Closed and Exact Forms

Let ω be a C^1 differential k-form defined on the open set U in $\mathbb{R}^n$. Then ω is called **closed** (on U) if $d\omega = 0$. Also, ω is called **exact** (on U) if there exists a $(k-1)$-form α on U such that $d\alpha = \omega$.

Theorem 1 (Poincare Lemma)**.** Every closed C^1 differential k-form defined on a star-shaped open subset U of $\mathbb{R}^n$ is exact.

Theorem 2. Let F be a C^1 vector field on the star-shaped open set $U \subset \mathbb{R}^3$. Then
(a) curl $F = \mathbf{0}$ if and only if there exists $f\colon U \to \mathbb{R}$ such that $F = \text{grad } f$,
(b) div $F = 0$ if and only if there exists $G\colon U \to \mathbb{R}^3$ such that $F = \text{curl } G$.