Spherical Coordinates

The spherical coordinates of a point are (ρ, φ, θ).

Example 1. Let $a > 0$. Graph the surface given by the equation

$$\rho = 2a \cos \varphi.$$

SOLUTION: The trick is to manipulate the equation to get $\rho \cos \varphi$. To do this, we multiply both sides by ρ. The surface is given by the equation

$$\rho^2 = 2a\rho \cos \varphi.$$

Now we can convert to Cartesian coordinates. In terms of Cartesian coordinates, the surface is given by the equation

$$x^2 + y^2 + z^2 = 2az.$$

Now we subtract $2az$ from both sides and complete the square:

$$\begin{aligned} x^2 + y^2 + z^2 - 2az &= 0 \\ x^2 + y^2 + z^2 - 2az + a^2 &= a^2 \\ x^2 + y^2 + (z - a)^2 &= a^2 \end{aligned}$$

From the last equation, we see that the surface is the sphere of radius a and center $(0, 0, a)$.

Example 2. Find an equation for the plane $z = -2$ in spherical coordinates.

SOLUTION: Since $z = \rho \cos \theta$, we get

$$\rho \cos \varphi = -2,$$

and since $\varphi \neq \dfrac{\pi}{2}$ we can write this as

$$\rho = \frac{-2}{\cos \varphi}, \quad \text{or} \quad \rho = -2 \sec \varphi.$$