Chain Rule Practice

1) Find the derivative of

$$y = \sqrt{3x + \sqrt{4x + \sqrt{5x}}}$$

If
$h(x) = f(g(x))$
then
$h'(x) = f'(g(x)) \cdot g'(x)$

$$y = \left(3x + \sqrt{4x + \sqrt{5x}}\right)^{1/2}$$

$$y' = \frac{1}{2}\left(3x + \sqrt{4x + \sqrt{5x}}\right)^{-1/2} \cdot \frac{d}{dx}\left(3x + \sqrt{4x + \sqrt{5x}}\right)$$

$$y' = \frac{1}{2}\left(3x + \sqrt{4x + \sqrt{5x}}\right)^{-1/2}\left[3 + \frac{d}{dx}\sqrt{4x + \sqrt{5x}}\right]$$

$y = x^{1/2}$

$y' = \frac{1}{2}x^{-1/2}$

$= \frac{1}{2} \cdot \frac{1}{\sqrt{x}}$

$= \frac{1}{2\sqrt{x}}$

$$\left[\frac{1}{2\sqrt{4x + \sqrt{5x}}} \cdot \frac{d}{dx}\left(4x + \sqrt{5x}\right)\right]$$

$$\left[\frac{1}{2\sqrt{4x + \sqrt{5x}}}\left(4 + \sqrt{5} \cdot \frac{1}{2\sqrt{x}}\right)\right]$$

$\sqrt{5x} = \sqrt{5}\sqrt{x}$

$$y' = \frac{1}{2\sqrt{3x + \sqrt{4x + \sqrt{5}}}}\left[3 + \frac{1}{2\sqrt{4x + \sqrt{5}}}\left(4 + \frac{\sqrt{5}}{2\sqrt{x}}\right)\right]$$

2) (a) Find the equation of the tangent line to the graph of $f(x) = \sqrt{x^2+3}$ at the point $(-1, 2)$.

(b) Find the equation of the normal line to the graph of $f(x) = \sqrt{x^2+3}$ at the point $(-1, 2)$.

perpendicular to tangent line

(c) Where does the normal line in (b) intersect the x-axis?

(a) $f(x) = \sqrt{x^2+3}$

$$f'(x) = \frac{1}{2\sqrt{x^2+3}} \cdot \frac{(2x+0)}{1} = \frac{\cancel{2}x}{\cancel{2}\sqrt{x^2+3}}$$

$$= \frac{x}{\sqrt{x^2+3}}$$

$$f'(-1) = \frac{-1}{\sqrt{(-1)^2+3}} = \frac{-1}{\sqrt{1+3}} = \frac{-1}{\sqrt{4}} = -\frac{1}{2}$$

m

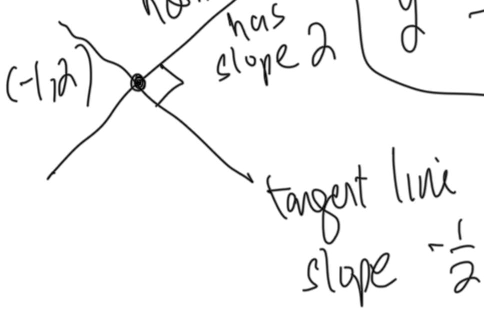
$(x_0, y_0) = (-1, 2)$

eq'n of tangent line

$$y - 2 = -\frac{1}{2}(x - (-1))$$

$$y - 2 = -\frac{1}{2}(x + 1)$$

$$y = 2 - \frac{1}{2}x - \frac{1}{2} = \boxed{-\frac{1}{2}x + \frac{3}{2} = y}$$



(b) Since tangent line has slope $-\frac{1}{2}$, the normal line has slope $-\frac{1}{\left(-\frac{1}{2}\right)} = -(-2) = 2$.

$(x_0, y_0) = (-1, 2)$

$$y - 2 = 2(x - (-1))$$

$$y - 2 = 2(x + 1)$$

$$y - 2 = 2x + 2$$

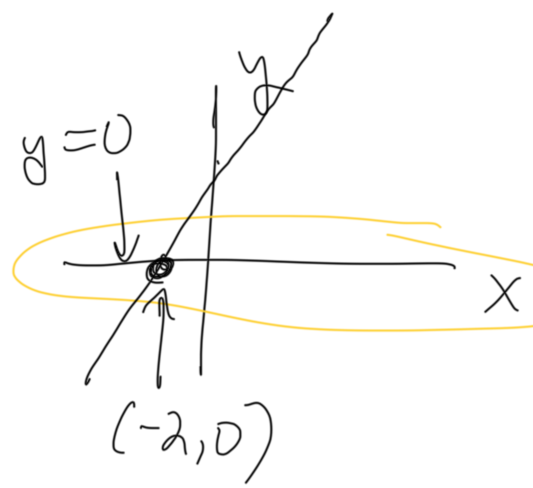
$$\boxed{y = 2x + 4}$$

(c)
$$2x + 4 = 0$$

$$2x = -4$$

$$x = -2$$

$$\boxed{\text{at } (-2, 0)}$$



③ Find the 30th derivative of

$$y = \cos(2x)$$

$$y' = -\sin(2x)(2) = \underbrace{-2}\,\underbrace{\sin(2x)}$$

$$y'' = \underbrace{-2}\cos(2x)\underbrace{(2)}$$

$$y'' = \underbrace{-4}\,\underbrace{\cos(2x)}$$

$$y''' = y^{(3)} = -4(-2)\sin(2x) = 8\sin(2x)$$

y	$1\cos 2x$
$-\ y'$	$-2\sin 2x$
$-\ y''$	$-4\cos 2x$
$+\ y^{(3)}$	$+8\sin(2x)$
$+\ y^{(4)}$	$+\ \underline{\text{const}}\cos$
$-\ y^{(5)}$	$-$
$-\ y^{(6)}$	$-$
$y^{(7)}$	$+$

$$y^{(30)} = -2^{30}\cos(2x)$$

1 2	$-$
3 4	$+$
5 6	$-$
7 8	$+$
... 29 30	$-$
31 32	$+$