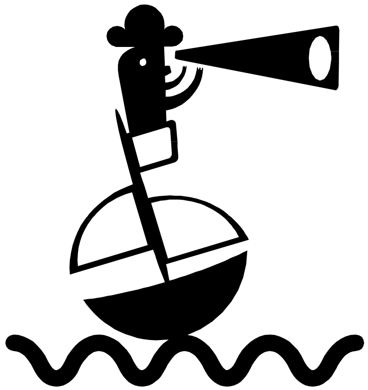
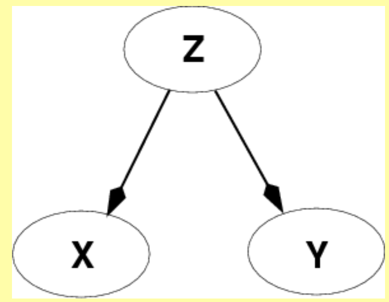
CS 151: Reasoning with Knowledge and Probability Theory (Review?)

Techniques for Implementing Policies

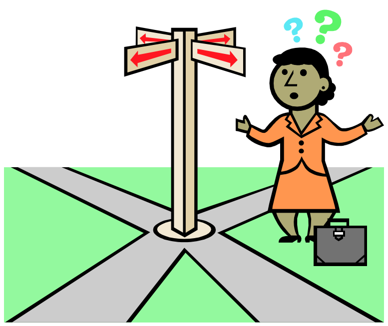


Search

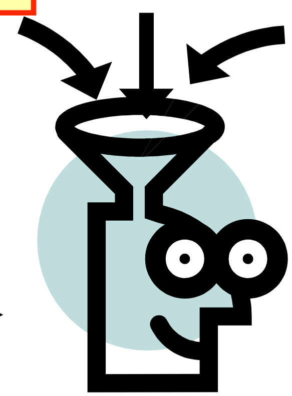


Reasoning with knowledge and uncertainty

At the core of this class will be several techniques for Policy design and implementation

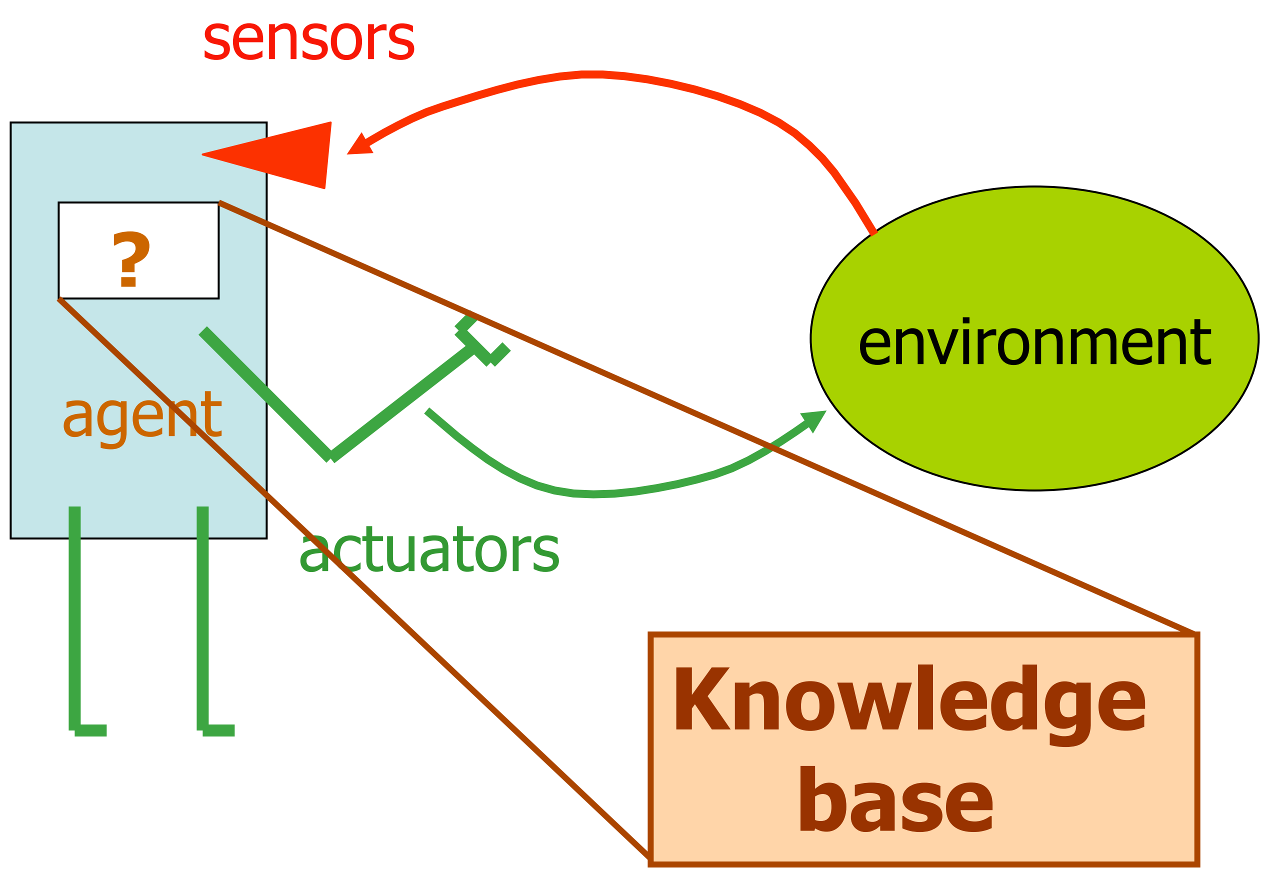


Reasoning with Utility



Learning

Knowledge-Based Agent



How do we represent knowledge?

- Procedurally (HOW):
  - Write methods that encode how to handle specific situations in the world
    - **`chooseMoveMancala()`**
    - **`driveOnHighway()`**
- Declaratively (WHAT):
  - Specify facts about the world
    - Two adjacent regions must have different colors
    - If the lights on the modem are off, it is not sending a signal

Logic for Knowledge Representation

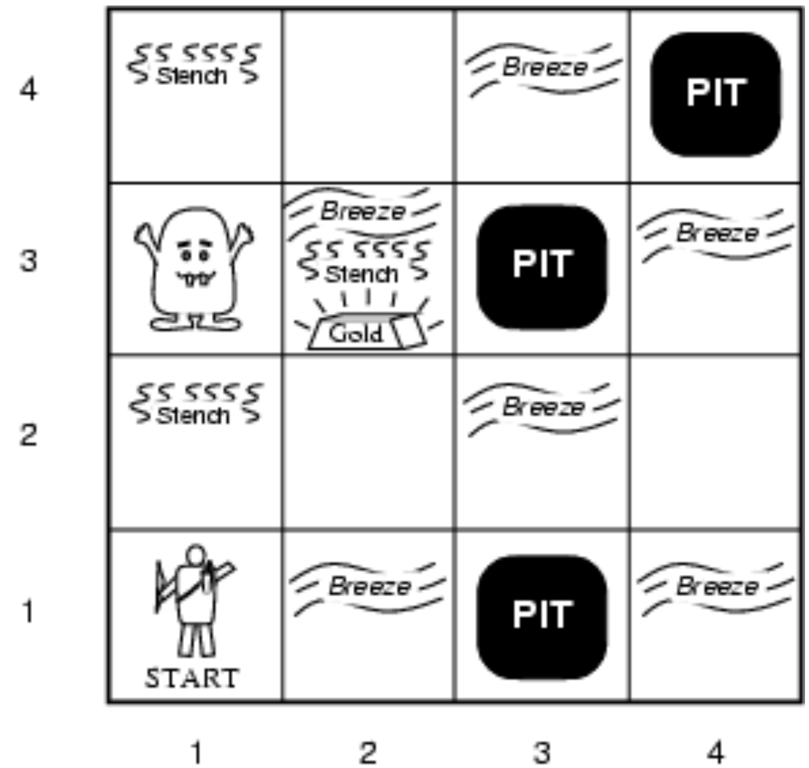
Logic is a declarative language to:

- Assert sentences representing facts that hold in a world W (these sentences are given the value true)

- Deduce the true/false values to sentences representing other aspects of W

The Wumpus World

- Performance measure
  - gold +1000, death -1000 (falling into pit or eaten by wumpus)
  - -1 per step, -10 for using the arrow
- Environment
  - 4x4 grid of rooms
  - Agent starts in [1,1] facing right
  - gold/wumpus squares randomly chosen
  - Any other room can have a pit (prob = 0.2)
- Sensors: Stench, Breeze, Glitter, Bump, Scream
- Actuators: Left turn, Right turn, Forward, Grab, Release, Shoot



Reasoning with Uncertainty

- Pure logic sometimes (often) fails:
  - Because the wumpus may not eat you
  - Because resetting the modem is not always reliable
  - Because the alarm might be caused by an earthquake or a burglar
  - Because John said it rained yesterday and Beth said it didn't

Because the real world does not conform to logic

Basic Probability

- Probability theory enables us to make rational decisions.
- Which mode of transportation is safer:
  - Car or Plane?
  - What is the probability of an accident?

Basic Probability Theory

- An **experiment** has a set of potential outcomes, e.g., throw a dice
- The **sample space** of an experiment is the set of all possible outcomes, e.g., {1, 2, 3, 4, 5, 6}
  - A **random variable** can take on any value in the sample space
- An **event** is a subset of the sample space.
  - {2}
  - {3, 6}
  - even = {2, 4, 6}
  - odd = {1, 3, 5}

Probability as Relative Frequency

Total Flips: 10
Number Heads: 5
Number Tails: 5

Probability of Heads:
Number Heads / Total Flips = 0.5

Probability of Tails:
Number Tails / Total Flips = 0.5 = 1.0 – Probability of Heads

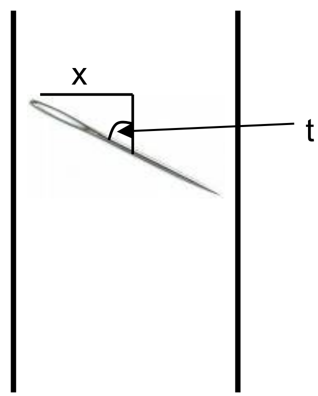
The experiments, the sample space and the events must be defined clearly for probability to be meaningful

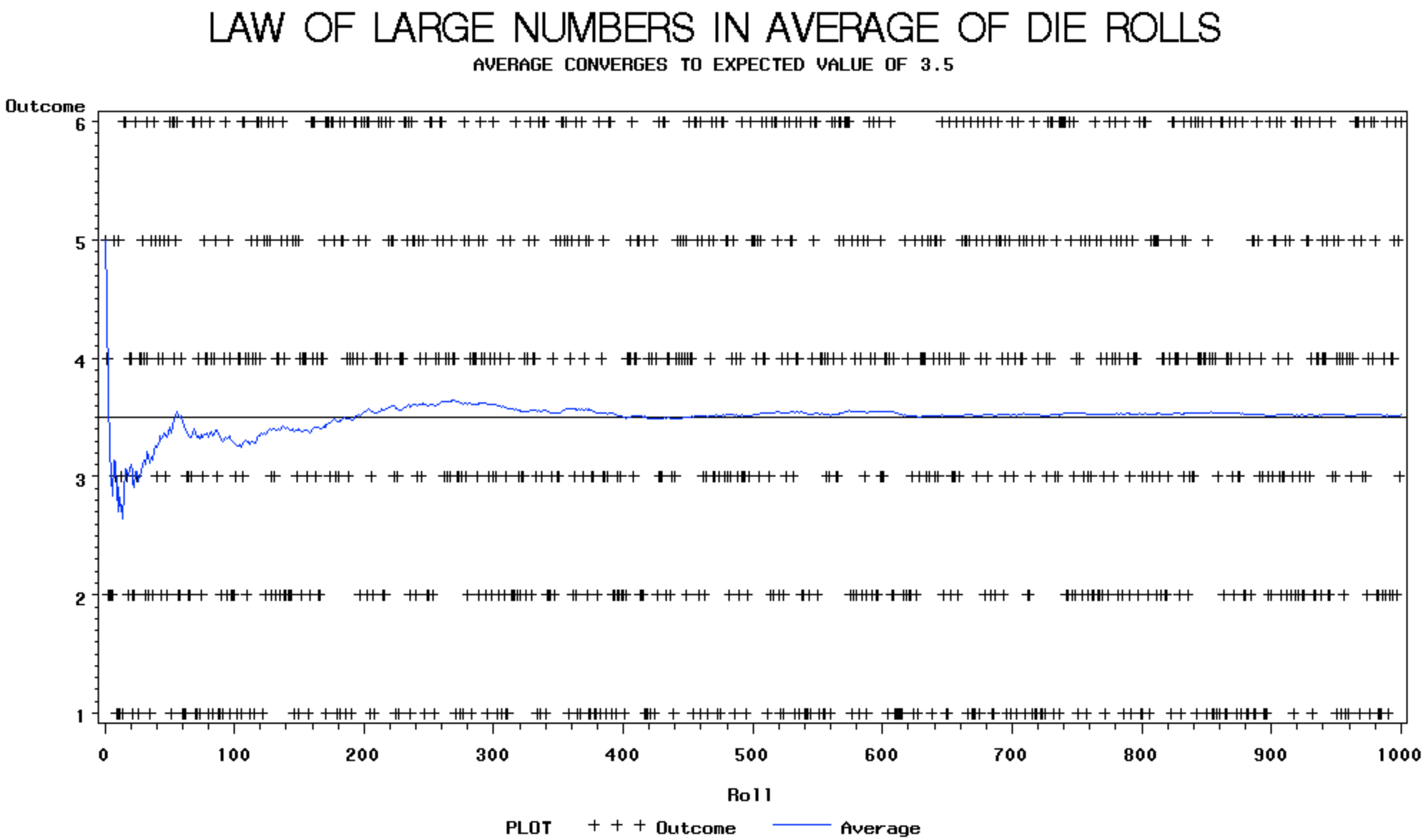
Theoretical Probability

- **Principle of Indifference**—Alternatives are always to be judged equi-probable if we have no reason to expect or prefer one over the other.
- Each outcome in the sample space is assigned equal probability.
- Example: throw a dice
  - $P(\{1\})=P(\{2\})= \ldots =P(\{6\})=1/6$

Law of Large Numbers

- As the number of experiments increases the relative frequency of an event more closely approximates the theoretical probability of the event.
  - if the theoretical assumptions hold.
- Buffon's Needle for Computing π





LAW OF LARGE NUMBERS IN AVERAGE OF DIE ROLLS
AVERAGE CONVERGES TO EXPECTED VALUE OF 3.5
Outcome
6
5
4
3
2
1
0
100
200
300
400
500
600
700
800
900
1000
Roll
PLOT
Outcome
Average

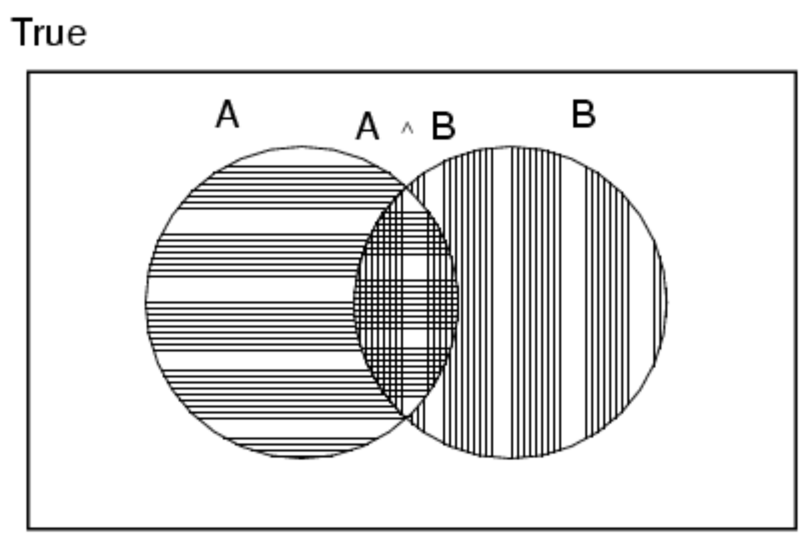
Large Number Reveals Untruth in Assumptions

Results of 1,000,000 throws of a die

Number	1	2	3	4	5	6
Fraction	.155	.159	.164	.169	.174	.179

Axioms of probability

- For any propositions *A, B*
  - $0 \le P(A) \le 1$
  - $P(\mathit{true}) = 1$ and $P(\mathit{false}) = 0$
  - $P(A \vee B) = P(A) + P(B) - P(A \wedge B)$



Prior probability

- Prior or unconditional probabilities of propositions
  e.g., P(*Cavity* = true) = 0.1 (also written as P(cavity)) and P(*Weather* = sunny) = 0.72 correspond to belief prior to arrival of any (new) evidence

- Probability distribution gives values for all possible assignments:
  P(*Weather*) = <0.72,0.1,0.08,0.1> (normalized, i.e., sums to 1)
  P(Dice) = <0.167, 0.167, 0.167, 0.167, 0.167, 0.167>

- Joint probability distribution for a set of random variables gives the probability of every atomic event on those random variables
  P(*Weather,Cavity*) = a 4 × 2 matrix of values:

Weather =	sunny	rainy	cloudy	snow
Cavity = true	0.144	0.02	0.016	0.02
Cavity = false	0.576	0.08	0.064	0.08

- Every question about a domain can be answered by the joint distribution

Properties of Probability

1. $P(\neg E) = 1 - P(E)$
2. If E1 and E2 are logically equivalent, then

   $P(E1)=P(E2)$.

   – E1: Not all philosophers are more than six feet tall.

   – E2: Some philosopher is not more that six feet tall.

     Then $P(E1)=P(E2)$.
3. $P(E1, E2) \leq P(E1)$.

Conditional Probability

- The probability of an event may change after knowing another event.

  The probability of A given B is denoted by P(A|B).

A: the top card of a deck of poker cards is a king of spades

P(A) =

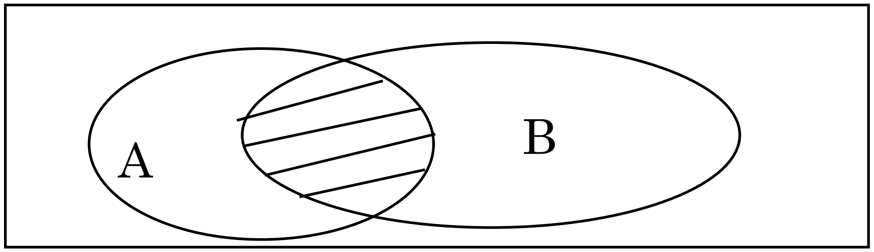
However, if we know

B: the top card is a king

then, the probability of A given B is true is

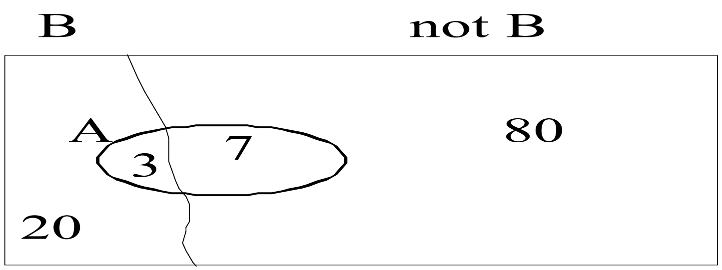
P(A|B) =

How to Compute P(A|B)?



Business Students

Of 100 students completing a course, 20 were business majors. Ten students received As in the course, and three of these were business majors., suppose A is the event that a randomly selected student got an A in the course, B is the event that a randomly selected event is a business major. What is the probability of A? What is the probability of A after knowing B is true?



Inference by enumeration

- Start with the joint probability distribution:

	toothache		*¬ toothache*	
	catch	*¬ catch*	*catch*	*¬ catch*
cavity	**.108**	**.012**	**.072**	**.008**
¬ cavity	**.016**	**.064**	**.144**	**.576**

- Sum out true events

Inference by enumeration

- Start with the joint probability distribution:

	toothache		*¬ toothache*	
	catch	*¬ catch*	*catch*	*¬ catch*
cavity	.108	.012	.072	.008
¬ cavity	.016	.064	.144	.576

- Sum out true events
- P(*toothache*) = 0.108 + 0.012 + 0.016 + 0.064 = 0.2

Inference by enumeration

- Start with the joint probability distribution:

	toothache		$\neg$ *toothache*	
	catch	$\neg$ *catch*	*catch*	$\neg$ *catch*
cavity	.108	.012	.072	.008
$\neg$ *cavity*	.016	.064	.144	.576

- Can also compute conditional probabilities:

$$P(\neg cavity \mid toothache) = \frac{P(\neg cavity \wedge toothache)}{P(toothache)}$$

$$= \frac{0.016+0.064}{0.108 + 0.012 + 0.016 + 0.064}$$

$$= 0.4$$

Normalization

	toothache		*¬ toothache*	
	catch	*¬ catch*	*catch*	*¬ catch*
cavity	.108	.012	.072	.008
¬ cavity	.016	.064	.144	.576

- Denominator can be viewed as a normalization constant α

$$\mathbf{P}(Cavity \mid toothache) = \alpha\, \mathbf{P}(Cavity, toothache)$$
$$= \alpha\, [\mathbf{P}(Cavity, toothache, catch) + \mathbf{P}(Cavity, toothache, \neg\, catch)]$$
$$= \alpha\, [\langle 0.108, 0.016\rangle + \langle 0.012, 0.064\rangle]$$
$$= \alpha\, \langle 0.12, 0.08\rangle = \langle 0.6, 0.4\rangle$$

General idea: compute distribution on query variable by fixing evidence variables and summing over hidden variables

Probabilistic Reasoning

- Evidence
  - What we know about a situation.
- Hypothesis
  - What we want to conclude.
- Compute
  - P(Hypothesis | Evidence)

Credit Card Authorization

- E is the data about the applicant's age, job, education, income, credit history, etc,
- H is the hypothesis that the credit card will provide positive return (for ccard company).
- The decision of whether to issue the credit card to the applicant is based on the probability $P(H|E)$.

Medical Diagnosis

- E is a set of symptoms, such as, coughing, sneezing, headache, ...
- H is a disorder, e.g., common cold, cancer, swine flu.
- The diagnosis problem is to find an H (disorder) such that $P(H|E)$ is maximum.

The Three-Card Problem

Three cards are in a hat. One is red on both sides (the red-red card). One is white on both sides (the white-white card). One is red on one side and white on the other (the red-white card). A single card is drawn randomly and tossed into the air.

a. What is the probability that the red-red card was drawn? (RR)

b. What is the probability that the drawn cards lands with a white side up? (W-up)

c. What is the probability that the red-red card was not drawn, assuming that the drawn card lands with the a red side up. (not-RR|R-up)

Fair Bets

- A bet is fair to an individual I if, according to the individual's probability assessment, the bet will break even in the long run.
- The following three bets are fair to a naïve (typical?) individual:

Bet (a): Win $4.20 if RR;
lose $2.10
otherwise. [since they believe P(RR)=1/3]

Bet (b): Win $2.00 if W-up;
lose $2.00
otherwise. [since they believe P(W-up)=1/2]

Bet (c): Win $4.00 if R-up and not-RR;
lose $4.00 if R-up and RR;
neither win nor lose if not-R-up.
[since they believe P(not-RR|R-up)=1/2]

Dutch Book

- The bets that this person accepted have an interesting property:

  No matter what card is drawn in the three-card problem, and no matter how it lands, you are guaranteed to lose money.

- This is called a Dutch Book

Verification

there are six possible outcomes

1. RR drawn, R-up (side 1)
2. RR drawn, R-up (side 2)
3. WR drawn, R-up
4. WR drawn, W-up
5. WW drawn, W-up (side 1)
6. WW drawn, W-up (side 2)

	1	2	3	4	5	6
a.	$4.20	$4.20	-$2.10	-$2.10	-$2.10	-$2.10
b.	-$2.00	-$2.00	-$2.00	$2.00	$2.00	$2.00
c.	-$4.00	-$4.00	$4.00	$0.00	$0.00	$0.00
Total	-$1.80	-$1.80	-$0.10	-$0.10	-$0.10	-$0.10

The Dutch Book Theorem

- Suppose that an individual I is willing to accept any bet that is fair for I. Then a Dutch book can be made against I if and only if I's assessment of probability violates Bayesian axiomatization.

Another Example

A patient takes a lab test and the result comes back positive. The test has a false negative rate of 2% and false positive rate of 2%. Furthermore, 0.5% of the entire population have this cancer.

What is the probability of cancer if we know the test result is positive?

Bayes Theorem

Independence: Intuition

- Events are independent if one has nothing whatever to do with others. Therefore, for two independent events, knowing one happening does not change the probability of the other event happening.
  - one toss of coin is independent of another coin toss (assuming it is a regular coin).
  - price of tea in England is independent of the result of general election in Canada.

Independent or Dependent?

- Getting a cold and getting a cat-allergy
- Mile Per Gallon and acceleration.
- Size of a person's vocabulary and the person's shoe size.

Independence: Definition

- Events A and B are independent iff:

$$P(A, B) = P(A) \times P(B)$$

which is equivalent to

$$P(A|B) = P(A) \text{ and}$$

$$P(B|A) = P(B)$$

when $P(A, B) > 0$.

T1: the first toss is a head.

T2: the second toss is a tail.

$$P(T2|T1) = P(T2)$$

Conditional Independence

- Dependent events can become independent given certain other events.
- Example,
  - Size of shoe
  - Size of vocabulary
  - ??
- Two events A, B are conditionally independent given a third event C iff

  $P(A|B, C) = P(A|C)$

Conditional Independence: Definition

- Let E1 and E2 be two events, they are conditionally independent given E iff

  $P(E1|E, E2)=P(E1|E)$,

  that is the probability of E1 is not changed after knowing E2, given E is true.
- Equivalent formulations:

  $P(E1, E2|E)=P(E1|E)\ P(E2|E)$

  $P(E2|E, E1)=P(E2|E)$

Bayes' Rule and conditional independence

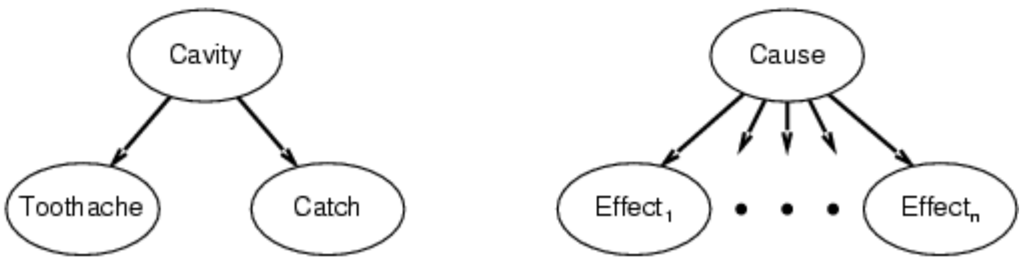
$\mathbf{P}(Cavity \mid toothache \wedge catch)$

$= \alpha\mathbf{P}(toothache \wedge catch \mid Cavity)\, \mathbf{P}(Cavity)$

$= \alpha\mathbf{P}(toothache \mid Cavity)\, \mathbf{P}(catch \mid Cavity)\, \mathbf{P}(Cavity)$

- This is an example of a naïve Bayes model:

$\mathbf{P}(\text{Cause},\text{Effect}_1, \dots ,\text{Effect}_n) = \mathbf{P}(\text{Cause})\, \pi_i \mathbf{P}(\text{Effect}_i|\text{Cause})$

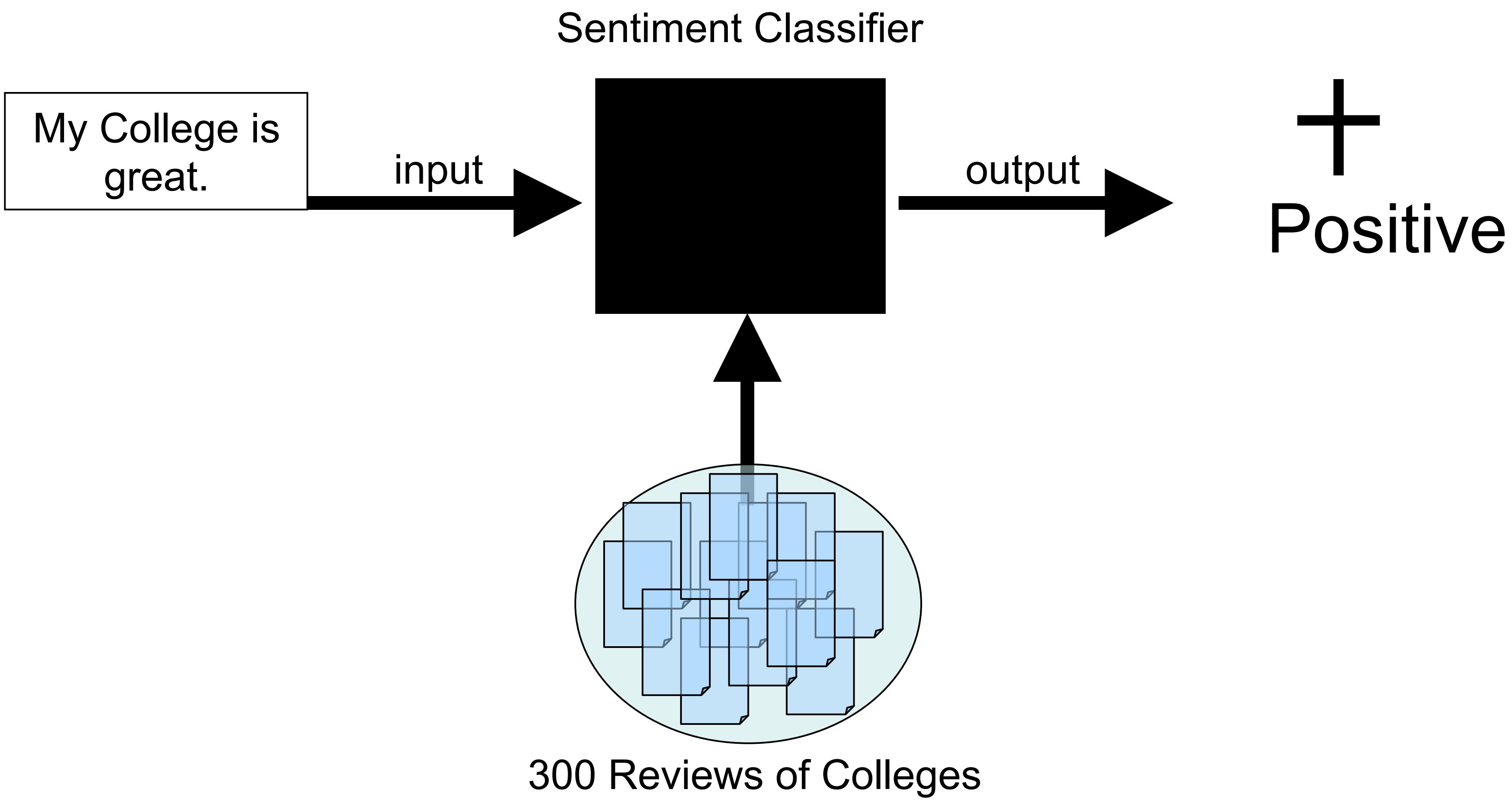


- Total number of parameters is linear in *n*

Summary

- Probability is a rigorous formalism for uncertain knowledge
- Joint probability distribution specifies probability of every atomic event
- Queries can be answered by summing over atomic events
- For nontrivial domains, we must find a way to reduce the joint size
- Independence and conditional independence provide the tools

Naïve Bayes Classification



Words	Positive Doc. Count	Negative Doc. Count	Neutral Doc. Count
my	6	5	5
college	100	100	100
great	40	1	2
the	100	100	100
bad	2	30	2
is	98	99	98
Total count	5000	5000	5000

P(pos|features)

= P(pos)* product of probabilities P(feature|pos)

= P(pos) * P("my"|pos) * P("college"|pos)
* P("is"|pos) * P("great"|pos)

=0.333 * 6/5000 * 100/5000 * 98/5000 * 40/5000