CS7646 Spring 2022 Project 1 Martingale Report

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1 Answer to Question 1-7

1.1 Question 1: In Experiment 1, based on the experiment results calculate and provide the estimated probability of winning $80 within 1000 sequential bets. Thoroughly explain your reasoning for the answer using the experiment output. Your explanation should NOT be based on estimates from visually inspecting your plots, but from analyzing any output from your simulation.

The probability of winning $80 within 1000 sequential bets is very close to 1.
Probability of winning $80 within 1000 sequential bets = 1 – probability of NOT winning $80
= 1 - (nCr(1000,79) * (18/38)^79 * (20/38)^921 + nCr(1000,78) * (18/38)^78 * (20/38)^922 + … + nCr(1000,0) * (18/38)^0 * (20/38)^1000) ~= 0.99999999
From the output of our simulator, all 10 episodes reach $80 within 1000 sequential bets, which matches with the above calculations.

1.2 Question 2: In Experiment 1, what is the estimated expected value of winnings after 1000 sequential bets? Thoroughly explain your reasoning for the answer.

Since the probability of winning $80 within 1000 bets is nearly equal to 1 (calculated from question 1), the estimated expected value of winnings after 1000 sequential bets is $80. Figure 2 which is generated by our simulator also reaches $80 within 1000 spins.

1.3 Question 3: In Experiment 1, do the upper standard deviation line (mean + stdev) and lower standard deviation line (mean – stdev) reach a maximum (or minimum) value and then stabilize? Do the standard deviation lines converge as the number of sequential bets increases? Thoroughly explain why it does or does not.

No, the upper and lower standard deviation lines do not reach a maximum or minimum and then stabilize. The lines do not converge as the number of sequential bets increases. Since the gambler can use an unlimited bankroll, he can lose unlimited bet amounts within 1000 spins. He can also win all the money he lost in a single spin by the martingale strategy. Since the probability of winning and losing are approximately equal (18/38 for win while 20/38 for loss) in each spin, some episodes can happen to be losing a large amount of money which makes large discrepancies from central tendency. Thus, the standard deviation lines neither converge nor stabilize as the number of bets increase.

1.4 Question 4: In Experiment 2, based on the experiment results calculate and provide the estimated probability of winning $80 within 1000 sequential bets. Thoroughly explain your reasoning for the answer using the experiment output. Your explanation should NOT be based on estimates from visually inspecting your plots, but from analyzing any output from your simulation.

According to the simulation outputs, the probability of winning $80 is 63.7%. By printing the 1000 episodes into an array (size 1000x1000), 637 out of 1000 episodes reached $80 at last while 363 out of 1000 episodes reached -$256 at last. Therefore, probability of winning = 637/1000 = 63.7%

1.5 Question 5: In Experiment 2, what is the estimated expected value of winnings after 1000 sequential bets? Thoroughly explain your reasoning for the answer.

Based on the winning probability stated in question 4, the expected value of winnings after 1000 sequential bets = 63.7% * $80 + 36.3% * (-$256) = -$41.968

1.6 Question 6: In Experiment 2, do the upper standard deviation line (mean + stdev) and lower standard deviation line (mean – stdev) reach a maximum (or minimum) value and then stabilize? Do the standard deviation lines converge as the number of sequential bets increases? Thoroughly explain why it does or does not.

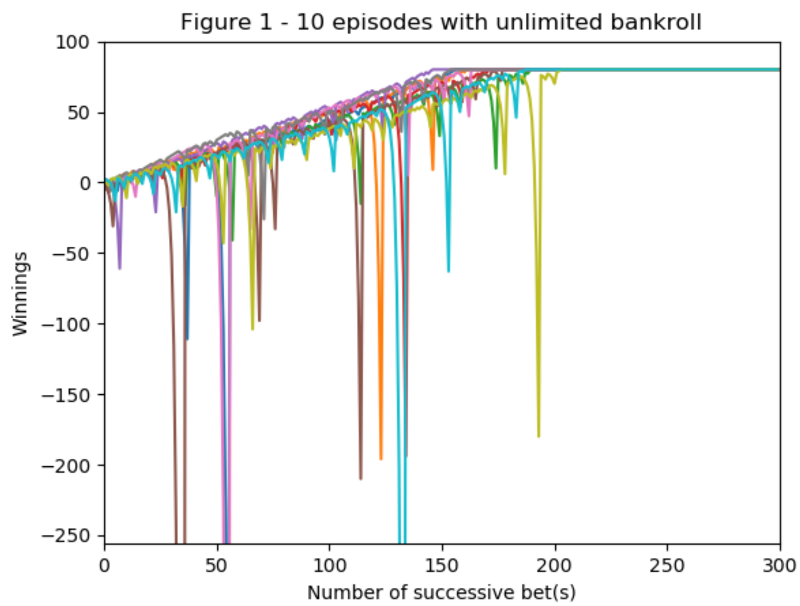
The standard deviation lines become stable and stay parallel (but not converge). Since experiment 2 limits the bankroll to $256, the amount of winnings will stay at -$256 when the gambler loses all his money. The amount of winnings will also stay at $80 when the gambler reaches the winning cap. The winning amounts either towards $80 or -$256 and then stays there for the rest of the spinning rounds. No large amount of losing bets in the sample data so no large discrepancy from central tendency. Thus, the standard deviation lines reach a maximum (or minimum) and then stabilize.

1.7 Question 7: What are some of the benefits of using expected values when conducting experiments instead of simply using the result of one specific random episode?

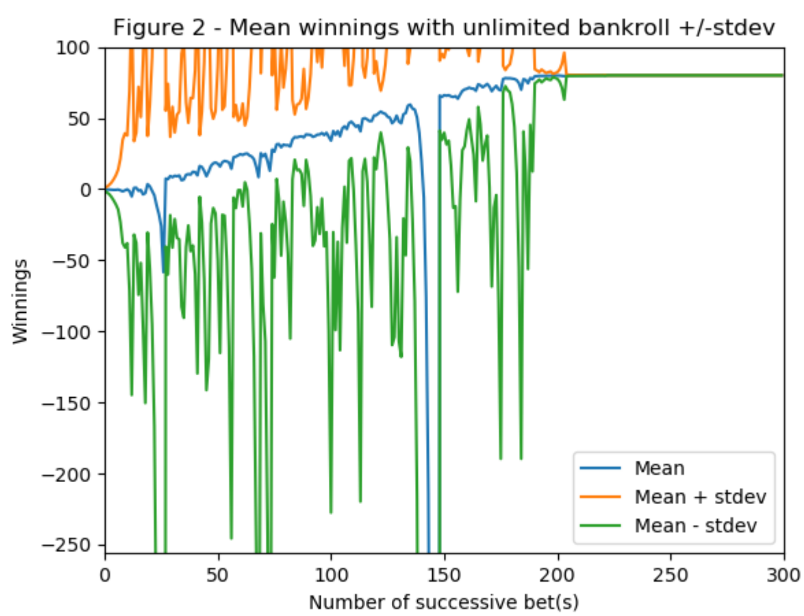
Using expected values means more sample data can be included to calculate mean/ median so that the mean can be more precise, and the error margin can be smaller. Using the result of one random episode could be **un**representative since that specific result could be an outlier from a large sampling which does not represent the correct mean value.

2 Figures generated from simulator

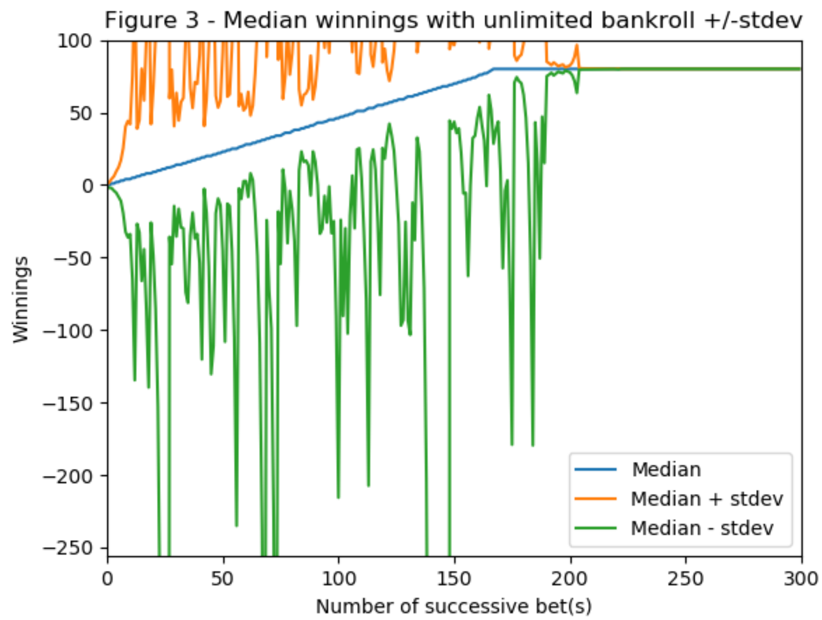
2.1 Figure 1 - 10 episodes with unlimited bankroll



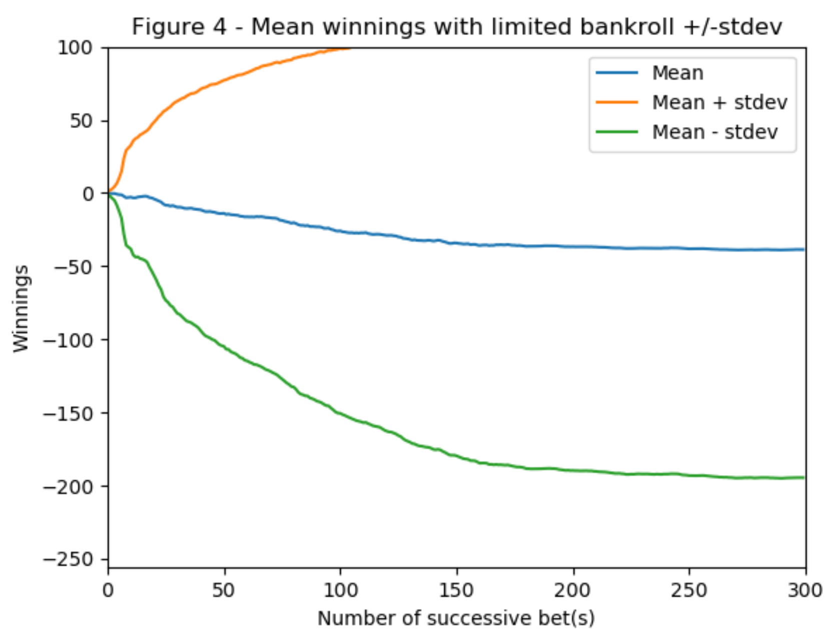
2.2 Figure 2 - Mean winnings with unlimited bankroll +/-stdev



2.3 Figure 3 - Median winnings with unlimited bankroll +/-stdev



2.4 Figure 4 - Mean winnings with limited bankroll +/-stdev



2.5 Figure 5 - Median winnings with limited bankroll +/-stdev