

Statement you want to prove:

(*) For every $\varepsilon > 0$, (phrase) $< \varepsilon$.

Statement you proved:

(**) For every $\varepsilon > 0$, (phrase) $< r\varepsilon$
(for some $r > 0$).

"Thm": $(**) \Rightarrow (*)$

"Proof": Let $\varepsilon > 0$.
Let $\varepsilon' = \frac{\varepsilon}{r} > 0$. Then by (**),
(phrase) $< r\varepsilon' = r\left(\frac{\varepsilon}{r}\right) = \varepsilon$.