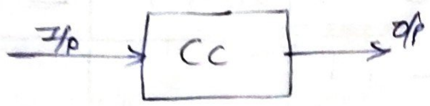
UNIT-2 {Combinational circuit}

» Combinational circuits are those circuits whose output depends upon present sequence of Input.

» NO feedback present so no need of memory.



Eg: Half adder, full adder, , decoder , etc.

→ Procedure for design of Combinational circuit

Step.①:- Identify input & output

Step.②:- Construct the truth table

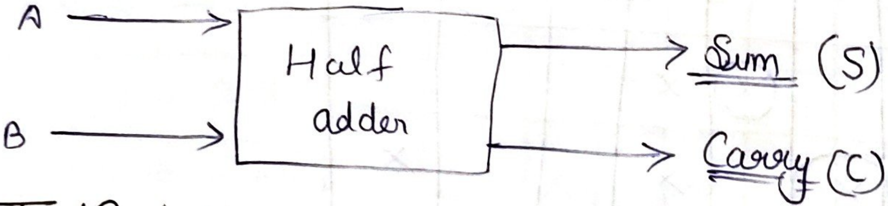
Step-③- Write logical expression in SOP & POS form

Step-④:- Minimise the logical Expression by K-Map

Step.⑤:- Implement the logic circuit.

Half adder :-

⊙ Block diagram :-



⊙ Truth table

A	B	S	C
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

⊙ $S(A,B) = \Sigma m(1,2)$

$C(A,B) = \Sigma m(3)$

Combinatio

Cir

⊙ minimise

K-map for S

A \ B	$\overline{B}$ 0	B 1
$\overline{A}$ 0		1
A 1	1	

$S = \overline{A}B + A\overline{B}$

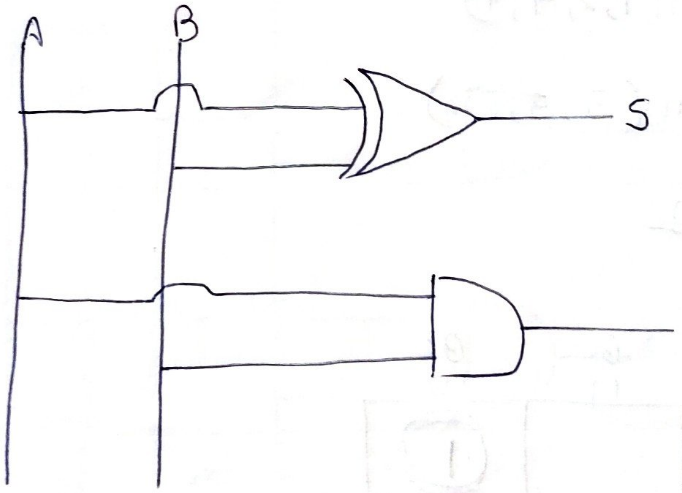
$\boxed{S = A \oplus B}$

K-map for Carry

A \ B	0	1
0		
1		1

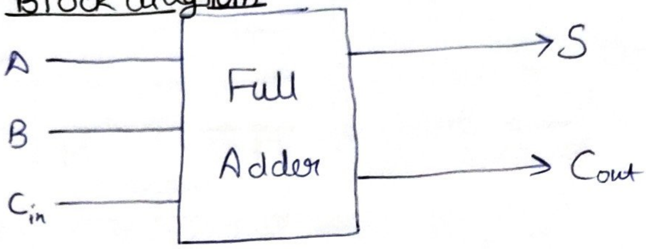
$\boxed{C = AB}$

⊙ Circuit diagram



Full adder :-

- Block diagram



- Truth table

A	B	C_{in}	S	C_{out}
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

⇒ SOP

$$S(A,B,C) = \sum m(1,2,4,7)$$

$$C(A,B,C) = \sum m(3,5,6,7)$$

→ K-map for Sum

A \ BC	$\bar{B}\bar{C}$ 00	$\bar{B}C$ 01	BC 11	$B\bar{C}$ 10
$\bar{A}$ 0		1		1
A 1	1		1	

$$S = A\bar{B}\bar{C} + \bar{A}\bar{B}C + ABC + \bar{A}B\bar{C}$$

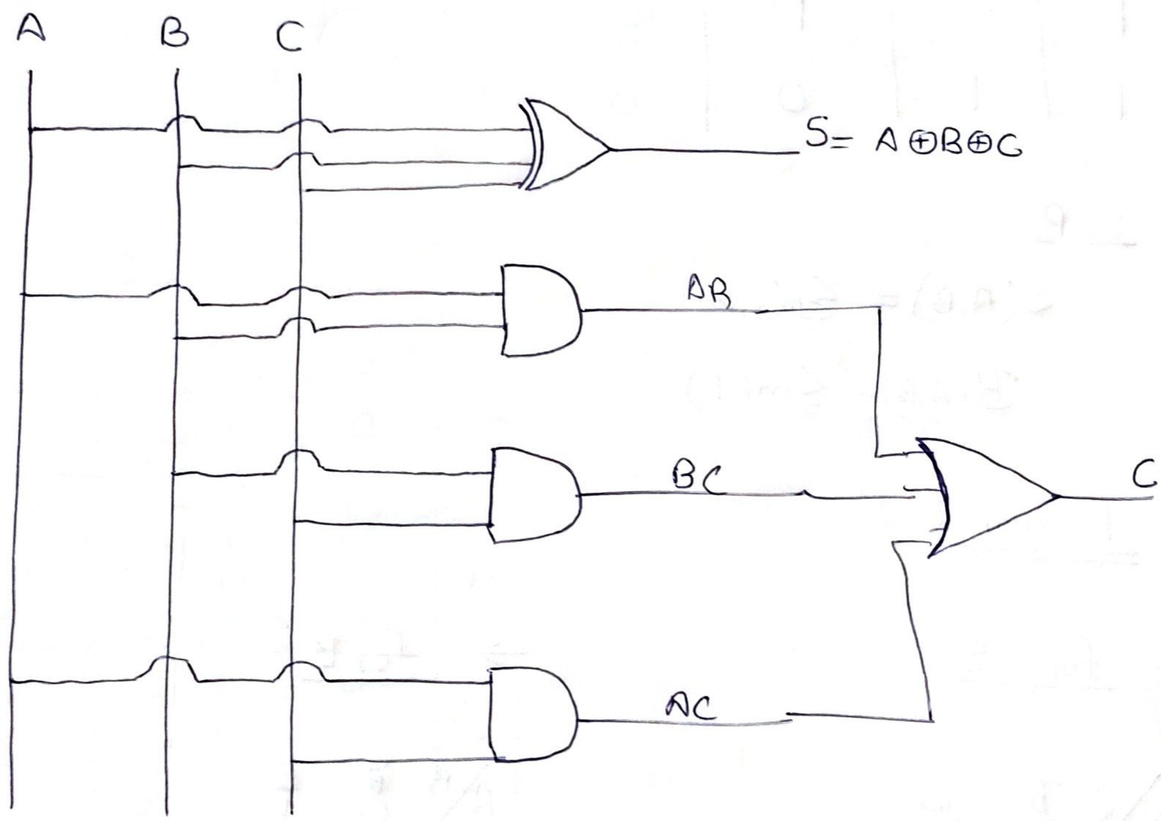
$$= A \oplus B \oplus C$$

→ K-map for Carry

A \ BC	00	01	11	10
0			1	
1		1	1	1

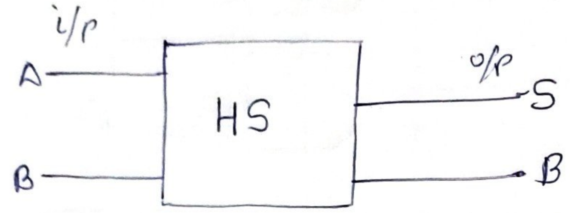
$C = AC +$

Circuit diagram



Half substractor :-

Block diagram



Truth table

A	B	S	B
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0

SoP

$$S(A,B) = \Sigma m(1,2)$$

$$B(A,B) = \Sigma m(1)$$

K-map

→ for S

A \ B	$\bar{B}$ 0	B 1
$\bar{A}$ 0		1
A 1	1	

$S = \bar{A}B + \bar{B}A$

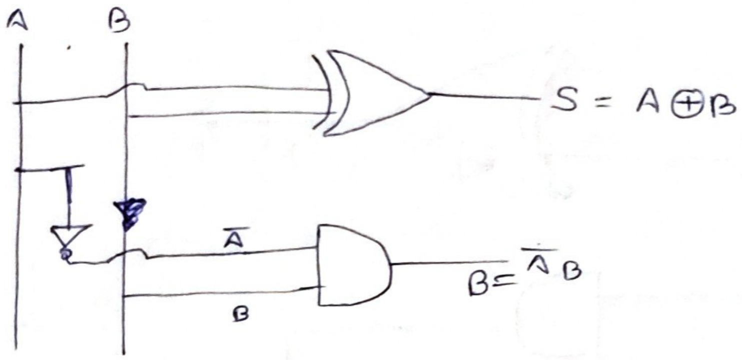
$\boxed{S = A \oplus B}$

→ for B

A \ B	$\bar{B}$ 0	B 1
$\bar{A}$ 0		1
A 1		

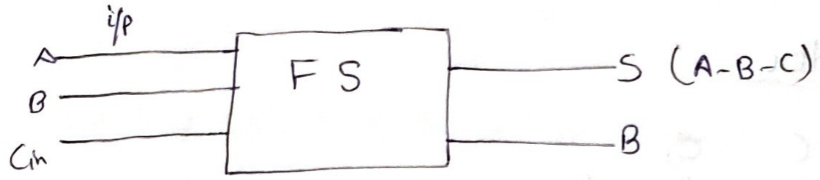
$\boxed{B = \bar{A}B}$

⊙ Circuit diagram



* Full subtractor :-

⊙ Block diagram



$2-1$

$0-1$ ⊙ Truth table

A	B	C	S	B
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	0	1
1	0	0	1	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

⊙ Sop

$S = \sum m(1,2,4,7)$

$B = \sum m(1,2,3,7)$

A \ BC	$\bar{B}\bar{C}$ 00	$\bar{B}C$ 01	BC 11	$B\bar{C}$ 10
$\bar{A}$ 0		1	1	1
A 1			1	~~1~~

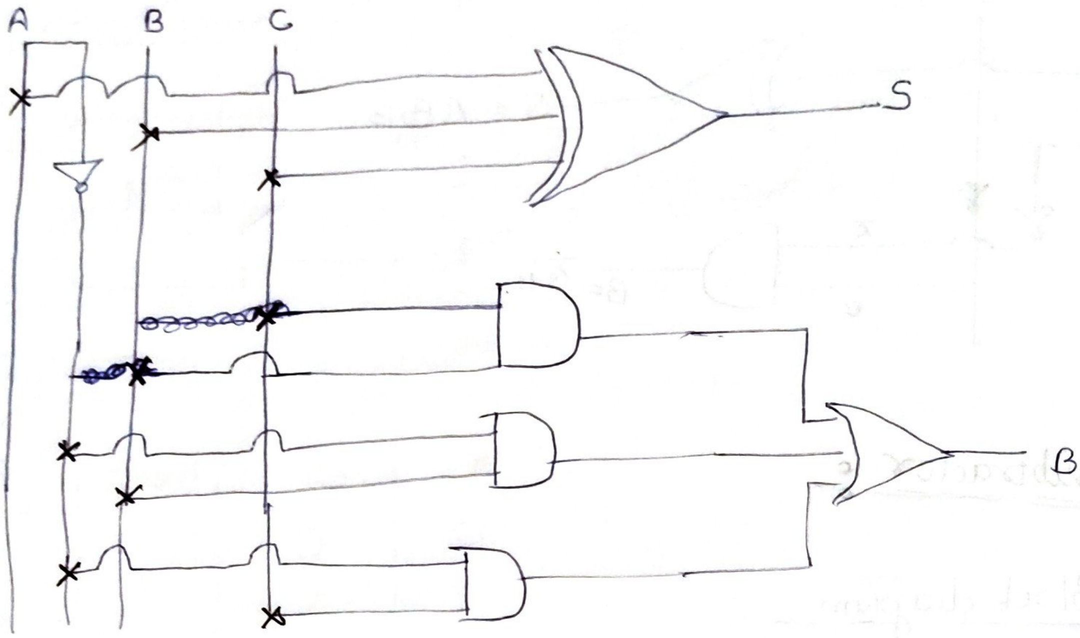
$B = \bar{A}C + \bar{A}B + BC$

⊙

A \ BC	$\bar{B}\bar{C}$ 00	$\bar{B}C$ 01	BC 11	$B\bar{C}$ 10
$\bar{A}$ 0		1		1
A 1	1		1	

$S = \bar{A}\bar{B}C + A\bar{B}\bar{C} + ABC + \bar{A}B\bar{C}$

⊙ Circuit diagram



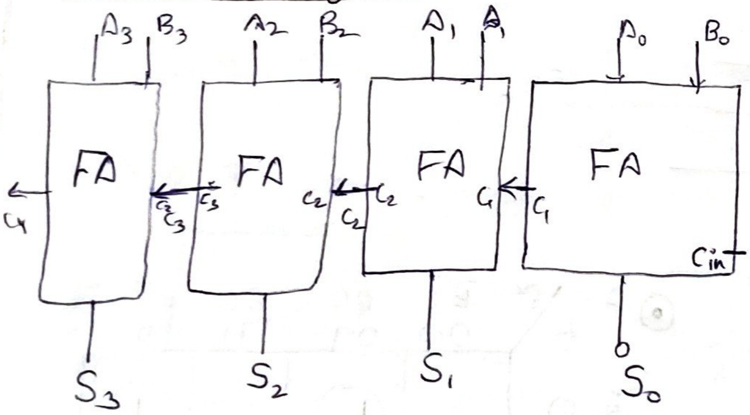
4-bit Parallel adder

5 Marks.

	C_4	C_3	C_2	C_1	
$A =$		A_3	A_2	A_1	A_0
$B =$		B_3	B_2	B_1	B_0
$S =$	C_4	S_3	S_2	S_1	S_0

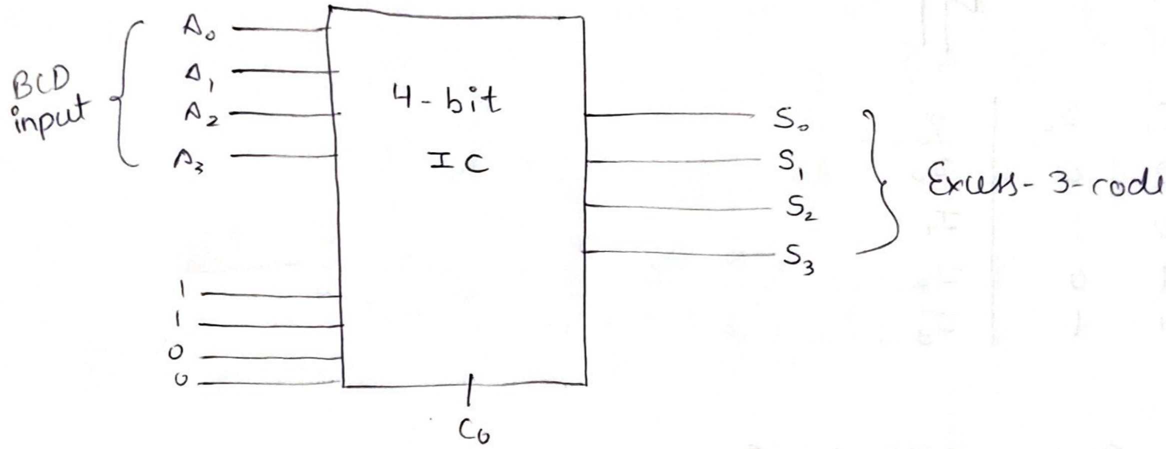
Ripple carrier adder

Circuit diagram



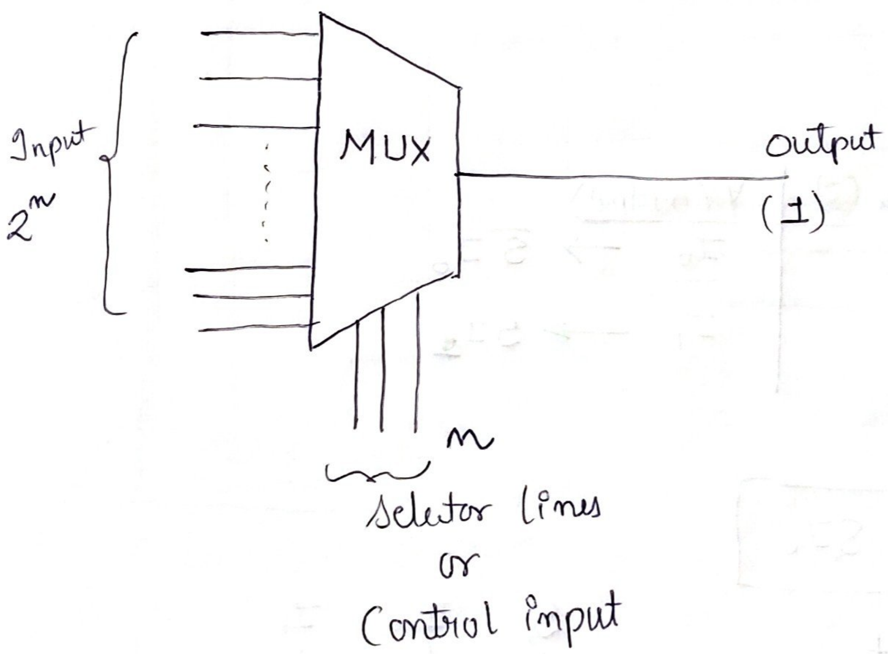
⇛ for n-bit binary parallel adder We need n-full adder input carry C_i in the least significant position must be zero. S_0, S_1, S_2, S_3 are output terminal of 4-bit binary - parallel adder.

Ques:- Design BCD to Excess-3 adder using 4-bit binary parallel adder?



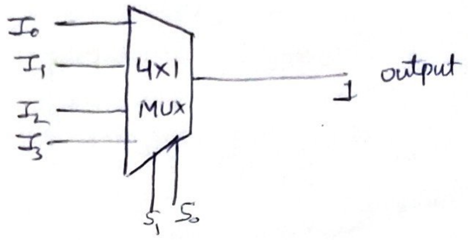
Multiplexer :-

- It is a combinational circuit which have many data inputs and single output.
- depending on selector or control input, one of the data input is transfer to the output.



→ It is a digital circuit which have 2^n input, one output and n - selection lines on the basis of selection lines it is decided which input will appea at output.

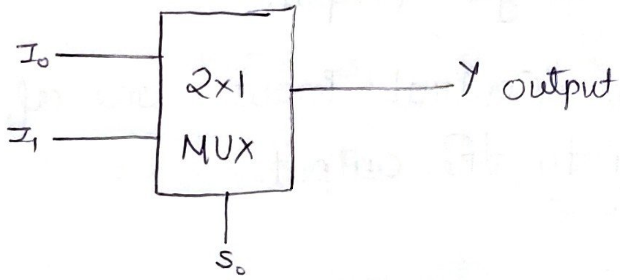
Eg. 4x1 Mux



S_1	S_0	Y
0	0	I_0
0	1	I_1
1	0	I_2
1	1	I_3

~~*~~ 2x1 or 2:1 MUX :-

(i) block diagram



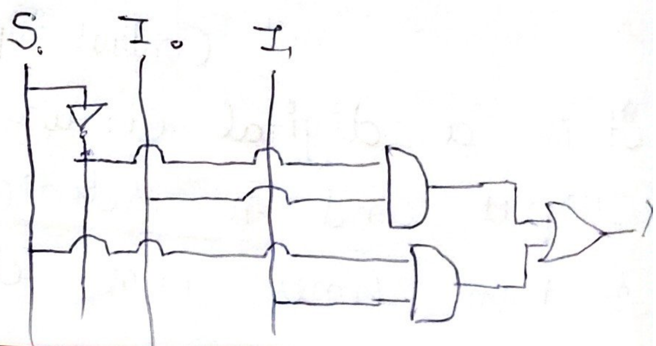
(ii) Truth table

Selection line (S)	Y (output)
0	$I_0 \rightarrow \bar{S} I_0$
1	$I_1 \rightarrow S I_1$

» output

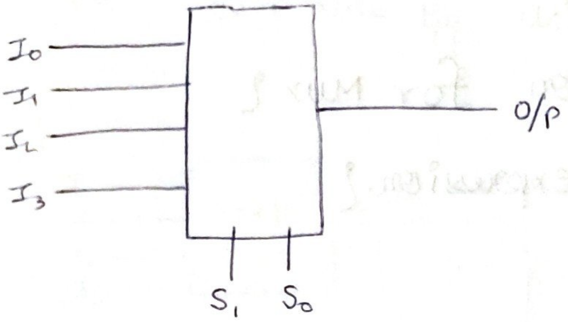
$$\boxed{Y = \bar{S}_0 I_0 + S_0 I_0}$$

> Logical circuit



* 4×1 MUX

(i) block diagram



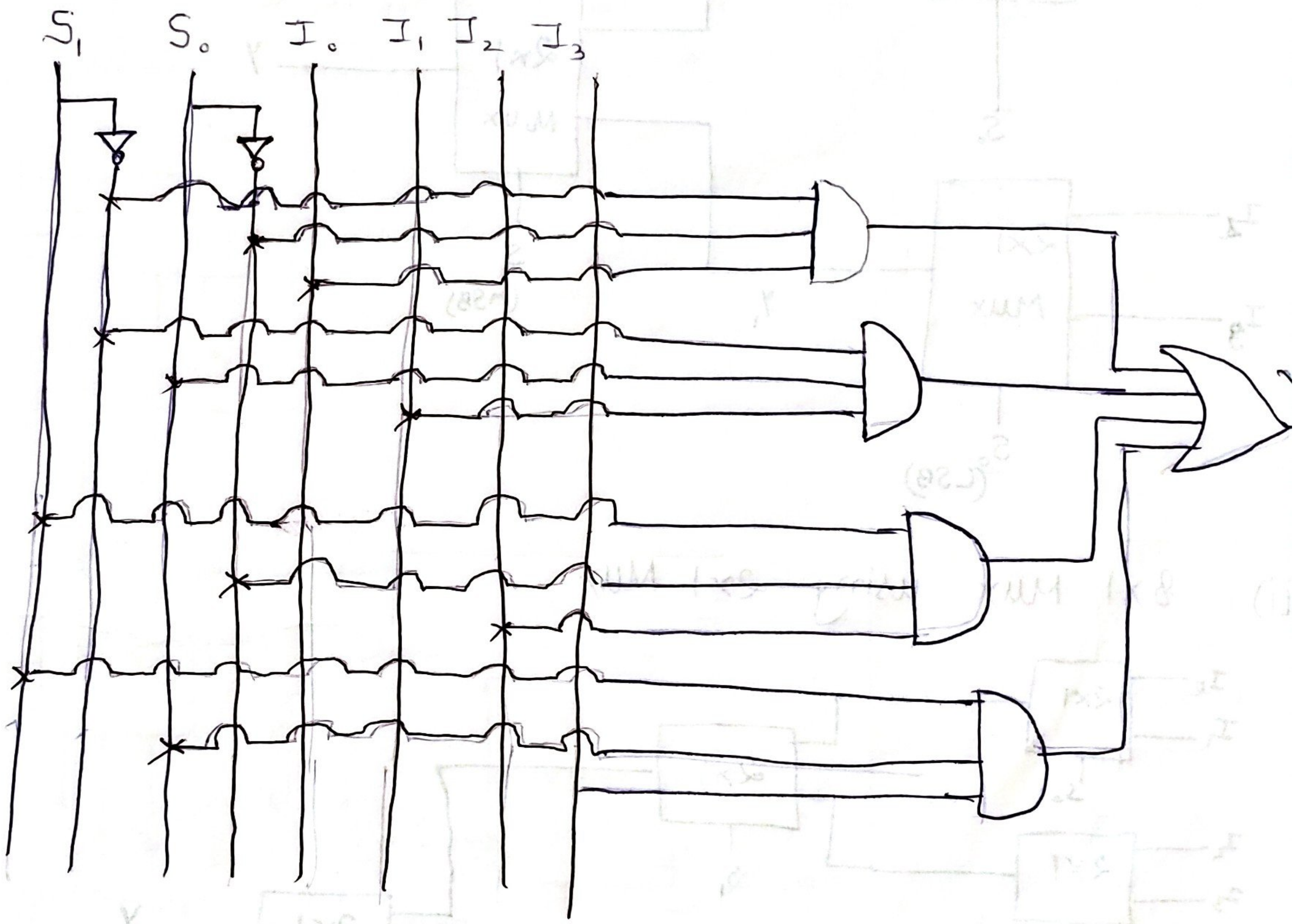
(ii) Truth table

S_1	S_0	Y
0	0	$I_0 = \bar{S}_1 \bar{S}_0 I_0$
0	1	$I_1 = \bar{S}_1 S_0 I_1$
1	0	$I_2 = S_1 \bar{S}_0 I_2$
1	1	$I_3 = S_1 S_0 I_3$

(iii) Output

$$\boxed{Y = \bar{S}_1 \bar{S}_0 I_0 + \bar{S}_1 S_0 I_1 + S_1 \bar{S}_0 I_2 + S_1 S_0 I_3}$$

(iv) Circuit



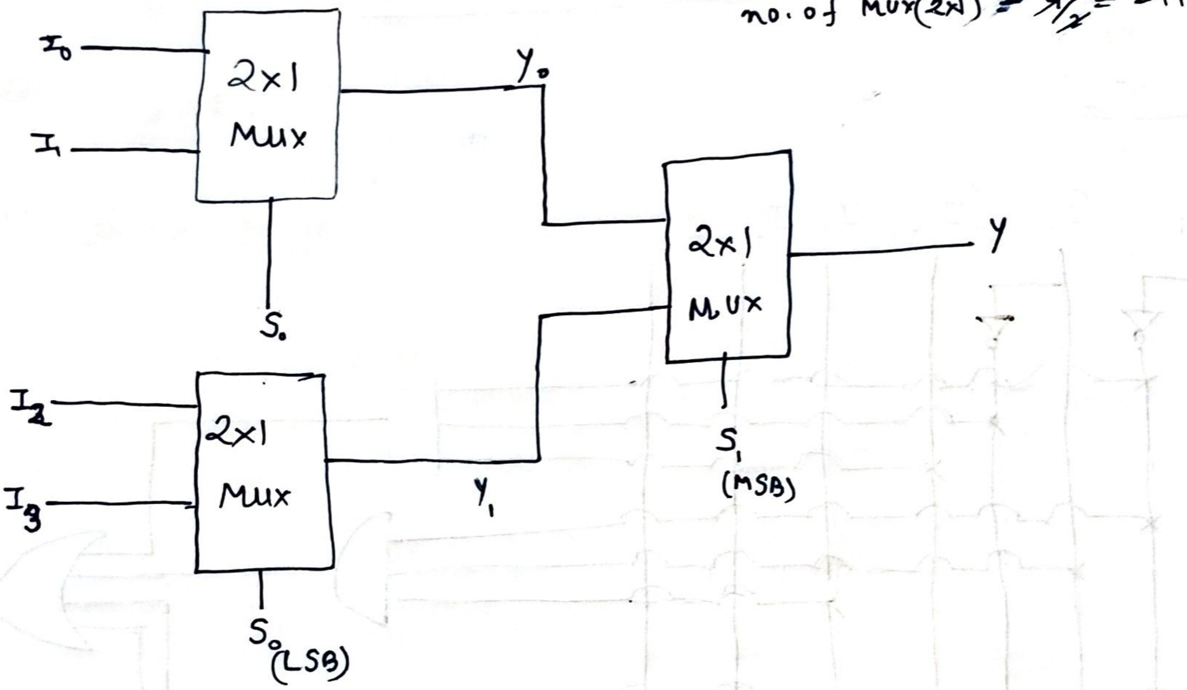
Ques:- Imp. questions (10 Marks)

1. Implementation of higher order mux using low order mux)
2. MUX as universal circuit ?
3. Determine minimised Expression for MUX ?
4. Implementation of Logic expression ?

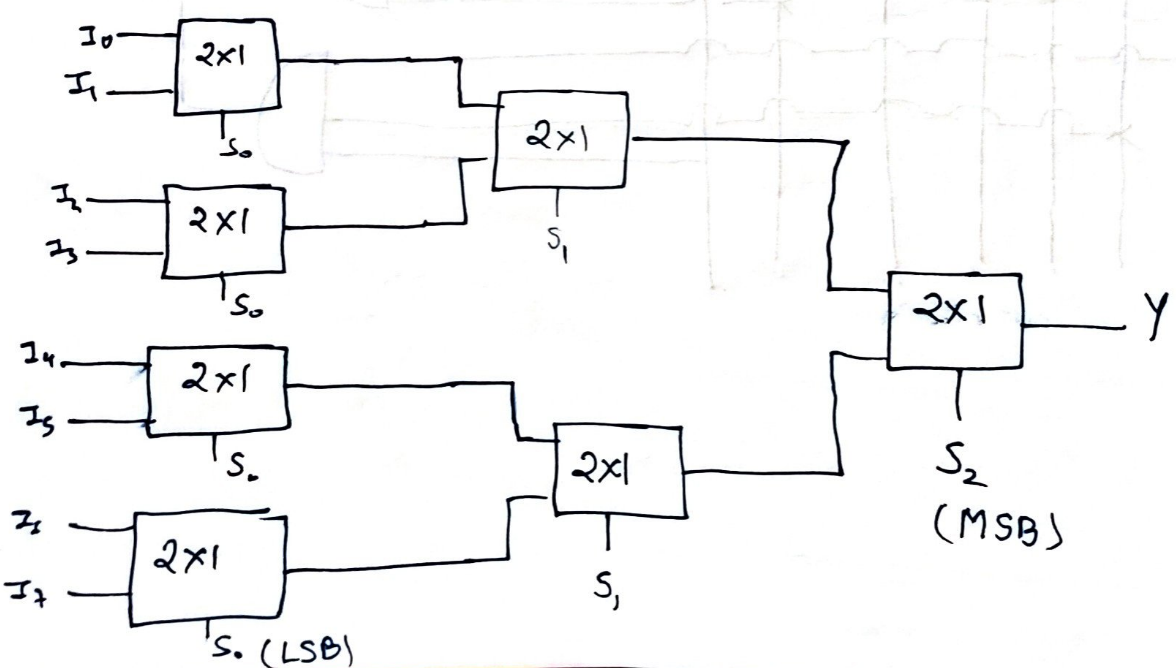
***1. Implementation H.O. using L.O. MUX ?**

(i) 4x1 mux using 2x1 MUX

no. of MUX(2x1) = $\frac{4}{2}$ = 2+1



(ii) 8x1 MUX using 2x1 MUX

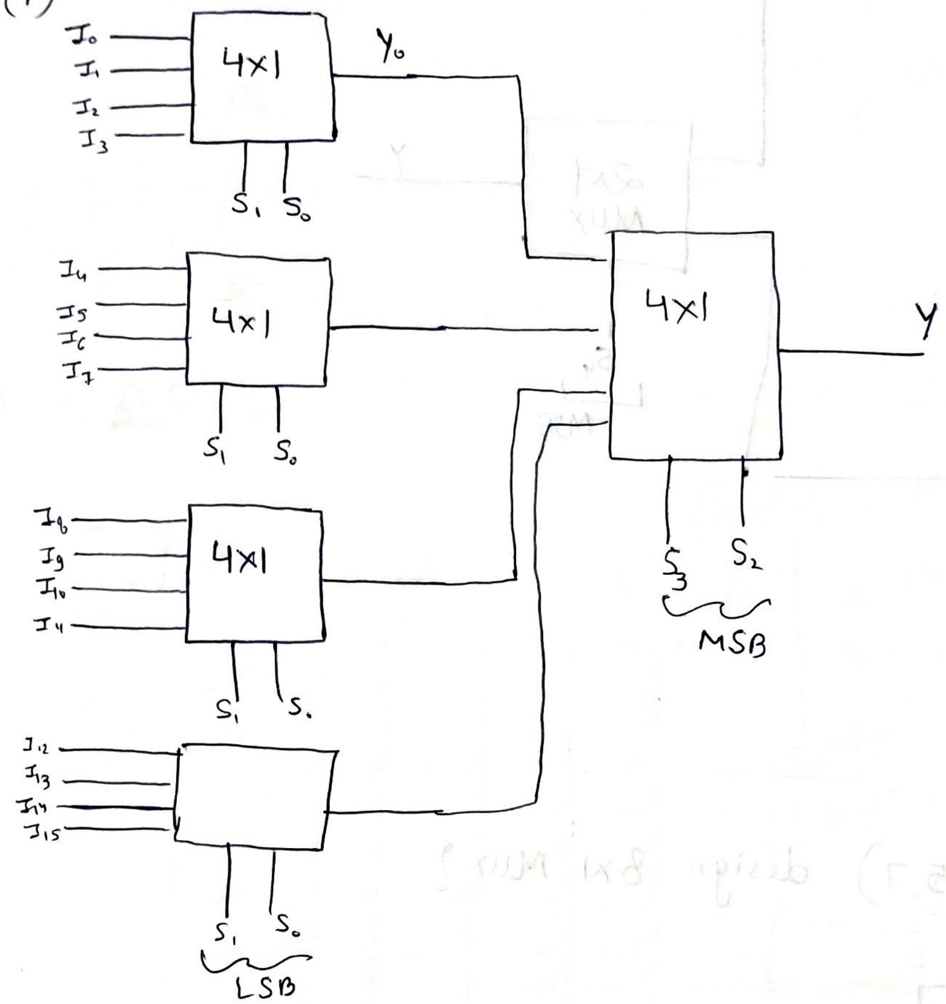


Ques: (1) 16×1 MUX using 4×1 mux.

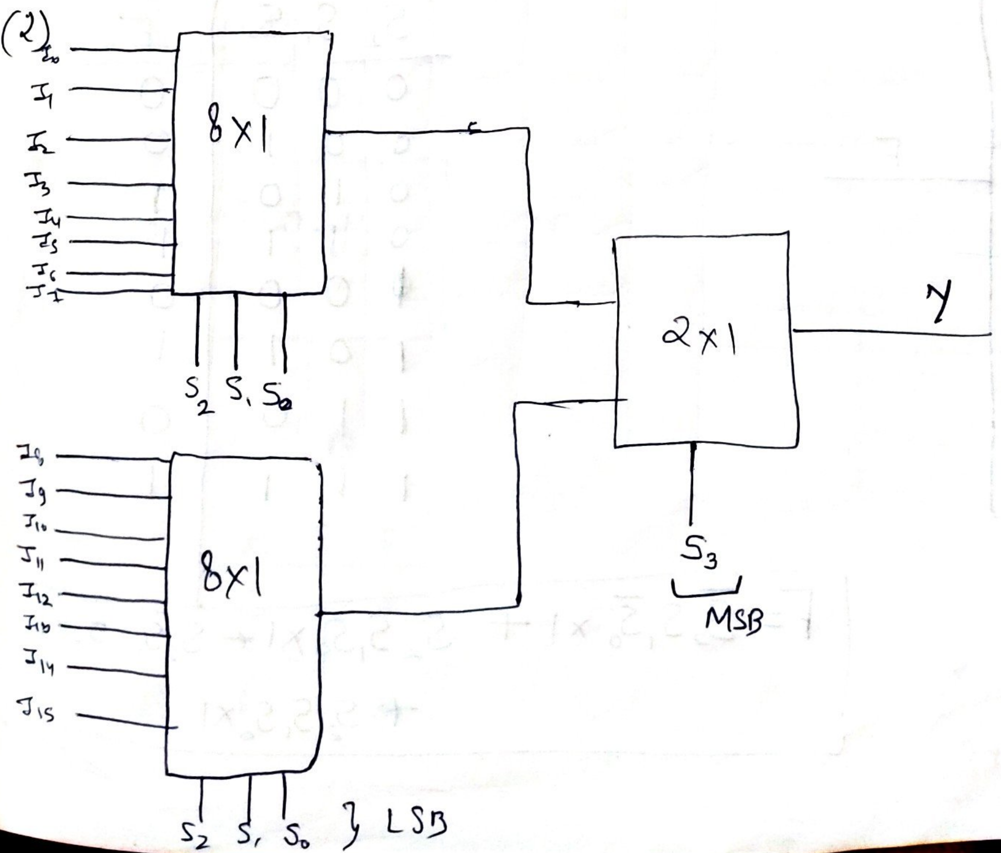
(2) 16×1 MUX by 8×1 and 2×1 Mux

(3) 32×1 MUX by using 16×1 & 2×1 MUX.

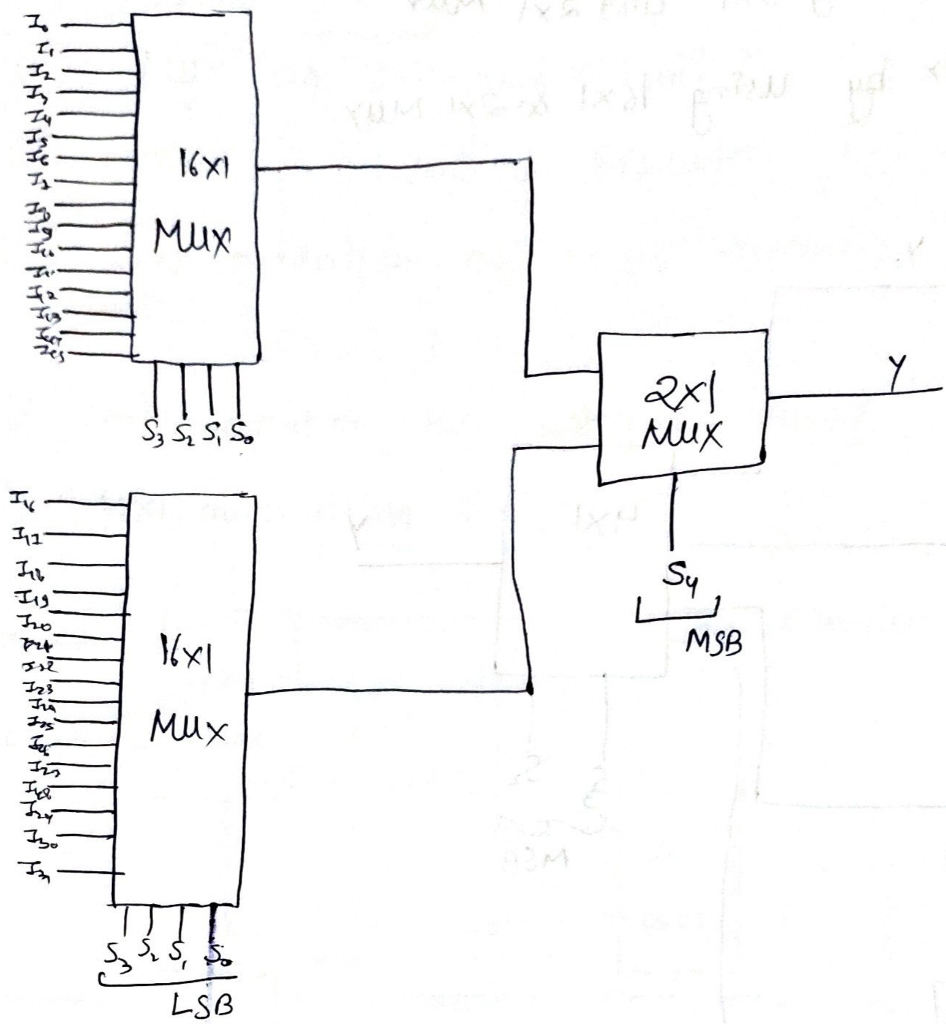
(1)



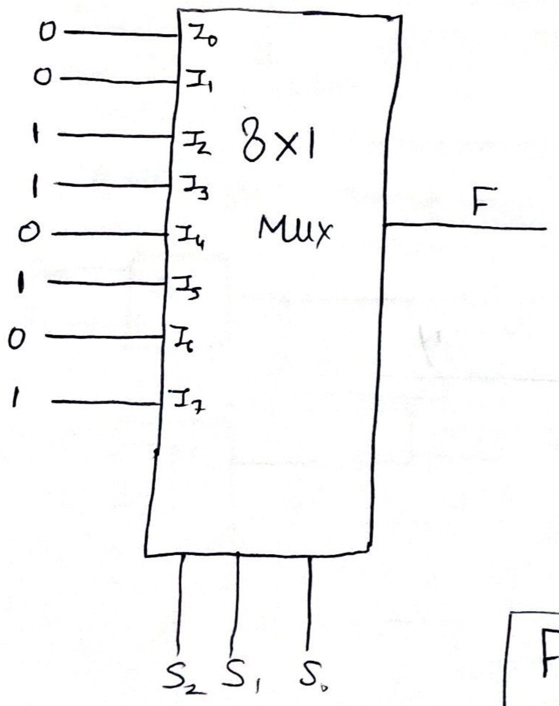
(2)



③ 32x1 using 16x1 & 2x1



Q:- $F = \sum m(2,3,5,7)$ design 8x1 Mux ?



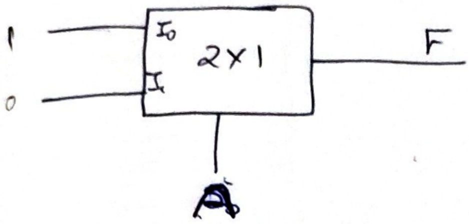
S_2	S_1	S_0	F
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

$$F = \bar{S}_2 S_1 \bar{S}_0 \times 1 + \bar{S}_2 S_1 S_0 \times 1 + S_2 \bar{S}_1 S_0 \times 1 + S_2 S_1 S_0 \times 1$$

Imp.

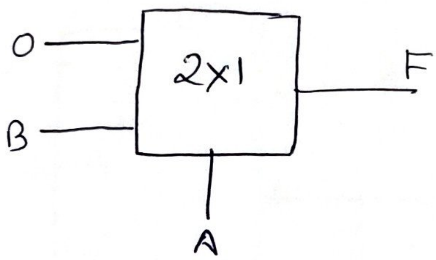
★ MUX as a Universal circuit

(1) NOT Gate



A	F
0	1
1	0

(2) AND Gate



Truth table

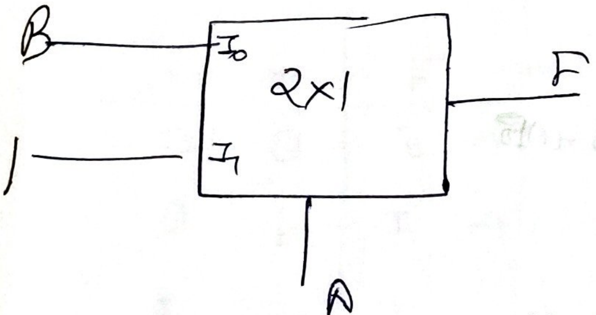
	A	F
$\bar{A}$	← 0	I_0
A	← 1	I_1

$F = I_0\bar{A} + I_1 A$

$F = 0\times\bar{A} + A.B$

$\boxed{F = A.B}$

(3) OR Gate



$F = A+B$

$I_0 = B$
$I_1 = 1$

	A	F
$\bar{A}$	0	I_0
A	1	I_1

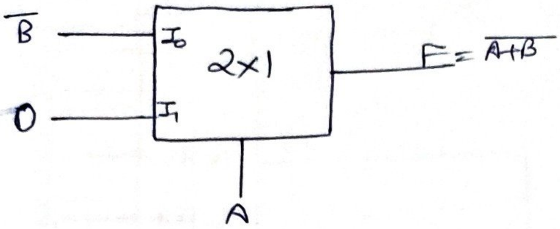
$F = \bar{A}.I_0 + A\,I_1$

$F = \bar{A}.B + A$

$F = (A+\bar{A}).(A+B)$

$\boxed{F = A+B}$

(4) NOR Gate

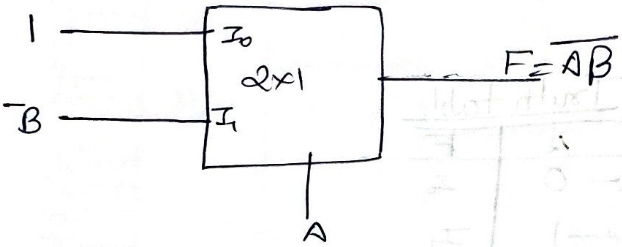


$$F = \bar{A}\bar{B} + A \times 0$$

$$F = \bar{A}\bar{B}$$

$$F = \overline{(A+B)}$$

(5) Nand Gate



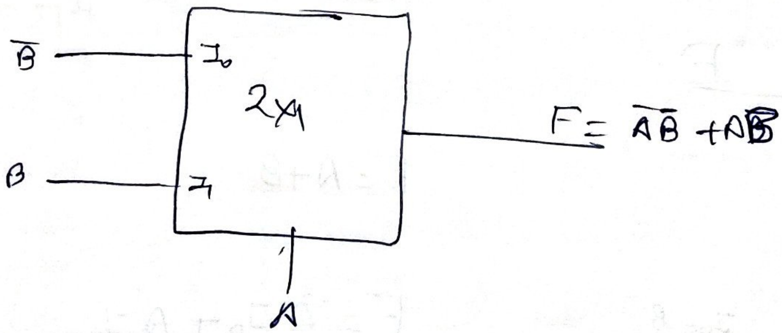
$$F = A \times \bar{B} + \bar{A} \times 1$$

$$= A.\bar{B} + \bar{A}.1$$

$$= (\bar{A} + \bar{A}).(\bar{A} + \bar{B})$$

$$= \bar{A} + \bar{B} = \overline{AB}$$

(6) X-NOR

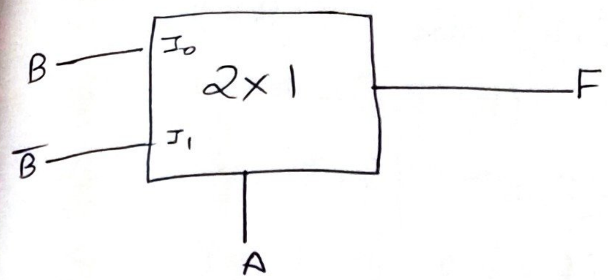


$$F = \bar{A}\bar{B} + AB$$

$$F = A \odot B$$

$$\rightarrow \boxed{F = \overline{A \oplus B}}$$

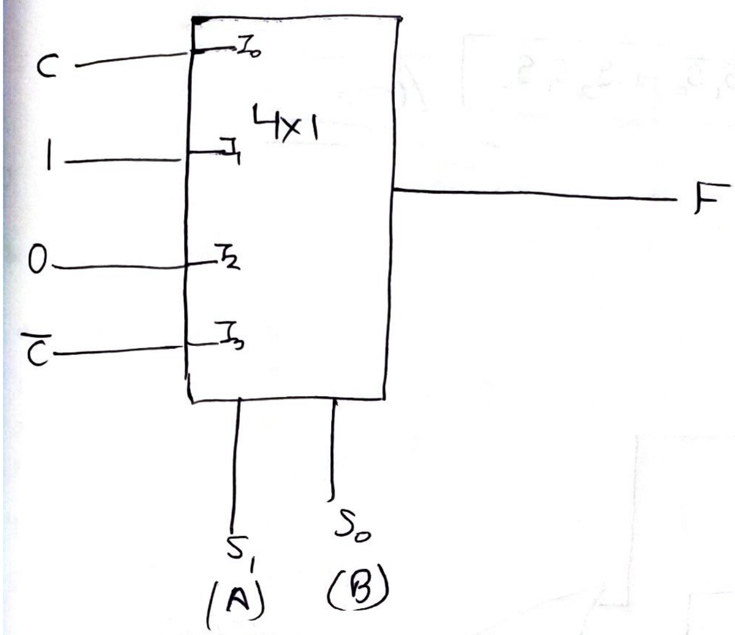
⑦ X-OR Gate



$F = \bar{A}B + A\bar{B}$

$\boxed{F = A \oplus B}$

Minimised Expression for Mux



S_1	S_0	F	
0	0	$I_0 \to C$	$\bar{A}\bar{B}$
0	1	$I_1 \to 1$	$\bar{A}B$
1	0	$I_2 \to 0$	$A\bar{B}$
1	1	$I_3 \to \bar{C}$	AB

$F = \bar{A}\bar{B} \times C + \bar{A}B \times 1 + A\bar{B} \times 0 + AB\bar{C}$

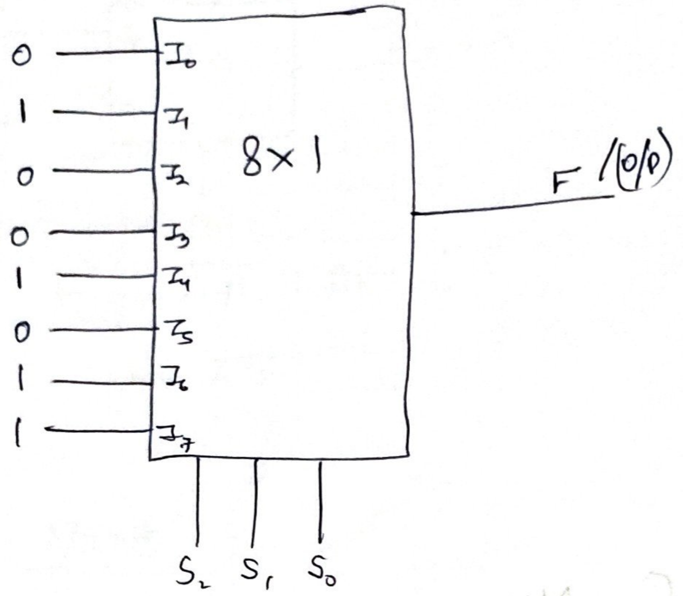
$F = \bar{A}\bar{B}C + \bar{A}B + AB\bar{C}$

in canonical form

$F = \bar{A}\bar{B}C + \bar{A}BC + \bar{A}B\bar{C} + AB\bar{C}$

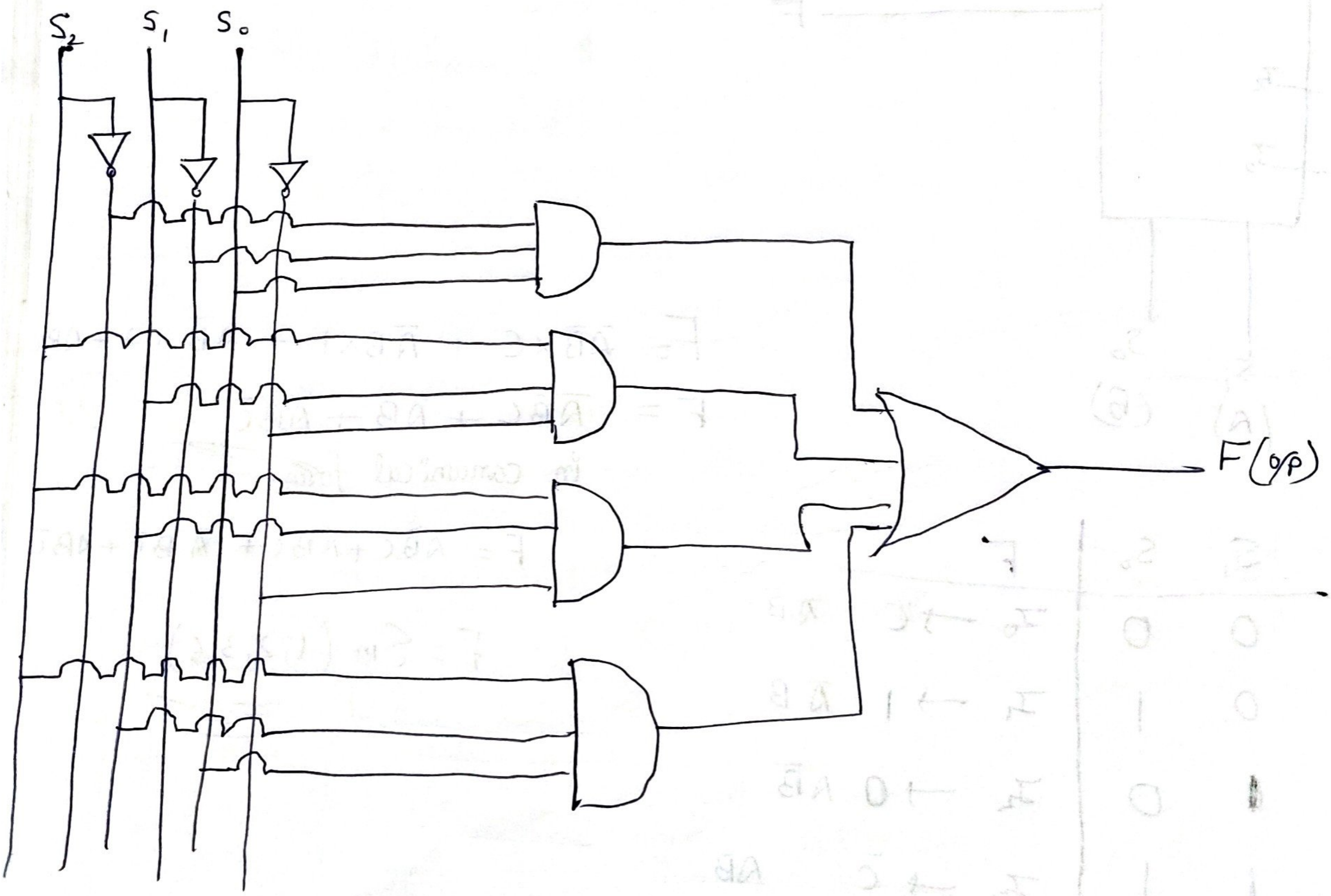
$F = \Sigma m\,(1, 2, 3, 6)$

Q. Implement Given Logic $F(A,B,C) = (1,4,6,7)$ by using 8×1 MUX



$$F = \bar{S}_2 \bar{S}_1 S_0 \times 1 + S_2 \bar{S}_1 \bar{S}_0 \times 1 + S_2 S_1 \bar{S}_0 \times 1 + S_2 S_1 S_0 \times 1$$

$$\Rightarrow \boxed{F = \bar{S}_2 \bar{S}_1 S_0 + S_2 \bar{S}_1 \bar{S}_0 + S_2 S_1 \bar{S}_0 + S_2 S_1 S_0}$$



Ques: Implement of given logic expression by using 4x1 MUX.

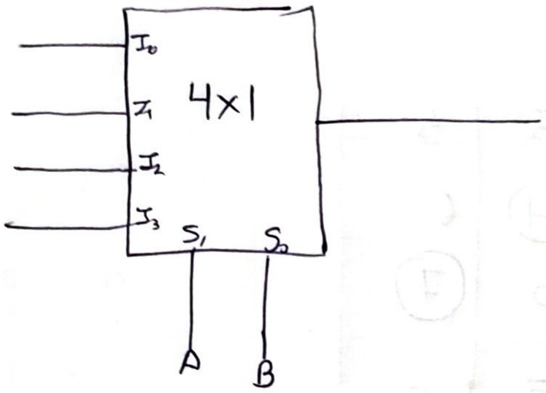
1. $F(A,B,C) = \Sigma m(2,3,5,6,7)$
2. $Y = \Sigma m(1,2,4,7)$
3. $Y = \Sigma m(3,5,6,7)$
4. $Y = \pi M(3,5,6,7)$
5. $F = \bar{A} + BC$

Solⁿ.

$$F(A,B,C) = \Sigma m(2,3,5,6,7)$$

①.

(a)



(b)

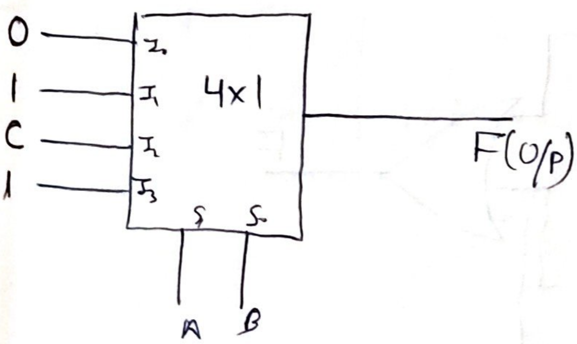
	MSB A	B	LSB C	
0	0	0	0	→ $\bar{C}$
1	0	0	1	→ C
2	0	1	0	→ $\bar{C}$
3	0	1	1	→ C
4	1	0	0	→ $\bar{C}$
5	1	0	1	→ C
6	1	1	0	→ $\bar{C}$
7	1	1	1	→ C

[both same/common = 1
not common = 0]

(c)

Min	I_0	I_1	I_2	I_3
$\bar{C}$	0	2	4	6
C	1	3	5	7
Inputs ⇒	0	1	C	1

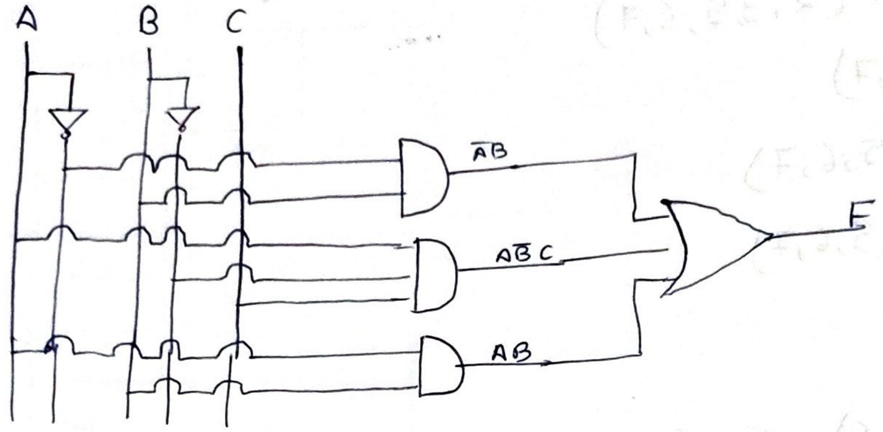
(d)



$$F = \bar{A}\bar{B}I_0 + \bar{A}BI_1 + A\bar{B}I_2 + ABI_3$$

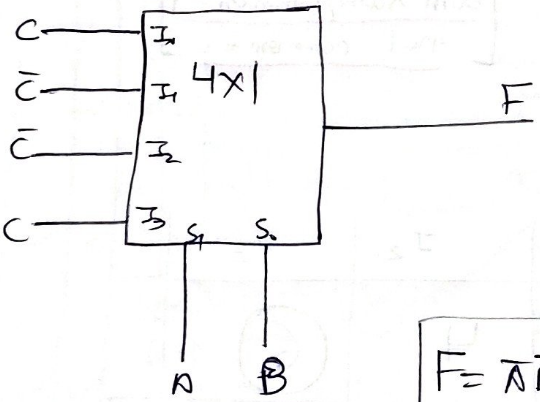
$$= \bar{A}B + A\bar{B}C + AB$$

Circuit design

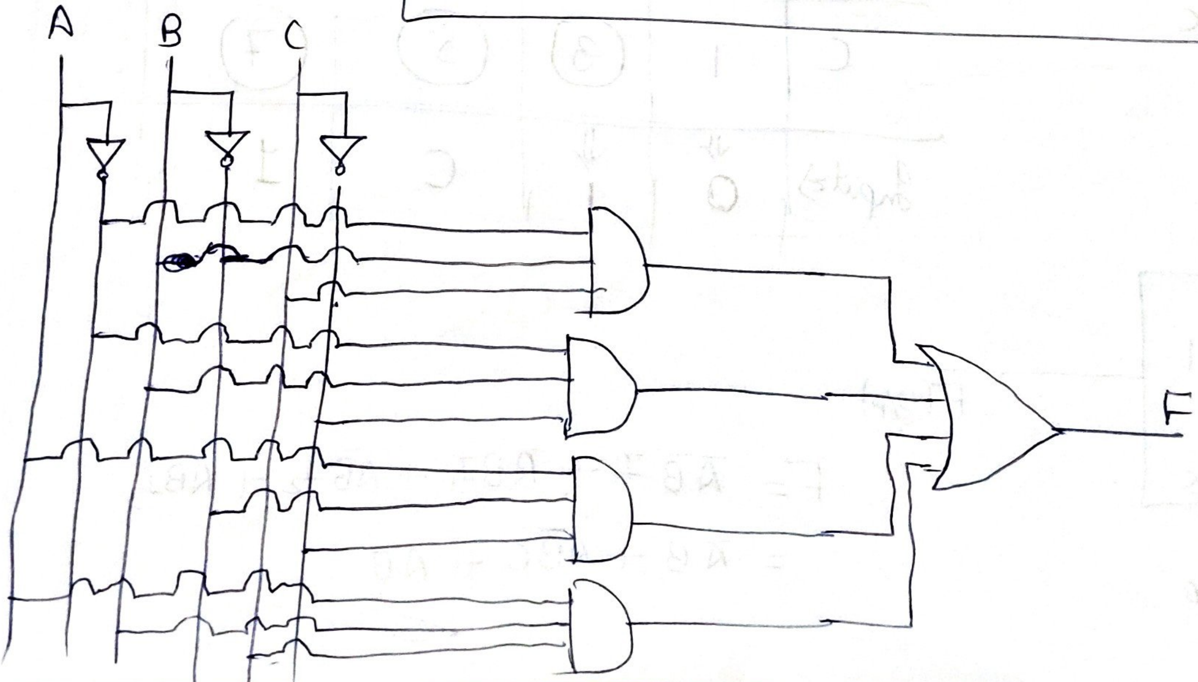


(11) $Y = \Sigma m(1,2,4,7)$

Mx	I_0	I_1	I_2	I_3
$\bar{C}$	0	2	4	6
C	1	3	5	7
Input	C	$\bar{C}$	$\bar{C}$	C

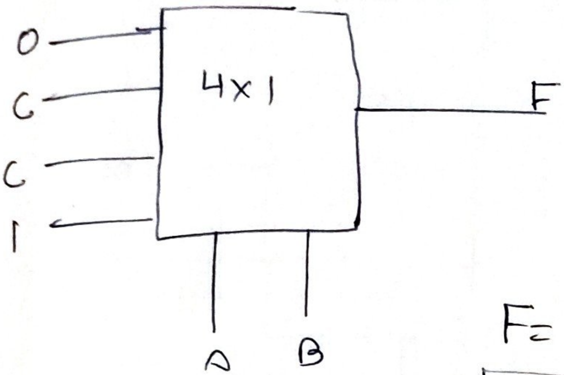


$$\boxed{F = \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C} + ABC}$$



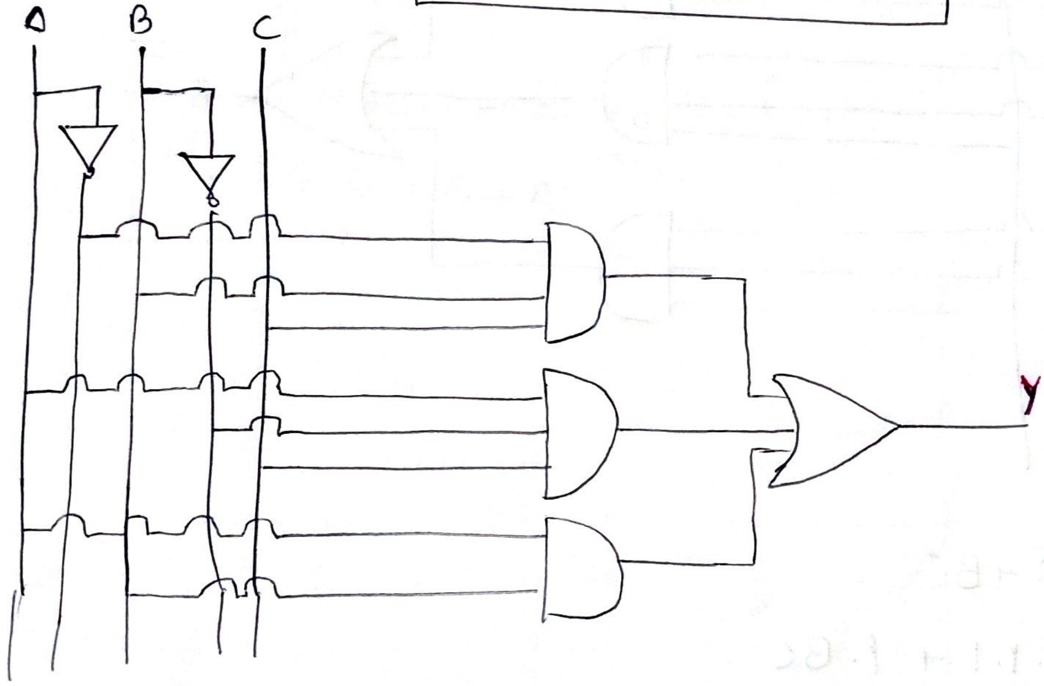
(III) $Y = \Sigma m(3,5,6,7)$

Min	I_0	I_1	I_2	I_3
$\bar{C}$	0	2	4	6
C	1	3	5	7
Input	0	C	C	1



$F = \bar{A}\bar{B}\times 0 + \bar{A}BC + A\bar{B}C + AB\times 1$

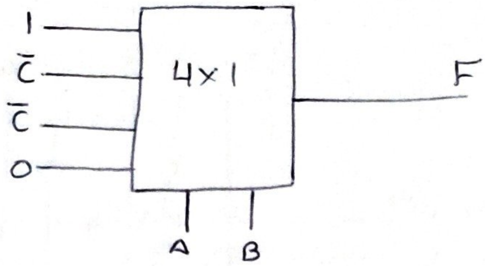
$$\boxed{F = \bar{A}BC + A\bar{B}C + AB}$$



(IV). $Y = \pi M(3,5,6,7)$

$Y = \Sigma m(0,1,2,4)$

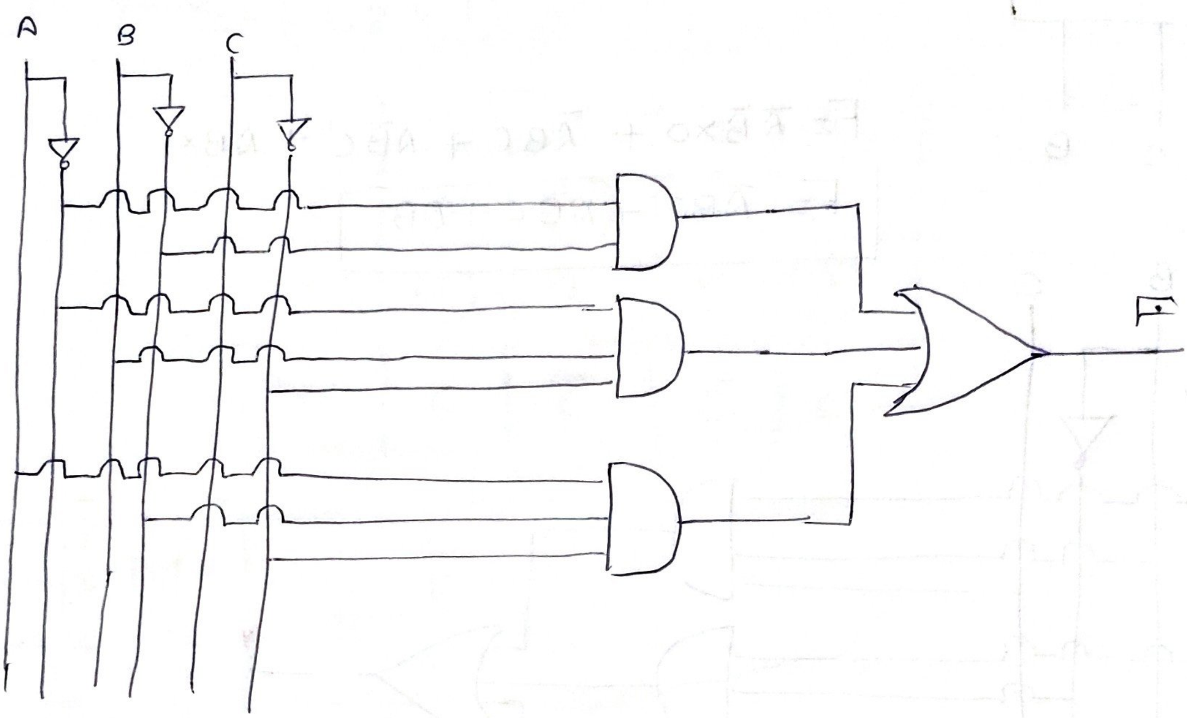
Min	I_0	I_1	I_2	I_3
$\bar{C}$	0	2	4	6
C	1	3	5	7
Input →	1	$\bar{C}$	$\bar{C}$	0



$$F = \bar{A}\bar{B}\times 1 + \bar{A}B\bar{C} + A\bar{B}\bar{C} + AB\times 0$$

$$\boxed{F = \bar{A}\bar{B} + \bar{A}B\bar{C} + A\bar{B}\bar{C}}$$

Circuit



(v) $F = \bar{A} + BC$

$= \bar{A}.1.1 + 1.BC$

$= \bar{A}.(B+\bar{B}).(C+\bar{C}) + (A+\bar{A}).BC$

$= (\bar{A}B+\bar{A}\bar{B}).(C+\bar{C}) + (A+\bar{A}).BC$

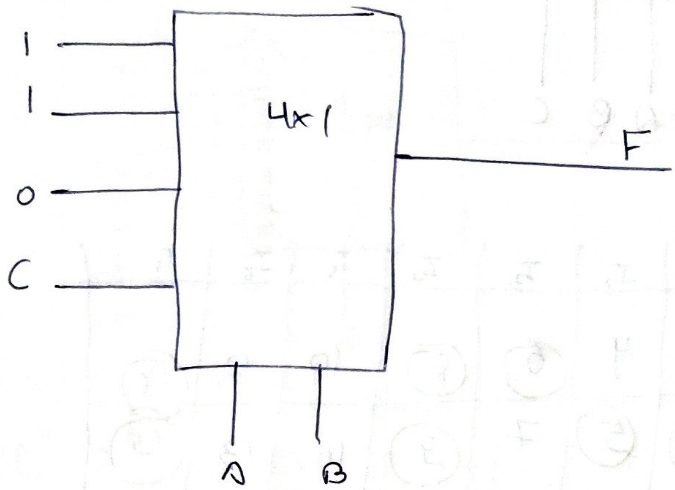
$= \bar{A}BC + \bar{A}B\bar{C} + \bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C} + ABC + \bar{A}BC$

$\Rightarrow \bar{A}BC + \bar{A}B\bar{C} + \bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C} + ABC$

$\bar{A}BC$	$\bar{A}B\bar{C}$	$\bar{A}\bar{B}C$	$\bar{A}\bar{B}\bar{C}$	ABC
011	010	001	000	111
↓	↓	↓	↓	↓
3	2	1	0	7

$F = \sum m(0,1,2,3,7)$

	I_0	I_1	I_2	I_3
$\bar{C}$	0	2	4	6
C	1	3	5	7
	1	1	0	C



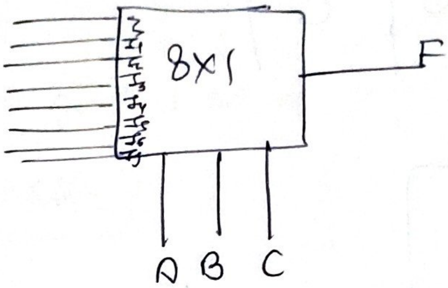
$$F = \bar{A}\bar{B}\times 1 + \bar{A}B\times 1 + A\bar{B}\times 0 + AB\times C$$

$$\boxed{F = \bar{A}\bar{B} + \bar{A}B + ABC}$$

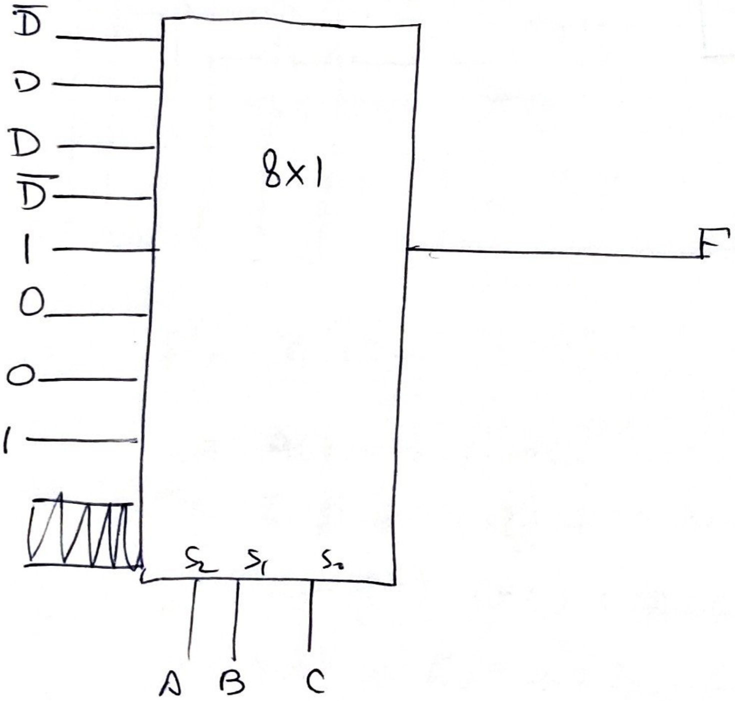
Q. Implement of given Logic expression by using 8x1 Mux?

$$F = \Sigma m(0, 3, 5, 6, 8, 9, 14, 15)$$

A	B	C	D	
0	0	0	0	$\bar{D}$
0	0	0	1	D
0	0	1	0	$\bar{D}$
0	0	1	1	D
0	1	0	0	$\bar{D}$
0	1	0	1	D
0	1	1	0	$\bar{D}$
0	1	1	1	D
1	0	0	0	$\bar{D}$
1	0	0	1	D
1	0	1	0	$\bar{D}$
1	0	1	1	D
1	1	0	0	$\bar{D}$
1	1	0	1	D
1	1	1	0	$\bar{D}$
1	1	1	1	D



	I_0	I_1	I_2	I_3	I_4	I_5	I_6	I_7
$\bar{D}$	0	2	4	6	8	10	12	14
D	1	3	5	7	9	11	13	15
Input	$\bar{D}$	D	D	$\bar{D}$	1	0	0	1



$$F = \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}CD + \bar{A}B\bar{C}D + \bar{A}BC\bar{D} + A\bar{B}\bar{C}$$

Q.* $F = \sum m(0,1,3,4,7,8,9,11,14,15)$

using:

(i) 8x1 Mux

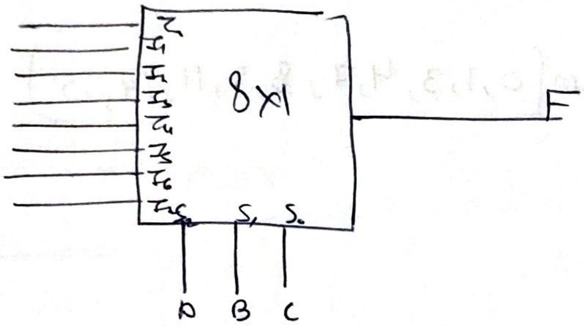
(ii) 4x1 Mux

(iii) 2x1 Mux

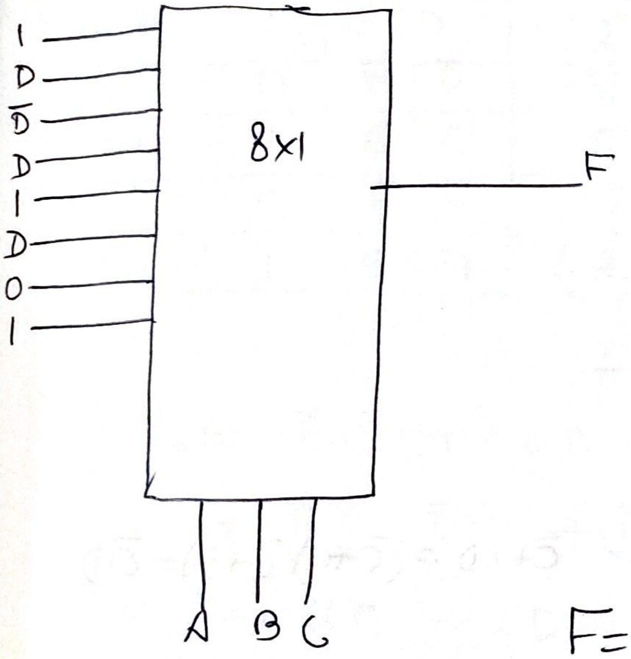
(iv) 16x1 Mux

Sol

(i) 8x1 Mux



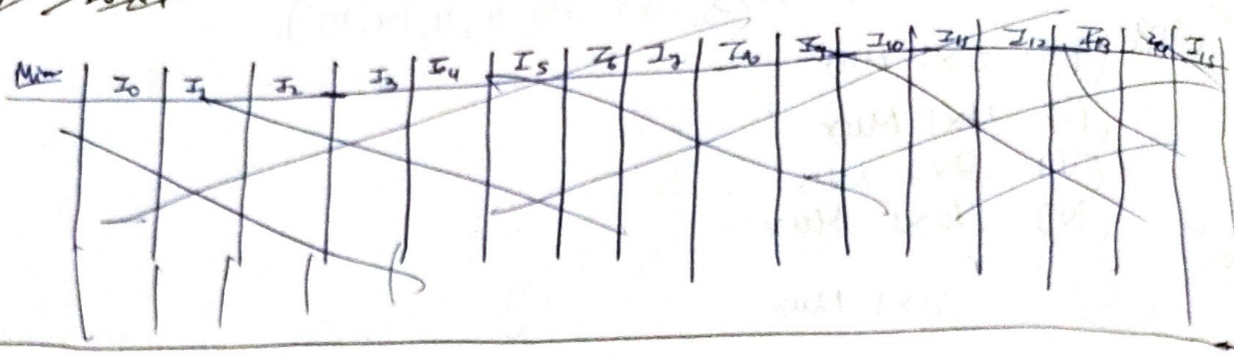
	I_0	I_1	I_2	I_3	I_4	I_5	I_6	I_7
$\bar{D}$	0	2	4	6	8	10	12	14
D	1	3	5	7	9	11	13	15
	1	D	$\bar{D}$	D	1	D	0	1



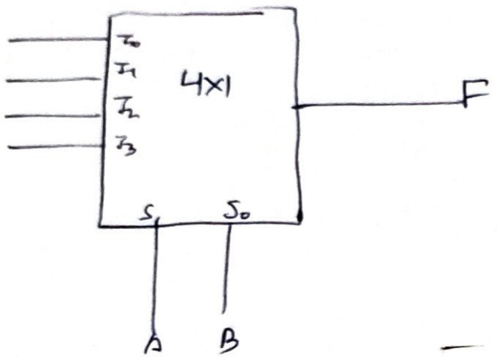
$F = \bar{A}\bar{B}\bar{C}\times 1 + \bar{A}\bar{B}C\times D + \bar{A}B\bar{C}\bar{D} + \bar{A}BCD + A\bar{B}\bar{C}\times 1 + ABCD + AB\bar{C}\times 0 + ABC\times 1$

$F = \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}CD + \bar{A}B\bar{C}\bar{D} + \bar{A}BCD + A\bar{B}\bar{C} + ABCD + ABC$

~~(iv) 16x1~~



(11) 4x1 $F(A,B,C,D) = \Sigma m(0,1,3,4,7,8,9,11,14,15)$



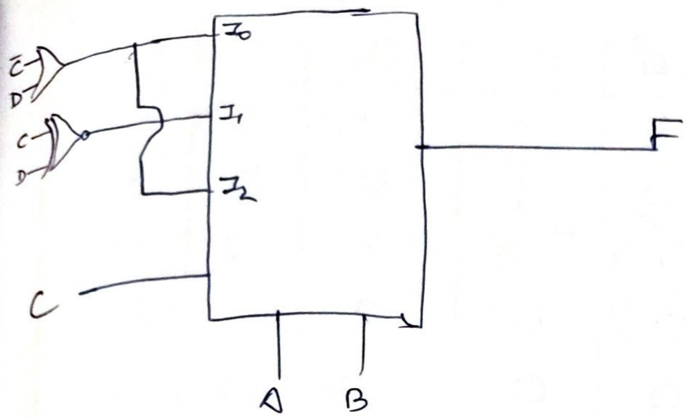
		00 I_0	01 I_1	10 I_2	11 I_3
00	$\bar{C}\bar{D}$	0	4	8	12
01	$\bar{C}D$	1	5	9	13
10	$C\bar{D}$	2	6	10	14
11	CD	3	7	11	15
Input →					

$I_0 = \bar{C}\bar{D} + \bar{C}D + CD = \bar{C}(D+\bar{D}) + CD = \bar{C} + CD = (\bar{C}+C)(\bar{C}+D) = \bar{C}+D$

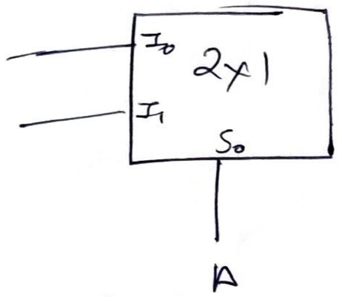
$I_1 = \bar{C}\bar{D} + \bar{C}D = C \odot D = \overline{C \oplus D}$

$I_2 = \bar{C}\bar{D} + \bar{C}D + CD = \bar{C} + CD = \bar{C}+D \quad \Rightarrow (I_2 = I_0)$

$I_3 = C\bar{D} + CD = C(D+\bar{D}) = C$



(III) 2x1 MUX



	B C D		I_0 (0)	I_1 (1)
0 →	0 0 0	→ $\bar{B}\bar{C}\bar{D}$	0	8
1 →	0 0 1	→ $\bar{B}\bar{C}D$	1	9
2 →	0 1 0	→ $\bar{B}C\bar{D}$	2	10
3 →	0 1 1	→ $\bar{B}CD$	3	11
4 →	1 0 0	$B\bar{C}\bar{D}$	4	12
5 →	1 0 1	$B\bar{C}D$	5	13
6 →	1 1 0	$BC\bar{D}$	6	14
7 →	1 1 1	BCD	7	15

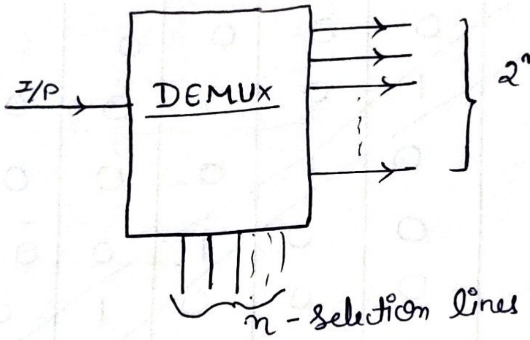
$$I_0 = \bar{B}\bar{C}\bar{D} + \bar{B}\bar{C}D + \bar{B}CD + B\bar{C}\bar{D} + BCD$$

$$= \bar{B}\bar{C}(\bar{D}+D) + (\bar{B}+B)CD + B\bar{C}\bar{D}$$

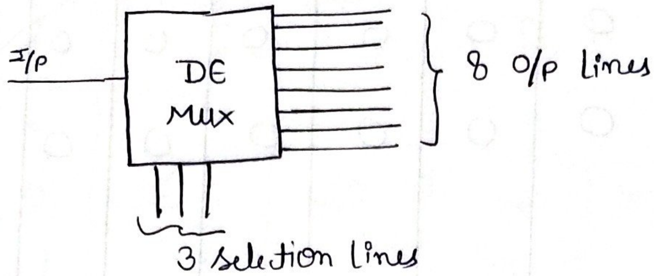
$$\Rightarrow \bar{B}\bar{C} + CD + B\bar{C}\bar{D}$$

Demultiplexer (DE-MUX)

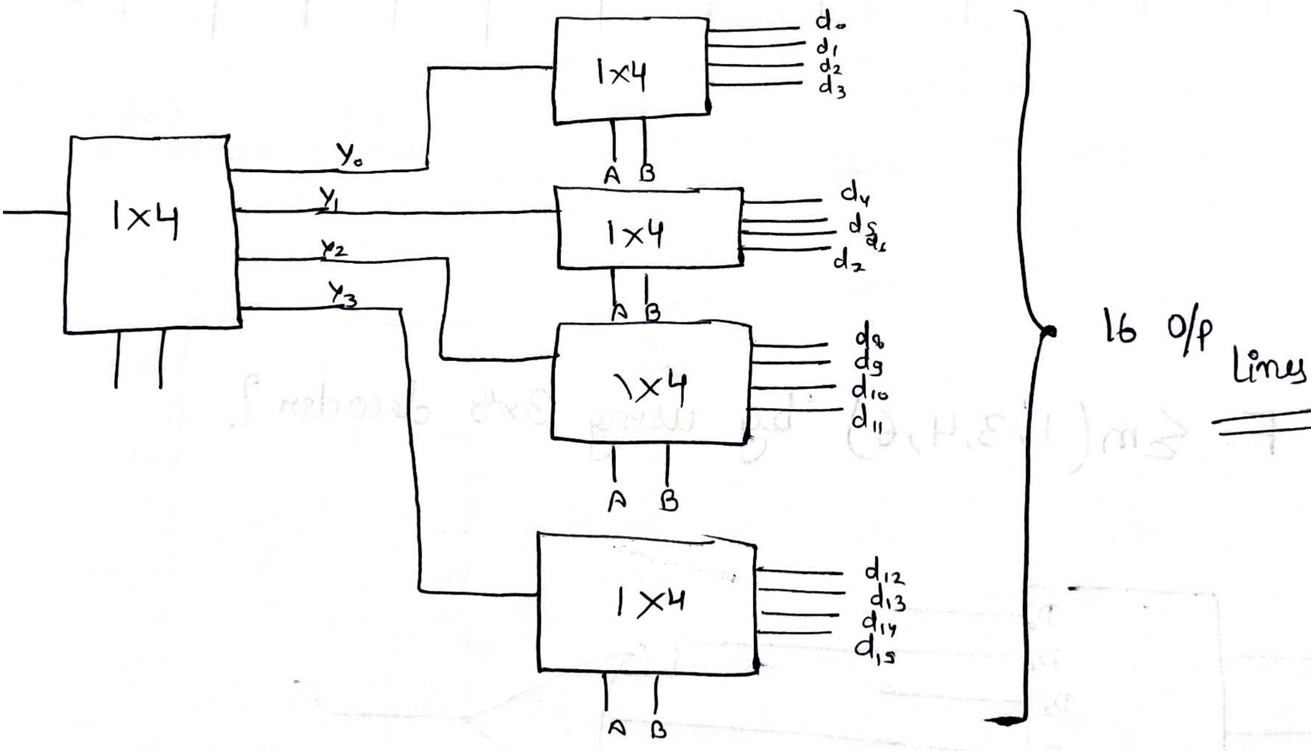
◎ It is a circuit which is having one input, 2^n output and n - selection lines.



Eg. $n=3$.



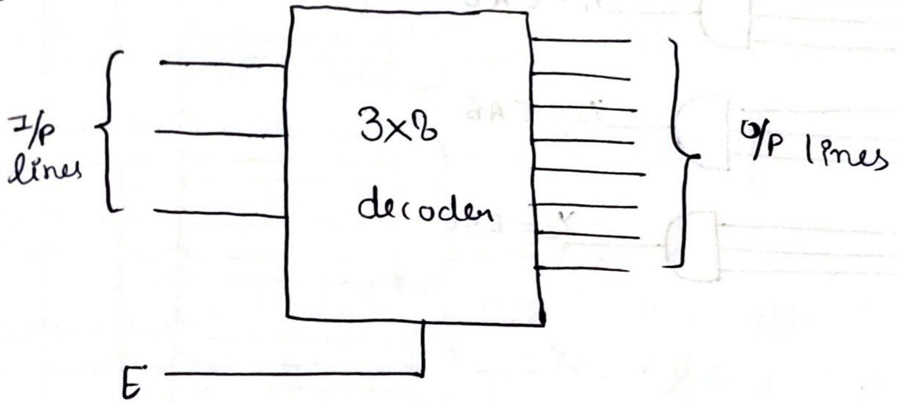
Ques:- Implement 1x16 demux using 1x4 demux?



Decoder – It is a combinational circuit which has many inputs and many outputs. It is used to convert binary to other code

such as – → binary to octal. (3x8)
→ BCD to decimal (4x10)

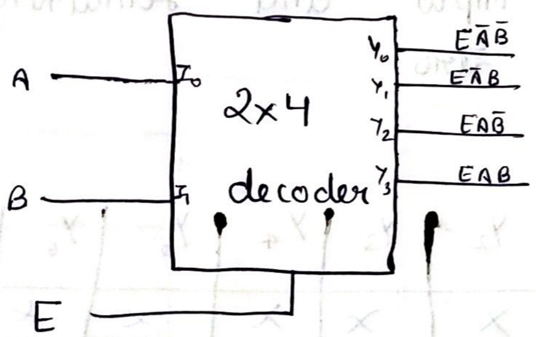
as –



→ decoder have n-inputs and less than equal to 2^n output. and which can be used to generate mean term.

basic configuration $= n \times 2^n$

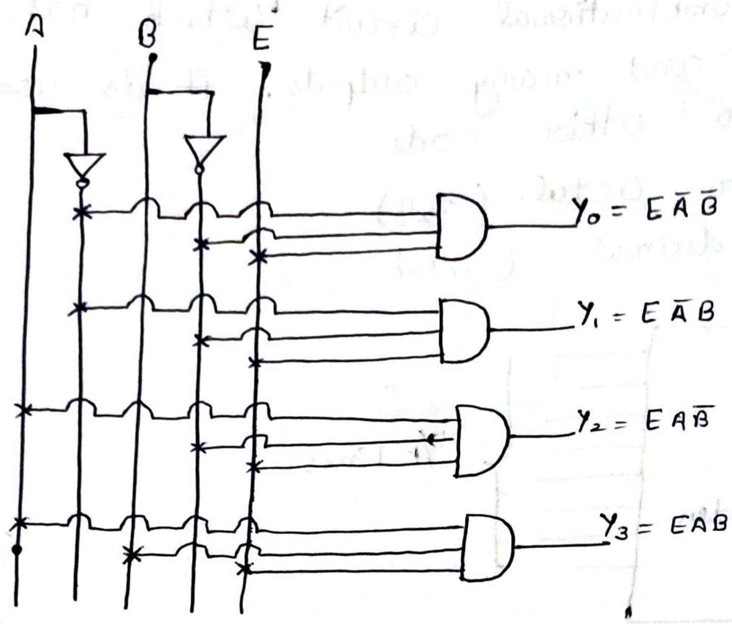
Eg. 2x4 decoder



Truth table.

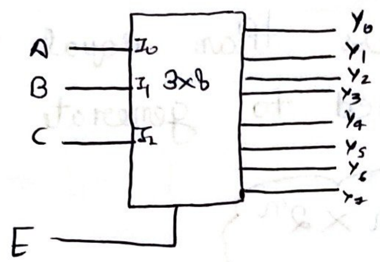
E	A	B	Y_0	Y_1	Y_2	Y_3
0	x	x	x	x	x	x
1	0	0	1	0	0	0
1	0	1	0	1	0	0
1	1	0	0	0	1	0
1	1	1	0	0	0	1

Circuit diagram :-



Imp.

3×8 decoder



Internal circuit of decoder is design in such a manner that for any particular input, output will be 1 according to decimal value of that input and remaining output terminal will have value zero.

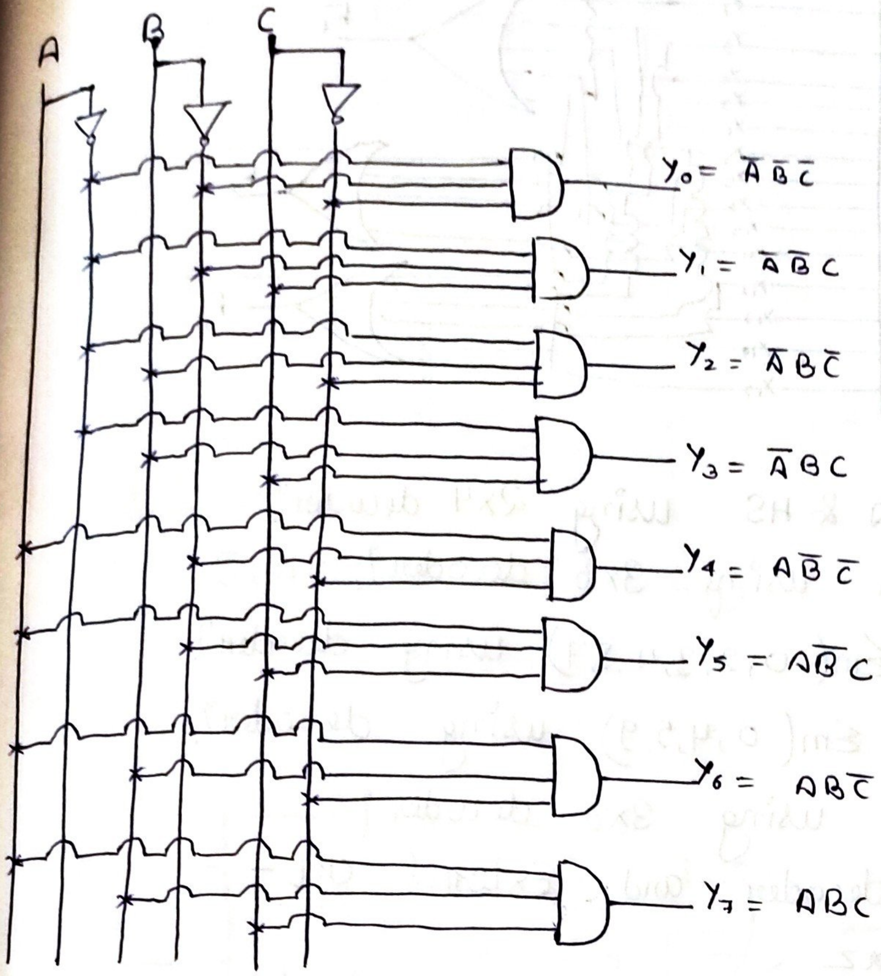
Truth table

E	A	B	C	Y_0	Y_1	Y_2	Y_3	Y_4	Y_5	Y_6	Y_7
0	X	X	X	X	X	X	X	X	X	X	X
1	0	0	0	1	0	0	0	0	0	0	0
1	0	0	1	0	1	0	0	0	0	0	0
1	0	1	0	0	0	1	0	0	0	0	0
1	0	1	1	0	0	0	1	0	0	0	0
1	1	0	0	0	0	0	0	1	0	0	0
1	1	0	1	0	0	0	0	0	1	0	0
1	1	1	0	0	0	0	0	0	0	1	0
1	1	1	1	0	0	0	0	0	0	0	1

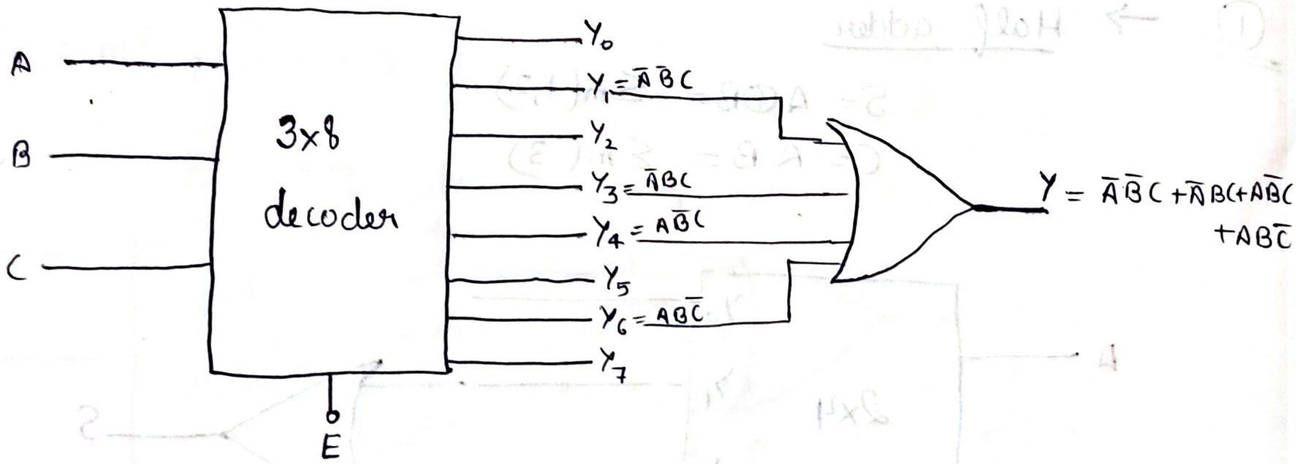
Circuit diagram

let $E = 1$ (high)

so



Ques:- Implement $F = \Sigma m(1,3,4,6)$ using 3×8 decoder?



Ques:- A Combinational circuit is specified by three boolean functio

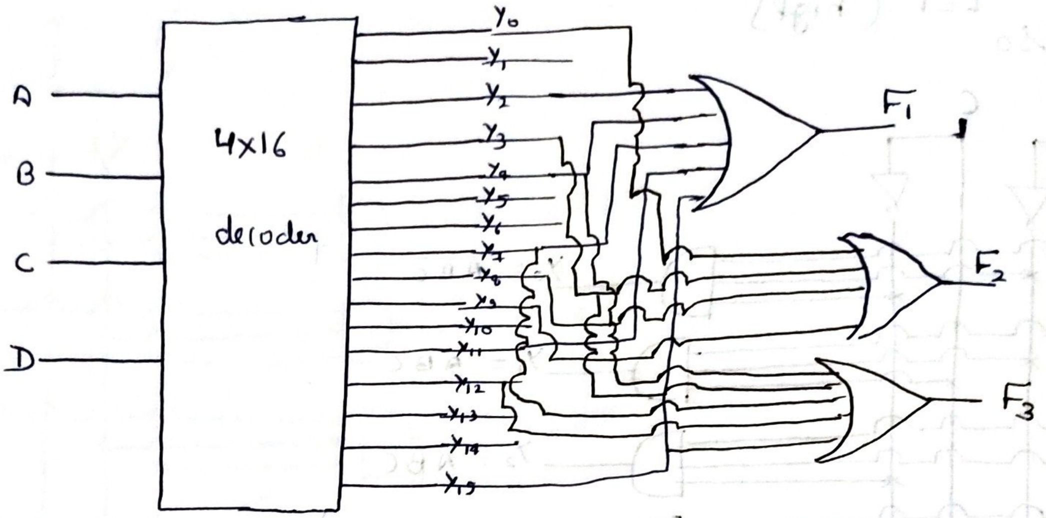
(i) $F_1(A,B,C,D) = \Sigma m(2,4,7,11,15)$

(ii) $F_2(A,B,C,D) = \Sigma m(0,3,8,9)$

(iii) $F_3(A,B,C,D) = \Sigma m(3,4,7,12,13,15)$

• Implement using the decoder?

Ans :-



Imp. Ques. (10 Marks)

Ques :- ① Implement HA & HS using 2x4 decoder?

② Implement FA using 3x8 decoder?

③ Implement $F = \Sigma m(0,2,3,4,5,7)$ using decoder?

④ Implement $F = \Sigma m(0,4,5,9)$ using decoder?

⑤ Design 4x16 using 3x8 decoder?

⑥ Using a decoder and External ckt -

$F_1 = \bar{x}y\bar{z} + xz$

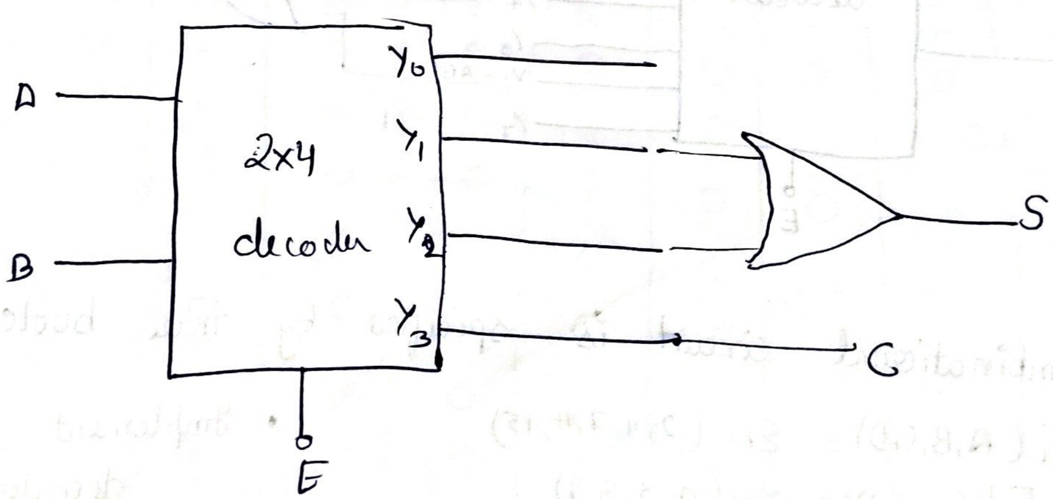
$F_2 = x\bar{y}\bar{z} + yz$

$F_3 = \bar{x}\bar{y}\bar{z} + xy$

①. → Half adder

$S = A \oplus B = \Sigma m(1,2)$

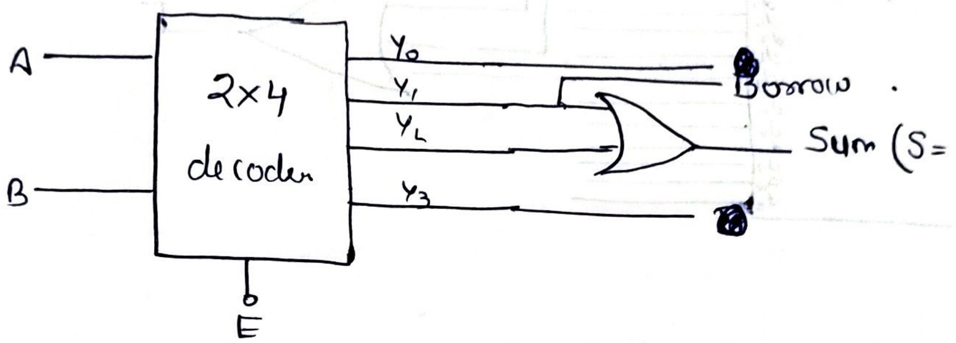
$C = A.B = \Sigma m(3)$



Half Subtractor

$S = \sum m(1,2) \rightarrow A \oplus B$

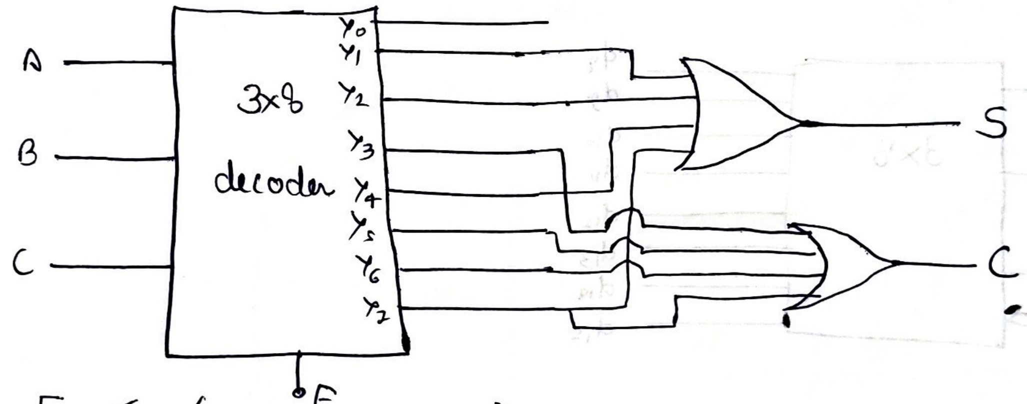
$B = \sum m(1) = \bar{A}B$



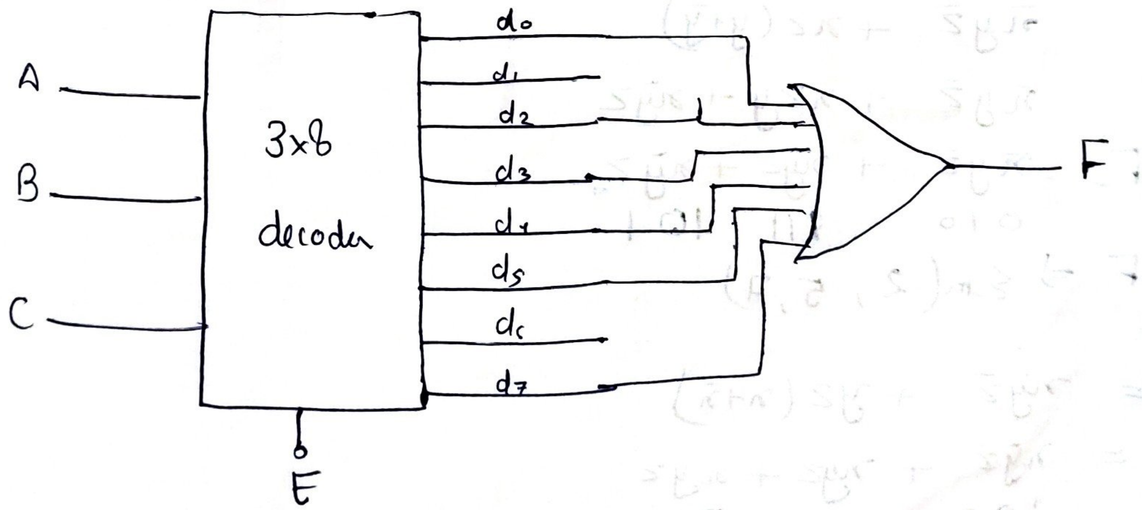
(2). Full adder

$S(A,B,C) = \sum m(1,2,4,7)$

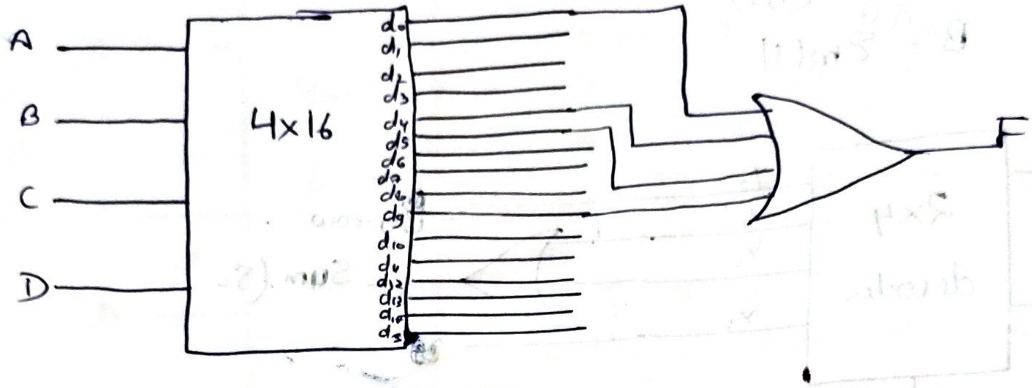
$C(A,B,C) = \sum m(3,5,6,7)$



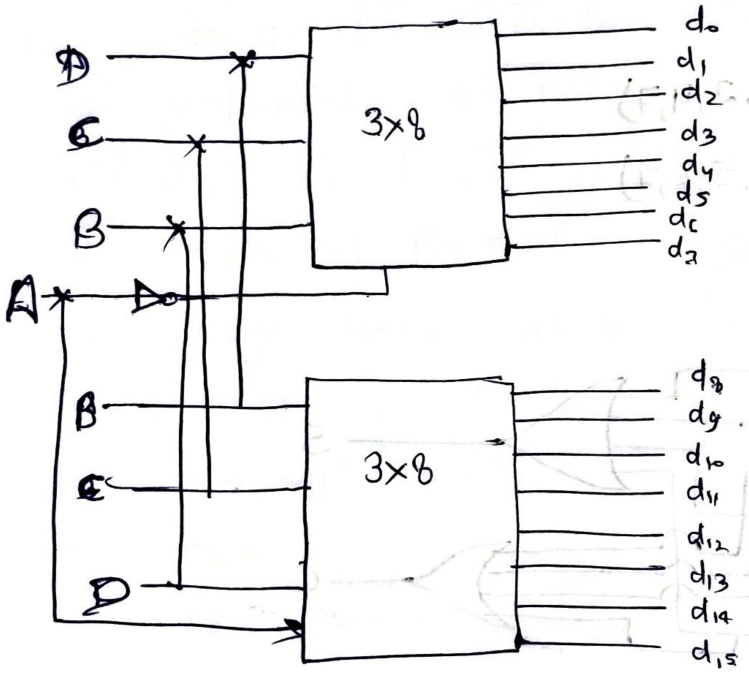
(3). $F = \sum m(0,2,3,4,5,7)$



(4)



(5)



(6)

$$F_1 = \bar{x}y\bar{z} + xz$$

$$\bar{x}y\bar{z} + xz(y+\bar{y})$$

$$\bar{x}y\bar{z} + xzy + x\bar{y}z$$

$$F = \bar{x}y\bar{z} + xyz + x\bar{y}z$$

$$\quad 010 \qquad 111 \qquad 101$$

$$F \Rightarrow \Sigma m(2, 5, 7)$$

$$F_2 = x\bar{y}\bar{z} + yz(x+\bar{x})$$

$$= x\bar{y}\bar{z} + xyz + \bar{x}yz$$

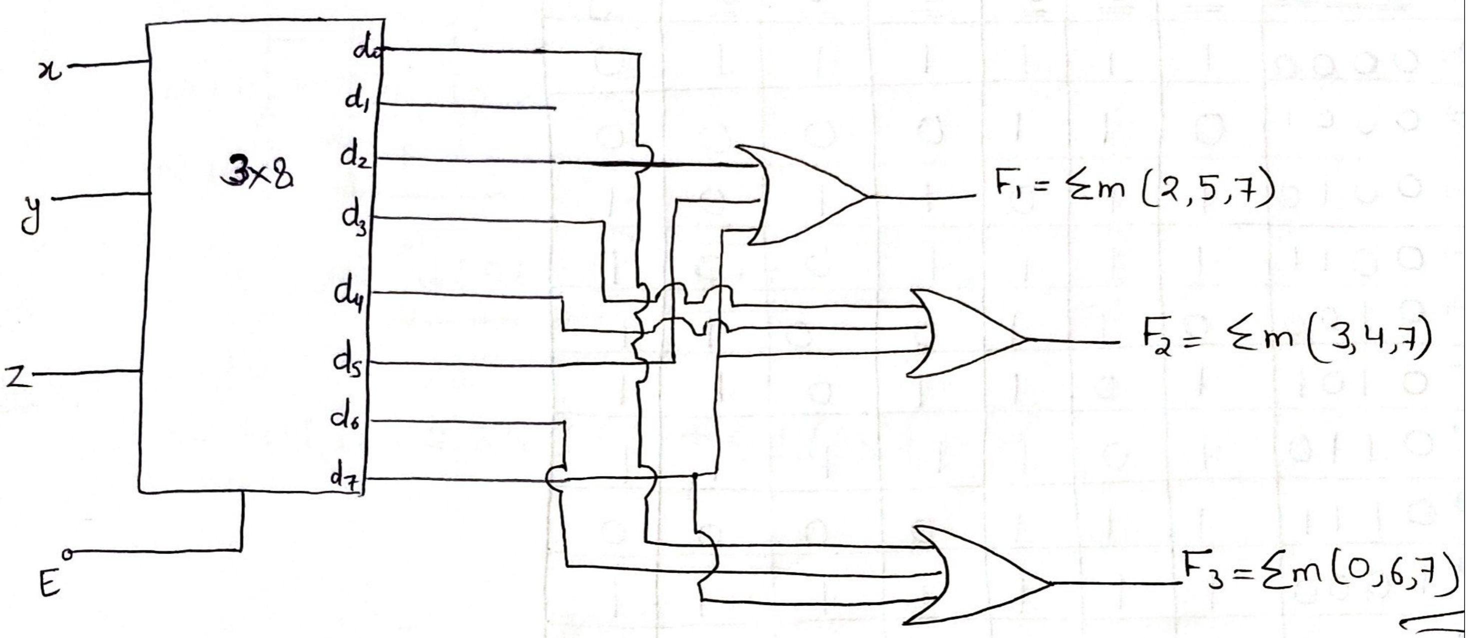
$$\quad 100 \qquad 111 \qquad 011$$

$$F \Rightarrow \Sigma m(4, 7, 3)$$

$F_3 = \bar{x}\bar{y}\bar{z} + xyz + xy\bar{z}$
(000) (111) (110)

$F_3 = \Sigma m(0, 6, 7)$

Then,



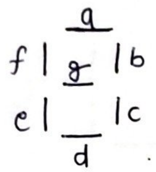
*

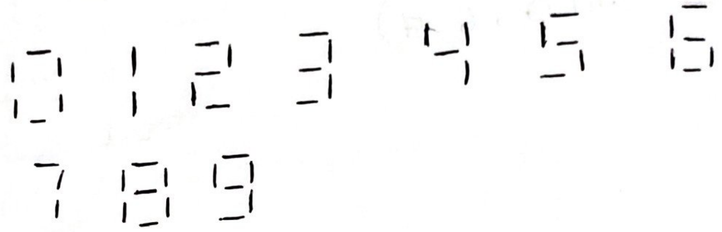
Most Important :-

** (10 Marks)

Ques. (i) Design a BCD to 7 segment circuit?
(ii) Design a BCD to Excess-3-code?

Ans.
(i)





BCD	a	b	c	d	e	f	g
0→ 0000	1	1	1	1	1	1	0
1→ 0001	0	1	1	0	0	0	0
2→ 0010	1	1	0	1	1	0	1
3→ 0011	1	1	1	1	0	0	1
4→ 0100	0	1	1	0	0	1	1
5→ 0101	1	0	1	1	0	1	1
6→ 0110	1	0	1	1	1	1	1
7→ 0111	1	1	1	0	0	0	0
8→ 1000	1	1	1	1	1	1	1
9→ 1001	1	1	1	1	0	1	1
10→ 1010	X	X	X	X	X	X	X
11→ 1011	X	X	X	X	X	X	X
12→ 1100	X	X	X	X	X	X	X
13→ 1101	X	X	X	X	X	X	X
14→ 1110	X	X	X	X	X	X	X
15→ 1111	X	X	X	X	X	X	X

$a = \sum m(0,2,3,5,6,7,8,9) + \sum d(10,11,12,13,14,15)$

$b = \sum m(0,1,2,3,4,7,8,9) + \sum d(10,11,12,13,14,15)$

$C = \sum m(0,1,3,4,5,6,7,8,9) + \sum d($ " " $)$

$d = \sum m(0,2,3,5,6,8,9) + \sum d($ " " $)$

$e = \Sigma m(0,2,6,8) + \Sigma d(\text{" "})$

$f = \Sigma m(0,4,6,5,8,9) + \Sigma d(\text{" "})$

$g = \Sigma m(2,3,4,5,6,8,9) + \Sigma d(\text{" "})$

for a.

$$a = \Sigma m(0,2,3,5,6,7,8,9) + \Sigma d(10,11,12,13,14,15)$$

AB \ CD	$\bar C\bar D$ 00	$\bar C D$ 01	CD 11	$C\bar D$ 10
$\bar A\bar B$ 00	1		1	1
$\bar A B$ 01		1	1	1
AB 11	x	x	x	x
$A\bar B$ 10	1	1	x	x

$$\Rightarrow \boxed{a(A,B,C,D) = A + C + \bar B\bar D + BD}$$

for b.

$$b = \Sigma m(0,1,2,3,4,7,8,9) + \Sigma d(10,11,12,13,14,15)$$

g_2 g_2

AB \ CD	$\bar C\bar D$ 00	$\bar C D$ 01	CD 11	$C\bar D$ 10
$\bar A\bar B$ 00	1	1	1	1
$\bar A B$ 01	1		1	
AB 11	X	X	X	X
$A\bar B$ 10	1	1	X	X

g_1

$$\Rightarrow \boxed{b(A,B,C,D) = \bar C\bar D + CD + A + \bar B}$$

for c

$$C = \Sigma m(0,1,3,4,5,6,7,8,9) + \Sigma d(10,11,12,13,14,15)$$

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	1	1	1	
$\bar{A}B$ 01	1	1	1	1
AB 11	X	X	X	X
$A\bar{B}$ 10	1	1	X	X

g_1 g_2 g_3

$$\Rightarrow \boxed{C(A,B,C,D) = B + D + \bar{C}}$$

for d.

$$d = \Sigma m(0,2,3,5,6,8,9) + \Sigma d(10,11,12,13,14,15)$$

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	1		1	1
$\bar{A}B$ 01		1		1
AB 11	X	X	X	X
$A\bar{B}$ 10	1	1	X	X

$$\Rightarrow \boxed{d = A + \bar{B}\bar{D} + \bar{B}C + C\bar{D}} + B\bar{C}D$$

for e.

$$e = \Sigma m(0,2,6,8) + \Sigma d \cdot (10,11,12,13,14,15)$$

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	1			1
$\bar{A}B$ 01				1
AB 11				X
$A\bar{B}$ 10	1			X

$$\Rightarrow \boxed{e = \bar{B}\bar{D} + C\bar{D}}$$

for f.

$f = \sum m(0,4,5,6,8,9) + \sum d(10,11,12,13,14,15)$

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	1			
$\bar{A}B$ 01	1	1		1
AB 11	X	X	X	X
$A\bar{B}$ 10	1	1	X	X

$\Rightarrow \boxed{f(A,B,C,D) = \bar{C}\bar{D} + B\bar{D} + A + B\bar{C}}$

for g:

$g = \sum m(2,3,4,5,6,8,9) + \sum d(10,11,12,13,14,15)$

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00			1	1
$\bar{A}B$ 01	1	1		1
AB 11	X	X	X	X
$A\bar{B}$ 10	1	1		X

$\Rightarrow \boxed{g = \cancel{B\bar{D}}\ A + B\bar{C} + \bar{B}C + C\bar{D}}$

(ii). Design BCD to Excess-3-code?

BCD	a	b	c	d
0→0000	0	0	1	1
1→0001	0	1	0	0
2→0010	0	1	0	1
3→0011	0	1	1	0
4→0100	0	1	1	1
5→0101	1	0	0	0
6→0110	1	0	0	1
7→0111	1	0	1	0
8→1000	1	0	1	1
9→1001	1	1	0	0
10→1010	X	X	X	X
11→1011	X	X	X	X
12→1100	X	X	X	X
13→1101	X	X	X	X
14→1110	X	X	X	X
15→1111	X	X	X	X

$a = \Sigma m(5,6,7,8,9) + \Sigma d(10,11,12,13,14,15)$

$b = \Sigma m(1,2,3,4,9) + \Sigma d($ " " "$)$

$C = \Sigma m(0,3,4,7,8) + \Sigma d($ " " "$)$

$d = \Sigma m(0,2,4,6,8) + \Sigma d($ " " "$)$

for a.

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00				
$\bar{A}B$ 01		1	1	1
AB 11	X	X	X	X
$A\bar{B}$ 10	1	1	X	X

$a = A + BD + BC$

for b

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00		1	1	1
$\bar{A}B$ 01	1			
AB 11	X	X	X	X
$A\bar{B}$ 10		1	X	X

$b = B\bar{C}\bar{D} + ~~[illegible]~~ + \bar{B}D + \bar{B}C$

for c,

AB \ CD	00	01	11	10
00	1		1	
01	1		1	
11	X	X	X	X
10	1		X	X

$C = \bar{C}\bar{D} + CD$

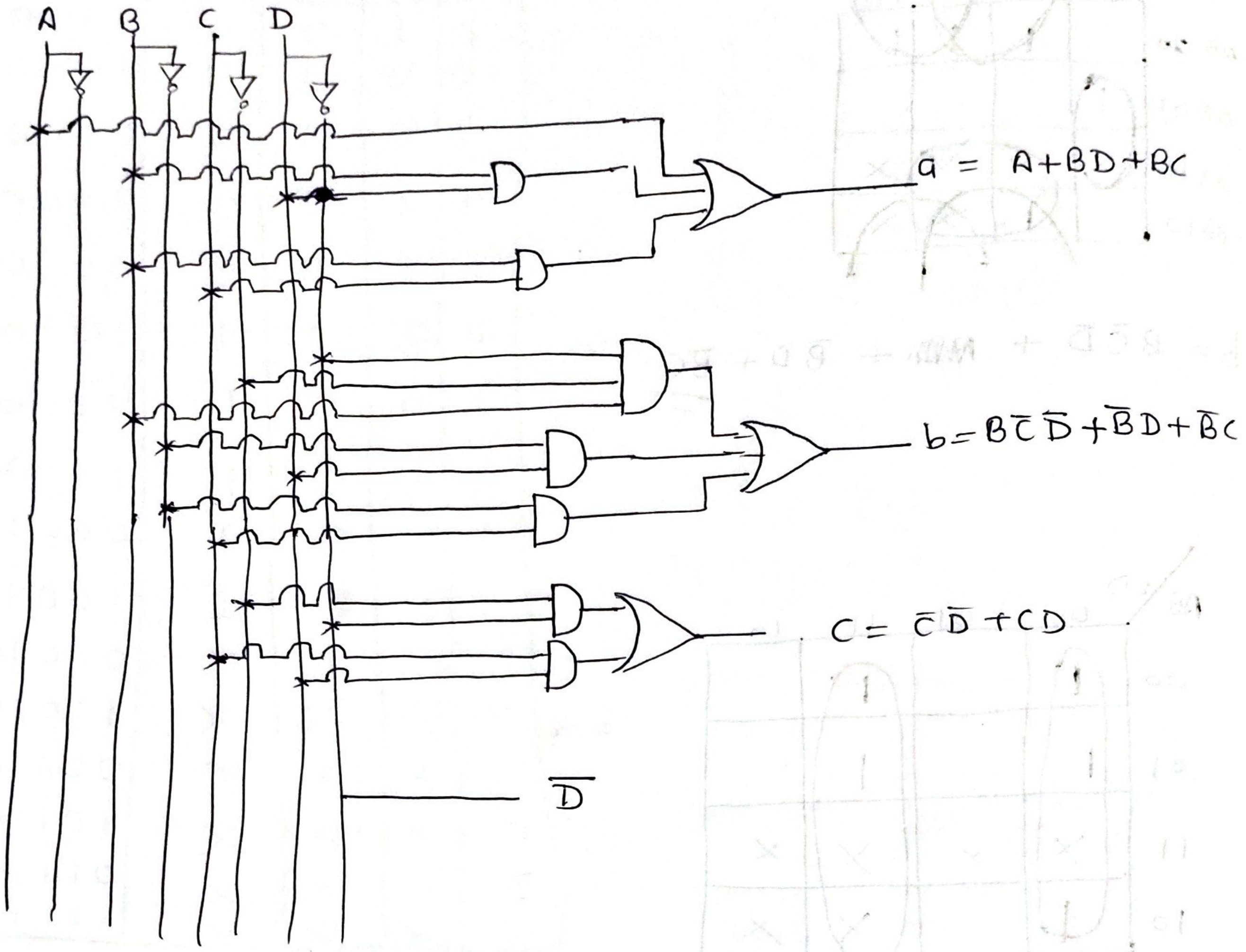
for d

AB \ CD	00	01	11	10
00	1			1
01	1			1
11	X	X	X	X
10	1		X	X

$d = \bar{D}$

Ckt diagram

for a, b, c, d



ENCODER :-

↳ Encoder perform operation of decoder

↳ Encoder is a combination circuit. Which will convert other codes to binary codes.

Such as -

(i) Octal to Binary (8x3).

(ii) Decimal to BCD (10x4)

(iii) Hex to Binary Code (16x4)

$8=2^3$

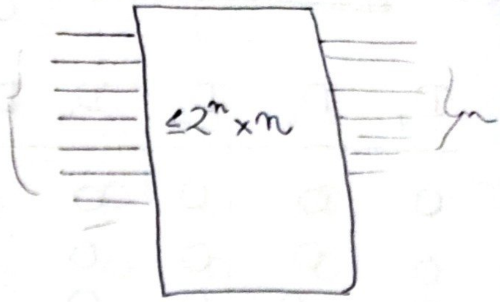
Imp.

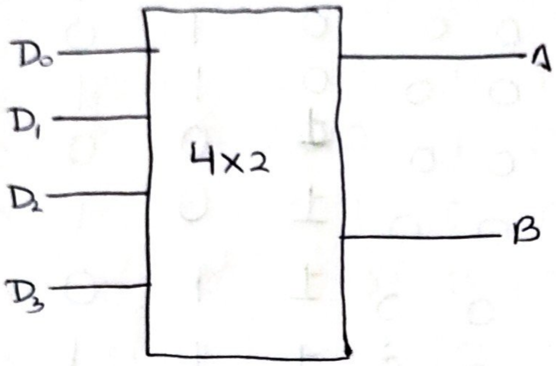
A digital ckt which has $\leq 2^n$ input and n output.

Eg. $n = 2$

Then $4 \rightarrow$ input $\leq 2^n$

$2 \rightarrow$ output



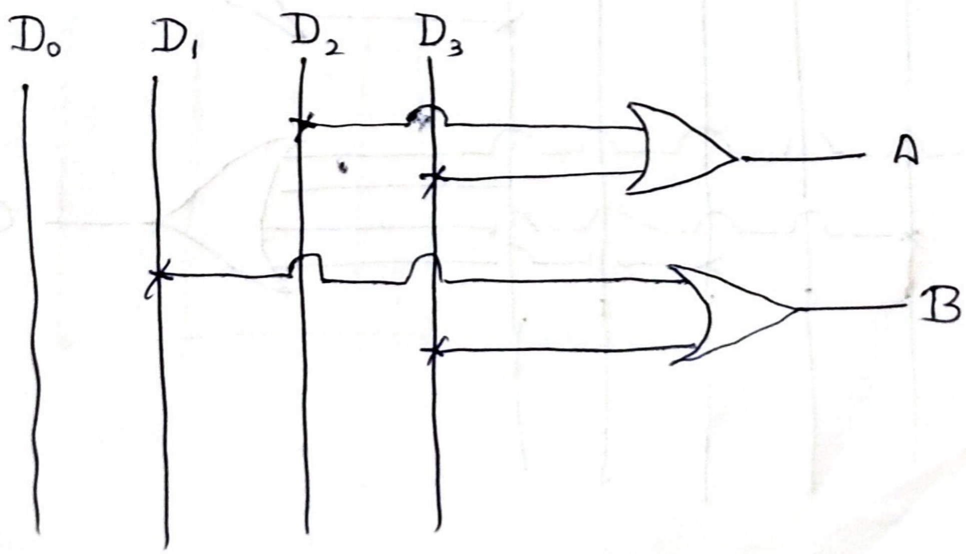


Truth table

D_3	D_2	D_1	D_0	A	B
0	0	0	1	0	0
0	0	1	0	0	1
0	1	0	0	1	0
1	0	0	0	1	1

$A = D_2 + D_3$ $\qquad$ $B = D_1 + D_3$

Ckt diagram



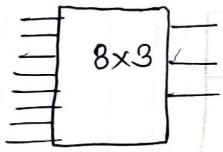
(i) Octal to binary (8x3)

D_7	D_6	D_5	D_4	D_3	D_2	D_1	D_0	A	B	C
0	0	0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	1	0	0	0	1
0	0	0	0	0	1	0	0	0	1	0
0	0	0	0	1	0	0	0	0	1	1
0	0	0	1	0	0	0	0	1	0	0
0	0	1	0	0	0	0	0	1	0	1
0	1	0	0	0	0	0	0	1	1	0
1	0	0	0	0	0	0	0	1	1	1

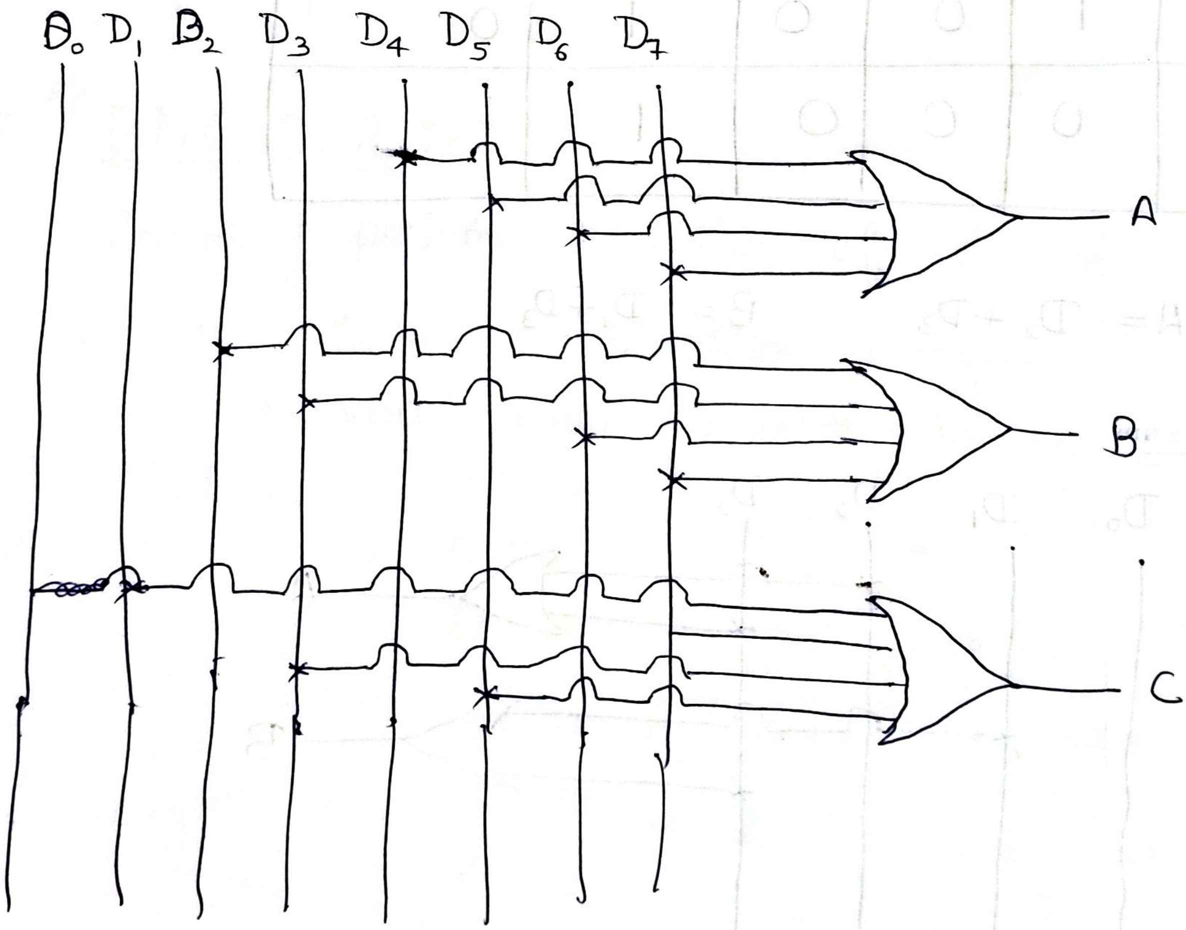
$A = D_4 + D_5 + D_6 + D_7$

$B = D_2 + D_3 + D_6 + D_7$

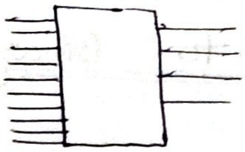
$C = D_1 + D_3 + D_5 + D_7$



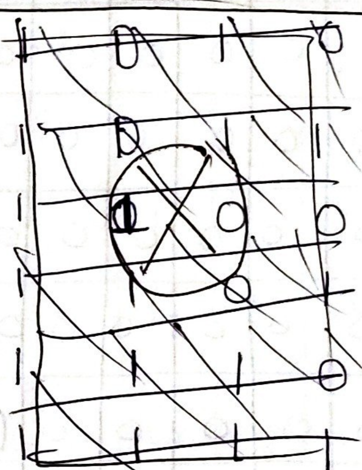
CKt diagram



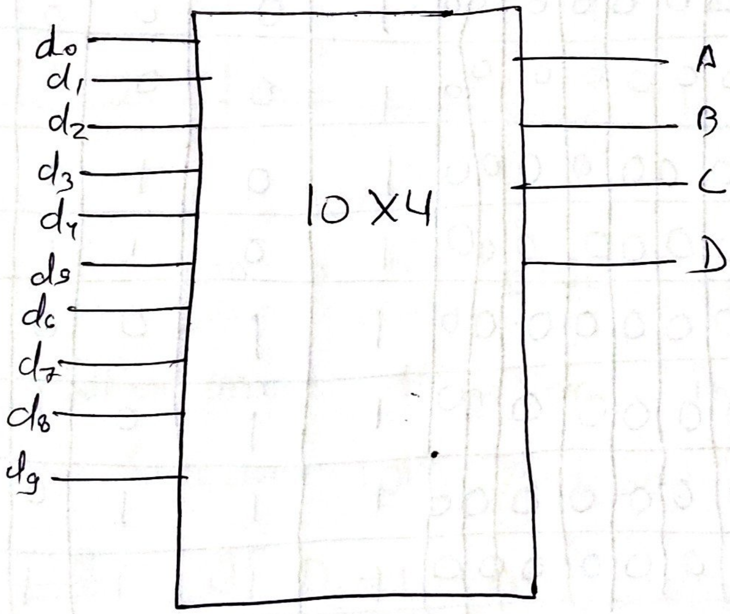
(ii) Decimal to BCD (10x4)



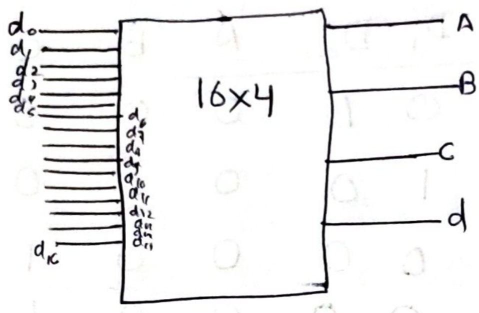
D_9	D_8	D_7	D_6	D_5	D_4	D_3	D_2	D_1	D_0	A	B	C	D
0	0	0	0	0	0	0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0	1	0	0	0	0	1
0	0	0	0	0	0	0	1	0	0	0	0	1	0
0	0	0	0	0	0	1	0	0	0	0	0	1	1
0	0	0	0	0	1	0	0	0	0	0	1	0	0
0	0	0	0	1	0	0	0	0	0	0	1	0	1
0	0	0	1	0	0	0	0	0	0	0	1	1	0
0	0	1	0	0	0	0	0	0	0	0	1	1	1
0	1	0	0	0	0	0	0	0	0	1	0	0	0
1	0	0	0	0	0	0	0	0	0	1	0	0	1



circuit diagram



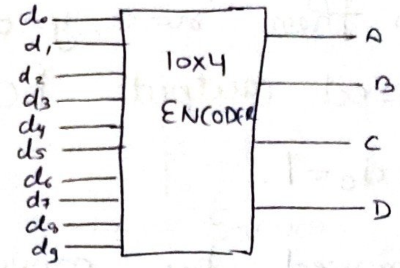
(iii) Hex to Binary Encoder



D_{15}	D_{14}	D_{13}	D_{12}	D_{11}	D_{10}	D_9	D_8	D_7	D_6	D_5	D_4	D_3	D_2	D_1	D_0	A	B	C	D
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1
0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0
0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	1
0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0	0
0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	1
0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	1	0
0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	1	1
0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0	1
0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	1	0
0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	1	1
0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0
0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	1
0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1

Decimal to BCD

(10x4)



D_9	D_8	D_7	D_6	D_5	D_4	D_3	D_2	D_1	D_0	A	B	C	D
0	0	0	0	0	0	0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0	1	0	0	0	0	1
0	0	0	0	0	0	0	1	0	0	0	0	1	0
0	0	0	0	0	0	1	0	0	0	0	0	1	1
0	0	0	0	0	1	0	0	0	0	0	1	0	0
0	0	0	0	1	0	0	0	0	0	0	1	0	1
0	0	0	1	0	0	0	0	0	0	0	1	1	0
0	0	1	0	0	0	0	0	0	0	0	1	1	1
0	1	0	0	0	0	0	0	0	0	1	0	0	0
1	0	0	0	0	0	0	0	0	0	1	0	0	1

Imp.
Priority based
(10 Marks)

Ques:- Problem with design of Encoder :-

① When more than one input are 1 than output of Encoder will not be correct –

Eg If $D_3 = 1$ and also $D_6 = 1$ at the same time than output will be get $x=1$, $y=1$ and $z=1$ i.e. output $= 111$ which is incorrect.

O/P according to $D_3=1$, $D_6=1$

(ii) When all input are zero then $x=0, y=0, z=0$ i.e. O/P $= 000$. This is incorrect output because O/P should be 000 only for $d_0=1$.

» This problem can be removed by providing one more output to indicate that atleast one input.

» Above two problems can be removed using "priority Encoder".

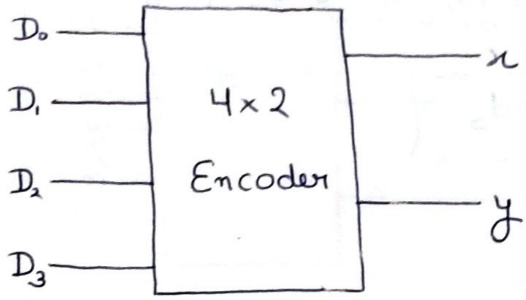
» In priority Encoder if ~~one~~ more than one input is equal to 1 than input with higher subscript will be given preference so if $D_3=1$ & $D_6=1$ at the same time $D_6=1$ will be given preference and output will be 110 for $D_6=1$.

» In priority Encoder, we never give all input equal to zero, if all input is equal to zero than this input is treated as invalid input.

Priority Encoder :-

→ The opr^n of priority Encoder is such that if two or more inputs are equal to 1. at the same time, then input having highest priority will be given preference. i.e input having highest subscript will be given preference and output will be obtain according to preference.

* 4:2 Priority Encoder :-



Truth table

D_3	D_2	D_1	D_0	x	y	V
0	0	0	1	0	0	1
0	0	1	X	0	1	1
0	1	X	X	1	0	1
1	X	X	X	1	1	1

	D_3	D_2	D_1	D_0	x	y	V
0→	0	0	0	0	X	X	0
1→	0	0	0	1	0	0	1
2→	0	0	1	0	0	1	1
3→	0	0	1	1	0	1	1
4→	0	1	0	0	1	0	1
5→	0	1	0	1	1	0	1
6→	0	1	1	0	1	0	1
7→	0	1	1	1	1	0	1
8→	1	0	0	0	1	1	1
9→	1	0	0	1	1	1	1
10→	1	0	1	0	1	1	1
11→	1	0	1	1	1	1	1
12→	1	1	0	0	1	1	1
13→	1	1	0	1	1	1	1
14→	1	1	1	0	1	1	1
15→	1	1	1	1	1	1	1

$x = \sum m(4,5,6,7,8,9,10,11,12,13,14,15) + \sum d(0)$

$y = \sum m(2,3,8,9,10,11,12,13,14,15) + \sum d(0)$

$$\boxed{V = d_0 + d_1 + d_2 + d_3}$$

for K-Map

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00				
$\bar{A}B$ 01	1	1	1	1
AB 11	1	1	1	1
$A\bar{B}$ 10	1	1	1	1

$$\boxed{x = A + B}$$

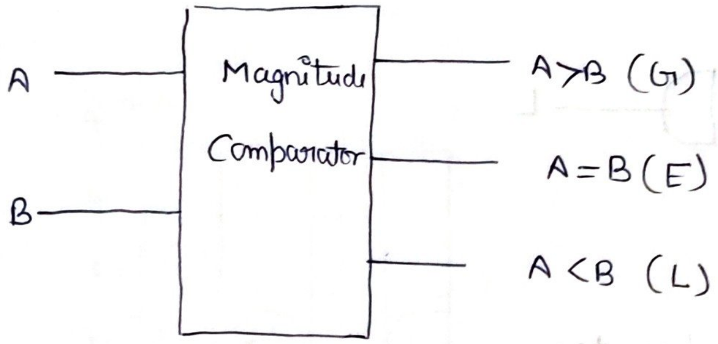
AB \ CD	00	01	11	10
00	X		1	1
01				
11	1	1	1	1
10	1	1	1	1

$y = A + \bar{B}C$

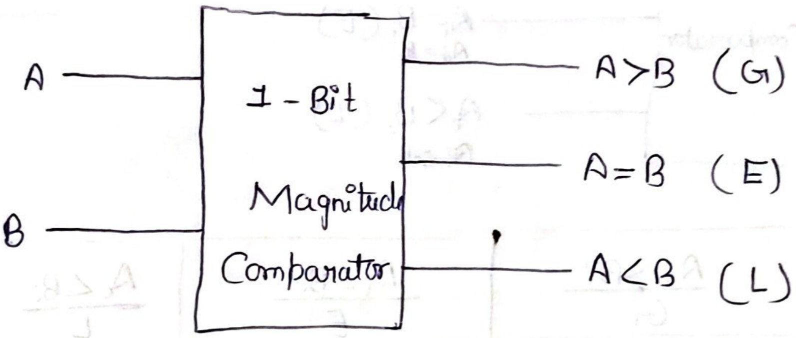
Magnitude Comparator :-

» Whenever two quantities (A & B) are compared three output possible.

(i) A>B (G)
(ii) A=B (E)
(iii) A<B (L)



» 1-bit magnitude Comparator :-



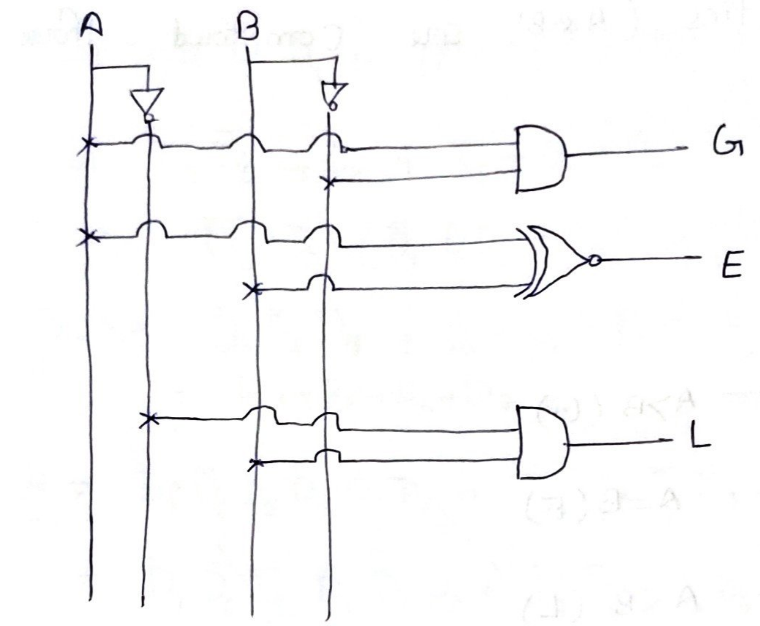
Truth table -

A	B	A>B (G)	A=B (E)	A<B (L)
0	0	0	1	0
0	1	0	0	1
1	0	1	0	0
1	1	0	1	0

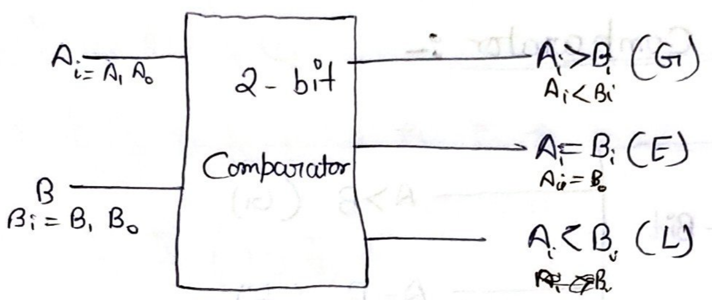
$G = A\bar{B}$ $\qquad$ $L = \bar{A}B$

$E = \bar{A}\bar{B} + AB = A \odot B$

Ckt diagram



» 2-bit magnitude Comparator



Truth table.

A_i		B_i		$A_i > B_i$	$A_i = B_i$	$A_i < B_i$
A_1	A_0	B_1	B_0	G	E	L
0	0	0	0	0	1	0
0	0	0	1	0	0	1
0	0	1	0	0	0	1
0	0	1	1	0	0	1
0	1	0	0	1	0	0
0	1	0	1	0	1	0
0	1	1	0	0	0	1
0	1	1	1	0	0	1
1	0	0	0	1	0	0
1	0	0	1	1	0	0
1	0	1	0	0	1	0
1	0	1	1	0	0	1
1	1	0	0	1	0	0
1	1	0	1	1	0	0
1	1	1	0	1	0	0
1	1	1	1	0	1	0

$G = \bar{A}_1 A_0 \bar{B}_1 \bar{B}_0 + A_1 \bar{A}_0 \bar{B}_1 \bar{B}_0 + A_1 \bar{A}_0 \bar{B}_1 B_0 + A_1 A_0 \bar{B}_1 \bar{B}_0 + A_1 A_0 \bar{B}_1 B_0$

$E =$ –

$G = \Sigma m(4, 8, 9, 12, 13, 14)$

$E = \Sigma m(0, 5, 10, 15)$

$L = \Sigma m(1, 2, 3, 6, 7, 11)$

for G

$A_1A_0 \backslash B_1B_0$	00	01	11	10
00				
01	1			
11	1	1		1
10	1	1		

$$\boxed{G = A_0 \bar{B}_1 \bar{B}_0 + A_1 \bar{B}_1 + A_1 A_0 \bar{B}_0}$$

for E

$A_1A_0 \backslash B_1B_0$	00	01	11	10
00	1			
01		1		
11			1	
10				1

$E = \bar{A}_1 \bar{A}_0 \bar{B}_1 \bar{B}_0 + \bar{A}_1 A_0 \bar{B}_1 B_0 + A_1 A_0 B_1 B_0 + A_1 \bar{A}_0 B_1 \bar{B}_0$

$E = A_1 \odot A_0 \odot B_1 \odot B_0$ $\Rightarrow \bar{A}_0 \bar{B}_0 (\bar{A}_1 \bar{B}_1 + A_1 B_1) + A_0 B_0 (\bar{A}_1 \bar{B}_1 + A_1 B_1)$

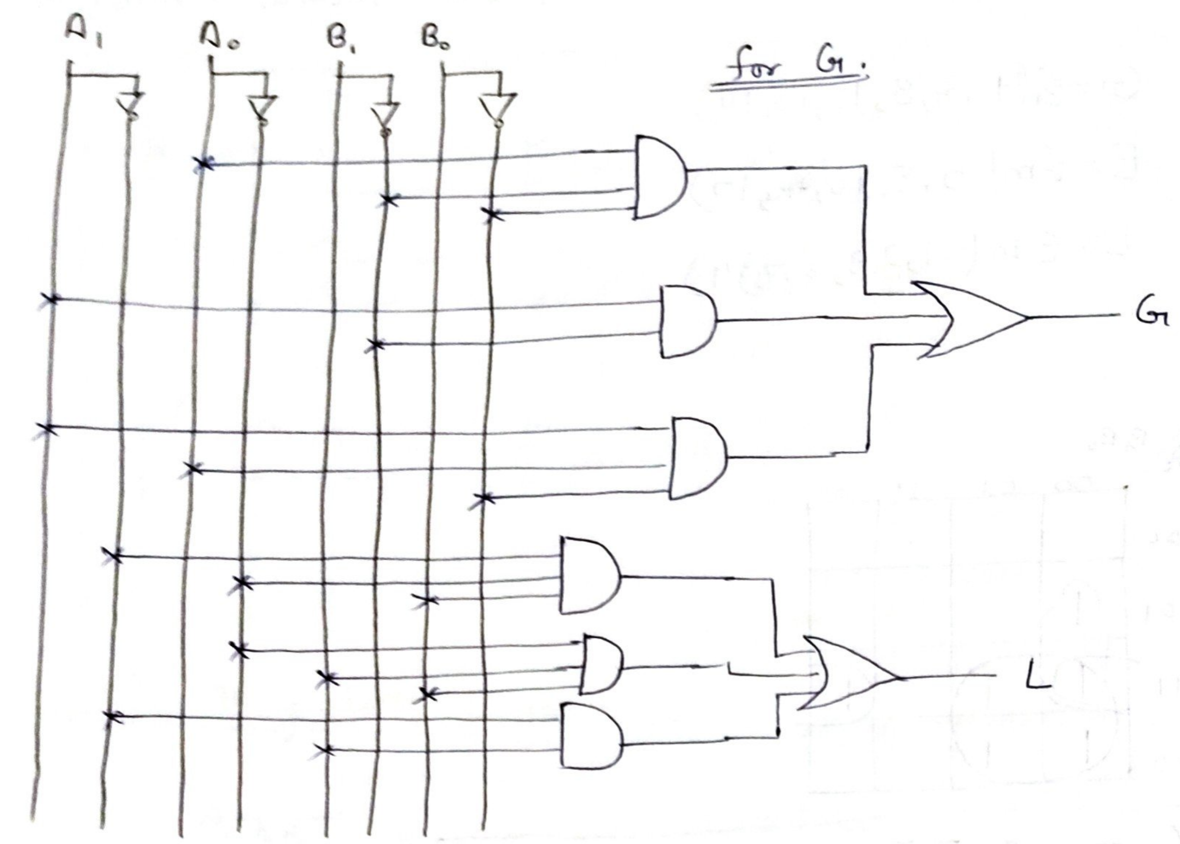
$\Rightarrow \bar{A}_0 \bar{B}_0 (A_1 \odot B_1) + A_0 B_0 (A_1 \odot B_1)$

for L

$A_1A_0 \backslash B_1B_0$	00	01	11	10
00		1	1	1
01			1	1
11				
10			1	

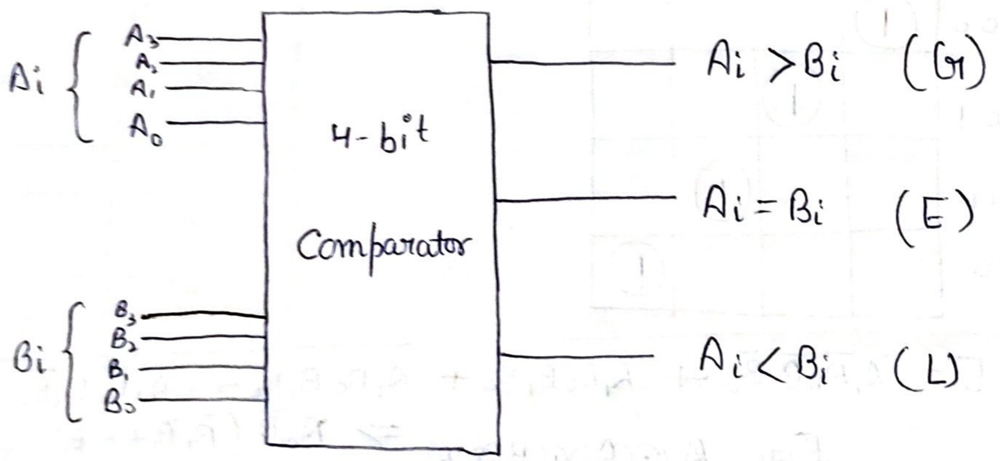
$$\boxed{L = \bar{A}_1 \bar{A}_0 B_0 + \bar{A}_0 B_1 B_0 + \bar{A}_1 B_1}$$

Circuit diagram



Imp. (10 Marks)

4-bit magnitude Comparator



Case (I) $\boxed{A_i = B_i}$ (E)

$$\begin{array}{ccccccc} A_i & = & A_3 & A_2 & A_1 & A_0 \\ & & \parallel & \parallel & \parallel & \parallel \\ B_i & = & B_3 & B_2 & B_1 & B_0 \end{array}$$

(d) $A_3 = B_3$ and $A_2 = B_2$ and $A_1 = B_1$ and $A_0 = B_0$

$$E = (\bar{A}_3\bar{B}_3 + A_3B_3).(\bar{A}_2\bar{B}_2 + A_2B_2).(\bar{A}_1\bar{B}_1 + A_1B_1).(\bar{A}_0\bar{B}_0 + A_0B_0)$$

$$\boxed{E = (A_3 \odot B_3).(A_2 \odot B_2).(A_1 \odot B_1).(A_0 \odot B_0)}$$

Case (ii). $A_i > B_i$ (G)

$$\begin{array}{cccccc} & A_3 & A_2 & A_1 & A_0 \\ A_i = & 1 & 0 & 1 & 0 \\ B_i = & 0 & 1 & 1 & 1 \\ & B_3 & B_2 & B_1 & B_0 \end{array}$$

(a) $A_3 > B_3$

$$\boxed{G_3 = A_3\bar{B}_3}$$

$A_i = 1011$
$B_i = 1010$

(b) $A_3 = B_3$ and $A_2 > B_2$

$$\boxed{G_2 = (\bar{A}_3\bar{B}_3 + A_3B_3).(A_2\bar{B}_2)}$$

(c) $A_3 = B_3$ and $A_2 = B_2$ and $A_1 > B_1$

$A_i = 1101$
$B_i = 1101$

$$G_1 = (\bar{A}_3\bar{B}_3 + A_3B_3).(\bar{A}_2\bar{B}_2 + A_2B_2)(A_1\bar{B}_1)$$

$$\boxed{G_1 = (A_3 \odot B_3)(A_2 \odot B_2)(A_1\bar{B}_1)}$$

(d) $A_3 = B_3$ and $A_2 = B_2$ and $A_1 = B_1$ and $A_0 > B_0$

$A_i = 1111$
$B_i = 1110$

$$G_0 = (\bar{A}_3\bar{B}_3 + A_3B_3).(\bar{A}_2\bar{B}_2 + A_2B_2).(\bar{A}_1\bar{B}_1 + A_1B_1).(A_0\bar{B}_0)$$

$$\boxed{G_0 = (A_3 \odot B_3).(A_2 \odot B_2).(A_1 \odot B_1)(A_0\bar{B}_0)}$$

then

$G = G_0$ or G_1 or G_2 or G_3

$$G = G_0 + G_1 + G_2 + G_3$$

$$G = (A_3 \odot B_3).(A_2 \odot B_2).(A_1 \odot B_1)(A_0\bar{B}_0) + (A_3 \odot B_3)(A_2 \odot B_2)(A_1\bar{B}_1) + (\bar{A}_3\bar{B}_3 + A_3B_3)(A_2\bar{B}_2) + A_3\bar{B}_3$$

Case iii). $A_i < B_i$ (L)

ⓐ $A_3 < B_3$

$A_i = 0111$

$B_i = 1010$

$$\boxed{L_3 = \bar{A}_3 B_3}$$

ⓑ $A_3 \neq B_3$ and $A_2 < B_2$

$A_i = 0010$

$B_i = 0101$

$$\boxed{L_2 = (A_3 \odot B_3)(\bar{A}_2 B_2)}$$

ⓒ $A_3 = B_3$ and $A_2 = B_2$ and $A_1 < B_1$

$A_i = 0001$

$B_i = 0010$

$$\boxed{L_1 = (A_3 \odot B_3).(A_2 \odot B_2).(\bar{A}_1 B_1)}$$

ⓓ $A_3 = B_3$ and $A_2 = B_2$ and $A_1 = B_1$, $A_0 < B_0$

$A_i = 1110$

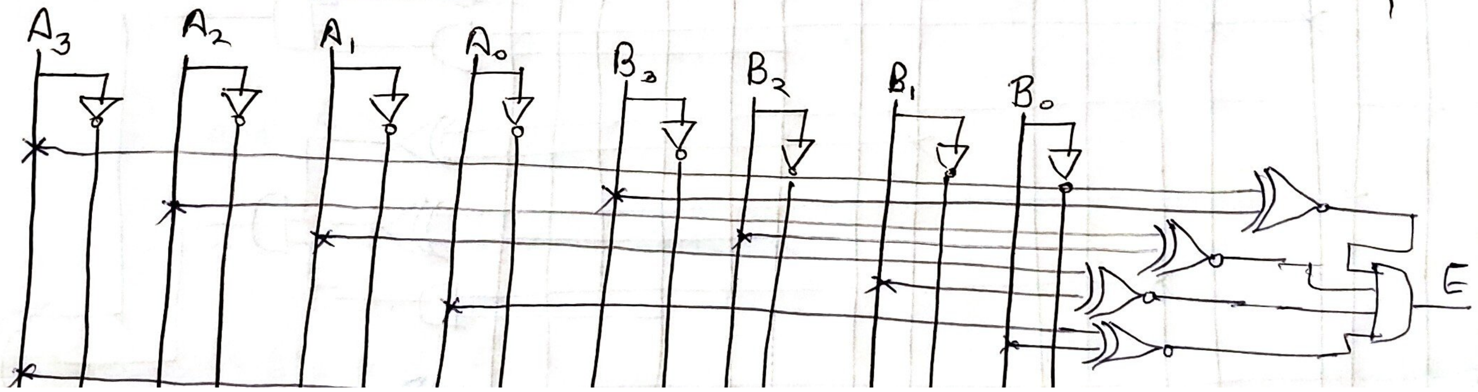
$B_i = 1111$

$$\boxed{L_0 = (A_3 \odot B_3).(A_2 \odot B_2).(A_1 \odot B_1)(\bar{A}_0 B_0)}$$

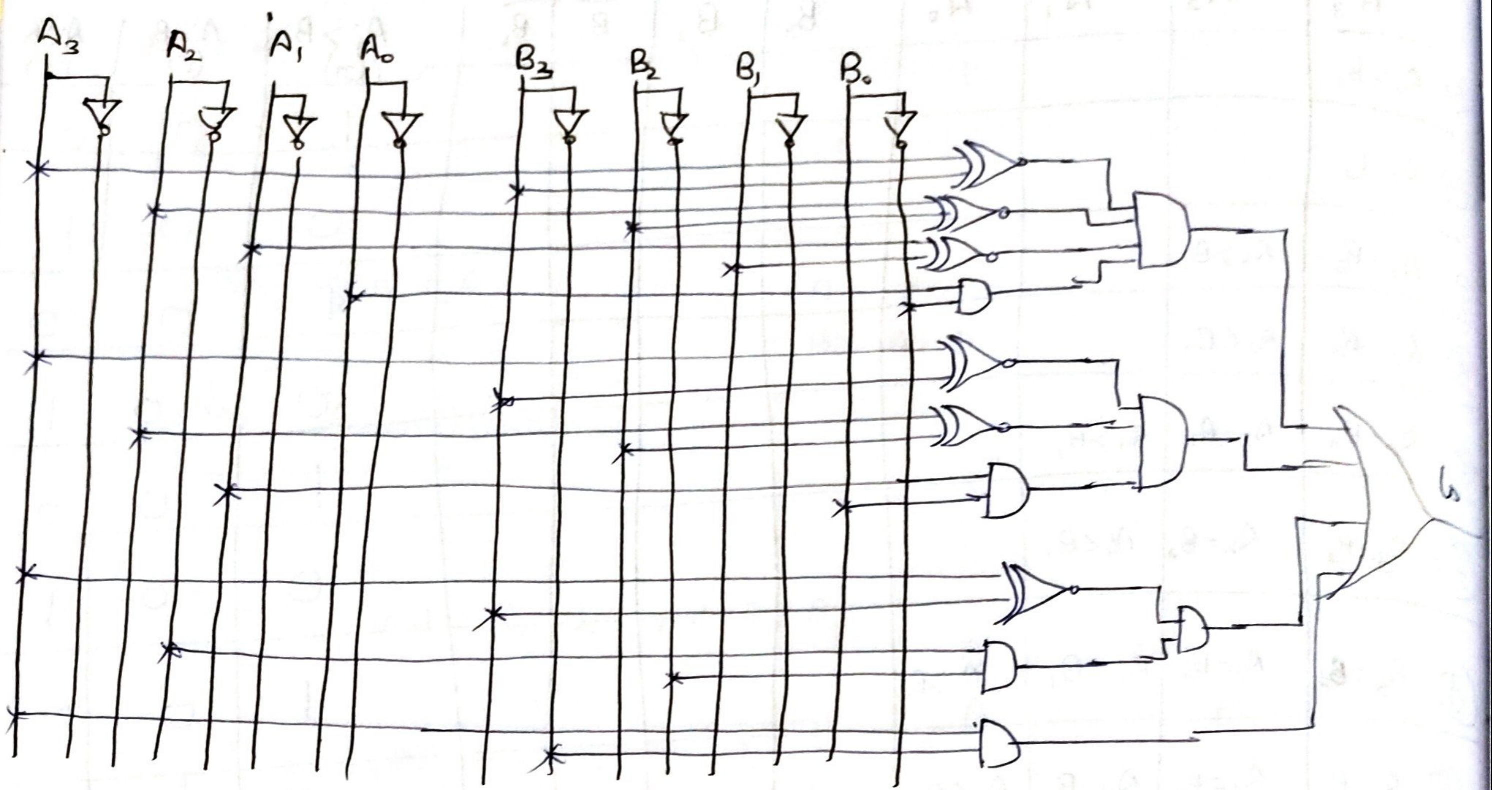
Truth table

	A_3	A_2	A_1	A_0	B_3	B_2	B_1	B_0	$A_i > B_i$ (G)	$A_i = B_i$ (E)	$A_i < B_i$ (L)
(a)	$A_3 > B_3$								1	0	0
(b)	$A_3 < B_3$								0	0	1
(c)	$A_3 = B_3$	$A_2 > B_2$							1	0	0
(d)	$A_3 = B_3$	$A_2 < B_2$							0	0	1
(e)	$A_3 = B_3$	$A_2 = B_2$	$A_1 > B_1$						1	0	0
(f)	$A_3 = B_3$	$A_2 = B_2$	$A_1 < B_1$						0	0	1
(g)	$A_3 = B_3$	$A_2 = B_2$	$A_1 = B_1$	$A_0 > B_0$					1	0	0
(h)	$A_3 = B_3$	$A_2 = B_2$	$A_1 = B_1$	$A_0 < B_0$					0	0	1
(i)	$A_3 = B_3$	$A_2 = B_2$	$A_1 = B_1$	$A_0 = B_0$					0	1	0

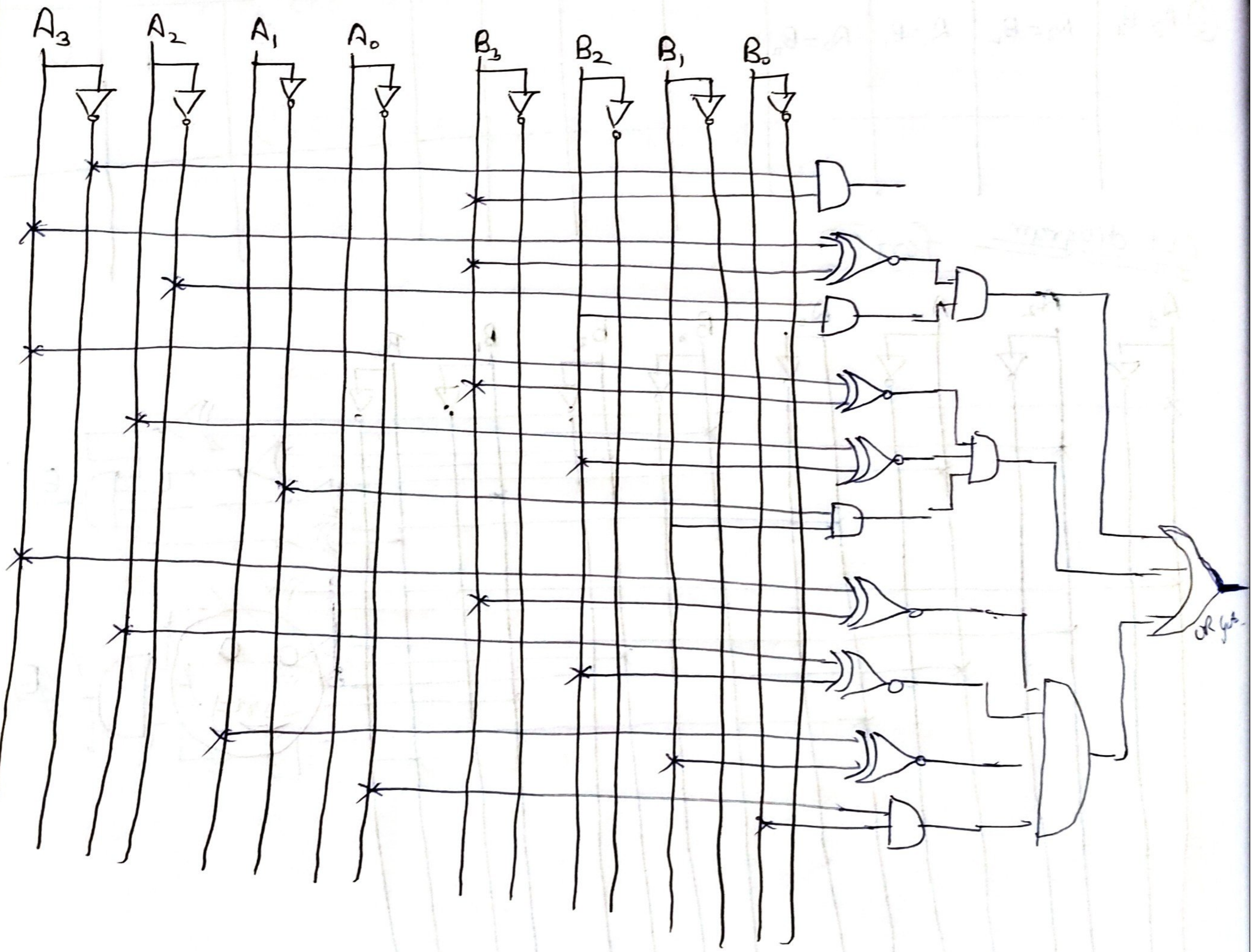
CKt diagram for (E)



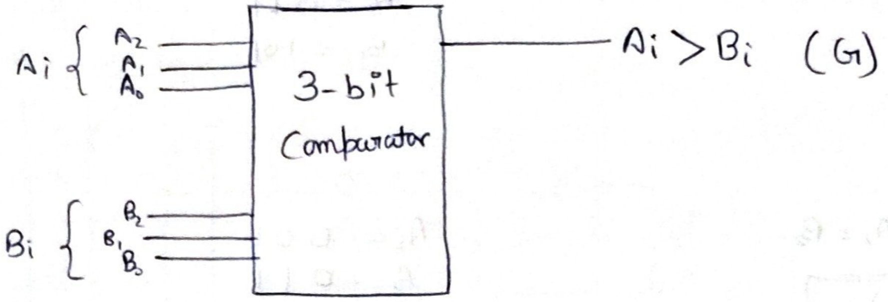
for G



for L.



3- bit Comparator



Case (I). $\boxed{A_i = B_i}$ (E)

$$A_i = A_2 \quad A_1 \quad A_0$$
$$\quad\;\; \| \quad\;\; \| \quad\;\; \|$$
$$B_i = B_2 \quad B_1 \quad B_0$$

(a) $A_2 = B_2$ and $A_1 = B_1$ and $A_0 = B_0$ ~~and $A_0 \neq B_0$~~

$$E = (\bar{A}_2 \bar{B}_2 + A_2 B_2) \cdot (\bar{A}_1 \bar{B}_1 + A_1 B_1) \cdot (\bar{A}_0 \bar{B}_0 + A_0 B_0)$$

$$\boxed{E = (A_2 \odot B_2) \cdot (A_1 \odot B_1) \cdot (A_0 \odot B_0)}$$

Case (II) $A_i > B_i$ (G)

$A_i = 1\ 0\ 1$
$B_i = 0\ 1\ 1$

(a) $A_2 > B_2$

$$\boxed{G_2 = A_2 \bar{B}_2}$$

(b) $A_2 = B_2$ and, $A_1 > B_1$

$$G_1 = (A_2 \odot B_2) \cdot (A_1 \bar{B}_1)$$

(c) $A_2 = B_2$ and $A_1 = B_1$ and $A_0 > B_0$

$$G_0 = (A_2 \odot B_2)(A_1 \odot B_1) \cdot (A_0 \bar{B}_0)$$

$$G = G_0 \text{ or } G_1 \text{ or } G_2$$
$$G = G_0 + G_1 + G_2$$

$$\boxed{G = (A_2 \odot B_2) \cdot (A_1 \odot B_1) A_0 \bar{B}_0 + (A_2 \odot B_2) A_1 \bar{B}_1 + A_2 \bar{B}_2}$$

Case (III) $A_i < B_i$ (L)

(a) $A_2 < B_2$

$A_i = 011$
$B_i = 101$

$$\boxed{L_2 = \bar{A}_2 B_2}$$

(b) $A_1 < B_1$ and $A_2 = B_2$

$A_i = 001$
$B_i = 011$

$$\boxed{L_1 = (A_2 \odot B_2) \cdot (\bar{A}_1 B_1)}$$

(c) $A_0 < B_0$ and $A_1 = B_1$ and $A_2 = B_2$

$A_i = 100$
$B_i = 101$

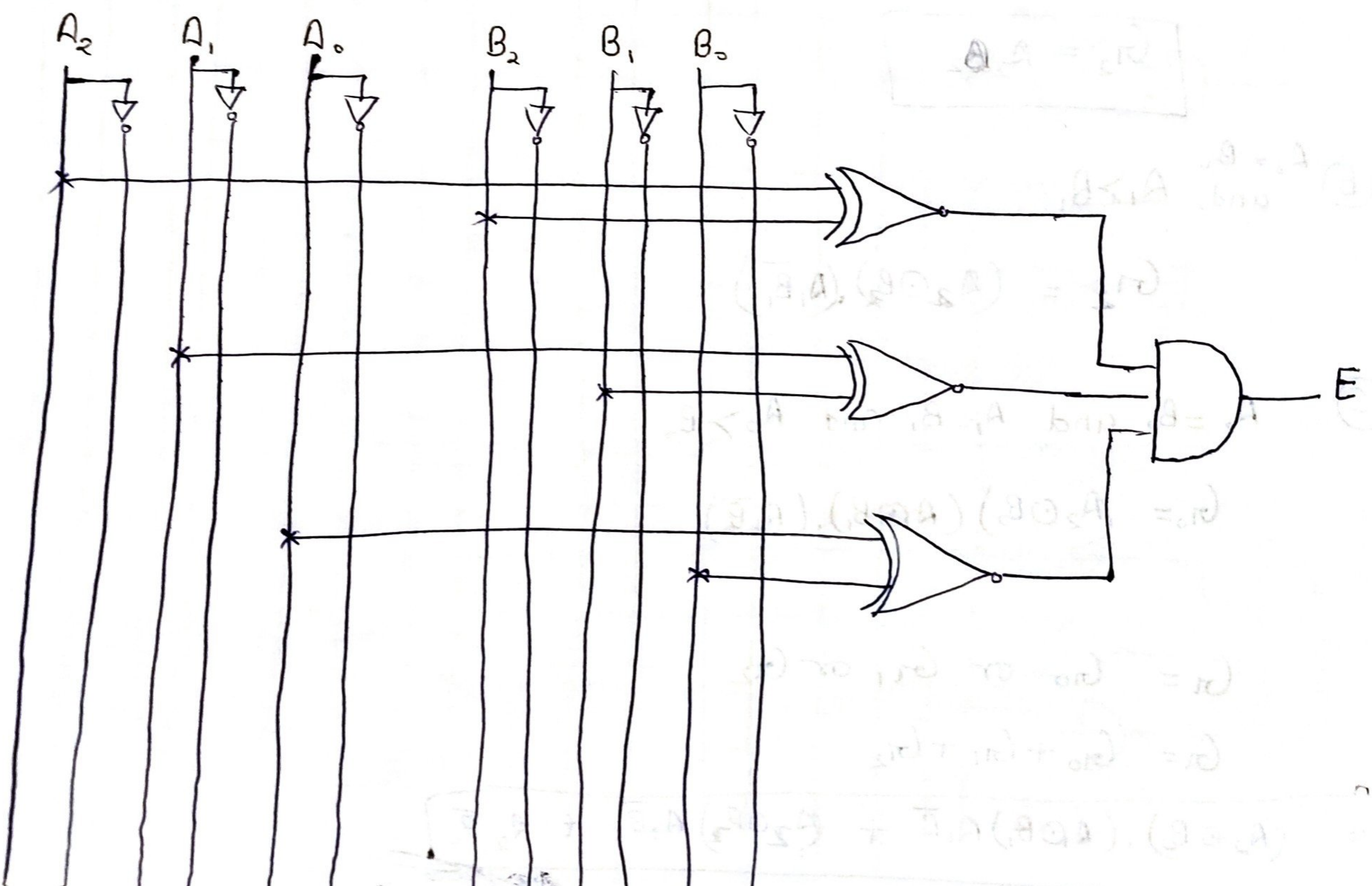
$$\boxed{L_0 = (A_2 \odot B_2) \cdot (A_1 \odot B_1) \cdot \bar{A}_0 B_0}$$

$L = L_0 + L_1 + L_2 \Rightarrow$ $\boxed{L_0 = (A_2 \odot B_2)(A_1 \odot B_1)\bar{A}_0 B_0 + (A_2 \odot B_2)\bar{A}_1 B_1 + \bar{A}_2 B_2}$

Circuit diagram

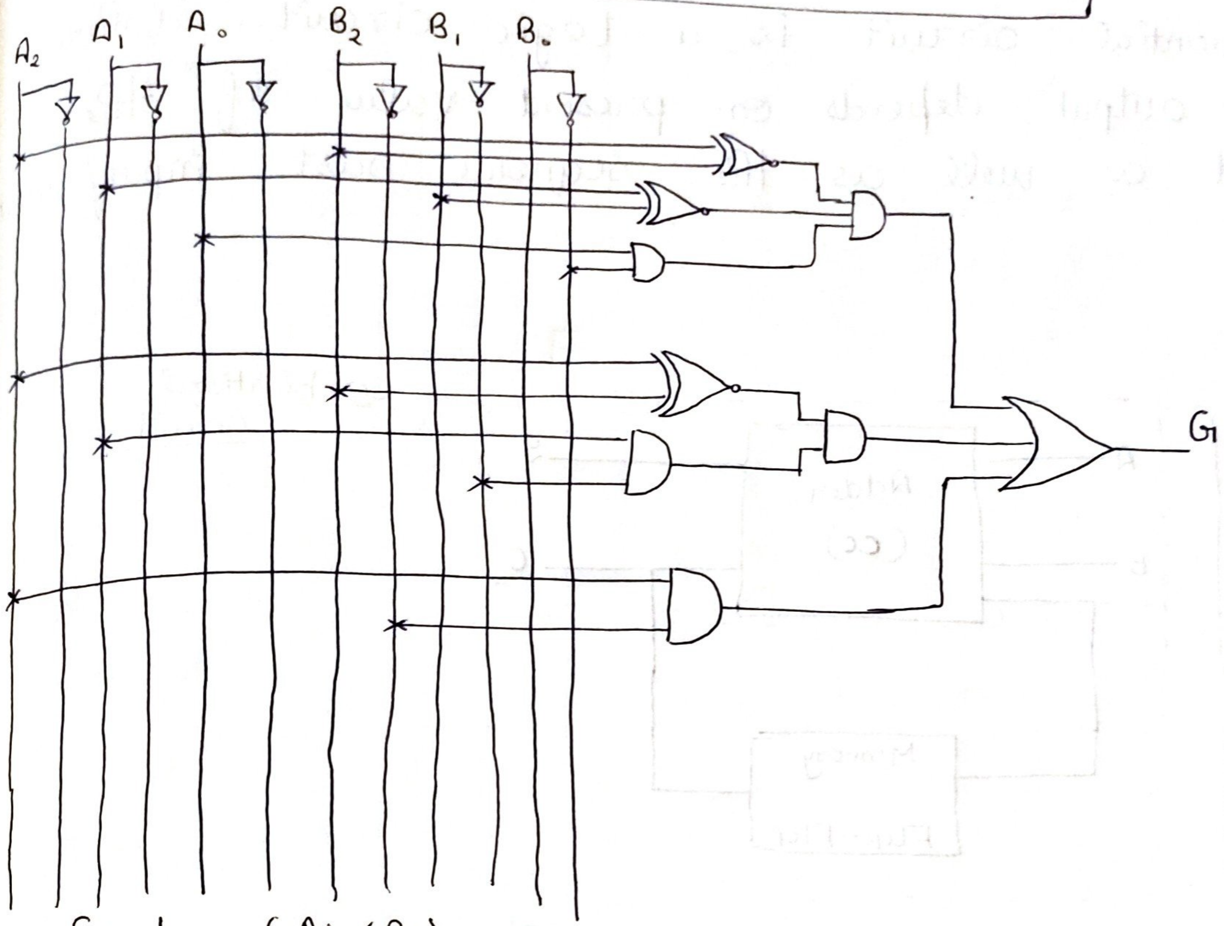
→ for E ($A_i = B_i$)

$$\boxed{E = (A_2 \odot B_2) \cdot (A_1 \odot B_1) \cdot (A_0 \odot B_0)}$$



→ for G ($A_i > B_i$)

$$\boxed{G = (A_2 \odot B_2)(A_1 \odot B_1) A_0 \overline{B_0} + (A_2 \odot B_2) A_1 \overline{B_1} + A_2 \overline{B_2}}$$



→ for L ($A_i < B_i$)

$$\boxed{L = (A_2 \odot B_2)(A_1 \odot B_1) \overline{A_0} B_0 + (A_2 \odot B_2) \overline{A_1} B_1 + \overline{A_2} B_2}$$