Find the minimum and maximum of the function $f(x,y) = x + y^2$ subject to the constraint $g(x,y) = 2x^2 + y^2 = 1$.

$$\nabla f = \lambda \nabla g$$
$$2x^2 + y^2 = 1$$

$$\nabla f = \langle 1, 2y \rangle$$
$$\nabla g = \langle 4x, 2y \rangle.$$

$$\begin{cases} (1) & 1 = \lambda 4x \\ (2) & 2y = \lambda 2y \\ (3) & 2x^2 + y^2 = 1 \end{cases}$$

$$\lambda 2y = \lambda 8xy$$
$$\lambda y = 4\lambda x y$$

$\lambda \neq 0$ because then (1) implies $1 = 0$.

$$\Rightarrow \quad y = 4xy$$
$$0 = 4xy - y$$
$$4xy - y = 0$$

$$y(4x-1)=0$$

$y=0$ $4x-1=0$

$x = 1/4$

$y=0: \quad 2x^2+0^2=1$

$$2x^2=1$$

$$x^2 = 1/2$$

$$x = \pm\frac{1}{\sqrt{2}}$$

$\left(\frac{1}{\sqrt{2}}, 0\right), \left(\frac{-1}{\sqrt{2}}, 0\right)$

$x=1/4: \quad 2\left(\frac{1}{4}\right)^2+y^2=1$

$\left(\frac{1}{4}, \sqrt{\frac{7}{8}}\right), \left(\frac{1}{4}, -\sqrt{\frac{7}{8}}\right)$

$$\frac{1}{8}+y^2=1$$

$$y^2=\frac{7}{8}$$

$$y=\pm\sqrt{\frac{7}{8}}$$

(a,b)	$f(a,b)=a+b^2$
$\left(\frac{1}{\sqrt{2}}, 0\right)$	$\frac{1}{\sqrt{2}}<1$
$\left(-\frac{1}{\sqrt{2}}, 0\right)$	$-1/\sqrt{2}$
$\left(\frac{1}{4}, \sqrt{\frac{7}{8}}\right)$	$1/4+7/8 = 9/8$
$\left(\frac{1}{4}, -\sqrt{\frac{7}{8}}\right)$	$1/4+7/8 = 9/8$

maximum of $f(x,y)$ is $9/8$

minimum of $f(x,y)$ is $-1/\sqrt{2}$