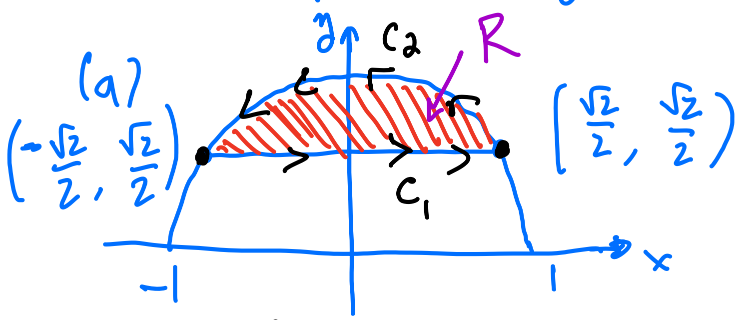
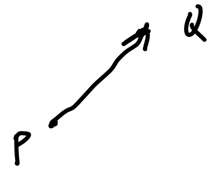
Use a line integral on the boundary to find the area of the regions.

(a)



$C = C_1 \cup C_2$



area of $R = \frac{1}{2}\oint_C x\,dy - y\,dx$

$\vec{r}(t) = \vec{OP} + t(\vec{PQ})$

$0 \le t \le 1$

C_1: $\vec{r}(t) = \left\langle -\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right\rangle + t\left\langle \sqrt{2}, 0 \right\rangle$

$0 \le t \le 1$ $\quad = \left\langle \underbrace{-\frac{\sqrt{2}}{2} + \sqrt{2}\,t}_{x(t)}, \underbrace{\frac{\sqrt{2}}{2}}_{y(t)} \right\rangle$

$dx = \sqrt{2}\,dt$

$dy = 0$

$\frac{1}{2}\int_{C_1}(x\,dy - y\,dx) = \frac{1}{2}\int_0^1 -\frac{\sqrt{2}}{2} \cdot \sqrt{2}\,dt$

$= -\frac{1}{2}\int_0^1 1\,dt = -\frac{1}{2}$

so

$\frac{1}{2}\int_C x\,dy - y\,dx$

$= \frac{\pi}{4} - \frac{1}{2}$

C_2: $x(t) = \cos t$, $y(t) = \sin t$, $\frac{\pi}{4} \le t \le \frac{3\pi}{4}$

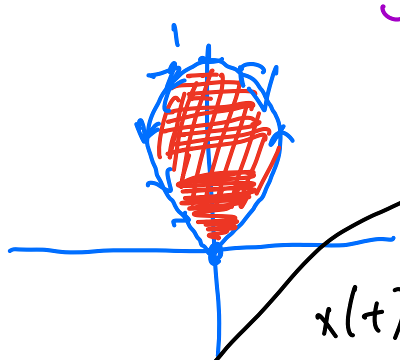
$dx = -\sin t\,dt$, $dy = \cos t\,dt$

$\frac{1}{2}\int_{C_2}(\cos^2 t + \sin^2 t)\,dt = \frac{1}{2}\int_{\pi/4}^{3\pi/4} 1\,dt = \frac{1}{2}\left(\frac{3\pi}{4} - \frac{\pi}{4}\right)$

$= \pi/4$

(b) The region bounded by the curve

$\vec{r}(t) = \langle \underbrace{t(1-t^2)}_{x(t)}, \underbrace{1-t^2}_{y(t)} \rangle$, for $-1 \le t \le 1$



use param from 1 to -1 to make curve go CCW.

$$\frac{1}{2} \int_C x\,dy - y\,dx$$

$x(t) = t - t^3$ $\qquad$ $y(t) = 1 - t^2$

$dx = (1 - 3t^2)\,dt$ $\qquad$ $dy = -2t\,dt$

$$= -\frac{1}{2} \int_{-1}^{1} \underbrace{\left[(-2t)(t-t^3) - (1-t^2)(1-3t^2)\right]}\,dt$$

$$-2t^2 + 2t^4 - (1 - 4t^2 + 3t^4)$$

$$= -2t^2 + 2t^4 - 1 + 4t^2 - 3t^4$$

$$= -t^4 + 2t^2 - 1$$

$$= -\frac{1}{2} \int_{-1}^{1} (-t^4 + 2t^2 - 1)\,dt = \frac{1}{2}\left[-\frac{t^5}{5} + \frac{2}{3}t^3 - t\right]_{-1}^{1}$$

$$= -\frac{1}{2}\left[\left(-\frac{1}{5} + \frac{2}{3} - 1\right) - \left(\frac{1}{5} - \frac{2}{3} + 1\right)\right]$$

$$= -\frac{1}{2}\left[-\frac{2}{5} + \frac{4}{3} - 2\right]$$

$$= \ominus \frac{1}{2}\left[-\frac{6}{15}+\frac{20}{15}-\frac{30}{15}\right]$$

$$= -\frac{1}{2}\left[-\frac{16}{15}\right] = +\frac{8}{15}$$