slide 8 <u>Prob</u> <u>Inference</u>

def of cond. prob.

$$P(A|B) = P(A \wedge B)/P(B)$$

product rule

$$P(A \wedge B) = P(A|B)\,P(B)$$

how many items in a full joint prob. dist. for 5 variables?

$$2^5 \Rightarrow \underline{\underline{32}}$$

given these causal relationships in a Bayesian network, we only need $\underline{\underline{10}}$ probabilities.

$$P(hb, do, lo, fo, bp) = P(hb|do, lo, fo, bp)\,P(do, lo, fo, bp)$$

ordered bottom to top b/c evidence matters

⋮ on slide

$$= P(hb|do, lo, fo, bp) \cdot P(do|lo, fo, bp) \cdot P(lo|fo, bp) \cdot P(fo|bp) \cdot P(bp)$$

$$= P(hb|do) \cdot P(do|fo, bp) \cdot P(lo|fo) \cdot P(fo) \cdot P(bp)$$

$$= 0.7 \cdot 0.99 \cdot 0.6 \cdot 0.15 \cdot 0.01$$

$$= 0.0006$$

slide 9 Compactness

- Bayes Net
  - 2^k
  - $n2^k$
- 2^n Full Joint Dist.

ex: $n=30$ $k=5$ (each node has 5 parents)

Bayes Net $= 30 \cdot 2^5 = 960$

Full Joint Dist $= 2^{30} \approx$ over a billion

Slide 24 Bayes nets represent joint probabilities

$$P(fo, \neg lo, do, hb, \neg bp) = P(fo) \cdot P(\neg lo|fo) \cdot P(do|fo, \neg bp) \cdot P(hb|do) \cdot P(\neg bp)$$

$$= 0.15 \cdot \boxed{0.4} \cdot 0.90 \cdot 0.7 \cdot \boxed{0.99}$$

$$= 0.37422$$

Slide 25 Inference In Bayesian Networks

$P(X|E)$

$P(Burglary \mid john, mary)$

$P(X=v \mid E)$

$P(burglary \mid john, mary)$

$P(X_i X_j \mid E)$

$P(Earthquake, Burglary \mid john, mary)$

slide 28 Inference By Enumeration

$P(B|j,m) = P(B,j,m) / P(j,m)$ def. of cond. prob.

$= \alpha\, P(B,j,m)$ α norm. constant b/c common denominator accross all probs

$= \alpha \sum_e \sum_a P(B,e,a,j,m)$ enumeration – sum $P(B,j,m)$ for all values of each hidden var

$= \alpha \sum_e \sum_a P(B) P(e) P(a|B,e) P(j|a) P(m|a)$

$= \alpha P(B) \sum_e P(e) \sum_a P(a|B,e) P(j|a) P(m|a)$

$\vdots$

$= \langle 0.28, 0.72 \rangle$

slide 30 Variable Elimination

$$f_4(A) = \begin{pmatrix} P(j|a) \\ P(j|\neg a) \end{pmatrix} = \begin{pmatrix} 0.90 \\ 0.05 \end{pmatrix}$$

$$\underset{f_m}{f_5}(A) = \begin{pmatrix} P(m|a) \\ P(m|\neg a) \end{pmatrix} = \begin{pmatrix} 0.70 \\ 0.01 \end{pmatrix}$$

$$f_3(A,B,E) = \begin{pmatrix} P(a|b,e) \\ P(a|b,\neg e) \\ P(a|\neg b,\neg e) \\ P(a|\neg b,e) \\ P(\neg a|b,e) \\ P(\neg a|b,\neg e) \\ P(\neg a|\neg b,\neg e) \\ P(\neg a|\neg b,e) \end{pmatrix} = \begin{pmatrix} 0.95 \\ \\ \\ \\ \\ \\ 0.999 \\ \\ \end{pmatrix}$$

slide 30 VE cont.

$$f_6(B,E) = \sum_a f_3(A,B,E) \times f_4(A) \times f_5(A)$$

AJM

$$= \big(f_3(a,B,E) \times f_4(a) \times f_5(a)\big) + \big(f_3(\neg a,B,E) \times f_4(\neg a) \times f(\neg a)\big)$$

$$f_7(B) = \sum_e f_2(E) \times f_6(B,E)$$

EAJM

$$= \big(f_2(e) \times f_6(B,e)\big) + \big(f_2(\neg e) \times f_6(B,\neg e)\big)$$

$$P(B \mid j,m) = \alpha \underset{\underset{B}{\uparrow}}{f_1}(B) \times \underset{\underset{EAJM}{\uparrow}}{f_7}(B)$$

Slide 33 Irrelevant Variables

$$P(J|b) = P(J,b)/P(b)$$
$$= \alpha P(J,b)$$
$$= \alpha \sum_e \sum_a \sum_m P(J,b,e,a,m)$$
$$= \alpha \sum_e \sum_a \sum_m P(J|a)P(b)P(e)P(a|b,e)P(m|a)$$
$$= \alpha P(b) \sum_e P(e) \sum_a P(a|b,e) \cancel{\sum_m} P(J|a) \underbrace{\sum_m P(m|a)}_{1 \text{ by def.}}$$

$$(P(m|a)+P(\neg m|a))=1$$

- remove any leaf node that is not a query or an evidence variable
- every variable that is not an ancestor of a query var or evidence var is irrelevant