Let $f(x,y,z) = \sin(xy-8) - \ln(z+1) + \frac{2x}{y-z}$.

(a) Compute the gradient ∇f.

(b) Find the equation of the tangent plane to the surface $f(x,y,z) = 4$ at $(4,2,0)$.

(c) Compute the directional derivative $D_{\vec{u}} f(4,2,0)$ where $\vec{u}$ is a unit vector in the direction of $\langle -2,1,0 \rangle$.

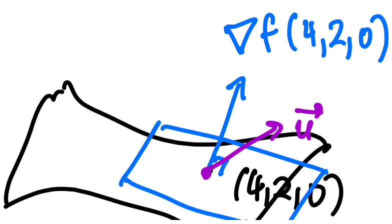
(a) $f_x = y\cos(xy-8) + \frac{2}{y-z}$

$f_y = x\cos(xy-8) - \frac{2x}{(y-z)^2}$

$f_z = \frac{-1}{z+1} + \frac{2x}{(y-z)^2}$

$$\boxed{\nabla f = \left\langle y\cos(xy-8) + \frac{2}{y-z},\ x\cos(xy-8) - \frac{2x}{(y-z)^2},\ \frac{-1}{z+1} + \frac{2x}{(y-z)^2} \right\rangle}.$$

(b)



$$\nabla f(4,2,0) = \left\langle 2\cos(8-8) + \frac{2}{2},\ \overset{4}{\cancel{2}}\cos(8-8) - \frac{2(4)}{2^2},\ \frac{-1}{1} + \frac{8}{2^2} \right\rangle$$

$$= \langle 3, \overset{2}{\cancel{0}}, 1 \rangle.$$

eq'n of plane: $3(x-4) \overset{+2(y-2)}{+} 1(z-0) = 0.$

$$\boxed{3(x-4) + 2(y-2) + z = 0}$$

(c) $|\langle -2, 1, 0\rangle| = \sqrt{(-2)^2 + 1^2 + 0^2} = \sqrt{5}$

$$\vec{u} = \left\langle \frac{-2}{\sqrt{5}}, \frac{1}{\sqrt{5}}, 0 \right\rangle.$$

$$D_{\vec{u}} f(4,2,0) = \nabla f(4,2,0) \cdot \vec{u}$$

$$= \langle 3,2,1 \rangle \cdot \left\langle \frac{-2}{\sqrt{5}}, \frac{1}{\sqrt{5}}, 0 \right\rangle$$

$$= \frac{-6}{\sqrt{5}} + \frac{2}{\sqrt{5}} + 1(0)$$

$$= \frac{-4}{\sqrt{5}}$$