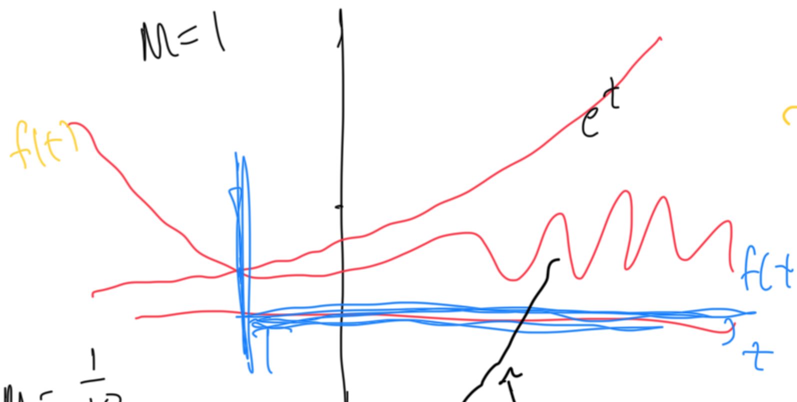
Exponential order

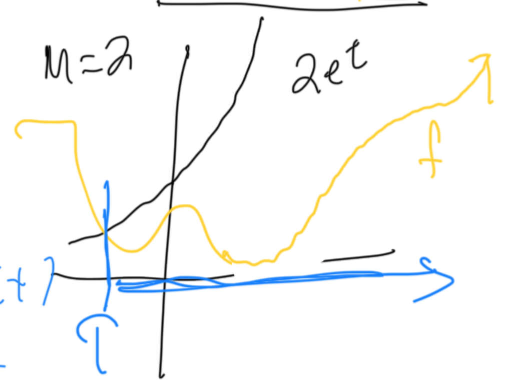
$f(t)$ is of <u>exponential order α</u> if there are constants T and M such that

$$|f(t)| \le M e^{\alpha t} \quad \text{for all } t \ge T.$$

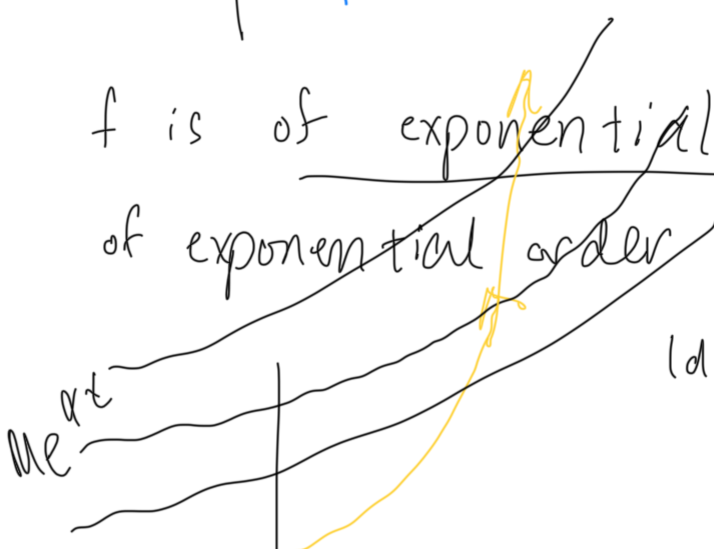
$\boxed{\alpha = 1}$ need $|f(t)| \le M e^{t}$ for all $t \ge T$

eventually this always holds

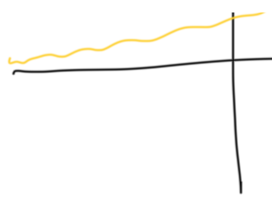




f is of <u>exponential order</u> if it's of exponential order for some α.



Idea: If f grows so fast that it exceeds every



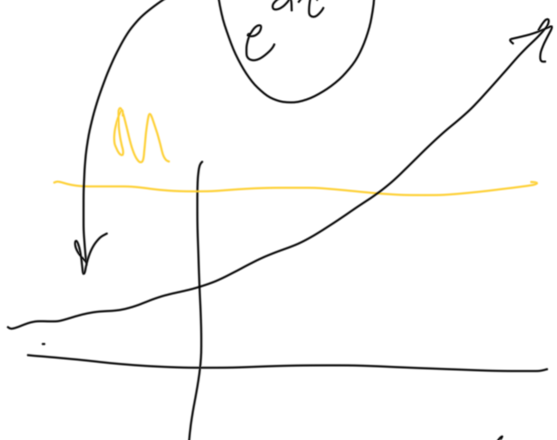
exponential function, then f is not of exponential order.

Ex: $f(t) = e^{t^2}$ is not of exponential order.

Need $|f(t)| \le M e^{\alpha t}$ in order for f to be of expon. order.

i.e. $\dfrac{|f(t)|}{e^{\alpha t}} \le M$

$$\frac{e^{t^2}}{e^{\alpha t}} = e^{t^2 - \alpha t} = e^{t(t-\alpha)} \longrightarrow \infty \text{ as } t \to \infty$$



Ex: $f(t) = e^{6t}$ is of exponential order 6.

Is it true that for some M, T,

$$|f(t)| \le M e^{6t} \quad \text{for all } t \ge T$$

Yes, because you can take $M=1$, $T=0$.

$$|e^{\alpha t}| \leq 1 e^{\alpha t} \quad \text{for all } t \geq 0.$$

Ex: t^n is of exponential order 1. $n \geq 0$ because

$$\frac{t^n}{e^t} \xrightarrow{t \to \infty} 0,$$

so you can take $M=1$.

Is it true that $|t^n| \leq e^t$ for all t past a certain T?

Yes, because eventually,

$$\left|\frac{t^n}{e^t}\right| \leq 1,$$

so $|t^n| \leq e^t$.

In general, if

$$\lim_{t \to \infty} \frac{|f(t)|}{e^{\alpha t}} \text{ exists},$$

then f is of exponential order α.