Linear Algebra Glossary

Here is a list of commonly used terms in linear algebra.

matrix – a rectangular array of numbers. An m by n matrix has m rows and n columns.

$\mathbb{R}^n$ - (n is a positive integer) the set of ordered lists of n numbers. Each element of $\mathbb{R}^n$ is called an n-vector.

vector - an n-vector for some positive integer n.

scalar - a real number. There are contexts in which scalars are taken to be complex numbers, or other kinds of numbers.

zero vector - a vector all of whose components are 0. There is a zero vector with n components for each positive integer n.

non-zero vector - a vector that is not the zero vector

linear equation in n unknowns - an equation of the form $a_1x_1 + \cdots + a_nx_n = b$

solution to a linear equation in n unknowns - an ordered list of n numbers that, when substituted in for the unknowns in the equation, produces a true statement

the (general) solution to a linear equation in n unknowns - the set of all solutions to the linear equation

elementary row operation - interchange two rows; multiply all entries in a row by a nonzero constant; replace one row by the sum of itself and a multiple of another row. When an elementary row operation is applied to the augmented matrix $[A|\mathbf{b}]$ to get $[\tilde{A}|\tilde{\mathbf{b}}]$, the solutions to $A\mathbf{x} = \mathbf{b}$ are the same as the solutions to $\tilde{A}\mathbf{x} = \tilde{\mathbf{b}}$.

row equivalent matrices - matrices that can be obtained from the other by elementary row operations

leading entry - the leading entry of a row of a matrix is the first non-zero entry of the row

(row) echelon form - a matrix is in row echelon form if (i) all zero rows are at the bottom, and (ii) the leading entries go to the right as you move down the rows.

pivot - a leading entry of a matrix in echelon form

reduced row echelon form - a matrix is in reduced row echelon form if it is in row echelon form, all pivots are 1, and all entries above the pivots are 0.

linear combination - if $\mathbf{v}_1, ..., \mathbf{v}_k$ are vectors in $\mathbb{R}^n$, a linear combination of $\mathbf{v}_1, ..., \mathbf{v}_k$ is a vector of the form $\mathbf{v} = c_1\mathbf{v}_1 + \cdots + c_k\mathbf{v}_k$ for scalars $c_1, ..., c_k$.

span of a (finite) set of vectors - the set of all linear combinations of the vectors in the set

linearly dependent set of vectors - a set of vectors $\mathbf{v}_1, ..., \mathbf{v}_k$ in $\mathbb{R}^n$ such that there exist scalars $c_1, ..., c_k$, not all 0, such that $c_1\mathbf{v}_1 + \cdots + c_k\mathbf{v}_k = \mathbf{0}$.

linearly independent set of vectors - a set of vectors $\mathbf{v}_1, ..., \mathbf{v}_k$ in $\mathbb{R}^n$ that is not linearly dependent. In other words, the only way $\mathbf{0}$ can be obtained as a linear combination of $\mathbf{v}_1, ..., \mathbf{v}_k$ is by setting all coefficients equal to 0:

$$c_1\mathbf{v}_1 + \cdots + c_k\mathbf{v}_k = \mathbf{0} \implies c_1 = \cdots = c_k = 0.$$

linear transformation - a function T from $\mathbb{R}^n$ to $\mathbb{R}^m$ with the property that for all vectors $\mathbf{v}$ and $\mathbf{w}$ in $\mathbb{R}^n$ and all scalars c and d, $T(c\mathbf{v} + d\mathbf{w}) = cT(\mathbf{v}) + dT(\mathbf{w})$

standard matrix of a linear transformation T - If $T\colon \mathbb{R}^n \to \mathbb{R}^m$ is a linear transformation, the standard matrix of T is the $m \times n$ matrix A whose jth column is $T(\mathbf{e}_j)$. The matrix A has the property that for every $x \in \mathbb{R}^n$, $T(\mathbf{x}) = A\mathbf{x}$.

square matrix - a matrix that has the same number of rows as columns

main diagonal of a square matrix - for an n by n matrix $A = (a_{ij})$, the list of entries a_{ii} for i from 1 to n

identity matrix - a square matrix that has 1's on the main diagonal and 0's everywhere else. There is an identity matrix I_n for each positive integer n.

diagonal matrix - a matrix such that every entry that is not on the main diagonal is 0. A diagonal matrix is both upper triangular and lower triangular. The identity matrix is a diagonal matrix.

upper triangular matrix - a matrix $A = (a_{ij})$ such that whenever $i < j$, $a_{ij} = 0$. In other words, all the entries below the main diagonal are 0.

lower triangular matrix - a matrix $A = (a_{ij})$ such that whenever $i > j$, $a_{ij} = 0$. In other words, all the entries above the main diagonal are 0.

triangular matrix - a matrix that is either upper triangular or lower triangular

invertible matrix - a square matrix A is invertible if there is a matrix B such that $AB = I_n$ and $BA = I_n$

two matrices are inverses of each other - their product is the identity matrix

a subset of $\mathbb{R}^n$ that is closed under addition - A subset of $\mathbb{R}^n$ is said to be closed under addition if, whenever two vectors are in the set, their sum is also in the set.

a subset of $\mathbb{R}^n$ that is closed under scalar multiplication - A subset of $\mathbb{R}^n$ is said to be closed under scalar multiplication if, whenever a vector is in the set, all of the scalar multiples of that vector are also in the set.

subspace of $\mathbb{R}^n$ - a nonempty subset of $\mathbb{R}^n$ that is closed under addition and closed under scalar multiplication. In other words, a nonempty subset with the following two properties: whenever two vectors are in the set, their sum is also in the set; and whenever a vector is in the set, all of the scalar multiples of that vector are also in the set.

basis for a subspace - a set of vectors in the subspace that is linearly independent and whose span is the subspace. Every subspace of $\mathbb{R}^n$ has a basis. The zero subspace is said to have the empty set as a basis.

standard basis for $\mathbb{R}^n$ - the set of vectors $\mathbf{e}_i$ in $\mathbb{R}^n$ (for $i = 1, ..., n$) with 1 as the ith component and 0 as the other components

dimension of a subspace - the number of vectors in a basis of a subspace. It doesn't matter which basis is used - it is a theorem that every basis for a subspace has the same number of elements.

nullspace of a matrix A - the set of solutions to the equation $A\mathbf{x} = \mathbf{0}$. If A is an $m \times n$ matrix, the nullspace of A is a subspace of $\mathbb{R}^n$.

column space of a matrix A - the span of the columns of A. If A is an $m \times n$ matrix, the column space of A is a subspace of $\mathbb{R}^m$.

row space of a matrix A - the span of the rows of A (the rows are viewed as vectors). If A is an $m \times n$ matrix, the row space of A is a subspace of $\mathbb{R}^n$.

rank of a matrix A - the dimension of the row space of A; the dimension of the column space of A; the number of pivots/pivot rows/pivot columns in the RREF of A; the maximum number of linearly independent columns of A; the number of non-zero rows in the RREF of A.

nullity of a matrix A - the dimension of the nullspace of A. It is equal to the number of free columns in the RREF of A.

determinant of a square matrix A - a number assigned to the matrix. It can be defined by the cofactor expansion. It is also characterized by a few axioms: linearity in each row; the determinant of I_n is 1; and swapping two rows multiplies the determinant by -1.

eigenvector of a square matrix A - a non-zero vector $\mathbf{x}$ such that $A\mathbf{x} = \lambda\mathbf{x}$ for some scalar λ

eigenvalue of a square matrix A - If $\mathbf{x}$ is an eigenvector of A, then $A\mathbf{x} = \lambda x$ for some scalar λ. The scalar λ is called an eigenvalue of A.

characteristic polynomial of a square matrix A - the polynomial $p(\lambda) = \det(A - \lambda I)$. The eigenvalues of A are the roots of the characteristic polynomial of A.

diagonalizable matrix - a square matrix A for which there exist a square matrix P and a square matrix D such that D is a diagonal matrix and $A = PDP^{-1}$. Every diagonal matrix is diagonalizable, because $D = IDI^{-1}$. When a matrix is diagonalizable, it is easy to calculate large powers of the matrix, because $A^k = PD^kP^{-1}$.