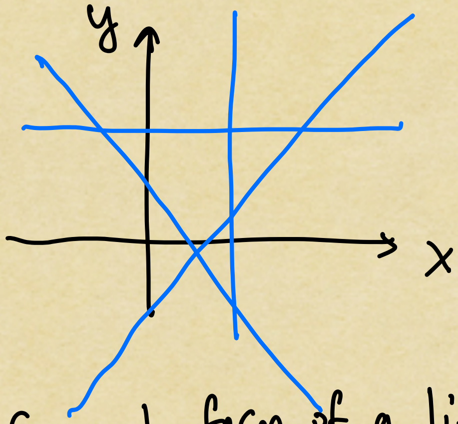
Review lines in 2-D space:



$$y = \underset{\text{slope}}{m}x + \underset{\substack{\text{y-int} \\ (0,b)}}{b}$$

$1x + 0y = c$

$x = c$ vertical lines

undefined slope

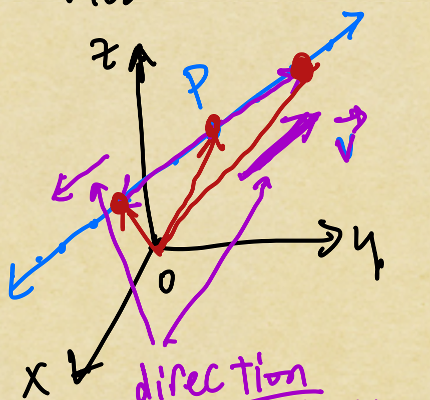
General form of a line

$ax + by = c$

$$\underset{a}{-m}x + \underset{b}{1}y = \underset{c}{b}$$

line is $\{(x,y) : ax + by = c\}$

Lines in 3-D space



Lines in 3D don't have a slope!

Instead, need parametric equation

$-\infty < t < \infty$

$$\boxed{\vec{r}(t) = \vec{OP} + t\vec{v}}$$

$t = 0$: $\vec{r}(0) = \vec{OP}$

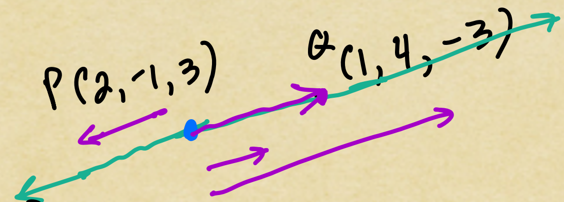
$t = 1$: $\vec{r}(1) = \vec{OP} + \vec{v}$

direction vectors of line

i.e. vector parallel to the line.

$t=-1$: $\vec{r}(-1) = \vec{OP} - \vec{v}$

Ex: Find an equation for the line that passes the points $P(2,-1,3)$ and $Q(1,4,-3)$.



$\vec{PQ} = \vec{v} = \langle 1,4,-3\rangle - \langle 2,-1,3\rangle = \langle -1,5,-6\rangle$.

$$\boxed{\begin{array}{l} \vec{r}(t) = \langle 2,-1,3\rangle + t\langle -1,5,-6\rangle \\ \text{(or)}\ \vec{r}(t) = \langle 2-t,\ -1+5t,\ 3-6t\rangle. \end{array}}$$

Note: We could have used $(1,4,-3)$ also. And could have used $\vec{QP}$, or could have used any nonzero scalar multiple of $\vec{v}$.

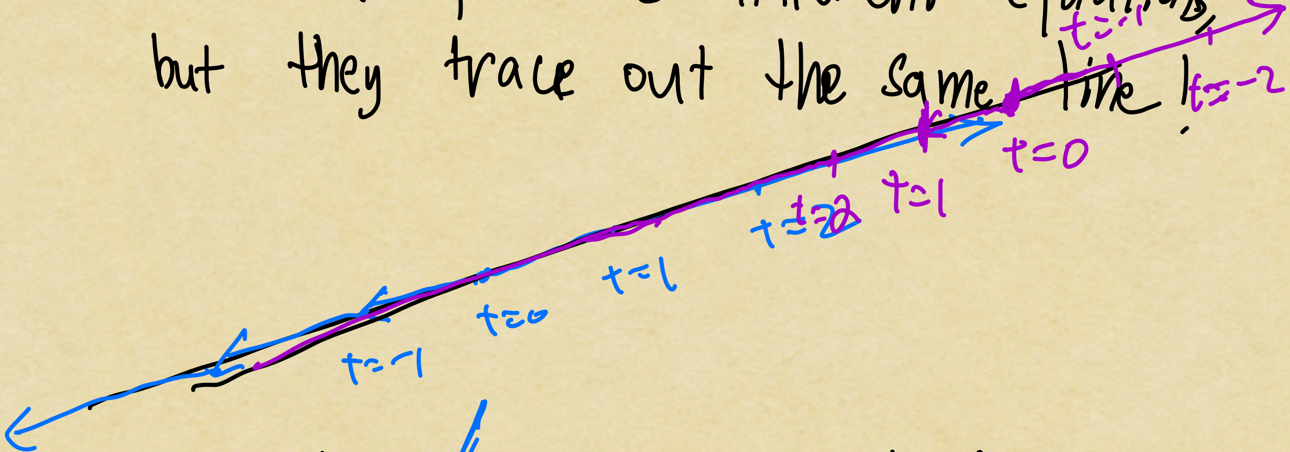
Ex: $\vec{OQ} = \langle 1,4,-3\rangle$

$\vec{w} = -2\vec{v} = \langle 2,-10,12\rangle$

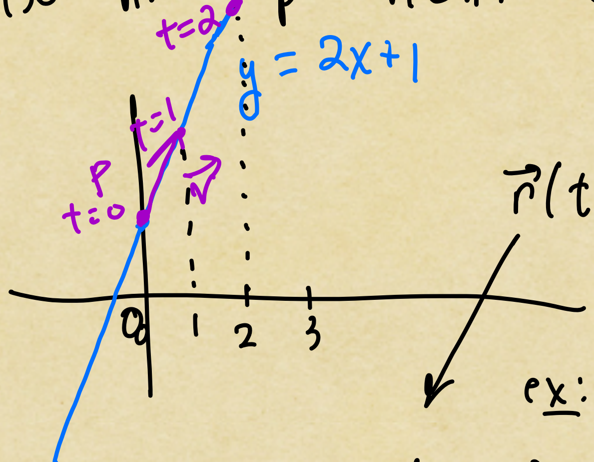
$\vec{r}_1(t) = \langle 1,4,-3\rangle + t\langle 2,-10,12\rangle$

$$\boxed{\vec{r}_1(t) = \langle \underbrace{1+2t}_{x(t)},\ \underbrace{4-10t}_{y(t)},\ \underbrace{-3+12t}_{z(t)} \rangle}$$

$\vec{r}(t)$ and $\vec{r}_1(t)$ are different equations, but they trace out the same line!



Also have parametric eqn's for lines in 2D.



$\vec{r}(t) = \langle t, 2t+1 \rangle$

$\quad\quad\quad\quad\; x(t) \quad y(t)$

ex: $\vec{r}(0) = \langle 0, 1 \rangle$

$\vec{r}(1) = \langle 1, 3 \rangle$

$\vec{r}(2) = \langle 2, 5 \rangle$

$\vdots$

$\vec{r}(t) = \underbrace{\langle 0, 1 \rangle}_{\vec{OP}} + t \underbrace{\langle 1, 2 \rangle}_{\vec{v}}$

$\vec{OP} + t\,\vec{v}$

In general, can parametrize $y = mx + b$ by $\vec{r}(t) = \langle t, mt + b \rangle$.

$ax + by + cz = d$ eq'n of a <u>plane</u>

in 3D