Absolute Value Inequalities

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Suppose that $|u| < 5$. What does that say about u? It says that u is between -5 and 5, not including -5 or 5. So

$$-5 < u < 5.$$

In general, to say that $|u| < a$ is the same as saying that

$$-a < u < a.$$

Similarly, $|u| \leq a$ is equivalent to

$$-a \leq u \leq a.$$

What if $|u| > a$? This says that the distance from u to 0 must be greater than a. This means that u must be greater than a, or u must be less than $-a$:

$$u < -a \quad \text{or} \quad u > a.$$

Similarly, $|u| \geq a$ is equivalent to

$$u \leq -a \quad \text{or} \quad u \geq a.$$

Example 1. Solve the inequality $|3 - x| > 6$.

SOLUTION: Let $u = 3 - x$. Then we are solving $|u| > 6$. This is equivalent to

$$u > 6 \quad \text{or} \quad u < -6.$$

Since $u = 3 - x$,

$$3 - x > 6 \quad \text{or} \quad 3 - x < -6.$$

Now we solve each inequality separately.

$$\begin{aligned} 3 - x &> 6 \\ 3 &> x + 6 \\ x &< -3 \end{aligned}$$

Also,

$$\begin{aligned} 3 - x &< -6 \\ 3 &< -6 + x \\ x &> 9 \end{aligned}$$

So $x > 9$ or $x < -3$. Translating this into interval notation, we get that the solution is

$$(-\infty, -3) \cup (9, \infty).$$

Example 2. Solve $|3x+3|+3 \leq 6$.

SOLUTION: To solve this problem, we want to rewrite the inequality in the form

$$|\text{something}| \leq \text{a number}.$$

In this example, we can subtract 3 from both sides of the inequality to get

$$|3x+3| < 3.$$

Now we proceed as before:

$$-3 < 3x+3 < 3.$$

For this compound inequality, we are trying to isolate x, so we subtract 3 from both sides to get

$$-6 < 3x < 0.$$

Finally, we divide both each side of the inequality by 3 to get

$$-2 < x < 0.$$

In interval notation, this is $(-2,0)$.

Example 3. Solve the inequality: $|-2x+5| \leq 6$.

SOLUTION: We have

$$-6 \leq -2x+5 \leq 6.$$

Subtracting 5 on each side gives us

$$-11 \leq -2x \leq 1.$$

Dividing each side by -2 gives us

$$\frac{11}{2} \geq x \geq \frac{1}{2}.$$

This is equivalent to

$$\frac{1}{2} \leq x \leq \frac{11}{2}.$$

In interval notation, this is

$$(1/2, 11/2).$$

Example 4. Solve the inequality $|\frac{2x-3}{2}| < \frac{1}{3}$.

SOLUTION: We have

$$-\frac{1}{3} < \frac{2x-3}{2} < \frac{1}{3}.$$

Multiply all sides by the LCM of 3 and 2, which is 6, to give

$$-2 < 3(2x-3) < 2,$$

which is equivalent to

$$-2 < 6x-9 < 2.$$

Adding 9 to both sides, we get

$$7 < 6x < 11.$$

Dividing both sides by 6, we get

$$\frac{7}{6} < x < \frac{11}{6}.$$

In interval notation this is

$$(7/6, 11/6).$$

Example 5. Solve the inequality $|\frac{2x-3}{2}| > \frac{1}{3}$.

SOLUTION: This is equivalent to

$$\frac{2x-3}{2} > \frac{1}{3}$$

or

$$\frac{2x-3}{2} < -\frac{1}{3}$$

For the first inequality we multiply both sides by 6 to get

$$3(2x-3) > 2.$$

This is equivalent to

$$6x - 9 > 2,$$

which is equivalent to

$$x > \frac{11}{6}.$$

For the second inequality we multiply both sides by 6 to get

$$3(2x-3) < -2.$$

Solving, we get

$$6x < 7$$

which is equivalent to

$$x < 7/6.$$

So the answer is

$$(-\infty, 7/6) \cup (11/6, \infty).$$

Example 6. A machine must produce a bearing that is within 0.01 inches of the correct diameter 4.1 inches. Using x as the diameter of the bearing, write this statement using absolute value notation.

SOLUTION: $|x - 4.1| < 0.01$

Example 7. Students who score within 16 points of 84 points will pass a particular test. Write this statement using absolute value notation and use the variable x for the score.

SOLUTION: This says that the distance from the score from 84 must be less than or equal to 16. The distance from x to 84 is $|x - 84|$, so we have

$$|x - 84| \leq 16.$$