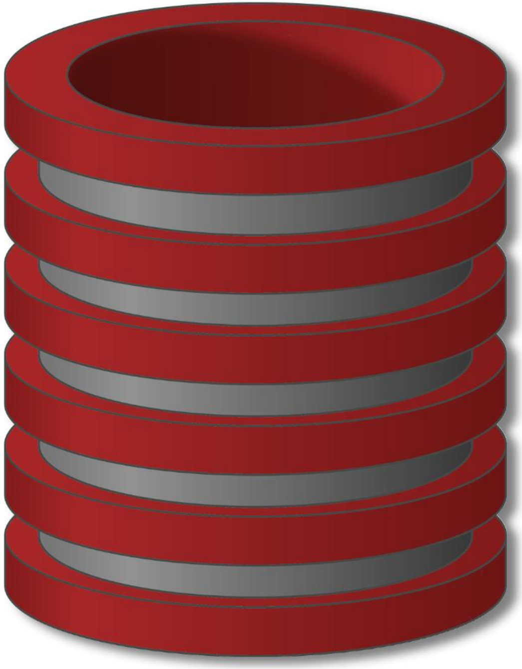

Relational Design Theory

Multivalued Dependencies & 4th Normal Form

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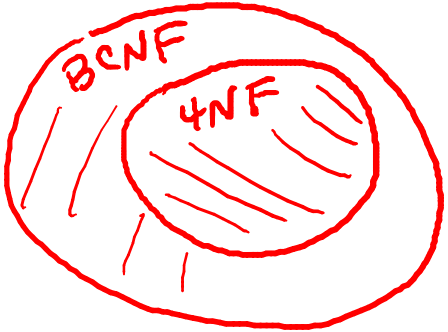
Relational design by decomposition

- "Mega" relations + properties of the data
- System decomposes based on properties
- Final set of relations satisfies normal form
  - No anomalies, no lost information
- Functional dependencies ⇒ Boyce-Codd Normal Form
- Multivalued dependences ⇒ Fourth Normal Form



Example: College application info.

`Apply(SSN, cName, hobby)`

FDs? No.

Keys? All attrs.

BCNF? Yes.

Good design? No.

5 colleges, 6 hobbies → 30 tuples.

Multivalued Dependency

- Based on knowledge of real world
- All instances of relation must adhere

$R \qquad \underline{\bar{A} \twoheadrightarrow \bar{B}} \qquad A_1, \ldots, A_n \qquad B_1, \ldots, B_n$

$\forall t, u \in R: \ t[\bar{A}] = u[\bar{A}]$ then

$\exists v \in R: \ v[\bar{A}] = t[\bar{A}]$ and

$v[\bar{B}] = t[\bar{B}]$ and

$v[rest] = u[rest]$

	$\bar{A}$	$\bar{B}$	rest
t	$\bar{a}$	$\bar{b}_1$	$\bar{r}_1$
u	$\bar{a}$	$\bar{b}_2$	$\bar{r}_2$
v	$\bar{a}$	$\bar{b}_1$	$\bar{r}_2$
w	$\bar{a}$	$\bar{b}_2$	$\bar{r}_1$

tuple-generating dependencies

Apply(SSN, cName, hobby)

SSN →→ cName   SSN →→ hobby

	SSN	cName	hobby
t	123	Stanford	trumpet
u	123	Berkeley	tennis
v	123	Stanford	tennis
w	123	Berkeley	trumpet
	⋮	⋮	⋮

Modified example

`Apply(SSN, cName, hobby)` ☆

Reveal hobbies to colleges selectively ☆

MVDs? None

Good design? Yes.

Expanded example

Apply(SSN, cName, date, major, hobby)

Reveal hobbies to colleges selectively ✓
Apply once to each college one day
May apply to multiple majors ✓

SSN, cName → date

SSN, cName, date ↠ major   "rest" hobby

Trivial Multivalued Dependency

$\bar{A} \twoheadrightarrow \bar{B}$ $\quad \bar{B} \subseteq \bar{A}$ or $\bar{A} \cup \bar{B}$ = all attributes

Nontrivial MVD

otherwise.

no "rest"

$\bar{A}$	$\bar{B}$
a	b

Rules for Multivalued Dependencies

FD-is-an-MVD rule

$\bar{A} \to \bar{B}$ then $\bar{A} \twoheadrightarrow \bar{B}$

$\bar{b}_1 = \bar{b}_2$

	$\bar{A}$	$\bar{B}$	rest
t	$\bar{a}$	$\bar{b}_1$	$\bar{r}_1$
u	$\bar{a}$	$\bar{b}_2$	$\bar{r}_2$
→ v	$\bar{a}$	$\bar{b}_1 = \bar{b}_2$	$\bar{r}_2$
		⋮	

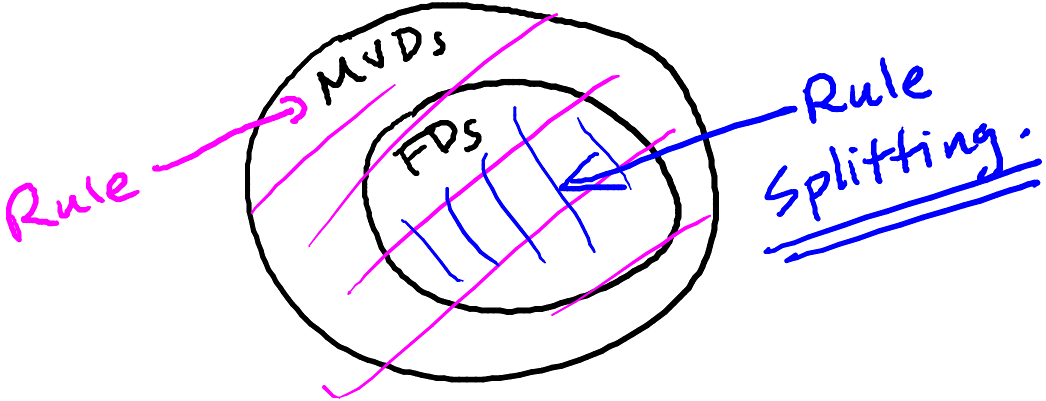
Rules for Multivalued Dependencies

Intersection rule

$\bar{A} \twoheadrightarrow \bar{B}$ $\quad \bar{A} \twoheadrightarrow \bar{C}$ then $\bar{A} \twoheadrightarrow \bar{B} \cap \bar{C}$

Transitive rule

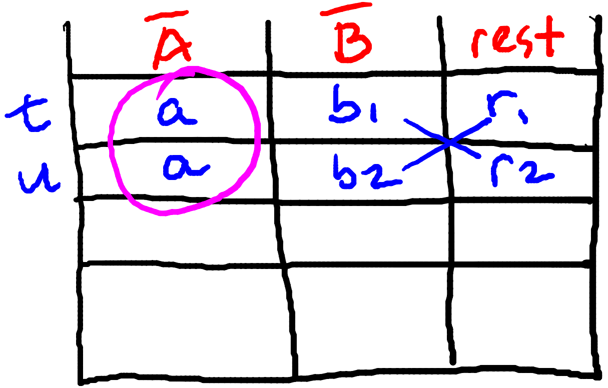
$\bar{A} \twoheadrightarrow \bar{B}$ $\quad \bar{B} \twoheadrightarrow \bar{C}$ then $\underline{\bar{A} \twoheadrightarrow \bar{C}} - \underline{\underline{\bar{B}}}$



Fourth Normal Form

Relation R with MVDs is in 4NF if:

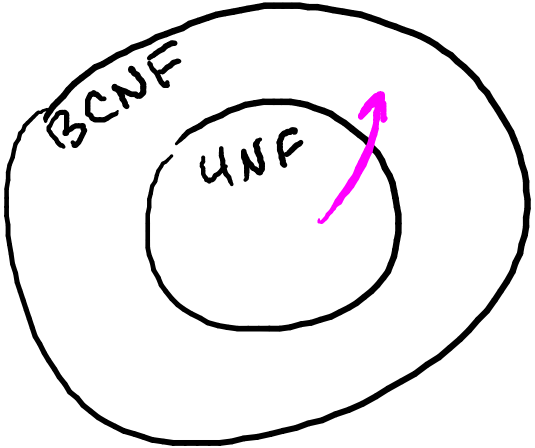
For each nontrivial $A \twoheadrightarrow B$, A is a key



Fourth Normal Form $\Rightarrow$ BCNF

Relation R with MVDs is in 4NF if:

For each nontrivial $A \twoheadrightarrow B$, A is a key



$A \rightarrow B$ A is a key

$\hookrightarrow A \twoheadrightarrow B$

4NF decomposition algorithm

Input: relation R + FDs for R + MVDs for R

Output: decomposition of R into 4NF relations with "lossless join"

Compute keys for R

Repeat until all relations are in 4NF:

- Pick any R' with nontrivial $A \twoheadrightarrow B$ that violates 4NF
- Decompose R' into $R_1(A, B)$ and $R_2(A, rest)$
- Compute FDs and MVDs for R_1 and R_2
- Compute keys for R_1 and R_2

4NF Decomposition Example #1

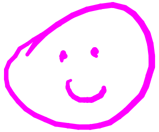
Apply(SSN, cName, hobby)

$SSN \twoheadrightarrow cName$ No keys.

A1(SSN, cName) No FDs

No MVDs

A2(SSN, hobby)



4NF Decomposition Example #2

Apply(SSN, cName, date, major, hobby)

SSN, cName → date ☆

No Keys.

✓ SSN, cName, date →→ major ✗

~~A1 (SSN, cName, date, major)~~

~~A2 (SSN, cName, date, hobby)~~

→ A3 (SSN, cName, date)

A4 (SSN, cName, major)

→ A5 (SSN, cName, hobby)

4NF

Relational design

- Functional dependencies & Boyce-Codd Normal Form

$R(A,B,C) \quad \underline{A} \rightarrow \underline{B}$

- Multivalued dependences & Fourth Normal Form

$R(A,B,C,D) \quad \bar{A} \twoheadrightarrow \bar{B}$