Unit-1 :

Basic Introduction
{ K-Map — etc. — }

Number system —

(i) Decimal number
(ii) Binary number
(iii) Octal number
(iv) Hexadecimal number

» Conversion :-

* Decimal to binary conversion :-

Q. $(32)_{10} = (?)_2$

2	32	0
2	16	0
2	8	0
2	4	0
2	2	0
	1	

$= (32)_{10} = (100000)_2$

Q. $(43.125)_{10} = (?)_2$

2	43	
2	21	1
2	10	1
2	5	0
2	2	1
	1	0

$.125 \times 2 = .250 \longrightarrow 0$

$.250 \times 2 = .500 \longrightarrow 0$

$.500 \times 2 = 1.000 \longrightarrow 1$

$= (101011.001)_2$

Q. $(58)_{10} = (?)_2$

2	58	
2	29	0
2	14	1
2	7	0
2	3	1
	1	1

$= (111010)_2$

* Decimal to octal :-

Q. $(9093)_{10} = (?)_8$

8	9093	
8	1129	1
8	141	1
8	17	5
	2	1

$= (21511)_8$

Q. $(533)_{10} = (?)_8$

8	533	
8	66	5
8	8	2
	1	0

$= (1025)_8$

* Decimal to Hexadecimal :-

Q. $(379)_{10} = (?)_{16}$

16	379	
16	23	11
	1	7

$= (17B)_{16}$

Base to decimal Conversion :-

Q. $(10101)_2 = (?)_{10}$

$(1\times2^4 + 0\times2^3 + 1\times2^2 + 0\times2^1 + 1\times2^0)$

$\Rightarrow 16+0+4+0+1$

$= (21)_{10}$

Q. $(0.101)_2 = (?)_{10}$

$\Rightarrow 1\times2^{-1} + 0\times2^{-2} + 1\times2^{-3}$

$= \frac{1}{2} + \frac{1}{8}$

$= \frac{8+2}{16}$

$= \frac{\cancel{10}\,5}{\cancel{16}\,8}$

$= (0.625)_{10}$

Q. $(735)_8 = (?)_{10}$

$(7\times8^2 + 3\times8^1 + 5\times8^0)$
$= 448 + 24 + 5$
$= (477)_{10}$

Q. $(246)_8 = (?)_{10}$

$(2\times8^2 + 4\times8^1 + 6\times8^0)$
$= 128 + 32 + 6$
$= (166)_{10}$

Q. $(BABA)_{16} = (?)_{10}$

A = 10
B = 11

so

$(11\times16^3 + 10\times16^2 + 11\times16^1 + 10\times16^0)$
$= (47802)_{10}$

Binary to Octal :-

Q. $(1101101)_2 = (?)_8$ $\quad 8 = 2^{③} \rightarrow$ pairing

$00(1101101) = (155)_8$

Octal to Binary :- $\quad (\ )_8 = (\ )_2$

$2^{③} = 8$

Q. $(542)_8 = (?)_2$

101 100 010

$\Rightarrow (101100010)_2$

Binary to Hexadecimal :-

Q. $(1101101)_2 = (\ )_{16}$ $\quad 2^{④} = 16$

$0(1101101)_2 \Rightarrow (6D)_{16}$

* Hexadecimal to Binary :-

$16 = 2^{4}$

Q. $(5B2)_{16} = (?)_2$

B = 11

$(5\ B\ 2)$

0101 1011 0010

$\Rightarrow (010110110010)_2$

Octal to Hexadecimal :-

Q. $(642)_8 = (?)_{16}$

first we convert in binary than in Hexadecimal

$(642)_8 = (?)_2$

110 100 010

$= (110100010)_2$

$\Downarrow$ Hexadecimal

$000(110100010)_2$

$\Rightarrow (1A2)_{16}$

Hexadecimal to Octal :-

Q. $(BC)_{16} = (?)_8$

$(BC)_{16}$

1011 1100

$= (10111100)_2$

$\Downarrow$

$0(10111100)_2 = (274)_8$

Q. $(26)_8 = (?)_2$

010 110

$= (010110)_2$

Numbers with different Base (radix(r))

Decimal Base 10	Binary Base. 2	Octal Base. 8	Hexadecimal Base. 16
0	0000	0	0
1	0001	1	1
2	0010	2	2
3	0011	3	3
4	0100	4	4
5	0101	5	5
6	0110	6	6
7	0111	7	7
8	1000	10	8
9	1001	11	9
10	1010	12	10 = A
11	1011	13	11 = B
12	1100	14	12 = C
13	1101	15	13 = D
14	1110	16	14 = E
15	1111	17	15 = F

Complement of radix (base) r :-

① Decimal (10)
- ↳ 9's Complement
- ↳ 10's Complement

② Binary (2)
- ↳ 1's Complement
- ↳ 2's Complement

③ Octal (8)
- ↳ 7's Complement
- ↳ 8's Complement

④ Hexadecimal (16)
- ↳ 15's complement
- ↳ 16's complement

Using 1's Complement

Q. $(10101)_M - (01010)_N = ?$

1's Complement of N

01010 = 10101

than,

M + 1's Complement of N

$$\begin{array}{r} 10101 \\ +\ 10101 \\ \hline \boxed{(1)01010} \end{array}$$

Carry → Add hoga ⇒

$$\begin{array}{r} 01010 \\ +1 \\ \hline \boxed{01011} \end{array}$$ Ans

Q. $(01010)_M - (10101)_N = ?$

1's Complement of N

= 01010

M + 1's Complement of N

$$= \begin{array}{r} 01010 \\ +01010 \\ \hline \boxed{10100} \end{array}$$

no carry i.e. -ve number

again, 1's Complement of Sum

= -(01011)

Important note :- * The process of finding 1's, 9's, 7's and 15's complements will remain same.

* The process of finding 2's, 10's, 8's, 16's complements will remain same.

Using's 2's Complement :-

Q. $(\underset{M}{01010}) - (\underset{N}{10101}) = ?$

1's complement of N = 01010

than

M as 2's complement = 1's +1

= 01011

M + 2's complement of N

= 01010
  01011
$\boxed{010101}$

no carry i.e. -ve number

so, again, 2's complement of sum

1's = 01010

2's = 1's +1 = $-(01011)$

Q. $(\underset{M}{10101}) - (\underset{N}{01010}) = ?$

1's complement of N = 10101

2's complement = 1's+1 = 10110

M + 2's complement of N = 10101
+10110
$\boxed{(1)01011}$

Carry, we discard carry & rest will be final answer.

= (01011)

ues :- Using 2's complement, solve it —

- $(\underset{M}{110000}) - (\underset{N}{10101}) = ?$

1's complement of N = 01010

2's complement of N = 1's +1 = 01011

than,

M + 2's complement of N

= 110000
  001011
no carry $\boxed{111011}$

again,

1's Complement of sum = 000100

2's Complement = 000101

Answer is = −(000101)

Q. Using 10's complement, solve it ?

- (9742) − (641) = ?
  M N

9's Complement of N

$$\begin{array}{r} 9999 \\ -\ 0641 \\ \hline 9358 \end{array}$$

10's complement = 9's + 1 = 9359

M + 10's Complement of N

$$\begin{array}{r} 9742 \\ +\ 9359 \\ \hline \boxed{(1)9101} \end{array}$$

Carry

in this we discard carry

so (9101) A

- (0641) − (9742) = ?
  M N

9's complement of N =

$$\begin{array}{r} 9999 \\ -\ 9742 \\ \hline 0257 \end{array}$$

10's Complement = 9's + 1 = 0258

than,

M + 10's Complement of N

$$\begin{array}{r} 0641 \\ 0258 \\ \hline \boxed{0899} \end{array}$$

no carry i.e. −ve number,

again 9's complement of sum =

$$\begin{array}{r} 9999 \\ -\ 0899 \\ \hline 9100 \end{array}$$

10's Complement = 9100 + 1

Answer = −(9101)

CODE :-

» A system of representation of Numeric, Alphabets or Special characters in binary form for processing and transmission using Digital techniques.

→ Types of Code

① **Binary Code** :- The number system with base 2 is Known as binary Code.

- These are 0 and 1.
- It is a weighted code.

E.g. $(8)_{10} = (1000)_2$

② **Binary Coded decimal** :- also Known as "BCD Code".

→ A Code for representing a decimal numbers in which each decimal digit is represented by 4 bits. binary code , also Known as natural BCD.

→ Decimal digit (0-9) are represented by their natural binary number equivalents using 4 bits.

→ Each decimal digit of decimal number is represented by this 4 bit code individually.

This code is also Known as - "8-4-2-1" Code or simply BCD. This is also a weighted code

Ex.

Decimal number	BCD Code
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001

Eg. BCD code of (12)

(12) → 0001, 0010 = (00010010)

③ **Excess - 3 code :-** BCD+3 = Excess-3 code

- A BCD code formed by adding 3 to the binary equivalent of the decimal number the code for each decimal digit is obtained by adding 3 to the natural BCD code of Digit.

Ex. 4096

Excess 3-Code =

$$\begin{array}{r} 4\,0\,9\,6 \\ +\,3\,3\,3\,3 \\ \hline 7\,3\,12\,9 \end{array}$$

0111 0011 1100 1001 = (0111 0011 1100 1001)

④ **Gray Code :-** A code in which only one bit changes between successive number.

Eg. $(10111000)_2$

↓

(11100100)

⑤ **Octal Code :-** A code in which each group of three bits starting from LSB is represented by its equivalent octal digit.

E.g. $(011\ 011)_2 = (\ )_8 \quad = 2^{③}$

$= (33)_8$

Hexadecimal Code :- A method of representing binary in which each group of 4 bit represented by Hex-digit

E.g. $(0110\ 0110)_2 = (?)_{16}$ $\quad 16 = 2^{④}$

$= (55)_{16}$

Various Binary Code :-

Decimal number	Binary $B_4 B_3 B_2 B_1$	BCD D C B A	Excess-3-code E_4 E_3 E_2 E_1	Gray $G_4 G_3 G_2 G_1$	Octal	Hexa decimal
0	0000	0000	0011	0000	00	0
1	0001	0001	0100	0001	01	1
2	0010	0010	0101	0011	02	9
3	0011	0011	0110	0010	03	3
4	0100	0100	0111	0110	04	4
5	0101	0101	1000	0111	05	5
6	0110	0110	1001	0101	06	6
7	0111	0111	1010	0100	07	7
8	1000	1000	1011	1100	10	8
9	1001	1001	1100	1101	11	9
10	1010			1111	12	10 = A
11	1011			1110	13	11 = B
12	1100			1010	14	12 = C
13	1101			1011	15	13 = D
14	1110			1001	16	14 = E
15	1111			1000	17	15 = F

Q. represent decimal numbers in binary form using —

(i) Binary Code

(ii) BCD code

(iii) Excess-3-code

(iv) Gray code

(v) Octal code

(vi) Hexadecimal code

ⓐ 27 ⓑ 396 ⓒ 4096

(a) 27

(i) $(27)_{10} = (11011)_2$

(ii) $(27)_{10} = (00100111)$

0010 0111

(iii)
$$\begin{array}{r} 27 \\ +33 \\ \hline 5\ 10 \end{array}$$

$5 \rightarrow 0101$, $10 \rightarrow 1010$ $\Rightarrow (01011010)$

(iv) (11011)

$\downarrow$

(10110)

(v) $(11011)_2 = (\ )_8$

$\Rightarrow (33)_8$

(vi) $(11011)_2 = (\ )_{16}$

$(1B)_{16}$

(b) 396

(i) $(396)_{10} = (110001100)_2$

(ii) 396 $= (001110010110)$

0011 1001 0110

(iii)
$$\begin{array}{r} 396 \\ +333 \\ \hline 6\ 12\ 9 \end{array}$$

$= (011011001001)$

0110 1100 1001

(iv) (10111000)

$\downarrow$

(11100100)

(v) in octal

$(10111000) = (570)_8$

(vi) $(10111000)_2 = (\ )_{16}$

$(178)_{16}$

(c) 4096

(i) $(4096)_{10} = (1000000000000)_2$

(ii) 4096 $= (0100000010010110)_2$

0100 0000 1001 0110

(iii)
$$\begin{array}{r} 4096 \\ +3333 \\ \hline 7\ 3\ 12\ 9 \end{array}$$

$\Rightarrow (0111001111001001)$

0111 0011 1100 1001

(iv) (1000000000000)

$\downarrow$

(1100000000000)

(vi) $(1000000000000)_2 = (\ )_{16}$

$(1000)_{16}$

(v) $(1000000000000)_2 = (\ )_8$

$(10000)_8$

Hamming Code :- Hamming code is used for error detection and Correction, It is also used Concept of Addition parity bits.

$$2^p \geq p+m+1$$

<u>Where</u> p = parity
m = message bits

- Length of Hamming Code = $m+p$
- Position of Hamming Code = 2^n (1, 2, 4, 8, 16 ...)

★ Imp. (10 Marks)

Parity
- Even parity
- Odd parity

"decided by no. of 1's"

Table :-

BCD D C B A	Even parity P	 D C B A	odd parity P	 D C B A
0 0 0 0 →	0	0 0 0 0 →	1	0 0 0 0
0 0 0 1 →	1	0 0 0 1 →	0	0 0 0 1
0 0 1 0 →	1	0 0 1 0 →	0	0 0 1 0
0 0 1 1 →	0	0 0 1 1 →	1	0 0 1 1
0 1 0 0 →	1	0 1 0 0 →	0	0 1 0 0
0 1 0 1 →	0	0 1 0 1 →	1	0 1 0 1
0 1 1 0 →	0	0 1 1 0 →	1	0 1 1 0
0 1 1 1 →	1	0 1 1 1 →	0	0 1 1 1
1 0 0 0 →	1	1 0 0 0 →	0	1 0 0 0
1 0 0 1 →	0	1 0 0 1 →	1	1 0 0 1

Ques :- Determine the hamming Code for 0110, using Even parity —

(i) 0110

MSB ↓ ↓ LSB

0110

→ $m = 4$

→ $2^p \geq p + m + 1$

$2^p \geq p + 5$

if $P = 1$ $2 \geq 6$ (F)

$P = 2$ (F)

$P = 3$ $8 \geq 8$ (valid)

$\boxed{P = 3} \rightarrow P_1, P_2, P_3$

→ Length of Hamming Code = $m + P = 4 + 3 = 7$

→ position of Hamming Code = $2^n (1, 2, 4, 8 \ldots)$
P_1, P_2, P_3

→ Hamming Code,

1	2	3	4	5	6	7
P_1	P_2	0	P_3	1	1	0

(0-7)	P_3	P_2	P_1
0 →	0	0	0
1 →	0	0	1
2 →	0	1	0
3 →	0	1	1
4 →	1	0	0
5 →	1	0	1
6 →	1	1	0
7 →	1	1	1

P_1 (1, 3, 5, 7)

for check :-

1	3	5	7
P_1	0	1	0

for even

$\boxed{P_1 = 1}$

* **for P_2 (2,3,6,7)**

for Check : 2 3 6 7

P_2	0	1	0

for even parity $\boxed{P_2 = 1}$

* **for P_3 (4,5,6,7)**

for Check: 4 5 6 7

P_3	1	1	0

for even $P_3 = 0$

than

Hamming Code :- 1100110

Ques :- find hamming code using Even parity :- (1101001)

→ $m = 7$

→ $2^p \geq p + 8$

if $2^4 \geq 12$

$16 \geq 12$ (valid) $\boxed{P = 4}$ → P_1, P_2, P_3, P_4

→ Length of Hamming code = 4+7 = 11

→ position of Hamming Code : 2^n (1, 2, 4, 8, 16 - -)

P_1 P_2 P_3 P_4

(0-11)	P_4	P_3	P_2	P_1
0	0	0	0	0
1	0	0	0	1
2	0	0	1	0
3	0	0	1	1
4	0	1	0	0
5	0	1	0	1
6	0	1	1	0
7	0	1	1	1
8	1	0	0	0
9	1	0	0	1
10	1	1	0	0
11	1	1	0	1

* **for P_1** = (1, 3, 5, 7, 9, 11)

for check : 1 3 5 7 9 11

P_1	1	1	1	0	1

for even $[P_1 = 0$

→ for P_2 (2,3,6,7,10,11)

To check:

	2	3	6	7	10	11
	P_2	1	0	1	0	1

for even $\boxed{P_2 = 1}$

→ for P_3 (4,5,6,7)

To check:

4	5	6	7
P_3	1	0	1

for even $\boxed{P_3 = 0}$

→ for P_4 (8,9,10,11)

To check:

8	9	10	11
P_4	0	0	1

for even $\boxed{P_4 = 1}$

therefore

for Hamming Code

1	2	3	4	5	6	7	8	9	10	11
P_1	P_2	1	P_3	1	0	1	P_4	0	0	1

Hamming Code :- 0110 1011001 Ans

Logic operations :-

→ Basic logic operation → NOT, AND, OR

→ Universal logic operation → NAND, NOR

→ Special logic operation → EX-OR, EX-NOR

Basic logic operation :-

- NOT
- AND
- OR

0 → Low
1 → High

Boolean Expression

SoP form (Sum of product)

- In SoP form input combination is taken corresponding to which output is 1.

A	F
0	$\overline{A}$
1	A

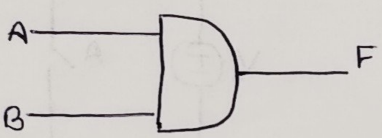
PoS form (Product of sum)

- In pos form Input Combination is taken corresponding to the which output is 0.

A	F
0	A
1	$\overline{A}$

* AND OPERATION :- (IC- 7408)

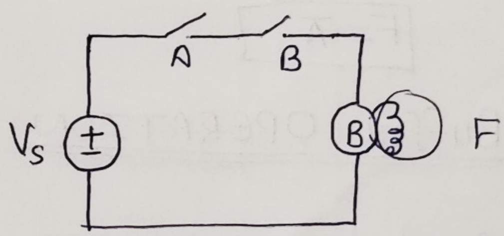
ⓐ Symbol



ⓑ Truth table

A	B	F
0	0	0
0	1	0
1	0	0
1	1	1

ⓓ switching circuit



ⓒ Algebric function

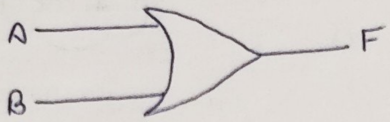
$\boxed{F = A.B}$ → (SoP)

↓

In PoS $\boxed{F = (A+B).(A+\overline{B}).(\overline{A}+B)}$

* OR OPERATION :- (IC-7432)

ⓐ Symbol



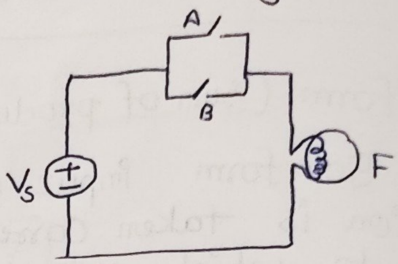
ⓑ Truth table

A	B	F
0	0	0
0	1	1
1	0	1
1	1	1

ⓒ Algebric function

$\boxed{F = A+B}$ (SOP or POS Both)

ⓓ Switching circuit



* NOT OPERATION :- (IC-7404) also known as "INVERTER"

ⓐ Symbol

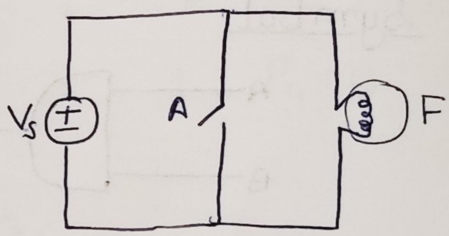
ⓑ Truth table

A	F
0	1
1	0

ⓒ Algebric function

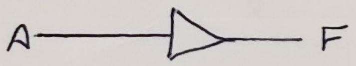
$\boxed{F = \overline{A}}$

ⓓ Switching Circuit



* Buffer OPERATION (It increase the magnitude of Input while provide output).

ⓐ Symbol



ⓑ Truth table

A	F
0	0
1	1

ⓒ Algebric function

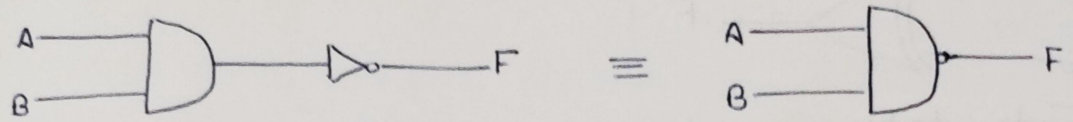
$\boxed{F = A}$

Universal Logic Operation

Imp * NAND OPERATION :- (IC - 7400)

(NAND = AND + NOT)

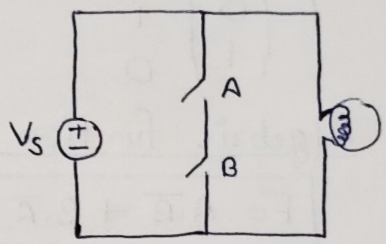
ⓐ Symbol



ⓑ Truth table

A	B	F
0	0	1
0	1	1
1	0	1
1	1	0

ⓓ Switching circuit



ⓒ Algebric function

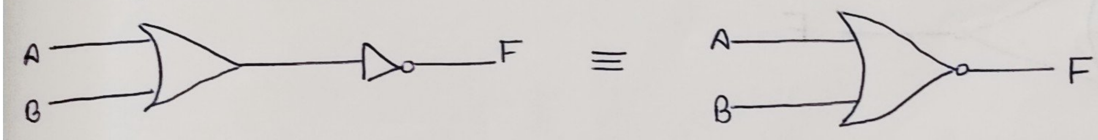
$\boxed{F = \overline{A.B}}$

Imp.

* NOR OPERATION :- (IC-7402)

(NOR = NOT + OR)

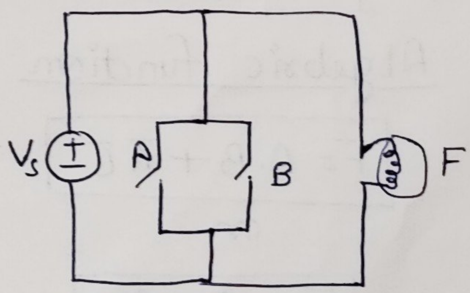
ⓐ Symbol



ⓑ Truth table

A	B	F
0	0	1
0	1	0
1	0	0
1	1	0

ⓓ Switching circuit



ⓒ Algebric function

$\boxed{F = \overline{A+B}}$

or

$\boxed{F = (A+B)'}$

Special logic operation :-

* EX-OR OPERATION :-

(a) Symbol

(b) Truth table

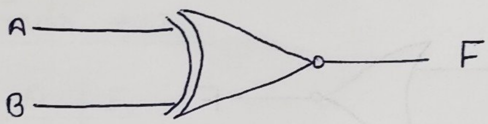
A	B	F
0	0	0
0	1	1
1	0	1
1	1	0

(c) Algebric function

$\boxed{F = A.\overline{B} + B.\overline{A}} = A \oplus B$

* EX-NOR OPERATION :-

(a) Symbol



(b) Truth table

A	B	F
0	0	1
0	1	0
1	0	0
1	1	1

(c) Algebric function

$\boxed{F = A.B + \overline{A}\,\overline{B}}$

or.

$\boxed{F = A \odot B}$

or

$\boxed{F = \overline{A \oplus B}}$

Boolean identity :-

Dual theorem –

(i) $0 \longleftrightarrow 1$, $1 \longleftrightarrow 0$

(ii) OR Logic $\longrightarrow$ AND logic ; AND Logic $\longrightarrow$ OR logic

Demorgan's theorem –

Demorgan's law
Change the sign, break the line

(i) $\overline{A+B+C} = \bar{A}.\bar{B}.\bar{C}$

(ii) $\overline{A.B.C} = \bar{A}+\bar{B}+\bar{C}$

OR		AND	
<i>	$A+0=A$	<i>	$A.0=0$
<ii>	$A+1=1$	<ii>	$A.1=A$
<iii>	$A+\bar{A}=1$	* <iii>	$A.\bar{A}=0$
<iv>	$A\oplus 0=A$	<iv>	$A\odot 0=\bar{A}$
<v>	$A\oplus A=0$	<v>	$A\odot\bar{A}=0$
<vi>	$A\oplus\bar{A}=1$	<vi>	$A\odot 1=A$
<vii>	$A\oplus 1=\bar{A}$	<vii>	$A\odot A=1$

$\therefore$ $1+\text{Anything}=1$

$\boxed{A+A=A}$

*** $\boxed{(A+BC) = (A+B)(A+C)}$

from R.H.S

$(A+B)(A+C)$

$= A.A + A.C + B.A + B.C$

$= A + AC + B.A + BC$

$= A(1+C+B) + BC$

$= A.1 + BC = A+BC = L.H.S$

*

→ $A+AB=A$

→ $A(A+B)=A$

→ $A+\bar{A}B = (A+\bar{A})(A+B) = A+B$

→ $A(\bar{A}+B) = AB$

→ $AB+A\bar{B} = A$

→ $(A+B)(A+\bar{B}) = A$

→ $AB+\bar{A}C = (A+C)(\bar{A}+B)$

→ $(A+B)(\bar{A}+C) = AC+\bar{A}B$

→ $AB+\bar{A}C+BC = AB+\bar{A}C$

→ $(A+B)(\bar{A}+C)(B+C) = (A+B)(\bar{A}+C)$

Minimisation of digital Expression—

(1) Boolean algebra

(2) K-Map

Imp. 10 Marks { (3) Tabular method (Qume - Mackluskey method).

* **Boolean algebra** :- We thought don't care condition it can be solved boolean algebra / identity is to remember, Expression can be minimisation is only in SOP form.

⊙ Least minimization is no any guranty.

Ques :- Simplify following :-

(i) $F(w,x,y,z) = \bar{y}\bar{z} + \bar{w}\bar{x}\bar{z} + \bar{w}xy\bar{z} + wy\bar{z}$

$\Rightarrow \bar{z}(\bar{y} + \bar{w}\bar{x} + \bar{w}xy + wy)$

$\Rightarrow \bar{z}[\bar{y} + \bar{w}(\bar{x} + xy) + wy]$

$\Rightarrow \bar{z}[\bar{y} + \bar{w}[(x+\bar{x}).(\bar{x}+y)] + wy]$

$\Rightarrow \bar{z}[\bar{y} + \bar{w}(\bar{x}+y) + wy]$

$\Rightarrow \bar{z}[\bar{y} + \bar{w}\bar{x} + y(w+\bar{w})$

$\Rightarrow \bar{z}[\bar{y} + \bar{w}\bar{x} + y]$

$\Rightarrow \bar{z}[\bar{y} + \bar{w}\bar{x} + y] \Rightarrow \bar{z} = [(y+\bar{y}) + \bar{w}\bar{x}]$

$\Rightarrow \bar{z}[1 + \bar{w}\bar{x}]$ let, $\therefore \bar{w}\bar{x} = A$

$\Rightarrow \bar{z}$

(ii) $Y(A,B,C,D) = (\bar{A}\bar{B}C\bar{D} + \bar{A}\bar{B}\bar{C}\bar{D})$

$\Rightarrow \bar{A}\bar{B}C\bar{D} + \bar{A}\bar{B}\bar{C}\bar{D}$

$\Rightarrow \bar{A}\bar{B}\bar{D}(C + \bar{C})$

$=$ $\therefore \{C + \bar{C} = 1\}$

$\Rightarrow \bar{A}\bar{B}\bar{D}$

(iii) $\bar{A}\bar{C} + ABC + A\bar{C} + A\bar{B} = ?$

$\bar{C}(\bar{A}+A) + A(\bar{B}+BC)$

$\bar{C} + A.(B+\bar{B})(C+\bar{B})$

$\bar{C} + A.(C+\bar{B})$

$\Rightarrow \bar{C} + AC + A\bar{B}$

$= \bar{C} + A(1) + (C+\bar{B})$

$\Rightarrow \bar{C} + AC + A\bar{B}$

$= (\bar{C}+A)(C+\bar{C}) + A\bar{B}$

$\bar{C} + A + A\bar{B}$

$\bar{C} + A(1+\bar{B})$

$\Rightarrow \bar{C} + A$

(iv) $(\bar{x}\bar{y}+z) + z + xy + wz = ?$

$\Rightarrow (z+\bar{x})(z+\bar{y}) + z + xy + wz$

$\Rightarrow z.z + z\bar{y} + z\bar{x} + \bar{x}\bar{y} + z + xy + wz$

$= z + z\bar{y} + z\bar{x} + \bar{x}\bar{y} + z + xy + wz$

$\Rightarrow z(1+\bar{y}+\bar{x}+1+w) + xy + \bar{x}\bar{y}$

$\Rightarrow z + xy + \bar{x}\bar{y}$

$\Rightarrow \bar{x}\bar{y} + z + z + wz + xy$

$z + wz + \bar{x}\bar{y} + xy$

$z(1+w) + \bar{x}\bar{y} + xy$

$\Rightarrow z + (\bar{x}\bar{y} + xy)$

$\Rightarrow z + x \odot y$

$= z + \overline{x \oplus y}$ Ans

* $\boxed{x \odot y = xy + \bar{x}\bar{y}}$

* $\boxed{x \oplus y = x\bar{y} + y\bar{x}}$

* $\boxed{x \odot y = \overline{x \oplus y}}$

Canonical Representation :- If an SOP/POS Expression is product/sum contain all the literals (with/without bar).

* every term should be present in every term

Ex.

$$F(A,B,C) = AB + BC$$

$\therefore A + \bar{A} = 1$

So in canonical form

$$F(A,B,C) = AB(C+\bar{C}) + (A+\bar{A})BC$$

$2^3 = 8$
$(0-7)$

$$= ABC + AB\bar{C} + ABC + \bar{A}BC$$

$$= ABC + AB\bar{C} + \bar{A}BC$$

111 110 011

7 6 3

$$\boxed{F(A,B,C) = \Sigma m(3,6,7)} \text{ — } \boxed{SOP}$$

$$\boxed{F(A,B,C) = \pi M(0,1,2,4,5)} \text{ — } \boxed{POS}$$

Eg: $F(A,B,C) = (A+B)\cdot(B+C)$

in canonical form

$$= (A+B+C\bar{C})\cdot(B+C+A\bar{A})$$

$$= (A+B+\bar{C})(A+B+C)(B+C+A)(B+C+\bar{A})$$

$$\Rightarrow (A+B+C)\cdot(A+B+\bar{C})\cdot(\bar{A}+B+C)$$

(000) (001) (100)

$$\boxed{F(A,B,C) = \pi M[0,1,4]} \rightarrow \boxed{POS}$$

$$\boxed{F(A,B,C) = \Sigma m(2,3,5,6,7)} \longrightarrow \boxed{SOP}$$

Ques:- Express into Canonical form in SOP & POS ?

<i> $F(x,y,z) = (xy+z)(y+xz)$

$= (x+z)(y+z)(x+y)(y+z)$

$= (x+z+0).(0+y+z).(x+y+0)(0+y+z)$

$= (x+y\bar{y}+z)(x\bar{x}+y+z)(x+y+z.\bar{z})(x\bar{x}+y+z)$

$\Rightarrow (x+y+z)(x+\bar{y}+z)(x+y+z)(\bar{x}+y+z)(x+y+z)(x+y+\bar{z})(x+y+z)(\bar{x}+y+$

$\Rightarrow (x+y+z)(x+\bar{y}+z)(\bar{x}+y+z)(x+y+\bar{z})$

000 010 100 001

$\Rightarrow$

$\boxed{F(x,y,z) = \pi M[0,2,4,1]}$ — (POS)

$F(x,y,z) = \Sigma m[3,5,6,7]$ — (SOP)

(ii) $y(A,B,C) = A + AB + BC$

$= A(B+\bar{B})(C+\bar{C}) + AB(C+\bar{C}) + (A+\bar{A})BC$

$= (AB+A\bar{B})(C+\bar{C}) + ABC + AB\bar{C} + ABC + \bar{A}BC$

$= ABC + AB\bar{C} + A\bar{B}C + A\bar{B}\bar{C} + ABC + AB\bar{C} + ABC + \bar{A}BC$

$= ABC + AB\bar{C} + A\bar{B}C + A\bar{B}\bar{C} + \bar{A}BC$

111 110 101 100 011

$\Rightarrow$

$y(A,B,C) \Rightarrow \Sigma m(7,6,5,4,3)$

$y(A,B,C) = \pi M[0,1,2]$

(iii) $y = A(A+\bar{B})(A+B+\bar{C})$

$= (A+B.\bar{B}+C.\bar{C})(A+\bar{B}+C.\bar{C})(A+B+C) = (A+B)(A+\bar{B})(A+B+\bar{C})$

$\Rightarrow (A+B+C)(A+\bar{B}+\bar{C})(A+\bar{B}+C)(A+\bar{B}+\bar{C})(A+B+\bar{C})$

$= (A+B+C)(A+\bar{B}+\bar{C})(A+\bar{B}+C)(A+B+\bar{C})$

$\quad 000 \quad 011 \quad 010 \quad 001$

~~$F(A,B,C) = \pi M[0,3,2]$~~ $\quad F(A,B,C) = \pi M[0,1,2,3]$

~~$F(A,B,C) = \Sigma m[1,2,4,5,6,7]$~~ $\quad f(A,B,C) = \Sigma m[4,5,6,7]$

Kernaugh Map (K-Map)

- Systematic method to minimise boolean expression, for an "n" variables no. of cells are 2^n.
- Adjacent 1/0 to be grouped. $1 \rightarrow$ SOP, $0 \rightarrow$ POS
- 2, 3, 4, 5 variable Expression can be minimised in K-map gray code representation is used.

$f(A,B) = 2^2 = 4$ cells

$f(A,B,C) = 2^3 = 8$ cells

$f(A,B,C,D) = 2^4 = 16$ cells

$f(A,B,C,D,E) = 2^5 = 32$ cells.

2 Variable K-Map

Q. $f(A,B) = 2^2 = 4$ cells

as -

A \ B	0	1
0	0 (0,0)	1 (0,1)
1	(1,0) 2	(1,1) 3

» 3- Variable

$f(A,B,C) = 2^3 = 8$ cells

A \ BC	$\bar{B}\bar{C}$ 00	$\bar{B}C$ 01	BC 11	$B\bar{C}$ 10
$\bar{A}$ 0	000 (0)	001 1	011 3	001 2
A 1	100 4	101 5	111 7	110 6

or

AB \ C	0	1
00	000 0	001 1
01	010 2	011 3
11	110 6	111 7
10	100 4	101 5

Imp.

» 4- Variable

$f(A,B,C,D) = 2^4 = 16$ cells

AB \ CD	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	12	13	5	14
10	8	9	11	10

Imp.

» 5- variable (5 or 10 Marks)

$f(A,B,C,D,E) = 2^5 = 32$ cells

10000

(16) $\leftarrow 2^4\, 2^3\, 2^2\, 2^1\, 2^0$

(i) $\boxed{\text{for } A=0}$

BC \ DE	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	12	13	15	14
10	8	9	11	10

(ii) $\boxed{\text{for } A=1}$

BC \ DE	00	01	11	10
00	16	17	19	18
01	20	21	23	22
11	28	29	31	30
10	24	25	27	26

*

AB \ CDE	000	001	011	010	110	111	101	100
00	00000 (0)	00100 (4)	01100 (12)	01000 8	11000 24	11100 28	10100 20	10000 16
01	00001 1	00101 5	01101 13	01001 9	11001 25	11101 29	10101 21	10001 17
11	00011 3	00111 7	01111 15	01011 11	11011 27	11111 31	10111 23	10011 19
10	00010 2	00110 6	01110 14	01010 10	11010 26	11110 30	10110 22	10010 18

Rules for solving K-Map :-

(i) First preference for octal (group of 8 cells)
(ii) Second preference for quard (group of 4 cells)
(iii) third preference for pair (group of 2 cells)
(iv) last preference for single

Q. ① $F(A,B) = \sum m(0,1,3) \longrightarrow$ SoP

A \ B		$\bar{B}$ 0	B 1
$\bar{A}$	0	1	1
A	1		1

$\boxed{f(A,B) = \bar{A} + B}$

② $F(A,B,C) = \sum m(0,1,2,4,5) \longrightarrow$ SoP

A \ BC		$\bar{B}\bar{C}$ 00	$\bar{B}C$ 01	BC 11	$B\bar{C}$ 10
$\bar{A}$	0	1	1		1
A	1	1	1		

$\boxed{f(A,B,C) = \bar{B} + \bar{A}\bar{C}}$

③ $F(A,B,C,~~D~~) = \sum m(0,1,2,4,7)$

AB \ CD	00	01	11	10
00	0	0		0
01	0		0	
11				
10				

A \ BC		$\bar{B}\bar{C}$ 00	$\bar{B}C$ 01	BC 11	$B\bar{C}$ 10
$\bar{A}$	0	1	1		1
A	1	1		1	

$F(A,B,C) = \bar{B}\bar{C} + \bar{A}\bar{C} + ABC + \bar{A}\bar{B}$

* **4-Variable K-Map Questions**

Q. $F(A,B,C,D) = \sum m(1,3,4,5,6,7,9,11,13,15)$

$2^4 = 16$ cells

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00		1	1	
$\bar{A}B$ 01	1	1	1	1
AB 11		1	1	
$A\bar{B}$ 10		1	1	

$f(A,B,C,D) = \bar{A}B + D$ Ans

Q. $f(A,B,C,D) = A\bar{B}C + \bar{A}\bar{B}\bar{C}D + A\bar{C}\bar{D} + ABCD + \bar{C}\bar{D} + B$

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	1	1		
$\bar{A}B$ 01	1	1	1	1
AB 11	1	1	1	1
$A\bar{B}$ 10	1		1	1

$$\boxed{f(A,B,C,D) = B + \bar{A}\bar{C} + A\bar{D} + AC}$$

Ans

es :- - - - - - - - - - - - - - - - - - -

.. $f(A,B,C,D) = \sum m(0,3,6,7,10,12,13)$

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	1		1	
$\bar{A}B$ 01			1	1
AB 11	1	1		
$A\bar{B}$ 10				1

Q. $F(A,B,C) = \pi M(0,1,3,5,7)$

A \ BC	$(B+C)$ 00	$(B+\bar{C})$ 01	$(\bar{B}+\bar{C})$ 11	$(\bar{B}+C)$ 10
A 0	0	0	0	
$\bar{A}$ 1		0	0	

$$F(A,B,C) = (A+B).(\bar{C})$$

Q. **K-Map with don't care :-**

* Some input conditions might not effect the system output and some input condition might never occur.

* These input conditions are refered as don't care condition can be use high/low output conditions as per our convinence.

Ques:- $f(A,B) = \Sigma m(0,3) + \Sigma d\ (2)$

A \ B	$\bar{B}$ 0	B 1
$\bar{A}$ 0	1	
A 1	X	1

$$f(A,B) = A+\bar{B}$$

A \ BC	$\bar{B}\bar{C}$ 00	$\bar{B}C$ 01	BC 11	$B\bar{C}$ 10
$\bar{A}\to 0$	1	1	X	
$A\to 1$		X	1	1

$f(A,B,C) = \bar{A}\bar{B} + AB$

$= A \odot B$ or $\overline{A \oplus B}$

Questions :-

(I) $F(A,B,C) = \Sigma m(0,1,6,7) + \Sigma d\ (3,5)$

(II) $F(A,B,C,D) = \Sigma m(0,1,6,7) + \Sigma d\ (3,4,5)$

(III) $F(w,x,y,z) = \Sigma m(1,3,7,11,15) + \Sigma d\ (0,2,5)$

Q. $F(A,B,C) = \Sigma m(0,1,6,7) + \Sigma d(3,5)$

A \ BC	$\bar{B}\bar{C}$ 00	$\bar{B}C$ 01	BC 11	$B\bar{C}$ 10
$\bar{A}$ 0	1	1	X	
A 1		X	1	1

$\Rightarrow AB + \bar{A}\bar{B}$

Q. $F(A,B,C,D) = \Sigma m(0,1,6,7) + \Sigma d(3,4,5)$

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	1	1		
$\bar{A}B$ 01	X	X	1	1
AB 11				
$A\bar{B}$ 10				

$f(A,B,C,D) = \bar{A}B + \bar{A}\bar{C}$

Q. $F(w,x,y,z) = \Sigma m(1,3,7,11,15) + \Sigma d(0,2,5)$

wx \ yz	$\bar{y}\bar{z}$ 00	$\bar{y}z$ 01	yz 11	$y\bar{z}$ 10
$\bar{w}\bar{x}$ 00	X	1	1	X
$\bar{w}x$ 01		X	1	
wx 11			1	
$w\bar{x}$ 10			1	

* We have to use/try to take less no. of don't care for minimisation

$f(A,B,C,D) = \Sigma m(\Rightarrow \bar{w}z + yz$

Ques: Simplify following by K-Map :-

(1) $\boxed{F(A,B,C,D) = \Sigma m(0,1,2,4,5,6,8,10,11,12,13,15)}$

①. $F(A,B,C,D,E) = \Sigma m(0,1,3,5,6,7,9,11,16,18,19,20,21,22,24,26)$

Ist method

for A

BC \ DE	$\bar{D}\bar{E}$ 00	$\bar{D}E$ 01	DE 11	$D\bar{E}$ 10
$\bar{B}\bar{C}$ 00	1	1	1	
$\bar{B}C$ 01		1	1	1
BC 11				
$B\bar{C}$ 10		1	1	

for $\bar{A}$

BC \ DE	$\bar{D}\bar{E}$ 00	$\bar{D}E$ 01	DE 11	$D\bar{E}$ 10
$\bar{B}\bar{C}$ 00	1		1	1
$\bar{B}C$ 01	1	1		1
BC 11				
$B\bar{C}$ 10	1			1

Other method →

② $Y = \Sigma m(0,4,7,8,9,10,11,16,24,25,28,27,29,31)$

③ $Y = \Sigma m(0,5,6,8,9,10,11,16,20,24,25,26,27,29,31)$

---

① Ans **Other method :-**

DE \ ABC	$\bar{A}\bar{B}\bar{C}$ 000	$\bar{A}\bar{B}C$ 001	$\bar{A}BC$ 011	$\bar{A}B\bar{C}$ 010	$AB\bar{C}$ 110	ABC 111	$A\bar{B}C$ 101	$A\bar{B}\bar{C}$ 100
$\bar{D}\bar{E}$ 00	1				1		1	1
$\bar{D}E$ 01	1	1		1			1	
DE 11	1	1		1				1
$D\bar{E}$ 10		1			1		1	1

$\cancel{f(A,B,C,D,E) = (DE\bar{A}\bar{B} + \bar{D}\bar{A}\bar{B}\bar{C} + D\bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C}E}$

$+ AB\bar{C}\cancel{\bar{E} + \bar{A}BC\bar{D}}$

$\cancel{+ A\bar{B}\bar{E}}$

$\cancel{+ DA\bar{B}\bar{C}}$

$= \bar{A}\bar{B}E + A\bar{B}\bar{E} + \bar{A}B\bar{C}E + AB\bar{C}\bar{E} + A\bar{B}C\bar{D} + A\bar{B}\bar{C}D$

$+ \bar{A}\bar{B}CD + \bar{A}B\bar{C}\bar{D}$

(2)

DE \ ABC	$\bar{A}\bar{B}\bar{C}$ 000	$\bar{A}\bar{B}C$ 001	$\bar{A}BC$ 011	$\bar{A}B\bar{C}$ 010	$AB\bar{C}$ 110	ABC 111	$A\bar{B}C$ 101	$A\bar{B}\bar{C}$ 100
$\bar{D}\bar{E}$ 00	1	1		1	1	1		1
$\bar{D}E$ 01				1	1	1		
DE 11		1		1	1	1		
$D\bar{E}$ 10				1				

$f(A,B,C,D,E) =$ ______

(3)

AB \ CDE	$\bar{C}\bar{D}\bar{E}$ 000	$\bar{C}\bar{D}E$ 001	$\bar{C}DE$ 011	$\bar{C}D\bar{E}$ 010	$CD\bar{E}$ 110	CDE 111	$C\bar{D}E$ 101	$C\bar{D}\bar{E}$ 100
$\bar{A}\bar{B}$ 00	1	1		1	1		1	1
$\bar{A}B$ 01		1		1	1	1		
AB 11				1	1	1		
$A\bar{B}$ 10		1		1	1			

$Y = \bar{A}\bar{B}\bar{D}\bar{E} + \bar{A}B\bar{C}\bar{D}E + A\bar{B}\bar{C}\bar{D}E + \bar{A}\bar{B}C\bar{D} + BCD + D\bar{E}$

Q. ③ $F(A,B,C,D,E) = \sum m(0,5,6,8,9,10,11,16,20,24,25,26,27,29,31)$.

DE \ ABC	000	001	011	010	110	111	101	100
$\bar{D}\bar{E}$ 00	1			1	1		1	1
$\bar{D}E$ 01		1		1	1	1		
DE 11				1	1	1		
$D\bar{E}$ 10		1		1	1			

$F(A,B,C,D,E) = B\bar{C} + ABE + \bar{B}\bar{C}\bar{D}\bar{E} + A\bar{B}\bar{D}\bar{E} + \bar{A}\bar{B}C\bar{D}E + \bar{A}\bar{B}CD\bar{E}$

Q. $F(A,B,C,D,E) = \sum m(8,9,10,11,13,15,16,18,21,24,25,26,27,30,31)$

DE \ ABC	000	001	$\bar{A}BC$ 011	$\bar{A}B\bar{C}$ 010	$AB\bar{C}$ 110	ABC 111	$A\bar{B}C$ 101	$A\bar{B}\bar{C}$ 100
$\bar{D}\bar{E}$ 00				1	1			1
$\bar{D}E$ 01			1	1	1		1	
DE 11			1	1	1	1		
$D\bar{E}$ 10				1	1	1		1

$F(A,B,C,D,E) = \bar{A}BE + B\bar{C} + DAB + A\bar{B}\bar{C}\bar{E} + A\bar{B}C\bar{D}E$

Q. $F(A,B,C,D,E) = \pi M(6, 9, 11, 13, 14, 17, 20, 25, 28, 29, 30)$

DE \ ABC	$(\bar{A}+\bar{B}+\bar{C})$ 000	$(\bar{A}+\bar{B}+C)$ 001	$(\bar{A}+B+C)$ 011	$(\bar{A}+B+\bar{C})$ 010	$(A+B+\bar{C})$ 110	$(A+B+C)$ 111	$(A+\bar{B}+C)$ 101	$(A+\bar{B}+\bar{C})$ 100
$(\bar{D}+\bar{E})$ 00						0	0	
$(\bar{D}+E)$ 01			0	0	0	0		0
$(D+E)$ 11				0				
$(D+\bar{E})$ 10		0	0			0		

$f(A,B,C,D,E) = (\bar{D}+E) * (\bar{A}+B+\bar{C}+E) * (C+\bar{D}+E) * (A+B+C+\bar{E})$

$* (\bar{D}+\bar{E}+A+C)$

$+ (\bar{A}+B+\bar{C}+\bar{D}+E)$

★ # Quine - Mcclusky method (tabular method)

10 Marks

Imp.

> It should the capability of Handeling large number of variables.

> It should ensures minimised Expression, It should be suitable for computer solution

Ques:-

$F(A,B,C,D) = \Sigma m(0,1,2,8,10,11,14,15)$ solve by tabular method.

Ans. Step-1

$0 \rightarrow 0000$

$1 \rightarrow 0001$

$2 \rightarrow 0010$

$8 \rightarrow 1000$

$10 \rightarrow 1010$

$11 \rightarrow 1011$

$14 \rightarrow 1110$

$15 \rightarrow 1111$

Table.1

Group		Step ① ABCD	Step ②	ABCD	Step-3	ABCD
G_0	0 ✓	0000	0,1	000_	0,2,8,10	–0–0
G_1	1 ✓	0001	0,2 ✓	00–0	0,8,2,10	–0–0
	2 ✓	0010	0,8 ✓	–000	10,11,14,15	1–1–
	8 ✓	1000	2,10 ✓	–010	10,14,11,15	1–1–
G_2	10 ✓	1010	8,10 ✓	10–0		
G_3	11 ✓	1011	10,11 ✓	101–		
	14 ✓	1110	10,14 ✓	1–10		
G_4	15 ✓	1111	11,15 ✓	1–11		
			14,15 ✓	111–		

Table-2

Min term		0	1	2	8	10	11	14	15
$\bar{A}\bar{B}\bar{C}$	0,1	×	×						
$\bar{B}\bar{D}$	0,2,8,10	×		×	×	×			
AC	10,11,14,15					×	×	×	×
		✓	✓	✓	✓	✓	✓	✓	✓

$\Rightarrow \bar{A}\bar{B}\bar{C} + \bar{B}\bar{D} + \bar{B}\bar{D} + AC + AC + AC$

$\Rightarrow \bar{A}\bar{B}\bar{C} + \bar{B}\bar{D} + AC$

Ques:- $F(w,x,y,z) = \Sigma m(0,3,5,6,7,10,12,13) + \Sigma d(2,9,15)$

Ans:- Table-1.

Group		Step-I $wxyz$	Step-II	$wxyz$	Step-III	
G_0	0 ✓	0000	0,2	00_0	2,3,6,7	0_1_
					2,6,3,7	0_1_
G_1	2 ✓	0010	2,3 ✓	001_		
G_2	3 ✓	0011	2,6 ✓	0_10	5,7,13,15	_1_1
	5 ✓	0101	2,10	_010	5,13,7,15	_1_1
	6 ✓	0110				
	9 ✓	1001	3,7 ✓	0_11		
	10 ✓	1010	5,7 ✓	01_1		
	12 ✓	1100				
G_3	7 ✓	0111	5,13 ✓	_101		
			6,7 ✓	011_		
	13 ✓	1101	9,13	1_01		
			12,13	110_		
G_4	15	1111	7,15 ✓	_111		
			13,15 ✓	11_1		

Table ②.

Min term	0	3	5	6	7	10	12	13
$\bar{w}\bar{x}\bar{z} \rightarrow 0,2$	X							
$\bar{x}y\bar{z} \leftarrow 2,10$						X		
$w\bar{y}z \leftarrow 9,13$								X
$wx\bar{y} \leftarrow 12,13$							X	X
$\bar{w}y \leftarrow 2,6,3,7$		X		X	X			
$xz \leftarrow 5,7,13,15$			X		X			X
	✓	✓	✓	✓	✓	✓	✓	✓

$f(w,x,y,z) = \bar{w}\bar{x}\bar{z} + \bar{w}y + xy + \bar{x}y\bar{z} + wx\bar{y}$

*