X has a Poisson distribution with parameter $\lambda > 0$ if

$$P(X = k) = e^{-\lambda} \frac{\lambda^k}{k!} \quad \text{for } k = 0, 1, 2, \ldots$$

Recall: $e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$

so $e^{\lambda} = \sum_{k=0}^{\infty} \frac{(\lambda)^k}{k!}$

$$\sum_{k=0}^{\infty} P(X = k) = \sum_{k=0}^{\infty} \left(e^{-\lambda} \frac{\lambda^k}{k!} \right)$$

$$= e^{-\lambda} \sum_{k=0}^{\infty} \frac{\lambda^k}{k!}$$

$$= e^{-\lambda} \cdot e^{\lambda} = 1.$$

Recall: If X has a discrete distribution, the expected value of X is

$$EX \stackrel{\text{def}}{=} \sum_{x} x \, P(X = x).$$

In the case of the Poisson distribution:

$$EX = \sum_{k=0}^{\infty} \left[k \, P(X=k) \right]$$

$$= \sum_{k=1}^{\infty} k \frac{e^{-\lambda} \lambda^{k}}{k!}$$

$$= \lambda \sum_{k=1}^{\infty} \cancel{k} \frac{e^{-\lambda} \lambda^{k-1}}{\cancel{k}(k-1)!}$$

$$= \lambda \underbrace{\sum_{u=0}^{\infty} \frac{e^{-\lambda} \lambda^{u}}{u!}}_{1}$$

$$= \lambda.$$