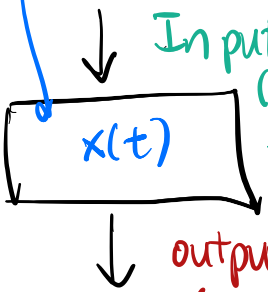
Mixing Problems

Amount of substance in tank at time t

Input Rate
(rate at which substance enters tank)



output rate
(rate at which substance leaves tank)

rate of change in the amt of substance in tank

$$\boxed{\frac{dx}{dt} = \text{input rate minus output rate}}$$

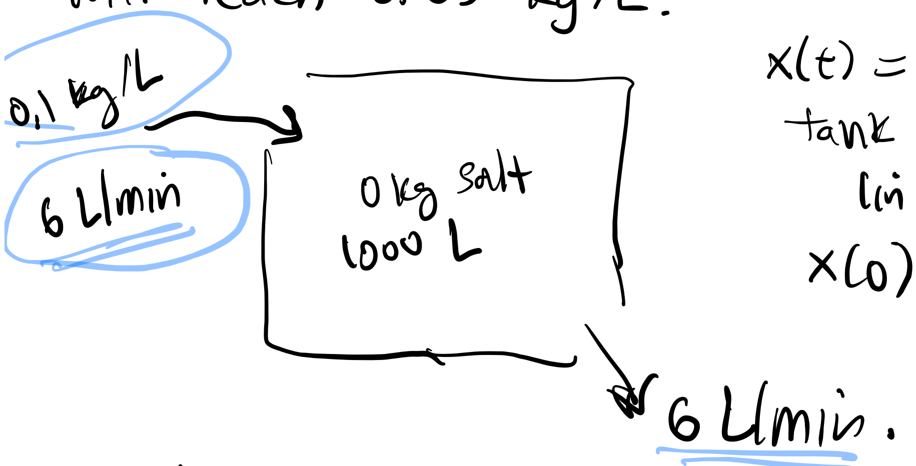
Input Rate: Flow rate (rate at which fluid containing the substance flows into tank) (volume / time)	times	Concentration of substance in fluid coming in (amt / volume)	=	Input Rate (amt / time)

Output Rate (if solution inside tank is kept well-stirred)

Concentration of substance in mixture in tank at time t $= \dfrac{x(t)}{\text{volume of mixture at time } t}$ ← amount of substance in tank at time t

Exit flow rate (exit rate of mixture of fluid in tank) times concentration of substance in mixture in tank at time t = Output Rate

Example: Consider a large tank holding 1000 L of pure water into which a brine solution of salt begins to flow at a constant rate of 6 L/min. The solution inside the tank is kept well stirred and is flowing out of the tank at a rate of 6 L/min. If the concentration of salt in the brine entering the tank is 0.1 kg/L, determine when the concentration of salt in the tank will reach 0.05 kg/L.



$x(t)$ = amt salt in tank at time t (min) (in kg)

$x(0) = 0$

$$\frac{dx}{dt} = \text{input rate} - \text{output rate}$$

$$= (0.1)(6) - (6)\left(\frac{x(t)}{1000}\right)$$

$$\frac{dx}{dt} = \frac{3}{5} - \frac{6}{1000}x \quad , \quad x(0)=0$$

$$\frac{dx}{dt} = \frac{600-6x}{1000}$$

$$\int \frac{1}{600-6x}\,dx = \int \frac{1}{1000}\,dt$$

$$-\frac{1}{6}\ln|600-6x| = \frac{1}{1000}t + C$$

$$\ln(600-6x) = \frac{-6}{1000}t + C'$$

$$600-6x = C''e^{\frac{-6t}{1000}}$$

$$600 - C''e^{\frac{-6t}{1000}} = 6\cancel{x}$$

$$x = 100 - Ce^{-6t/1000}$$

$$x(0)=0$$

$$0 = 100 - C \Rightarrow C = 100$$

$$x(t) = 100 - 100e^{-3t/500}$$

$$\text{conc. of salt} = \frac{100 - 100e^{-3t/500}}{1000}$$

$$= 0.1\left(1 - e^{-3t/500}\right)$$

$$0.1\left(1 - e^{-3t/500}\right) = 0.05$$

$$1 - e^{-3t/500} = \frac{1}{2}$$

$$\frac{1}{2} = e^{-3t/500}$$

$$e^{-3t/500} = \frac{1}{2}$$

$$\frac{-3t}{500} = \ln\left(\frac{1}{2}\right) = \ln 1 - \ln 2$$

$$= 0 - \ln 2$$

$$= -\ln 2$$

$$\Rightarrow \frac{3t}{500} = \ln 2$$

$$t = \frac{500 \ln 2}{3} \approx 115.52$$

$$\boxed{\text{After } \frac{500 \ln 2}{3} \text{ min.}}$$