\forall x\ At(x,NU) \Rightarrow Smart(x)$

- $\forall x\ P$ is true in a model *m* iff *P* is true with *x* being each possible object in the model

- Roughly speaking, equivalent to the conjunction of instantiations of *P*

$$\begin{array}{ll} & At(KingJohn,NU) \Rightarrow Smart(KingJohn) \\ \wedge & At(Richard,NU) \Rightarrow Smart(Richard) \\ \wedge & At(Jane,NU) \Rightarrow Smart(Bob) \\ \wedge & \ldots \end{array}$$

A common mistake to avoid

- Typically, $\Rightarrow$ is the main connective with $\forall$
- Common mistake: using $\wedge$ as the main connective with $\forall$:

  $\forall x\ At(x,NU) \wedge Smart(x)$

  means "Everyone is at NU and everyone is smart"

Existential quantification

- $\exists$<*variables*> <*sentence*>

- Someone at NU is smart:
- $\exists x$ At(x,NU) $\wedge$ Smart(x)

- $\exists x\, P$ is true in a model *m* iff *P* is true with *x* being some possible object in the model

- Roughly speaking, equivalent to the disjunction of instantiations of *P*

  At(KingJohn,NU) $\wedge$ Smart(KingJohn)
  $\vee$ At(Richard,NU) $\wedge$ Smart(Richard)
  $\vee$ At(Jane,NU) $\wedge$ Smart(NU)
  $\vee$ ...

Another common mistake to avoid

- Typically, $\wedge$ is the main connective with $\exists$
- Common mistake: using $\Rightarrow$ as the main connective with $\exists$:

$$\exists x \ \mathrm{At}(x,\mathrm{NUS}) \Rightarrow \mathrm{Smart}(x)$$

is true if there is anyone who is not at NU!

Properties of quantifiers

- $\forall x \, \forall y$ is the same as $\forall y \, \forall x$
- $\exists x \, \exists y$ is the same as $\exists y \, \exists x$
- $\exists x \, \forall y$ is not the same as $\forall y \, \exists x$
- $\exists x \, \forall y \, Loves(x,y)$
  - "There is a person who loves everyone in the world"
- $\forall y \, \exists x \, Loves(x,y)$
  - "Everyone in the world is loved by at least one person"
- Quantifier duality: each can be expressed using the other
- $\forall x \, Likes(x,IceCream)$ $\quad\quad$ $\neg \exists x \, \neg Likes(x,IceCream)$
- $\exists x \, Likes(x,Broccoli)$ $\quad\quad$ $\neg \forall x \, \neg Likes(x,Broccoli)$

Quantifiers

- Existential:
  - There is a Northwestern Student from Hawaii.
- Universal:
  - Northwestern students live in Evanston.

Examples

- All purple mushrooms are poisonous
- No purple mushroom is poisonous
- Every CS student knows a programming language.
- A programming language is known by every CS student

Equality

- $term_1 = term_2$ is true under a given interpretation if and only if $term_1$ and $term_2$ refer to the same object

- E.g., definition of *Sibling* in terms of *Parent*:

$$\forall x,y\ Sibling(x,y) \Leftrightarrow [\neg(x = y) \wedge \exists m,f\ \neg(m = f) \wedge Parent(m,x) \wedge Parent(f,x) \wedge Parent(m,y) \wedge Parent(f,y)]$$

Using FOL

The kinship domain:

- Brothers are siblings

  $\forall x,y\ Brother(x,y) \Leftrightarrow Sibling(x,y)$

- One's mother is one's female parent

  $\forall m,c\ Mother(c) = m \Leftrightarrow (Female(m) \wedge Parent(m,c))$

- "Sibling" is symmetric

  $\forall x,y\ Sibling(x,y) \Leftrightarrow Sibling(y,x)$

Interacting with FOL KBs

- Suppose a wumpus-world agent is using an FOL KB and perceives a smell and a breeze (but no glitter) at *t=5*:

  `Tell`(KB,Percept([Smell,Breeze,None],5))
  `Ask`(KB,∃a BestAction(a,5))

- I.e., does the KB entail some best action at *t=5*?

- Answer: *Yes*, {*a/Shoot*} ← substitution (binding list)

- Given a sentence *S* and a substitution σ,
- *S*σ denotes the result of plugging σ into *S*; e.g.,
  S = Smarter(x,y)
  σ = {x/Jane,y/Sue}
  *S*σ = Smarter(Jane,Sue)

- `Ask`(KB,S) returns some/all σ such that KB $\models$ σ

Knowledge base for the wumpus world

- Perception
  - $\forall t,s,b$ Percept([s,b,Glitter],t) $\Rightarrow$ Glitter(t)

- Reflex
  - $\forall t$ Glitter(t) $\Rightarrow$ BestAction(Grab,t)

Deducing hidden properties

- $\forall x,y,a,b$ *Adjacent*$([x,y],[a,b]) \Leftrightarrow$
  $[a,b] \in \{[x+1,y], [x-1,y],[x,y+1],[x,y-1]\}$

Properties of squares:

- $\forall s,t$ *At*(Agent,s,t) $\wedge$ Breeze(t) $\Rightarrow$ Breezy(s)

Squares are breezy near a pit:

– Diagnostic rule---infer cause from effect

  $\forall s$ Breezy(s) $\Rightarrow \exists r$ Adjacent(r,s) $\wedge$ Pit(r)

– Causal rule---infer effect from cause

  $\forall r$ Pit(r) $\Rightarrow$ [$\forall s$ Adjacent(r,s) $\Rightarrow$ Breezy(s)]

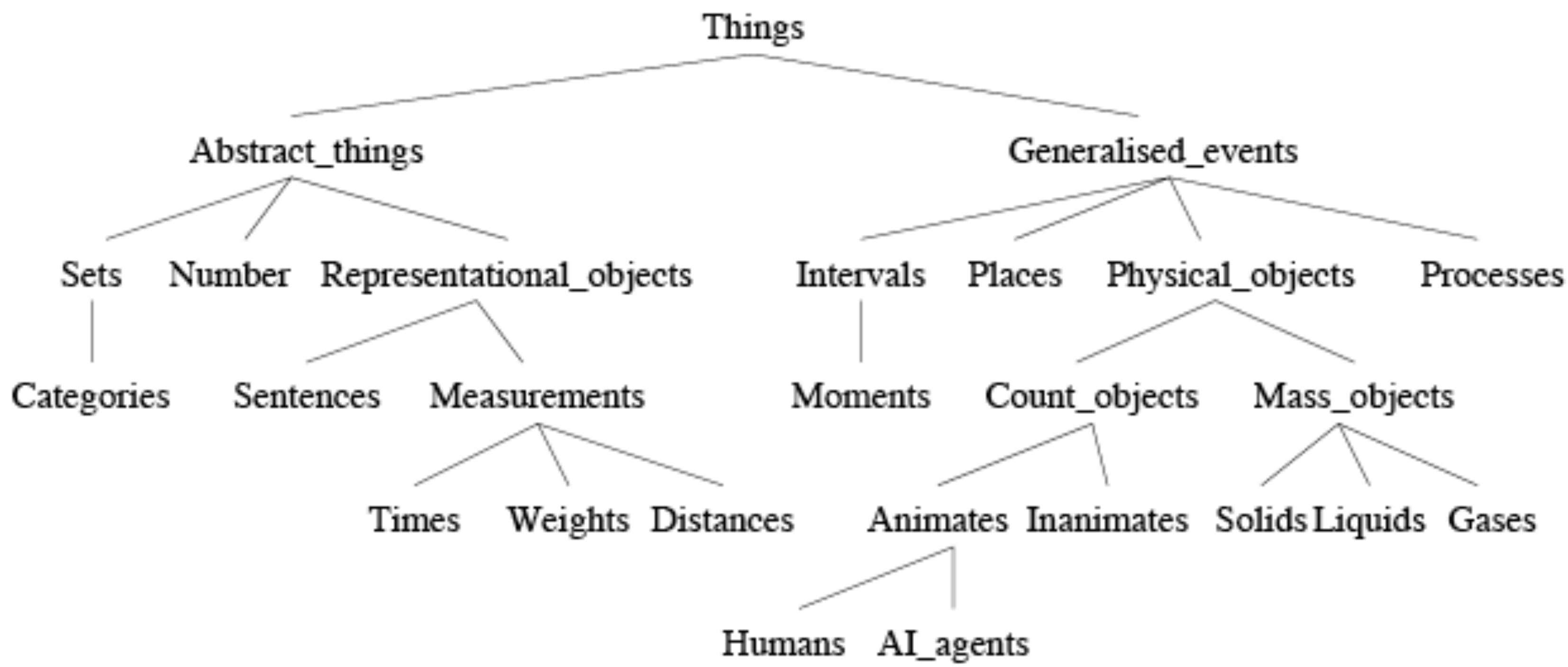
Knowledge engineering in FOL

1. Identify the task
2. Assemble the relevant knowledge
3. Decide on a vocabulary of predicates, functions, and constants
4. Encode general knowledge about the domain
5. Encode a description of the specific problem instance
6. Pose queries to the inference procedure and get answers
7. Debug the knowledge base

Knowledge Representation

- Representing general concepts
  - ACTIONS
  - TIME
  - PHYSICAL OBJECTS
  - BELIEFS
- Ontological Engineering versus Knowledge Engineering

Upper Ontology



Categories and Objects

- Predicates
  - Basketball(b)
- Objects
  - Basketballs
- Inheritance
  - Every Apple is edible
- Taxonomy/Taxonomic Hierarchy

Stating facts about categories

- An object is a member of a category
- A category is a subclass of another category
- All members of a category have some properties
- Members of a category can be recognized by some properties
- A category as a whole has some properties

Categories

- Disjoint
  - Disjoint({Animals, Vegetables})
- Exhaustive Decomposition
  - ExhaustiveDecomposition({Americans, Canadians, Mexicans}, NorthAmericans)
- Partition
  - Partition({Males, Females}, Animals)

Physical Composition

- PartOf relation to relate two things
  - PartOf(Bucharest, Romania)
  - PartOf(Romanai, Eastern Europe)
  - PartOf(EasternEurope, Europe)
  - PartOf(Europe, Earth)
  - Therefore PartOf(Bucharest, Earth)
- Composite Objects
  - Biped has two legs attached to a body
  - Biped(a) => ∃ l1, l2, b Body(b) ∩ Leg(l1) ∩ Leg(l2) ∩ PartOf(l1, a) ∩ PartOf(l2, a) ∩ PartOf(b, a) ∩ Attached(l1, b) ∩ Attached(l2, b) …

Measurements

- Units Functions
  - Length(L1) = Inches(1.5) = Centimeters(3.81)
- Conversion
  - Centimeters(2.54 x d) = Inches(d)
- More examples
  - Diameter(Basketballx) = Inches(9.5)
  - ListPrice(Basketballx) = $(19)
  - d E Days => Duration(d) = Hours(24)

Substances and objects

- Individuation
- Count nouns
  - One “cat” cut in two is not two “cats”
  - If it has any **extrinsic** qualities
- Mass nouns
  - One “butter-object” cut in half is two “butter-objects”
  - x E Butter ∩ PartOf(y, x) => y E Butter
  - x E Butter => MeltingPoint(x, Centigrade(30))
  - All qualities are **intrinsic**

Cyc

- CycL
  - Knowledge rep language
  - High level logic
- Constants
  - dog, BillClinton, ColorRed…
- Formula
  - (isa GeorgeBush person)
- Logical connectives
  - (and
    (performedBy GettysburgAddress Lincoln)
    (objectHasColor Rover TanColor))
- Quantifiers
  - (thereExistsExactly 12 ?ZOS
    (isa ?ZOS ZodiacSign))