Evaluate

$$\iiint_D \frac{x}{x^2+y^2}\, dV$$

where $D$ is the region in the first octant bounded by the sphere $x^2+y^2+z^2=9$ and the planes $x=0$, $y=0$, and $z=0$.

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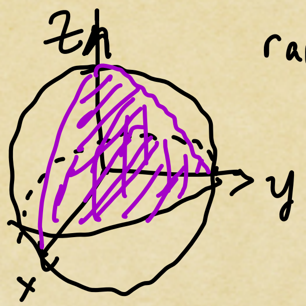
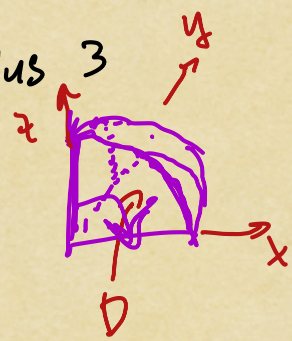



radius 3




$$x = \rho \sin\varphi \cos\theta$$

$$y = \rho \sin\varphi \sin\theta$$

$$dV = \rho^2 \sin\varphi \, d\rho \, d\varphi \, d\theta$$

$$\frac{x}{x^2+y^2} = \frac{\rho \sin\varphi \cos\theta}{\rho^2 \sin^2\varphi \cos^2\theta + \rho^2 \sin^2\varphi \sin^2\theta}$$

$$= \frac{\rho \sin\varphi \cos\theta}{\rho^2 \sin^2\varphi \, (\underbrace{\cos^2\theta + \sin^2\theta}_{1})}$$

$$= \frac{\cancel{\rho} \cancel{\sin\varphi} \cos\theta}{\cancel{\rho^2} \cancel{\sin^2}\varphi}$$

$$= \frac{\cos\theta}{\rho \sin\varphi}$$

$\pi/2$ $\pi/2$ 3

$$\int_0 \int_0 \int_0 \frac{\cos\theta}{\cancel{\rho \sin\varphi}} \, \rho^{\cancel{2}} \cancel{\sin\varphi} \, d\rho \, d\varphi \, d\theta$$

$$= \left[ \int_0^3 \rho \, d\rho \right] \left[ \int_0^{\pi/2} 1 \, d\varphi \right] \left[ \int_0^{\pi/2} \cos\theta \, d\theta \right]$$

$$= \frac{1}{2}\rho^2 \Big|_0^3 \cdot \varphi \Big|_0^{\pi/2} \cdot \sin\theta \Big|_0^{\pi/2}$$

$$= \frac{9}{2} \cdot \frac{\pi}{2} \cdot 1$$

$$= \boxed{\frac{9\pi}{4}}$$