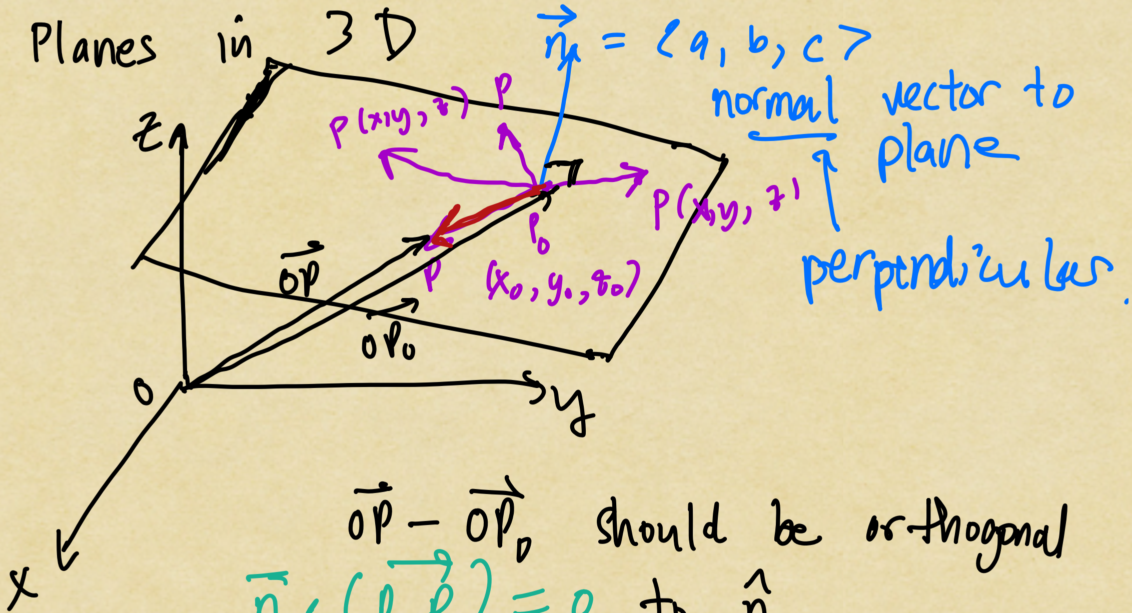
Planes in 3D



$\vec{OP} - \vec{OP_0}$ should be orthogonal to $\hat{n}$.

$\vec{n} \cdot (\vec{P_0P}) = 0$

$$\boxed{\vec{n} \cdot (\vec{OP} - \vec{OP_0}) = 0.}$$

$$\langle a, b, c \rangle \cdot \big(\langle x, y, z \rangle - \langle x_0, y_0, z_0 \rangle \big) = 0$$

$$\langle a, b, c \rangle \cdot \langle x - x_0, y - y_0, z - z_0 \rangle = 0.$$

$$\boxed{a(x - x_0) + b(y - y_0) + c(z - z_0) = 0.}$$

$$ax - ax_0 + by - by_0 + cz - cz_0 = 0.$$

$$\boxed{ax + by + cz = ax_0 + by_0 + cz_0}.$$

has the form $ax + by + cz = d$ for constants a, b, c, and d.

Conversely, any equation of the form

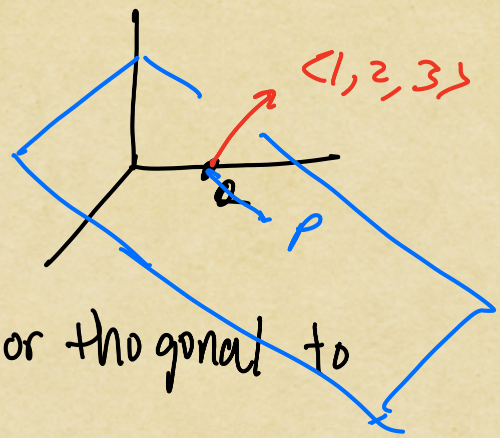
$$ax + by + cz = d$$

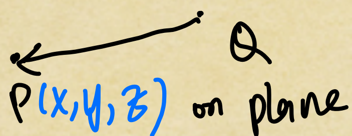
is the equation of a plane.

<u>Ex</u>: $\underline{1}x + \underline{2}y + \underline{3}z = 6$.

$\overset{(x,y,z)}{P\ (6,0,0)}$

$Q\ (0,3,0)$ on plane because they satisfy the equation.





$\overrightarrow{QP} = (-6,3,0)$ is orthogonal to

$\underline{\langle 1,2,3\rangle}$

In general, $\overrightarrow{QP} = \langle x, y-3, z\rangle$ is orthogonal to $\langle 1,2,3\rangle$ because

$$\langle 1,2,3\rangle \cdot \langle x, y-3, z\rangle$$

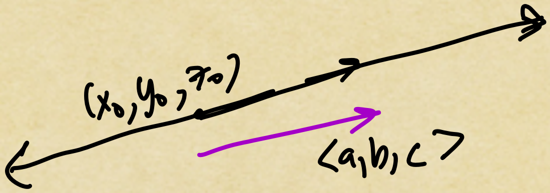
$$= x + 2y - 6 + 3z = 0.$$

Equation of a line

$\vec{r}(t) =$

$\langle x_0 + at,\ y_0 + bt,\ z_0 + ct \rangle$

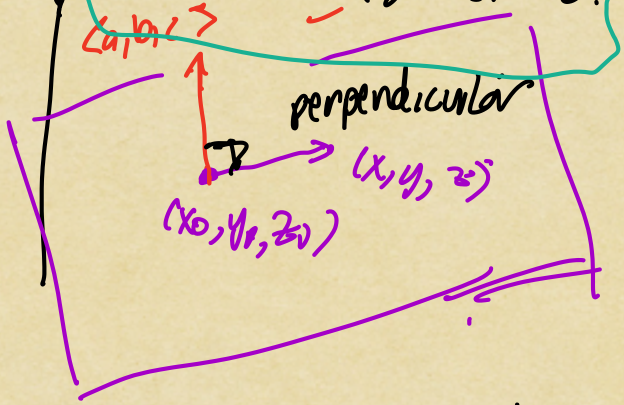
$\vec{r}(t) = \langle x_0, y_0, z_0 \rangle + t\langle a, b, c \rangle$



$\langle a,b,c\rangle$'s are unrelated

Equation of plane

$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0.$



$ax + by + cz = d$

plane w/ normal vector $\langle a, b, c \rangle$.

$$2x + 3y - z = 5$$

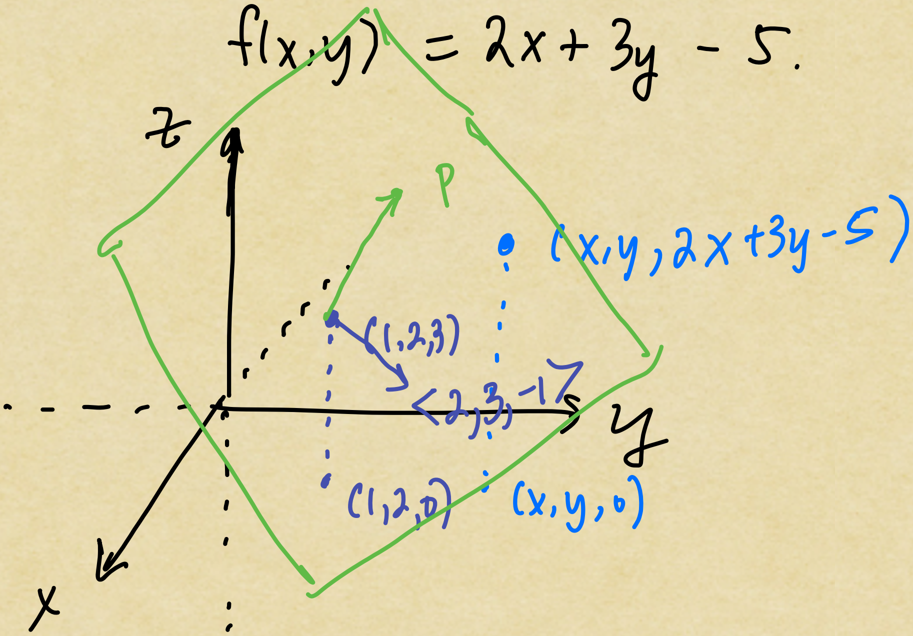
$$z = 2x + 3y - 5$$

$$f(x,y) = 2x + 3y - 5$$

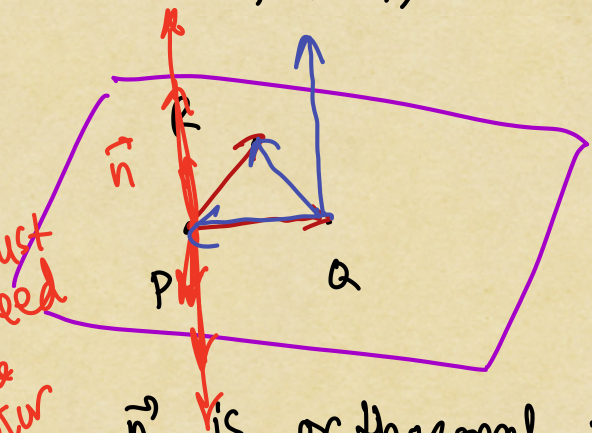
The plane

$\{(x,y,z) : 2x + 3y - z = 5\}$ is the graph of the function $n = \langle 2, 3, -1 \rangle$

$f(x,y) = 2x + 3y - 5.$



__Ex:__ Find an equation of the plane that contains $P(\overset{x_0}{1}, \overset{y_0}{-2}, \overset{z_0}{0})$, $Q(3, 1, 4)$, and $R(0, -1, 2)$.



We have a point on plane ; need a normal vector $\vec{n}$ to the plane.

just need one vector ⊥ to plane.

$\vec{n}$ is orthogonal to both $\overrightarrow{PQ}$ and $\overrightarrow{PR}$.

$\overrightarrow{PQ} = \langle 2, 3, 4 \rangle$ and $\overrightarrow{PR} = \langle -1, 1, 2 \rangle$.

We can take $\vec{n}$ to be $\vec{PQ} \times \vec{PR}$.

$$\vec{PQ} \times \vec{PR} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ -1 & 1 & 2 \end{vmatrix} = \hat{i}(2) - \hat{j}(8) + \hat{k}(5)$$

$$= \langle 2, -8, 5 \rangle = \vec{n}$$

Equation of the plane:

$$\langle 2, -8, 5 \rangle \cdot \langle x-1, y+2, z \rangle = 0$$

$$\boxed{2(x-1) - 8(y+2) + 5z = 0}.$$

$$2x - 2 - 8y - 16 + 5z = 0$$

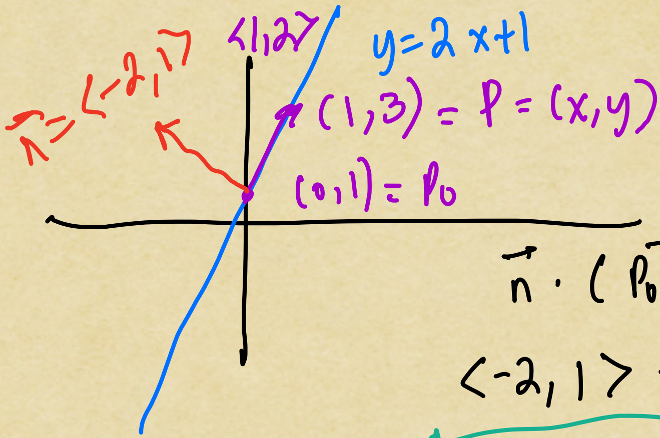
$$\boxed{2x - 8y + 5z = 18}$$

Line	Plane
in 2D	in 3D
$y = mx + b$ ⟷	$z = kx + ly + m$
$ax + by = c$ ⟷	$ax + by + cz = d$

$y - y_0 = m(x - x_0)$

$$m(x-x_0) - (y-y_0) = 0 \longleftrightarrow \boxed{a(x-x_0) + b(y-y_0) + c(z-z_0) = 0}$$

$$a(x-x_0) + b(y-y_0) = 0$$



$$\overrightarrow{P_0P} = \langle 1, 2\rangle$$

$$\vec{n} = \langle -2, 1\rangle$$

$$\overrightarrow{P_0P} = \langle x, y-1\rangle$$

$$\vec{n} \cdot (\overrightarrow{P_0P}) = 0$$

$$\langle -2, 1\rangle \cdot \langle x, y-1\rangle = 0.$$

$$\boxed{-2x + 1(y-1) = 0.}$$