Just One More Spin:
CS7646 Project 1 - Martingale

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***Abstract*—This report will begin to look at the Martingale strategy, and how effective it actually becomes over time through Monte Carlo simulation that simulates an American roulette wheel. The experiments will follow two different kinds:
1) An unlimited bankroll scenario where we can go until we win or reach the maximum 1,000 spins.
2) A realistic bankroll scenario, where the loss condition is going below the bankroll available.

1 INTRODUCTION

The Martingale system is a popular gambling strategy in probability theory that proves to be very successful given with "casinos without table limits and where the gambler has unlimited money" (Pflaumer, 2019), which is a bit unrealistic. The aim of this report is to see how well the martingale system under different assumptions.
To explore the martingale gambling system, this report will try to create a Monte Carlo simulation to simulate winning and losing on an American roulette wheel. On the roulette wheel, there are 38 numbers, two of which are 0 and 00, and the remaining 36 numbers are half red and half black (Casino.org,). This means that the probability of winning on the American style roulette wheel is 47.37%.

1.1 Experiment 1 Setup: Unlimited Resources

The Monte Carlo Simulation will be split into episodes. In each episode, the gambler will sit for a maximum of 1,000 simulations where his aim is to reach $80 (the end condition). The strategy that will be explored in this report follows this pseudo code:

```
episode_winnings = $0
while episode_winnings < $80:
    won = False
```

```
bet_amount = $1
while not won
    if spin > 1,000: exit while loop
    wager bet_amount on black
    won = result of roulette wheel spin
    if won == True:
        episode_winnings = episode_winnings + bet_amount
    else:
        episode_winnings = episode_winnings - bet_amount
        bet_amount = bet_amount * 2
```

Effectively, for every loss, the gambler will double their bet in order to win back their losses. Once they win, the gambler would go back to betting a singular dollar. Note, the effective gain of this strategy is $1 per win. This will continue until either the gambler reaches $80 or the simulation reaches 1,000 spins. Given unlimited money and no limits, a gambler should always gain. It's the equivalent of saying given no loss conditions and unlimited resources, the gambler will always win. For each experiment we will calculate the winning expectations:

$$E[X] = \sum_{x} x P[X = x]$$

Which translates to: given a distribution, X, the expectation is the value x multiplied by its probability of finding x in that distribution for all possible x values.[1] The reason for using expected values of episodes is preferred of simply using the result of one specific random episode is because the episode can vary greatly (as will be seen when looking at the realistic scenarios). That variability means that a single episode isn't representative of the overall experiment, and as such, measures such as mean and expectation are preferred. Also, "Expected value can help investors size up whether an investment's risk is worth the potential reward"(Kenton, 2023)

1.2 Experiment 2 Setup: Realistic Strategy

In reality, no one has unlimited resources, so the next experiment will take into consideration a few small changes to the pseudo code. The gambler will continue so long as they haven't reached $80 or hasn't exhausted his bankroll of $256. Since there is a bankroll, there is also the case that the gambler can not gamble more

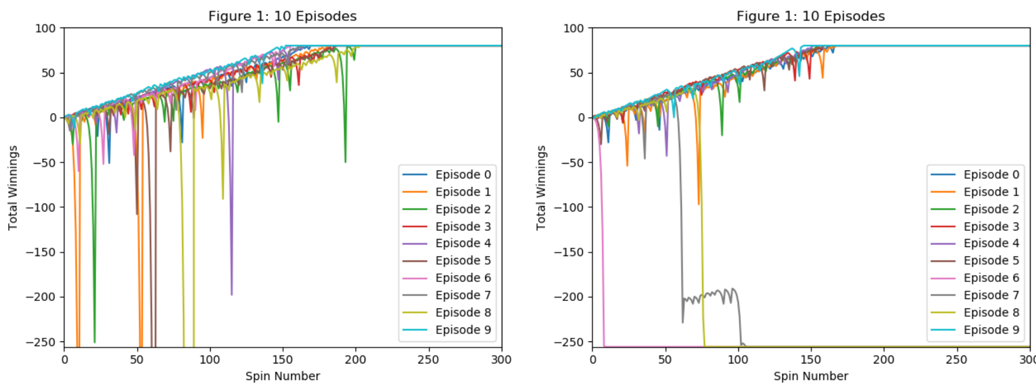
1 Taken from the project brief found on Canvas

than they have. This means that the bet is equal to the minimum between the bet amount doubled or whatever is remaining to ensure they don't bet more than they have.

2 EXPERIMENTAL RESULTS AND ANALYSIS

2.1 Starting Small: 10 Episodes

Taking a look at the Martingale Strategy through 10 episodes reveals why this strategy works well with unlimited money and no limits. Three of the episodes exceed the -256 limit of the chart. One of those three episodes even goes as low as -$16,000+.[2]



(a) Unrealistic Conditions

(b) Realistic Conditions

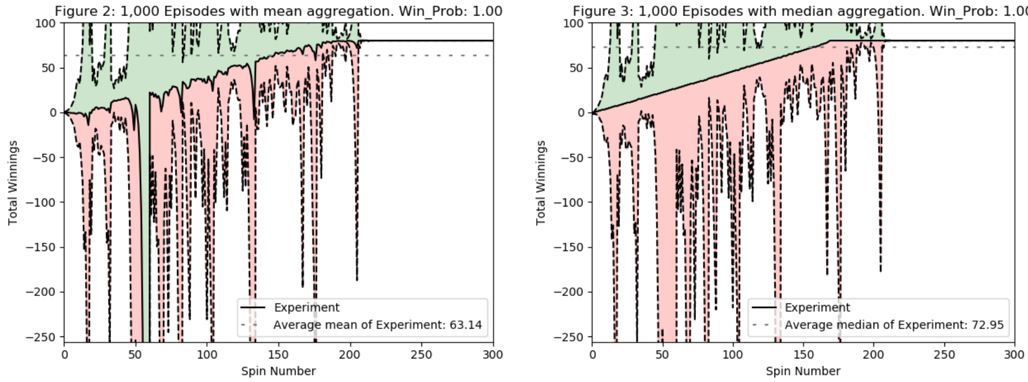
Figure 1—Monte Carlo Simulations with 10 Episodes.

Another interesting fact about the 10 episodes is that in the unrealistic scenario, the win rate is 100%. It makes sense since the only stop condition provided is that the gambler either reaches \$80 or 1,000 spins. On average it took 176.40 ± 16.99 spins to reach a win. Much less than the 1,000 spins. The realistic situation had a 70% chance of winning with an average spin count of 157.57 ± 7.17.

2.2 Unrealistic Scenario: 1,000 Spin

Increasing the amount of episodes to 1,000, we see that the win rate is still 100%. Again, this is due to the fact that there is no stop condition other than the gambler

2 Values calculated by martingale.py and output in p1_results. Tables found in p1_result



(a) Mean ***(b)*** Median

Figure 2—Experiment 1. 1,000 Episodes. Different aggregations of episodes

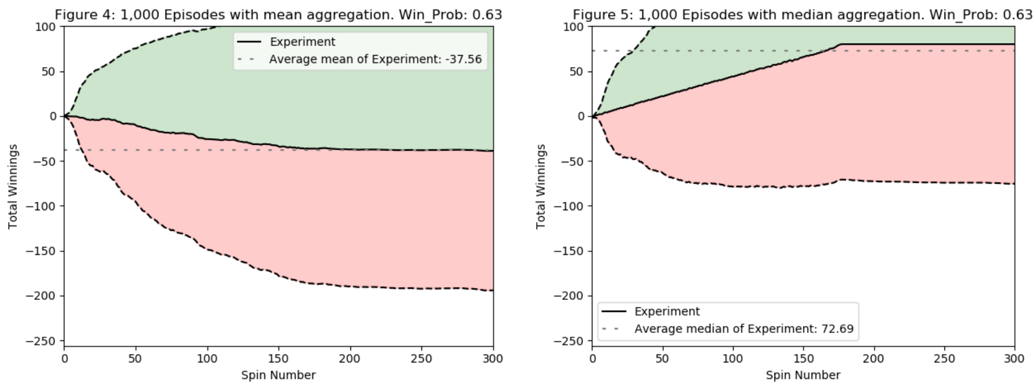
reaching \$80 or 1,000 spins. With an average of 169.23 ± 13.52 spins to reach a win, the gambler was going to win before reaching that 1,000 spin condition. This means with 100% win rate, the expectation of winnings is \$80. [1000 episodes, each ending in 80, means $\mathsf{E}[X] = \$80 * \frac{1000}{1000} = \$80 * 100\% = \$80$]
A visual indicator that the gambler wins 100% of the time would be to look standard deviation bands. Eventually, the bands converge to 0 and stabilize, meaning that there is a point where all episodes converge to \$80.
It also bodes well for the strategy that both the mean and median aggregations of the experiments trends upwards to \$80 meaning that the strategy will yield a winning of \$80 (given the unrealistic conditions of experiment 1). Some interesting facts:

- 203 of the 1,000 episodes went below the -256 limit of the chart. One of them even reaches around negative 4 million dollars before soaring back to a win. It is worth mentioning that the gain of winning a single spin is \$1. That means the gambler lost 4 million dollars before jumping back to a positive \$1 gain.
- That severe drop in winnings is also why there is a large drop in the mean aggregation (Figure 2 (a) Mean) around 50 spins. The median is unaffected by the large spike since the median value of the 1000 spins tends to be a slow and steady rise matching the \$1 gain per win.
- It also appears that the median values stabilize at 169 which is approximately 80/0.4737. The mean value at the same index is approximate \$75.

2.3 Realistic Scenario: 1,000 Spin

Unlike the previous example, here we see a steady rise in the standard deviation bands and what appears to be eventual stabilization, which implies that not all the episodes converged on the $80 win condition:

- 633 episodes reached the $80 win condition (a 63.3% probability of winning)
- 365 episodes lost all $256 in their bankroll
- 2 episodes reach the 1000 spin condition before either winning or losing.
  1 episode reached $45 and the last reached $67



(a) Mean

(b) Median

Figure 3—Experiment 2. 1,000 Episodes. Different aggregations of episodes

The last point that two of the 1000 episodes that don't reach either $80 or -$256 means that the standard deviation never truly stabilizes, but in practice, it levels out enough to be considered stable. If the last 2 are ignored, the standard deviation would ≈ 161.83.

Using the numbers listed above, the expectation for experiment 2 is -$42.688

$$
\begin{aligned}
\mathrm{E}[X] &= \$80 * \frac{633}{1000} + -\$256 * \frac{365}{1000} + \$45 * \frac{1}{1000} + \$67 * \frac{1}{1000} \\
&= \$80 * 63.3\% - \$256 * 36.5\% + \$45 * 0.1\% + \$67 * 0.1\% \\
&= \$50.6400 - \$93.4400 + \$0.0450 + \$0.0670 = -\$42.688
\end{aligned}
$$

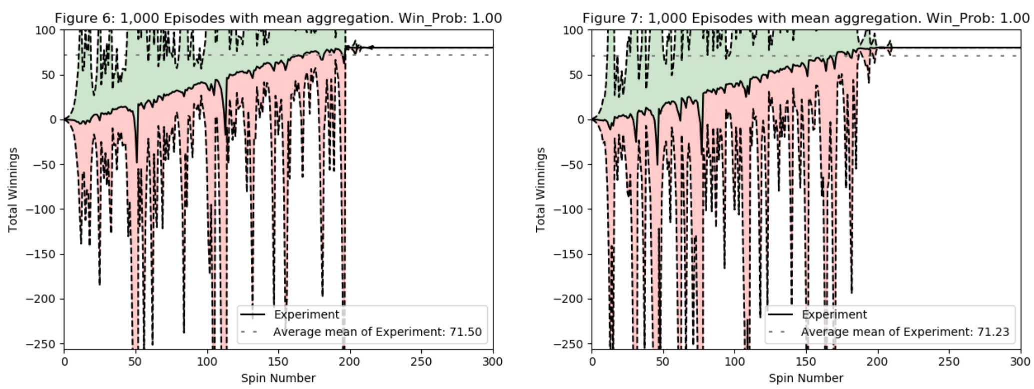
We do see that just like experiment 1, the median of the experiment will eventually stabilize to 80, but at a later spin count (177 instead of 169, approx 4% increase). It reaches 80 due to the almost double count of episodes that end in

\$80. It is unclear to the author why the median stabilizes at \$80 later between the two experiments, but a possible reason could be that when the winnings get closer to -\$256, there isn't enough money to double, and therefore it takes more spins to recover to the point where the bets can be doubled again comfortably, and not limited by the floor of the bankroll. That floor can lead to a situation where winning can no longer gains \$1 as in the unrealistic scenario, and may in fact lose the gambler money and having a harder time catching back up. That slow down could be the reason for that 4% increase in spins-to-win count.

Unlike experiment 1, the mean of the spins trends downward below 0. This is very much because that $\frac{1}{3}$ of the episodes end on -\$256 winnings. -\$256 is 3x more than \$80, and as such, those episodes can pull the mean down harder than the other episodes can pull the mean back up.

3 CONCLUSION

The martingale strategy has some merits, but under very unrealistic conditions. A gambler following the strategy in real life (if not very rich, and unable to find a very high limit table) will generally walk away more of a loser than a winner.



(a) 200 Thousand

(b) 4 Million

Figure 4—Experiment 1. 1,000 Episodes. Mean aggregations of episodes. High Bankroll

4 REFERENCES

[1] Casino.org (n.d.). *A guide to understanding roulette odds*. URL: https://www.casino.org/roulette/odds/.

[2] Kenton, Will (Aug. 2023). *Expected value definition, formula, and examples*. URL: https://www.investopedia.com/terms/e/expected-value.asp.

[3] Pflaumer, Peter (2019). "A statistical analysis of the roulette martingale system: examples, formulas and simulations with R". In.