NANYANG TECHNOLOGICAL UNIVERSITY

CZ4041 Machine Learning

Assignment 2

Iterated 3-Player Prisoner’s Dilemma

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Objective:

Design a competitive policy/strategy for an agent in an iterated 3-player prisoner's dilemma tournament and get a good ranking.

The pay-off given:

$$\mathrm{U(DCC)} = 8,$$

$$\mathrm{U(CCC)} = 6,$$

$$\mathrm{U(DDC)} = \ \mathrm{U(DCD)} \ = 5,$$

$$\mathrm{U(CDC)} = \ \mathrm{U(CCD)} = 3,$$

$$\mathrm{U(DDD)} = 2,$$

$$\mathrm{U(CDD)} \ = 0$$

Where the first argument refers to the decision - detect (D) or cooperate (C) - of the subject agent itself while the second and third are the decisions made by the two opponents.

Problem Solving Approach:

I decided to program explicitly what the agent is supposed to do under certain conditions. Multiple potential policies are implemented and compete with each other in a tournament with 3-player setting. All possibilities of 3-player combinations will be exhausted. The policy that wins this tournament will be adopted as my policy for submission.

Candidate policies:

Simple Strategies

The following strategies can all be categorised under 'simple strategy', in the sense that only tactic is carried out regardless of the situation.

1. The default (given) policies/strategies except the 'random' policy. It is because it is unlikely that any other submission will adopt the

‘random’ policy and it creates unnecessary and unwanted uncertainties in the tournament.

2. Variants from Tit-for-Tat strategy

Besides the classical Tit-for-Tat strategy given, there are several of its variants included:

a. T4TPlayer_1stD: same as Tit-for-Tat except defect at the first round (random opponent is chosen for evaluation).
b. T42TPlayer: retaliate only if the opponent kept defecting for the last two rounds (random opponent is chosen for evaluation).
c. T4TPlayer_both: defect only if both opponents defected for the last round consecutively. First round always cooperate.
d. T4TPlayer_either: defect only if any of the opponents defected for the last round consecutively. First round always cooperate.
e. T43TPlayer: defect only if the opponent chosen at random defected for the last 3 rounds consecutively. First 3 rounds always cooperate.
f. T4_3TPlayer_accumulate: defect only if sum of defections of both opponents reaches 3 since last time the agent itself defected.
g. T4_10TPlayer_accumulate: defect only if sum of defections of both opponents reaches 10 since last time the agent itself defected.
h. T4_3in10TPlayer_accumulate: defects only if the sum of defections of both opponents reaches 10 since for the last 10 consecutive rounds
i. T44TPlayer: defect only if the opponent chosen at random defect for the last 4 rounds consecutively. First 4 rounds always cooperate.
j. T44TPlayer_deter: defect only if any of the opponents defected for the last 4 rounds consecutively. First 4 rounds always cooperate.
k. T45TPlayer: defect only if the opponent chosen at random defect for the last 5 rounds consecutively. First 5 rounds always cooperate.

3. Variants from Joss strategy:

a. Joss: Each round a random opponent will be chosen to do the classical Tit-for-Tac but with 0.1 probability of defecting.

b. ForgivingJoss: have 0.1 probability of defection. If did not defect for that probability, choose an opponent at random and defect only if that opponent defected for the last two rounds consecutively.

4. Variants from UpToU strategy:

The UpToU strategy family is the make probabilistic decisions based on the histories of the opponents' decisions.

a. UpToU1: The probability of defection is computed by the ratio of the sum of the number of both opponents' defections over the sum of number of the decisions that both opponents have made so far. First round always cooperate.

b. UpToU1_forgiving: The probability of defection is computed by the way in UpToU1 then minus 0.2. If that result before subtraction is already less than 0.2, then cooperate. This means the agent is always less likely to defect.

c. UpToU1_discount1: Invoke the concept of discount factor in Markov decision process. When computing the sum for both of number defections and number of decisions made, for the history one step ahead of the current step, multiply by discount factor 0.99:

$$\mathrm{Sum}_{\mathrm{defection[current]}} = \mathrm{defection[current](if\ any)} + 0.99 * \mathrm{Sum\_defection[current} - 1\mathrm{](if\ any)}$$

$$\mathrm{Sum}_{\mathrm{cooperate[current]}} = \mathrm{cooperate[current](if\ any)} + 0.99 * \mathrm{Sum\_cooperate[current} - 1\mathrm{](if\ any)}$$

$$\mathrm{Sum\_decision[current]} = \mathrm{Sum\_defection[current]} + \mathrm{Sum\_cooperate[current]}$$

d. UpToU1_discount2: same as UpToU1_discount1 except changed discount factor to 0.9

e. UpToU1_discount3: same as UpToU1_discount1 except changed discount factor to 0.8

f. UpToU1_forgiving3_41: on top of UpToU1, there is a threshold set at $\frac{3}{4}$. If the ratio of defection calculated in UpToU1 is more than $\frac{3}{4}$, use that ratio as the agent's probability of defection. Otherwise, cooperate.

g. UpToU1_forgiving3_42: still use $\frac{3}{4}$ as the threshold, but now when the ratio of defection calculated in UpToU1 is more than $\frac{3}{4}$, always defect; otherwise, use that ratio as the probability of defection.

h. UpToU2: same as UpToU2 except first round always defect.

5. Variants from TolerantPlayer strategy:

The TolerantPlayer is basically saying that the agent decides to defect if the total number of defections of both opponents is more than half of the total number of decisions made by both opponents, otherwise cooperate. There are 37 variants from TolerantPlayer strategy. They are composed by different combinations of the following variations:

a. The threshold of defection.
   For the given example, the threshold of defection is det as 0.5 (ratio of no. of defections to no. of decisions). This threshold could be adjusted. $\frac{1}{3}, \frac{2}{3}, \frac{3}{4}$ are set as the threshold of defection as well.
b. On top of the thresholding being satisfied, either/both of the opponents need(s) to defect during the last round in order for agent to defect.
c. Account for the impact of the length of the history of decisions by computing weighted percentage. This is done by invoking the concept of discount factor (UpToU1_discount1 for details).
d. Compute the ratio of defection and cooperate history separately for different opponents. When either/both opponents' number of defections reached certain threshold, defect; otherwise cooperate.

Not all combinations of aforementioned conditions are tested exhaustively since that is rather impractical.

6. OneOff: Defect for the first 5 rounds. For the following 5 rounds, cooperate. If for the last 2 rounds of where the agent cooperated, if any

of the opponents defects, then for the rest of the rounds in this match always defect; otherwise always cooperate.

7. Variants from TrickyPlayer strategy:
   a. TrickyPlayer1: for the first 2 rounds cooperate. For the rest of the rounds, if so far for any single round both opponents defected at the same time, and if any of the opponents defected consecutively for 2 rounds before a particular round, then at round defect; otherwise that cooperate for that round.
   b. TrickyPlayer2: only different from TrickyPlayer1 in that both opponents needed to defect consecutively for 2 rounds before a particular round in order for the agent to defect (more lenient).
   c. TrickyPlayer3: only different from TrickyPlayer2 in that 'defected consecutively for 2 rounds before a particular round' becomes 'defected in any of the previous 2 rounds'.
   d. TrickyPlayer3_biased: differ from TrickyPlayer3 in that opponent2 needs to fulfil 'defected consecutively for 2 rounds before a particular round'.
   e. TrickyPlayer3_biased2: differ from TrickyPlayer3_biased in that only one of oppoent1's and opponent2's conditions needs to be fulfilled.
   f. TrickyPlayer4: differ from TrickyPlayer3 in that only one of oppoent1's and opponent2's conditions needs to be fulfilled.
   g. TrickyPlayer4_biased: differ from TrickyPlayer4 in that opponent2 needs to fulfil 'defected consecutively for 2 rounds before a particular round'.
   h. TrickyPlayer4_biased2: differ from TrickyPlayer4_biased in that both oppoent1's and opponent2's conditions need to be fulfilled.

8. a. Alternating1: Starting by cooperate, then change decision for each following round (i.e. cooperate, defect, cooperate, defect, …)
   b. Alternating2: Starting by defect, then change decision for each following round (i.e. defect, cooperate, defect, cooperate, …)

9. Adaptive: starting by cooperate. For the following rounds, if the last round lost (get points less than 6), change decision for this round (if the last round cooperated, this round defect and vice versa).

Patterns of the variants

It could be noticed that many of the variants of certain main strategies are trying to tackle the following issues:

1. In a 3-player based tournament, it is unclear whether the decision should be made based on one of the opponent's behaviour or both opponents' behaviours
2. It is unclear that, how important are certain parts of the history for decision making.

Experiments and Analysis

All the strategies (including each and every variant of any strategy) are put in the tournament and compete. There are 75 candidates involved. I set each match to approximately 100 rounds (not fixed, same as the actual tournament setting) and ran multiple times to make the ranking more stable. One instance of the results is as follows:

Strategies	Ranking
TolerantPlayer3_discount2: 16537.07 points.	1
TolerantPlayer4_discount1: 16524.195 points.	2
TrickyPlayer4_biased2: 16518.697 points.	3
TrickyPlayer3: 16518.34 points.	4
TrickyPlayer3_biased: 16512.502 points.	5
TolerantPlayer4: 16506.938 points.	6
T4TPlayer_both: 16503.125 points.	7
TolerantPlayer_individual_both_discount1_Bboth: 16500.492 points.	8
TolerantPlayer_individual_both_discount1: 16497.656 points.	9
TolerantPlayer_individual_both_Bboth: 16497.43 points.	10
TolerantPlayer_individual_both_discount2_Bboth: 16495.982 points.	11
TolerantPlayer_individual_both_discount2: 16494.385 points.	12
T45TPlayer: 16491.691 points.	13
T44TPlayer: 16490.512 points.	14
TolerantPlayer4_discount2: 16490.346 points.	15
TolerantPlayer_individual_both: 16490.018 points.	16
TrickyPlayer2: 16489.176 points.	17
TolerantPlayer4_Bboth: 16485.469 points.	18
T43TPlayer: 16468.26 points.	19
TolerantPlayer_discount1: 16464.645 points.	20
TolerantPlayer_discount2: 16455.562 points.	21

TolerantPlayer3_discount1: 16450.54 points.	22
TolerantPlayer3: 16450.129 points.	23
TolerantPlayer: 16443.078 points.	24
TolerantPlayer3_Bboth: 16437.945 points.	25
TrickyPlayer: 16435.275 points.	26
TolerantPlayer_Bboth: 16427.588 points.	27
T44TPlayer_deter: 16423.828 points.	28
TrickyPlayer3_biased2: 16409.465 points.	29
T42TPlayer: 16400.28 points.	30
TrickyPlayer4_biased: 16393.746 points.	31
T4_3TPlayer_accumulate: 16386.502 points.	32
TolerantPlayer_individual_either_discount1: 16384.418 points.	33
UpToU1: 16368.192 points.	34
TolerantPlayer_individual_either_Bboth: 16360.132 points.	35
T4_10TPlayer_accumulate: 16352.998 points.	36
UpToU1_forgiving3_41: 16352.558 points.	37
UpToU1_discount1: 16350.129 points.	38
TolerantPlayer_individual_either: 16345.833 points.	39
UpToU1_discount3: 16340.878 points.	40
UpToU1_forgiving: 16337.943 points.	41
NicePlayer: 16337.704 points.	42
TolerantPlayer_individual_either_discount2_Bboth: 16333.683 points.	43
TolerantPlayer_individual_either_discount1_Bboth: 16321.811 points.	44
TolerantPlayer_individual_either_discount2: 16320.553 points.	45
T4TPlayer: 16312.517 points.	46
T4_3in10TPlayer_accumulate: 16309.057 points.	47
UpToU1_discount2: 16288.017 points.	48
TolerantPlayer2_Bboth: 16274.993 points.	49
TolerantPlayer2_discount2: 16270.586 points.	50
TolerantPlayer2_discount1: 16269.109 points.	51
TolerantPlayer2: 16265.368 points.	52
TolerantPlayer3_Beither: 16244.584 points.	53
TolerantPlayer2_Beither: 16242.698 points.	54
TolerantPlayer_individual_both_Beither: 16240.456 points.	55
TolerantPlayer_individual_either_Beither: 16233.985 points.	56
TolerantPlayer4_Beither: 16227.711 points.	57
TolerantPlayer_individual_either_discount1_Beither: 16226.293 points.	58
TrickyPlayer4: 16226.096 points.	59
TolerantPlayer_Beither: 16226.044 points.	60
TolerantPlayer_individual_either_discount2_Beither: 16221.06 points.	61
T4TPlayer_either: 16220.864 points.	62
TolerantPlayer_individual_both_discount2_Beither: 16217.097 points.	63
TolerantPlayer_individual_both_discount1_Beither: 16200.504 points.	64
Adaptive: 15975.669 points.	65
ForgivingJoss: 15842.013 points.	66
Alternating1: 15479.032 points.	67

Joss: 15217.871 points.	68
Alternating2: 14831.701 points.	69
UpToU2: 14677.747 points.	70
T4TPlayer_1stD: 14494.979 points.	71
FreakyPlayer: 13994.21 points.	72
OneOff: 13610.425 points.	73
NastyPlayer: 11784.872 points.	74
UpToU1_forgiving3_42: 11569.974 points.	75

Table 1

Overall, the ones that consistently ranked at the top tier are some of the variants of TolerantPlayer, Tit-for-Tat and TrickyPlayer.

It is interesting to see TolerantPlayer based strategies played this well, especially when TolerantPlayer actually holds grudges rather than retaliating as soon as possible. My guess is that, sometimes taking reaction immediately (classical example being Tit-for-Tat) means short-sightedness. Always behaving like this can be susceptible to the deceptive moves by the opponents. Also, it might cause perpetual alternation between cooperation and detection (Tit-for-Tat and Tit-for-Tat starting with defection) or even worse: always defect (Tit-for-Tat and Joss). On the other hand, taking grudges sometimes can mean having memory and being calculative. This type of strategies can overcome the disadvantage of being too responsive and subsequently short-sighted.

TrickyPlayer based strategies differ from Tic-for-Tac based strategies in that they have one more consideration: the coalition between opponents. If by any means the two opponents defected at the same round, there is possibility that they have conspired against our agent. This more additional consideration might give TrickyPlayer an edge against certain players.

It took me by surprise that UpToU based strategies in general performed not as good as TolerantPlayer based strategies. It appeared to me initially UpToU is more intelligent since it does not depend on a fixed and rigid threshold. The agent can be more flexible in terms of decision making, sometimes might gaining more points by surprise attack. After further research, I came to know what the strategies which are stochastic in nature is not necessarily needed in finding an optimal strategy. Despite the fact that there is no optimal strategy in this iterated 3-player prisoner’s dilemma tournament, it somewhat suggests the possibility that UpToU based strategies are sub-optimal since it involves random process.

In general, it could be observed that cooperative strategies tend to have higher rankings. This could be due to the fact that the rule of the tournament is set in such a way that the win/loss of individual match does not contribute to the final ranking; rather it is the total points obtained by the agent. Therefore, when agents tend to be more cooperative, they can usually get more points not necessarily by winning the individual matches. For a simple illustration, when the agent candidates are nice player (always cooperates), a Tit-4-Tac agent and nasty player (always defects), nasty player will surely take advantage of the nice player, but when nasty player play with Tit-4-Tac and other nasty players, it will gain less points while nice player will gain more points when playing with Tit-4-Tac and other nice players (based on the code given, there will be the same strategy complete with itself scenario). Overall, nasty player will rank the last by losing a lot of points while nice play will rank the second by losing not as much as nasty player. However, for such pattern to occur, the underlying assumption is that there are more cooperative agents participating the tournament than aggressive agents, or at least the number of cooperative agents and neutral agents (e.g. Tit-4-Tac) are dominant over aggressive agents. Here, I choose to believe that this is indeed the case.

Hybrid Strategies

I now postulate regarding to the so-far-the-best strategy: tolerant player.

Since its decision making depends on a ratio, or a weighted percentage of the opponents history of decisions, so to speak, according to the intuition from the law of larger number, it will be more accurate as the history gets longer; conversely, when the history is too short, tolerant player's decision is not reliable. Hence, I try to optimize it with another exiting simple and competitive strategy: Tit-for-Tat.

More specifically, I set a certain round as threshold (20,30,40 rounds, etc), before which the agent uses Tit-for-Tat based strategy and after which, the agent uses tolerant based strategy. This change is applied only to the top tier of the tolerant players. These strategies are now called ‘hybrid strategies’.

On the other hand, I reckon that the last round will have no impact to any future round because by definition there is no future round. However, in the given environment, the exact number of rounds cannot be determined but I know that there will be at least 90 rounds. Therefore, I also let the agent behave as nasty play (always defect) for the rounds beyond 90/95/100/105. This change is applied only to the top tier of ‘hybrid strategies’ and the products are also considered ‘hybrid strategies’ themselves. One exception is given to ‘T4TPlayer_both’ which is a variant from Tit-for-Tat. It is also given the change of being nasty for the rounds beyond 90 since it also showed promising ranking.

Besides, a few Trickyplayers are also modified into hybrid strategies, combining with Tic-for-Tac and NastyPlayer ending.

Hybrid strategies have an obvious advantage over the simple strategies: it is hard to counter. To begin with, there is no single simple strategy that can counter a hybrid strategy since it is made of several simple strategies. Secondly, in my implementation, each simple strategy segment only functions for very limited steps. This will give rise to more challenges for the opponents to predict my move. If the opponents are programmed in such a way that it will predict my general strategy type first then act accordingly, its time to make prediction will be very limited, and when I change my strategy it will create further confusion for the opponents to predict my strategy correctly. In the event that the opponents find a strategy which can counter one of the base(simple) strategy I used, the damage is not as large as using that base(simple) strategy throughout the match.

Experiments and Analysis

The following shows one instance of the top-tier strategies in the ranking as now the number of strategies (153 in total, including both simple and hybrid strategies) are too many to display all.

Strategies	Ranking
T4TPlayer_both_and_TrickyPlayer3_nasty_30_biased2: 68761.47 points.	1
T4TPlayer_both_and_TrickyPlayer3_nasty_40_biased2: 68712.734 points.	2
T4TPlayer_both_and_TrickyPlayer3_nasty_20_biased2: 68703.89 points.	3
T4TPlayer_both_and_TolerantPlayer_discount2_ver3_nasty: 68700.516 points.	4
TrickyPlayer3_and_T4TPlayer_both_nasty_40_biased: 68699.78 points.	5
T4TPlayer_both_and_TrickyPlayer3_nasty_20: 68698.31 points.	6
T4TPlayer_both_and_TolerantPlayer_discount2_ver4_nasty: 68696.99 points.	7
T4TPlayer_both_and_TrickyPlayer3_nasty_50_biased2: 68696.61 points.	8
TrickyPlayer3_and_T4TPlayer_both_nasty_20_biased: 68693.82 points.	9
T4TPlayer_both_and_TrickyPlayer3_nasty_50_biased: 68692.5 points.	10
T4TPlayer_both_and_TolerantPlayer3_Bboth_30_nasty: 68687.84 points.	11
TrickyPlayer3_nasty: 68684.28 points.	12
T4TPlayer_both_and_TolerantPlayer3_20_nasty: 68683.875 points.	13
TrickyPlayer3_and_T4TPlayer_both_nasty_30_biased: 68677.34 points.	14
T4TPlayer_both_and_TolerantPlayer_discount2_nasty: 68669.21 points.	15
T4TPlayer_both_and_TrickyPlayer3_nasty_40_biased: 68667.56 points.	16
TrickyPlayer3_and_T4TPlayer_both_nasty_50_biased: 68667.375 points.	17
T4TPlayer_both_and_TrickyPlayer3_nasty_30: 68667.02 points.	18
T4TPlayer_both_and_TrickyPlayer3_nasty_20_biased: 68661.734 points.	19
T4TPlayer_both_and_TolerantPlayer3_Bboth_50_nasty: 68657.8 points.	20
TrickyPlayer3_and_T4TPlayer_both_nasty_50: 68650.0 points.	21
T4TPlayer_both_and_TrickyPlayer3_nasty_30_biased: 68648.695 points.	22
T4TPlayer_both_and_TolerantPlayer_discount2_ver2_nasty: 68641.805 points.	23
T4TPlayer_both_and_TolerantPlayer3_Bboth_40_nasty: 68639.35 points.	24
T4TPlayer_both_and_TrickyPlayer3_nasty_40: 68633.05 points.	25
TrickyPlayer3_and_T4TPlayer_both_nasty_30: 68629.766 points.	26
T4TPlayer_both_and_TrickyPlayer3_nasty_50: 68628.0 points.	27
TrickyPlayer3_and_T4TPlayer_both_nasty_40: 68618.195 points.	28
TrickyPlayer3_and_T4TPlayer_both_nasty_20: 68604.65 points.	29
T4TPlayer_both_nasty: 68601.67 points.	30

Table 2

During the multiple trail running, if in any instance the 1st strategies has a marginal condition (e.g. threshold is 20 from the given threshold 20, 30, 40, 50), widen the threshold to see this marginal value is really the optimal value (e.g. create another strategy with threshold 15 or 10 while keep the rest conditions the same and rerun the tournament). At the same time, the number of strategies needs to be cut down, since it is already way more than the expected number of submission (30-40). This might create another issue, especially when currently a larger portion of the strategies are spawned from the same based strategy: Tolerantplayer and Tit-for-Tac. This imbalance might cause overfit to those same based strategies and as the result the ranking might not be indicative. At the same time, if there exists one strategy that is particularly successful at countering those strategies, it will have tremendous advantage in the entire tournament as it will be facing its natural prey for a lot of matches. In fact, this happened when there are too many Tolerantplayer based strategies and it turns out that Alternating1 can perfectly counter Tolerantplayer because the probability of its defection is always about 0.5, and if the threshold of Tolerantplayer based strategies is more than that they are bound to lose. As the result, Alternating1 can be ranked 1st by winning the second position by a few hundred points consistently when there are too many Tolerantplayers. The following is one of the instances:

Strategies	Ranking
Alternating1: 77901.89 points.	1
T4TPlayer_both_and_TrickyPlayer3_nasty_40_biased2: 77787.54 points.	2
T4TPlayer_both_and_TrickyPlayer3_nasty_60_biased2: 77780.516 points.	3
T4TPlayer_both_and_TolerantPlayer_discount2_ver2_nasty: 77774.69 points.	4
T4TPlayer_both_and_TrickyPlayer3_nasty_40_biased: 77772.28 points.	5

Table 3

Finalizing the strategy for submission

The cut down starts from the strategies with the most same based strategies and the lowest ranking. For each cut-down candidate, the tournament will be run multiple times to make sure that the strategy to be taken off is consistently ranked low.

Eventually, there are 51 strategies remained, which is a more realistic number compared to the actual number of submissions.

To select the strategy for submission, run the tournament for 20 times:

Strategies	frequency
T4TPlayer_both_and_TolerantPlayer_discount2_ver4_nasty	6
T4TPlayer_both_and_TolerantPlayer_discount2_ver3_nasty2	5
T4TPlayer_both_and_TolerantPlayer_discount2_ver3_nasty	3
T4TPlayer_both_and_TolerantPlayer_discount2_ver4_nasty2	2
T4TPlayer_both_and_TolerantPlayer_discount2_nasty	2
T4TPlayer_both_and_TolerantPlayer_discount2_ver2_nasty	1

TolerantPlayer_individual_either_discount2_Bboth	1

Table 4

The most frequent Strategy and the second frequent strategy only differ by one time of appearance.

Run the tournament again to observe the stability. In another 10 tournaments, ‘T4TPlayer_both_and_TolerantPlayer_discount2_ver4_nasty’ was among the top 5 for 5 times and ‘T4TPlayer_both_and_TolerantPlayer_discount2_ver3_nasty2’ was among the top 5 for 3 times.

Therefore, strategy wins the 1st position most frequently and with higher stability of being among the top 5. In conclusion, ‘T4TPlayer_both_and_TolerantPlayer_discount2_ver4_nasty’ is adopted as the strategy for submission. The strategy was renamed as per required. Its code is shown below:

CODE:

```
class Zhang_Zeyu_Player extends Player {
        //hybrid strategy
        //composed from varients of Tic-for-Tac, Tolerant and Nasty Player
        int selectAction(int n, int[] myHistory, int[] oppHistory1, int[] oppHistory2) {

            //when this history length is within 10, play as Tic-for-Tac-both
            //(only if both opponents detected for the last round then defect
            //for this round) cooperate by default for the first round
            if (n<10) {
                if (n==0) return 0;
                if (oppHistory1[n-1] == 1 && oppHistory2[n-1] == 1) {
                    return 1;
                }
                else {
                    return 0;
                }
            }

            //when this history length larger than 95, play as Nastyplayer
            if (n>95) {
                return 1;
            }

            //when this history length larger than 10 and smaller or equal to 95,
            //play as TolerantPlayer with discount factor (value more recent
        //opponents' decisions) to compute the ratio of the weighted defection
        //history over the weighted cooperation history. Discount factor is
        //chosen to be 0.9.
        else  {
            double opponentCoop = 0;
            double opponentDefect = 0;
            for (int i=0; i<n; i++) {
                if (oppHistory1[i] == 0) {
                    opponentCoop = opponentCoop + Math.pow(0.9, (n-i));
                }
                else {
                    opponentDefect = opponentDefect + Math.pow(0.9, (n-i));
                }
            }

            for (int i=0; i<n; i++) {
                if (oppHistory2[i] == 0) {
                    opponentCoop = opponentCoop + Math.pow(0.9, (n-i));
                }
                else {
                    opponentDefect = opponentDefect + Math.pow(0.9, (n-i));
                }
            }

            //If the weighted defection history
            //is more than the weighted cooperation
            //history, then defect for this round.
            //Otherwise cooperate.
            if (opponentDefect > opponentCoop) {
                return 1;
            }
            else {
                return 0;
            }
        }
    }
}
```