Any RV (discrete or continuous)

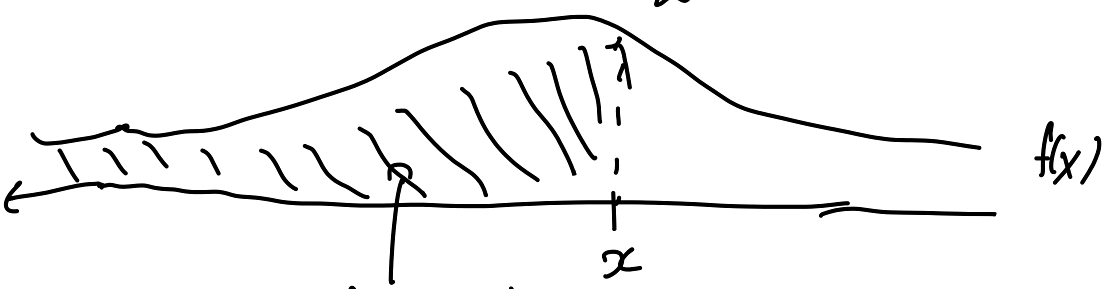
has a <u>distribution function</u>

$$\boxed{F(x) = P(X \le x).}$$

Mainly focus on when $X$ is continuous.

$X$ has a density function, $f(x)$.

$$F(x) = P(X \le x) = \int_{-\infty}^{x} f(t)\,dt$$




area is $F(x)$

So $F(x)$ is increasing (non-decreasing)

$$\lim_{x \to \infty} F(x) = 1.$$

observe: $P(a \le X \le b)$

$$= P(X \le b) - P(X \le a)$$

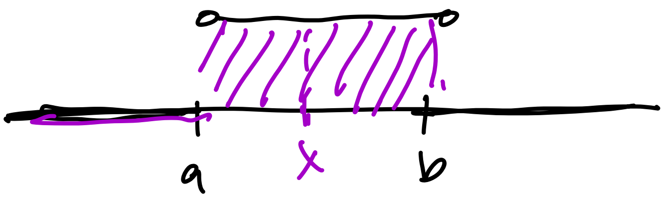
$$= F(b) - F(a).$$

so $$\int_a^b f(x)\,dx = F(b) - F(a)$$

Ex: Uniform dist on $(a, b)$.

$$f(x) = \begin{cases} \frac{1}{b-a}, & a < x < b \\ 0 & \text{otherwise.} \end{cases}$$

$$F(x) = P(X \le x) = \int_{-\infty}^{x} f(t)\,dt = \begin{cases} 0, & x \le a \\ \frac{x-a}{b-a} & a \le x \le b \\ 1 & x > b \end{cases}$$




Let $p > 1$.

Ex: $f(x) = (p-1)\,x^{-p}$, $x \ge 1$.

$$F(x) = \begin{cases} 0 & x \le 1 \\ 1 - x^{-p+1} & x \ge 1 \end{cases}$$

when $x \ge 1$

$$F(x) = \int_1^x (p-1)\,t^{-p}\,dt$$

$$= \frac{(\rho-1)\, t^{-\rho+1}}{-\rho+1} \Big|_1^x$$

$$= -t^{-\rho+1} \Big|_1^x$$

$$= -x^{-\rho+1} - (-(1)^{-\rho+1})$$

$$= 1 - x^{-\rho+1}$$

Ex: Exponential dist.

$$f(x) = \lambda e^{-\lambda x}, \quad x \geq 0.$$

$$F(x) = \begin{cases} 0, & x \leq 0 \\ 1 - e^{-\lambda x} & x > 0. \end{cases}$$

When $x>0$

$$F(x) = \int_0^x \lambda e^{-\lambda t}\, dt = -e^{-\lambda t} \Big|_0^x$$

$$= -e^{-\lambda x} - (-1)$$

$$= 1 - e^{-\lambda x}$$