Let A be the matrix

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 4 & -3 \\ 3 & 2 & -1 \end{bmatrix}$$

(a) Give all eigenvalues of A.

$$A - \lambda I = \begin{bmatrix} 1-\lambda & 0 & 0 \\ 2 & 4-\lambda & -3 \\ 3 & 2 & -1-\lambda \end{bmatrix}$$

$\det(A - \lambda I)$

$= (1-\lambda)\left[(4-\lambda)(-1-\lambda) - (2)(-3)\right]$

$= (1-\lambda)(-4-3\lambda+\lambda^2+6)$

$= (1-\lambda)(\lambda^2-3\lambda+2)$

$= \underbrace{(1-\lambda)(\lambda-1)}_{\text{alg multip. of 1 is } 2}\underbrace{(\lambda-2)}_{\text{alg mult of 2 is 1}} = 0$

$\boxed{\lambda = 1, 2}$

(b) For each (distinct) eigenvalue λ, give the dimension of the λ-eigenspace (the geometric multiplicity).

$\lambda = 1$:

$$A - 1I = \begin{bmatrix} 0 & 0 & 0 \\ 2 & 3 & -3 \\ 3 & 2 & -2 \end{bmatrix}$$

$\operatorname{rank}(A - I) = 2$

By rank-nullity,

nullity of $A - I$ is $3 - 2$

$= 1$

$\lambda = 2$:

$$A - 2I = \begin{bmatrix} -1 & 0 & 0 \\ 2 & 2 & -3 \\ 3 & 2 & -3 \end{bmatrix} \to \begin{bmatrix} -1 & 0 & 0 \\ 0 & 2 & -3 \\ 0 & 2 & -3 \end{bmatrix} \to \begin{bmatrix} -1 & 0 & 0 \\ 0 & 2 & -3 \\ 0 & 0 & 0 \end{bmatrix}$$

rank is 2

nullity is 1

(c) Is A diagonalizable? Answer YES or NO and explain briefly. DO NOT find matrices P, D etc.

No, because the alg. mult of 1 is not equal to the geom. mult of 1.