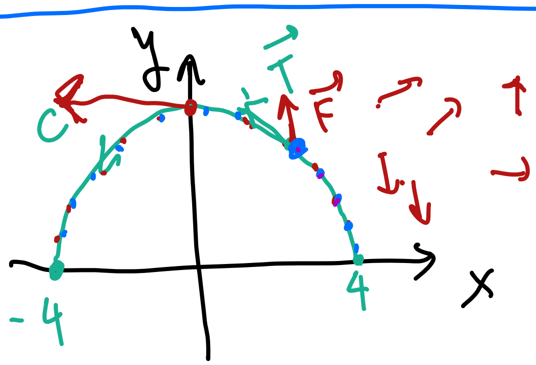
Given the following vector field and oriented curve C, evaluate $\int_C \vec{F}\cdot\vec{T}\,ds$.

function on C

$\vec{F} = \langle -y, x\rangle$ on the semicircle $\vec{r}(t) = \langle 4\cos t, 4\sin t\rangle$, for $0 \le t \le \pi$

$-4\sin t$ $\quad 4\cos t$

$x(t)$ $\quad y(t)$



$$\int_C \vec{F}\cdot\hat{T}\,ds$$

$$= \int_{t_0}^{t_1} \underbrace{\vec{F}\cdot\vec{r}\,'(t)}_{\text{function of } t}\,dt$$

$\vec{F} = \langle -4\sin t, 4\cos t\rangle$

$\vec{r}\,'(t) = \langle -4\sin t, 4\cos t\rangle$

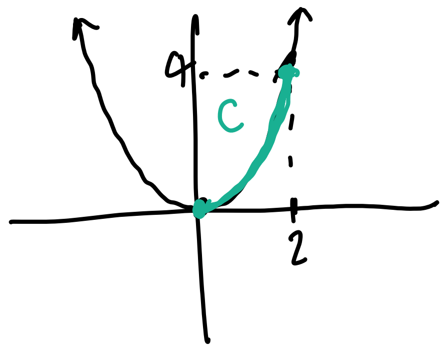
$$\begin{aligned}\vec{F}\cdot\vec{r}\,'(t) &= 16\sin^2 t + 16\cos^2 t\\ &= 16(\sin^2 t + \cos^2 t)\\ &= 16\cdot 1\\ &= 16\end{aligned}$$

$$\int_C \vec{F}\cdot\hat{T}\,ds = \int_0^{\pi} 16\,dt = \boxed{16\pi}$$

Same question for

$\vec{F} = \langle -y, x \rangle$ on the parabola $y = x^2$

from $(0,0)$ to $(2,4)$.



$$\int_C \vec{F} \cdot \hat{T}\, ds$$

$x(t)$ $y(t)$

$$\vec{r}(t) = \langle t, t^2 \rangle$$

$$0 \le t \le 2$$

$$\vec{F} = \langle -y, x \rangle = \langle -t^2, t \rangle.$$

$$\vec{r}\,'(t) = \langle 1, 2t \rangle.$$

$$\vec{F} \cdot \vec{r}\,'(t) = -t^2 + 2t^2$$

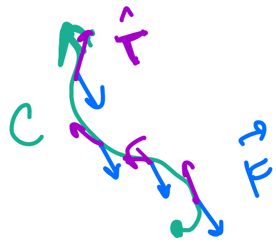
$$= t^2$$

$$\int_C \vec{F} \cdot \hat{T}\, ds = \int_0^2 t^2\, dt$$

$$= \frac{1}{3} t^3 \Big|_0^2$$

$$= \frac{1}{3}(2)^3 - \frac{1}{3}(0)^3$$

$$= \frac{8}{3}$$


$\hat{T}$
C
$\vec{F}$