This problem is about the following matrix:

$$A = \begin{bmatrix} 1 & 0 \\ 1 & 3 \end{bmatrix}$$

(a) Give the eigenvalues of A.

char poly is $\lambda^2 - 4\lambda + 3 = (\lambda-1)(\lambda-3) = 0$

$\boxed{\lambda = 1, 3}$

(b) Find a basis for $\mathbb{R}^2$ consisting of eigenvectors for A.

$\boxed{\left\{ \begin{bmatrix} 3 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}}$

$\lambda=1$: $A - I = \begin{bmatrix} 0 & 0 \\ 1 & 3 \end{bmatrix}$ $\quad \vec{v}_1 = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$

$\lambda=3$: $A - 3I = \begin{bmatrix} -2 & 0 \\ 1 & 0 \end{bmatrix}$ $\quad \vec{v}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

(c) Give a diagonal matrix D and invertible matrix P such that $A = PDP^{-1}$. Do NOT compute P^{-1} explicitly.

$D = \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}$ $\qquad D = \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}$

$P = \begin{bmatrix} 3 & 0 \\ -1 & 1 \end{bmatrix}$ $\qquad P = \begin{bmatrix} 0 & 3 \\ 1 & -1 \end{bmatrix}$

(d) Give a simple expression for A^{10} as a product of other matrices (not including A itself). Give the entries of your matrices explicitly and make your expression as simple as possible, without doing excessive arithmetic or finding inverses.

$A^{10} = PD^{10}P^{-1}$

$$\boxed{= \begin{bmatrix} 3 & 0 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 3^{10} \end{bmatrix} \begin{bmatrix} 3 & 0 \\ -1 & 1 \end{bmatrix}^{-1}}$$