Let A be a 3×5 matrix. Form a 3×6 matrix, B, by making the first 5 columns of B equal to the 5 columns of A, and making the 6th column of B equal to the sum of the columns of A.

Suppose that the nullity of A is 3. For each part, give the number that correctly completes the sentence.

(a) The nullity of B is 4

(b) The nullity of A^T is 1

(c) The nullity of B^T is 1

$$A = \begin{matrix} 3 \\ \\ \end{matrix} \begin{bmatrix} v_1 & v_2 & v_3 & v_4 & v_5 \end{bmatrix} \quad \begin{matrix} \\ \\ 5 \end{matrix}$$

$$B = \begin{bmatrix} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{bmatrix}$$

$v_6 = v_1 + v_2 + v_3 + v_4 + v_5$

nullity of A is 3

rank of A is $5 - 3 = 2$

rank of B is also 2

nullity of B is $6 - 2 = 4$

$$A^T = \begin{matrix} 3 \\ \\ 5 \end{matrix} \begin{bmatrix} - v_1 - \\ \vdots \\ - v_5 - \end{bmatrix}$$

B^T is a 6×3

rank of A^T = rank of A = 2

nullity of $A^T = 3 - 2 = 1$

rank of B^T = rank of B = 2

nullity of B^T is $3 - 2 = 1$