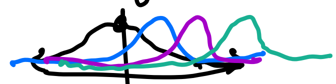
Let $f_n : \mathbb{R} \to \mathbb{R}$ be given by

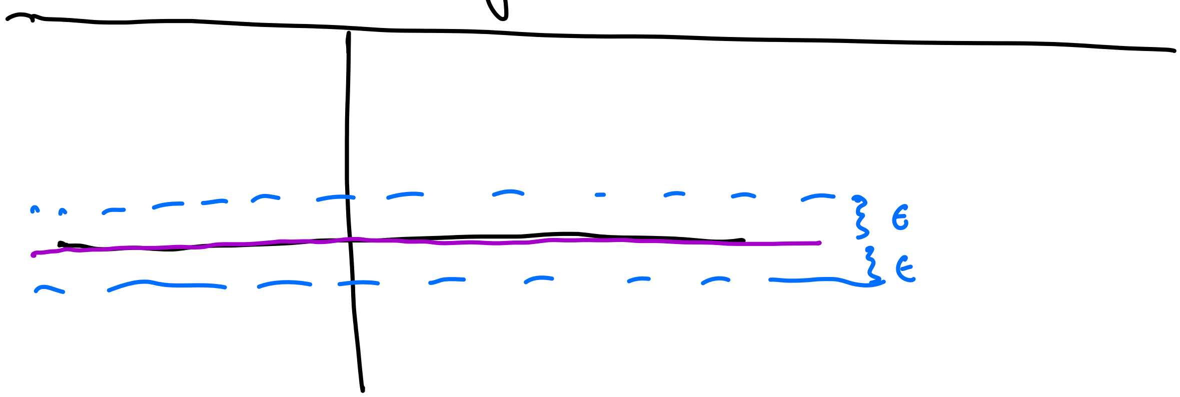
$$f_n(x) = \frac{1}{1+(x-n)^2}.$$

$g(x) = \frac{1}{1+x^2}$



The sequence $f_1, f_2, f_3, \ldots$ converges to the function f defined by $f(x) = 0$.

Is the convergence uniform?



We'll show that the convergence is not uniform. Suppose the convergence is uniform. Let $\varepsilon = 1/2$. Then $\exists N$ s.t.

$\forall n > N$, $\forall x \in \mathbb{R}$, $\quad |f_n(x) - f(x)| < \varepsilon$.

ie. $$\left| \frac{1}{1+(x-n)^2} \right| < \frac{1}{2}$$

But for any n, if $x = n$,

$$\left| \frac{1}{1+(x-n)^2} \right| = 1 > \frac{1}{2},$$

a contradiction. So the convergence is not uniform.