

Graphs and Level Sets

1. **Example:** Find the domain of

(a) $f(x, y) = \frac{x}{y}$

(b) $f(x, y) = \ln(x + y)$

(c) $g(x, y, z) = \frac{1}{\sqrt{4 - x^2 - y^2 - z^2}}.$

Solution:

(a) Need $y \neq 0$, so the domain is $\{(x, y) : y \neq 0\}$.

(b) Need $x + y > 0$, so the domain is $\{(x, y) : x + y > 0\}$. These are all the points above the line $y = -x$.

(c) Need $4 - x^2 - y^2 - z^2 > 0$ so the domain is $\{(x, y, z) : x^2 + y^2 + z^2 < 4\}$, which is the inside of a sphere of radius 2 centered at the origin.

Your turn: Find the domain of the following functions

(a) $f(x, y) = \frac{y - 3}{x^2 - 4}$

(b) $g(x, y) = \ln(x^2 - y)$

(c) $h(x, y) = \sqrt{x - 2y + 4}$

(d) $f(x, y, z) = \frac{3}{\sqrt{x^2 + y^2 + z^2 - 25}}$

2. **Example:** Sketch the level sets for the function $f(x, y) = 4x^2 + 4y^2 + 2$ which correspond to the function values 2, 4, and 10.

Your turn: Let $f(x, y) = 1/2x^2 - y$. Sketch the three level curves on which $f(x, y) = -1$ or 0 or 1

Your turn: Describe the level sets of $z = \frac{x}{x^2 + y}$.

3. **Example:** Let

$$f(x, y) = x\sqrt{x^2 - y}$$

(a) Sketch the domain of f .

(b) Sketch the 0-level set of f .

Solution:

(a) Need $x^2 - y \geq 0$, i.e. $y \leq x^2$

The domain is $\{(x, y) : y \leq x^2\}$. To sketch the domain, draw the x and y -axis and then the parabola $y = x^2$ (not dotted). Then shade everything below it.

(b) Set $x\sqrt{x^2 - y} = 0$

So either $x = 0$ or $\sqrt{x^2 - y} = 0$

i.e. $x = 0$ or $y = x^2$

So on the xy -plane, sketch the vertical line $x = 0$ and the parabola $y = x^2$. That is the 0-level set.