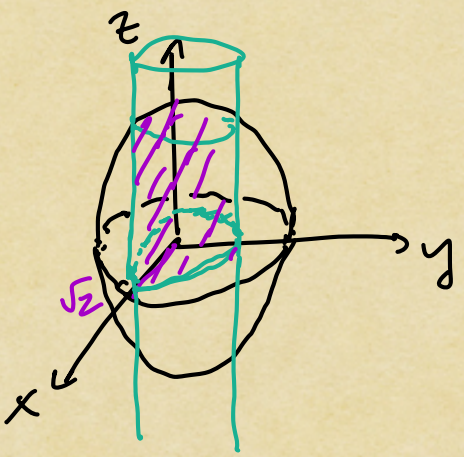
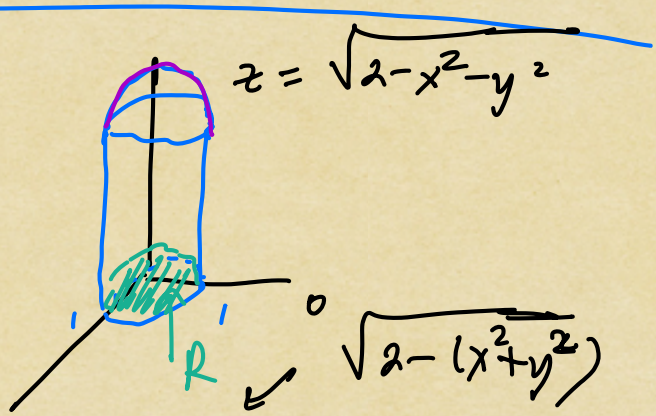
Compute $\iiint_A z\,dV$

where A is the solid inside the sphere $x^2+y^2+z^2=2$, inside the cylinder $x^2+y^2=1$, and above the xy-plane.





$$\iiint_A z\,dV = \int_0^{2\pi}\int_0^{1}\int_0^{\sqrt{2-r^2}} z\,r\,dz\,dr\,d\theta$$

inner: $$r\int_0^{\sqrt{2-r^2}} z\,dz = \frac{r}{2}\,z^2\Big|_0^{\sqrt{2-r^2}}$$

$$= \frac{r}{2}(2-r^2)$$

$$= r - \frac{r^3}{2}$$

middle: $\int_0^1 \left(r - \frac{r^3}{2}\right) dr$

$= \frac{r^2}{2} - \frac{r^4}{8}\Big|_0^1 = \frac{1}{2} - \frac{1}{8} = \frac{3}{8}$

outer: $\int_0^{2\pi} \frac{3}{8}\, d\theta = \frac{3}{8}(2\pi - 0)$

$= \frac{3\pi}{4}$