Thm: $A \overset{\text{open}}{\subset} \mathbb{R}^m$, $f: A \to \mathbb{R}^n$.

Suppose that the partial derivatives $D_j f_i(x)$ of the component functions of f exist at each point x of A, and are continuous on A. Then f is differentiable at each point of A.

Such fns f are called continuously differentiable (C^1).

$f: A \to \mathbb{R}^n$

$$f(x_1, \ldots, x_m) = \begin{bmatrix} f_1(x_1, \ldots, x_m) \\ \vdots \\ f_n(x_1, \ldots, x_m) \end{bmatrix}$$

Ex: $n=1$, $m=2$ $\quad f(x,y) = x^2 y - \sin(xy^3)$

is differentiable by the thm.

$D_j f_i : A \to \mathbb{R}$

will also have partial derivatives

$D_k D_j f_i$ "second-order" partial derivatives.

If all partial derivatives up through order r are continuous, then f is called <u>of class C^r</u>.

f is <u>of class C^∞</u> if all partial derivatives (of all orders) are continuous.

Ex: of thm below

$$f(x,y) = x^2y^3 + \sin(xy^2)$$

$$f_x = 2xy^3 + \cos(xy^2)\, y^2$$

$$f_{xy} = 6xy^2 + \cos(xy^2)\cdot 2y + -y^2 \sin(xy^2)\cdot 2xy$$

same

$$f_y = 3x^2y^2 + 2yx\cos(xy^2) \leftarrow \cancel{xy}$$

$$f_{yx} = 6xy^2 + 2y\left[-x\sin(xy^2)\cdot y^2 + \cos(xy^2)\right]$$

(Clairaut's thm)

<u>Thm</u>: Let A be open in $\mathbb{R}^m$; let $f: A \to \mathbb{R}$ be of class C^2. Then for each $a \in A$,

$$D_k D_j f(a) = D_j D_k f(a)$$

ie.

$$\boxed{\frac{\partial^2 f}{\partial x_k \partial x_j}(a) = \frac{\partial^2 f}{\partial x_j \partial x_k}(a)}$$