6.1: Game Theory – Multi-Agent Interaction

AI6125: Multi-Agent System

Assoc Prof Zhang Jie

Based on "An Introduction to MultiAgent Systems" by Michael Wooldridge, John Wiley & Sons, 2002/2009

Topics in this Module

- This module focuses on the issue of reaching agreement among self-interested agents
- 5.1. Multiagent Interaction
  - Basic concepts on multiagent encounters; how to model agents' decision making, how cooperation can be facilitated
- 5.2. Allocating Scarce Recourses
  - Auctions
- 5.3. Making Group Decisions (if time permits)
  - Social choice, or voting

5.1: MULTIAGENT INTERACTION

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Utilities and Preferences

- Assume we have just two agents: $Ag = \{i, j\}$
- Agents are assumed to be *self-interested*: they *have preferences over how the environment is*
- Assume $\Omega = \{\omega_1, \omega_2, \ldots\}$ is the set of "outcomes" that agents have preferences over
- We capture preferences by *utility functions*:

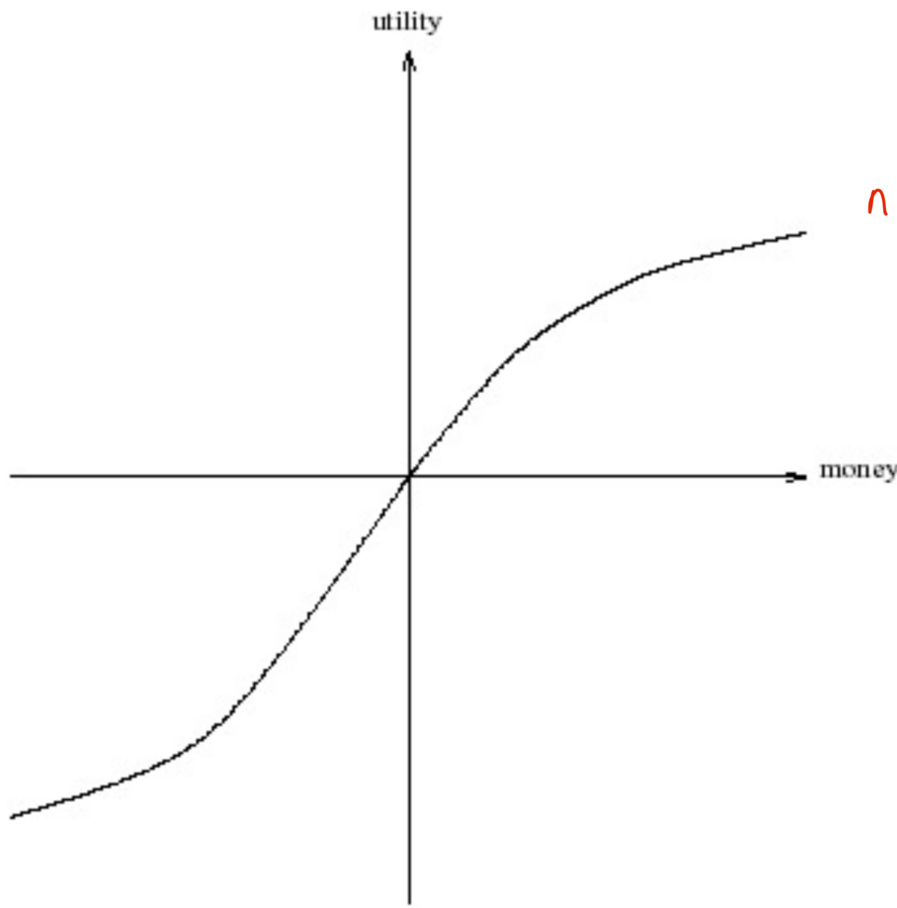
$$u_i = \Omega \to \Re$$
$$u_j = \Omega \to \Re$$

- Utility functions lead to *preference orderings* over outcomes:

$$\omega \geq_i \omega' \text{ means } u_i(\omega) \geq u_i(\omega')$$
$$\omega >_i \omega' \text{ means } u_i(\omega) > u_i(\omega')$$

What is Utility?

- Utility is *not* money (but it is a useful analogy)
- Typical relationship between utility & money:



not linear.

Multiagent Encounters

- We need a model of the environment in which these agents will act…
  - agents simultaneously choose an action to perform, and as a result of the actions they select, an outcome in Ω will result
  - the *actual* outcome depends on the *combination* of actions
  - assume each agent has just two possible actions that it can perform, C ("cooperate") and D ("defect")
- Environment behavior given by *state transformer function*:

$$\tau : \underbrace{Ac}_{\text{agent } i\text{'s action}} \times \underbrace{Ac}_{\text{agent } j\text{'s action}} \rightarrow \Omega$$

Multiagent Encounters

- Here is a state transformer function:

  defect cooperate.

  $$\tau(D, D) = \omega_1 \quad \tau(D, C) = \omega_2 \quad \tau(C, D) = \omega_3 \quad \tau(C, C) = \omega_4$$

  (This environment is sensitive to actions of both agents.)
- Here is another:

  $$\tau(D, D) = \omega_1 \quad \tau(D, C) = \omega_1 \quad \tau(C, D) = \omega_1 \quad \tau(C, C) = \omega_1$$

  (Neither agent has any influence in this environment.)
- And here is another:

  $$\tau(D, D) = \omega_1 \quad \tau(D, C) = \omega_2 \quad \tau(C, D) = \omega_1 \quad \tau(C, C) = \omega_2$$

  (This environment is controlled by *j*.)

Rational Action

- Suppose we have the case where *both* agents can influence the outcome, and they have utility functions as follows:

agent i → $u_i(\omega_1) = 1 \quad u_i(\omega_2) = 1 \quad u_i(\omega_3) = 4 \quad u_i(\omega_4) = 4$

agent j → $u_j(\omega_1) = 1 \quad u_j(\omega_2) = 4 \quad u_j(\omega_3) = 1 \quad u_j(\omega_4) = 4$

- With a bit of abuse of notation:

$$u_i(D,D) = 1 \quad u_i(D,C) = 1 \quad u_i(C,D) = \underline{4} \quad u_i(C,C) = \underline{4}$$
$$u_j(D,D) = 1 \quad u_j(D,C) = 4 \quad u_j(C,D) = 1 \quad u_j(C,C) = 4$$

- Then agent i's preferences are:

$$C,C \succeq_i C,D \quad \succ_i \quad D,C \succeq_i D,D$$

- "*C*" is the *rational choice* for i.
(Because i prefers all outcomes that arise through *C* over all outcomes that arise through *D*.)

Payoff Matrices

- We can characterize the previous scenario in a *payoff matrix*:

j \ i	defect	coop
defect	1 1	4 1
coop	1 4	4 4

。如果 i、j 都 D.
$U_i=1$ $U_j=1$

。如果 i C，j D.
$U_i=4$. $U_j=1$

- Agent i is the *column player*
- Agent j is the *row player*

Solution Concepts

- How will a rational agent behave in any given scenario?
- Answered in solution concepts:
  - dominant strategy;
  - Nash equilibrium strategy;
  - Pareto optimal strategies;
  - strategies that maximise social welfare.

Dominant Strategies

i . j 都C为 dominant strategy .

- Given any particular strategy (either *C* or *D*) of agent i, there will be a number of possible outcomes
- We say s_1 *dominates* s_2 if every outcome possible by i playing s_1 is preferred over every outcome possible by i playing s_2
- A rational agent will never play a dominated strategy
- So in deciding what to do, we can *delete dominated strategies*
- Unfortunately, there isn't always a unique undominated strategy

Nash Equilibrium

- In general, we will say that two strategies s_1 *and* s_2 are in Nash equilibrium if:
    1. under the assumption that agent i plays s_1, agent j can do no better than play s_2; and
    2. under the assumption that agent j plays s_2, agent i can do no better than play s_1.
- *Neither agent has any incentive to deviate from a Nash equilibrium*
- Unfortunately:
    1. *Not every interaction scenario has a Nash equilibrium*
    2. *Some interaction scenarios have more than one Nash equilibrium*

Matching Pennies

- Players i and j simultaneously choose the face of a coin, either “heads” or “tails”.
- If they show the same face, then i wins, while if they show different faces, then j wins.
- The Payoff Matrix:

	i heads	i tails
j heads	1 -1	-1 1
j tails	-1 1	1 -1

Mixed Strategies for Matching Pennies

- NO pair of strategies forms a pure strategy NE: whatever pair of strategies is chosen, somebody will wish they had done something else.
- The solution is to allow mixed strategies:
  - play “heads” with probability 0.5
  - play “tails” with probability 0.5.
- This is a NE strategy.

Mixed Strategies

- A mixed strategy has the form

  – play 1 with probability p_1

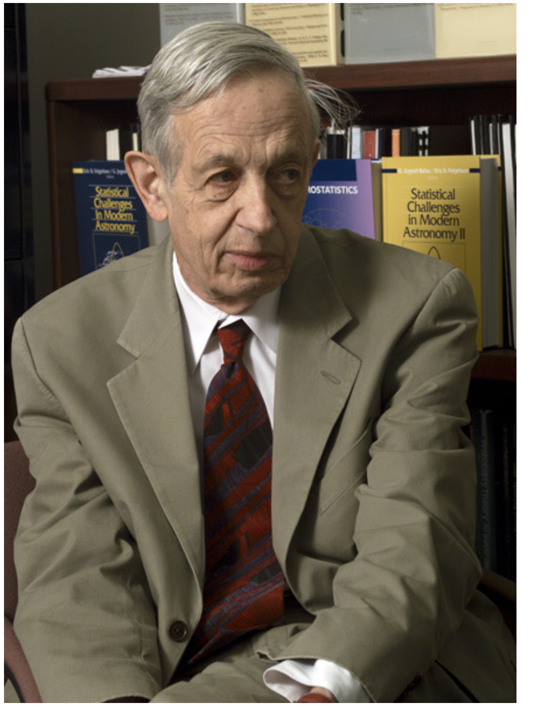
  – play 2 with probability p_2

  – . . .

  – play k with probability p_k.

  such that $p_1 + p_2 + \ldots + p_k = 1$.
- Nash proved that every finite game has a Nash equilibrium in mixed strategies.

Nash's Theorem



- John Forbes Nash
- Nash proved that every finite game has a Nash equilibrium in mixed strategies. (Unlike the case for pure strategies.)
- So this result overcomes the lack of solutions; but there still may be more than one Nash equilibrium...

Pareto Optimality

- An outcome is said to be Pareto optimal (or Pareto efficient) if there is no other outcome that makes one agent better off without making another agent worse off.
- If an outcome is Pareto optimal, then at least one agent will be reluctant to move away from it (because this agent will be worse off).
- If an outcome ω is *not Pareto optimal, then there is* another outcome ω' that makes *everyone as happy, if* not happier, than ω.
- "Reasonable" agents would agree to move to ω' in this case. (Even if I don't directly benefit from ω', you can benefit without me suffering.)

Social Welfare

- The social welfare of an outcome ω is the sum of the utilities that each agent gets from ω:

$$\sum_{i \in Ag} u_i(\omega)$$

- Think of it as the "total amount of money in the system".
- As a solution concept, may be appropriate when the whole system (all agents) has a single owner (then overall benefit of the system is important, not individuals).

Competitive and Zero-Sum Interactions

- Where preferences of agents are diametrically opposed we have *strictly competitive* scenarios
- Zero-sum encounters are those where utilities sum to zero:

$$u_i(\omega) + u_j(\omega) = 0 \qquad \text{for all } \omega \in \Omega$$

- Zero sum implies strictly competitive
- The best outcome for me is the *worst for you!*
- Zero sum encounters in real life are very rare … but people tend to act in many scenarios as if they were zero sum

The Prisoner’s Dilemma

- Two men are collectively charged with a crime and held in separate cells, with no way of meeting or communicating. They are told that:
  - if one confesses and the other does not, the confessor will be freed, and the other will be jailed for three years
  - if both confess, then each will be jailed for two years
- Both prisoners know that if neither confesses, then they will each be jailed for one year

The Prisoner's Dilemma

- Payoff matrix for prisoner's dilemma:

		i	
		defect	coop
j	defect	2 2	1 4
	coop	4 1	3 3

- Top left: If both defect, then both get punishment for mutual defection
- Top right: If i cooperates and j defects, i gets sucker's payoff of 1, while j gets 4
- Bottom left: If j cooperates and i defects, j gets sucker's payoff of 1, while i gets 4
- Bottom right: Reward for mutual cooperation

The Prisoner's Dilemma

- The *individual rational* action is *defect*
  This guarantees a payoff of no worse than 2, whereas cooperating guarantees a payoff of at most 1
- So defection is the best response to all possible strategies: both agents defect, and get payoff = 2
- But *intuition* says this is *not* the best outcome:
  Surely they should both cooperate and each get payoff of 3!
  - D is a dominant strategy
  - *(D;D) is the only Nash equilibrium.*
  - All outcomes *except (D;D) are Pareto optimal.*
  - *(C;C) maximises social welfare.*

The Prisoner’s Dilemma

- This apparent paradox is *the fundamental problem of multi-agent interactions*.
  It appears to imply that *cooperation will not occur in societies of self-interested agents*.
- Real world examples:
  - nuclear arms reduction (“why don’t I keep mine. . . ”)
- The prisoner’s dilemma is *ubiquitous*.
- Can we recover cooperation?

Arguments for Recovering Cooperation

- Conclusions that some have drawn from this analysis:
  - the game theory notion of rational action is wrong!
  - somehow the dilemma is being formulated wrongly
- Arguments to recover cooperation:
  - We are not all Machiavelli!
  - The other prisoner is my twin!
  - Program equilibria and mediators
  - The shadow of the future…

Program Equilibria

- The strategy you really want to play in the prisoner's dilemma is:

  I'll cooperate if he will.
- Program equilibria provide one way of enabling this.
- Each agent submits a program strategy to a mediator which jointly executes the strategies. Crucially, strategies can be conditioned on the strategies of the others.

Program Equilibria

- Consider the following program:

```
IF HisProgram == ThisProgram THEN
      DO(C);
ELSE
      DO(D);
END-IF.
```

  Here == is textual comparison.

- The best response to this program is to submit the same program, giving an outcome of (C;C)!
- You can't get the sucker's payoff by submitting this program.

The Iterated Prisoner's Dilemma

- One answer: *play the game more than once*
- If you know you will be meeting your opponent again, then the incentive to defect appears to evaporate
- *Cooperation is the rational choice in the infinititely repeated prisoner's dilemma*
  (Hurrah!)

Backwards Induction

- But…suppose you both know that you will play the game exactly n times
  On round $n - 1$, you have an incentive to defect, to gain that extra bit of payoff…
  But this makes round $n – 2$ the last “real”, and so you have an incentive to defect there, too.
  This is the *backwards induction* problem.
- Playing the prisoner’s dilemma with a fixed, finite, pre-determined, commonly known number of rounds, defection is the best strategy

Axelrod’s Tournament

- Suppose you play iterated prisoner’s dilemma against a *range* of opponents…
  What strategy should you choose, so as to maximize your overall payoff?
- Axelrod (1984) investigated this problem, with a computer tournament for programs playing the prisoner’s dilemma

Strategies in Axelrod's Tournament

- <u>ALLD</u>:
  - "Always defect" — the *hawk* strategy;
- <u>TIT-FOR-TAT</u>:
  1. On round $u = 0$, cooperate
  2. On round $u > 0$, do what your opponent did on round $u - 1$
- <u>TESTER</u>:
  - On 1st round, defect. If the opponent retaliated, then play TIT-FOR-TAT. Otherwise intersperse cooperation and defection.
- <u>JOSS</u>:
  - As TIT-FOR-TAT, except periodically defect

Recipes for Success in Axelrod's Tournament

- Axelrod suggests the following rules for succeeding in his tournament:
  - *Don't be envious*:
    Don't play as if it were zero sum!
  - *Be nice*:
    Start by cooperating, and reciprocate cooperation
  - *Retaliate appropriately*:
    Always punish defection immediately, but use "measured" force — don't overdo it
  - *Don't hold grudges*:
    Always reciprocate cooperation immediately

Game of Chicken

- Consider another type of encounter — the *game of chicken*:

		i	
		defect	coop
j	defect	j: 1, i: 1	j: 4, i: 2
	coop	j: 2, i: 4	j: 3, i: 3

- Difference to prisoner's dilemma:

  Mutual defection is most feared outcome.

- There is no dominant strategy (in our sense).
- Strategy pairs (*C;D)) and (D;C)) are Nash* equilibriums.
- All outcomes except (*D;D) are Pareto optimal.*
- All outcomes except (*D;D) maximise social welfare.*