The Classical Theorems of Vector Analysis

Theorem 1 (Divergence Theorem)**.** If $\mathbf{F}$ is a C^1 vector field defined on a neighborhood of the compact n-manifold with boundary $V \subset \mathbb{R}^n$, then

$$\int_V \text{div } \mathbf{F} = \int_{\partial V} \mathbf{F} \cdot \mathbf{N} \, dA,$$

where N and dA are the outer normal and surface area form of the positively-oriented boundary ∂V.

The integral $\displaystyle\int_{\partial V} \mathbf{F} \cdot \mathbf{N} \, dA$ is sometimes called the **flux** of the vector field $\mathbf{F}$ across the surface ∂V.

Theorem 2. Let D be an oriented compact 2-manifold with boundary in $\mathbb{R}^3$, and let $\mathbf{N}$ and $\mathbf{T}$ be the unit normal and unit tangent vector fields, on D and ∂D, respectively. If $\mathbf{F}$ is a C^1 vector field on an open set containing D, then

$$\int_D (\text{curl } \mathbf{F}) \cdot \mathbf{N} dA = \int_{\partial D} \mathbf{F} \cdot \mathbf{T} \, ds.$$