6.3: Game Theory – Making Group Decisions

AI6125: Multi-Agent System

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Based on "An Introduction to MultiAgent Systems" by Michael Wooldridge, John Wiley & Sons, 2002/2009

Social Choice

- In the previous sub-module, we learned how agents make decisions in two-agent games
  - Nash Equilibrum
- In this sub-module, we will learn about how a group of agents make decisions
  - Social choice theory is concerned with group decision making.
  - Classic example of social choice theory: voting.
  - Formally, the issue is combining preferences to derive a social outcome.

Components of a Social Choice Model

- Assume a set $Ag = \{1, \ldots, n\}$ of voters.

  These are the entities who will be expressing preferences.
- Voters make group decisions wrt a set

  $\Omega = \{\omega_1, \omega_2, \ldots\}$ of *outcomes*.

  Think of these as the *candidates*.
- If $|\Omega| = 2$, we have a *pairwise election*.

Preferences

- Each voter has preferences over Ω: an *ordering* over the set of possible outcomes Ω.
- Example. Suppose

  $$\Omega = \{gin, rum, brandy, whisky\}$$

  then we might have agent *mjw with preference order:*

  $$\varpi_{mjw} = (brandy; rum; gin; whisky)$$

  *m*eaning

  $$brandy \succ_{mjw} rum \succ_{mjw} gin \succ_{mjw} whisky$$

Preference Aggregation

- The fundamental problem of social choice theory:

  given a collection of preference orders, one for each voter, how do we combine these to derive a group decision, that reflects <u>as closely as possible</u> the preferences of voters?
- Two variants of preference aggregation:
  - *social welfare functions;*
  - *social choice functions.*

Social Welfare Functions

- Let $\pi(\Omega)$ be the set of preference orderings over Ω.
- A *social welfare function* takes the voter preferences and produces a *social preference order:*

$$f: \underbrace{\pi(\Omega) \times \ldots \times \pi(\Omega)}_{\text{n times}} \rightarrow \pi(\Omega)$$

- We let $\succ^*$ denote to the outcome of a social welfare function
- Example: beauty contest.

Social Choice Functions

- Sometimes, we just want to select *one* of the possible candidates, rather than a social order.
- This gives *social choice functions:*

$$f\colon\ \underbrace{\pi(\Omega) \times \ldots \times \pi(\Omega)}_{\text{n times}} \rightarrow \Omega$$

- Example: presidential election.

Voting Procedures: Plurality

- Social choice function: selects a single outcome.
- Each voter submits preferences.
- Each candidate gets one point for every preference order that ranks them first.
- Winner is the one with largest number of points.
- Example: Political elections in UK.
- If we have only two candidates, then plurality is a *simple majority election*.

Anomalies with Plurality

- Suppose $|Ag|$ = *100 and* $\Omega = \{\omega_1, \omega_2, \omega_3\}$ *with:*

  40% voters voting for ω_1

  30% of voters voting for ω_2

  30% of voters voting for ω_3
- With plurality, ω_1 gets elected even though a *clear majority* (60%) prefer another candidate!

Strategic Manipulation by Tactical Voting

- Suppose your preferences are

  $$\omega_1 \succ \omega_2 \succ \omega_3$$

  while you believe 49% of voters have preferences

  $$\omega_2 \succ \omega_1 \succ \omega_3$$

  and you believe 49% have preferences

  $$\omega_3 \succ \omega_2 \succ \omega_1$$

- You may do better voting for ω_2, *even though this is not your true preference profile.*
- This is *tactical voting: an example of strategic manipulation of the vote.*

Condorcet's Paradox

- Suppose *Ag = {1; 2; 3} and* $\Omega = \{\omega_1, \omega_2, \omega_3\}$ *with:*

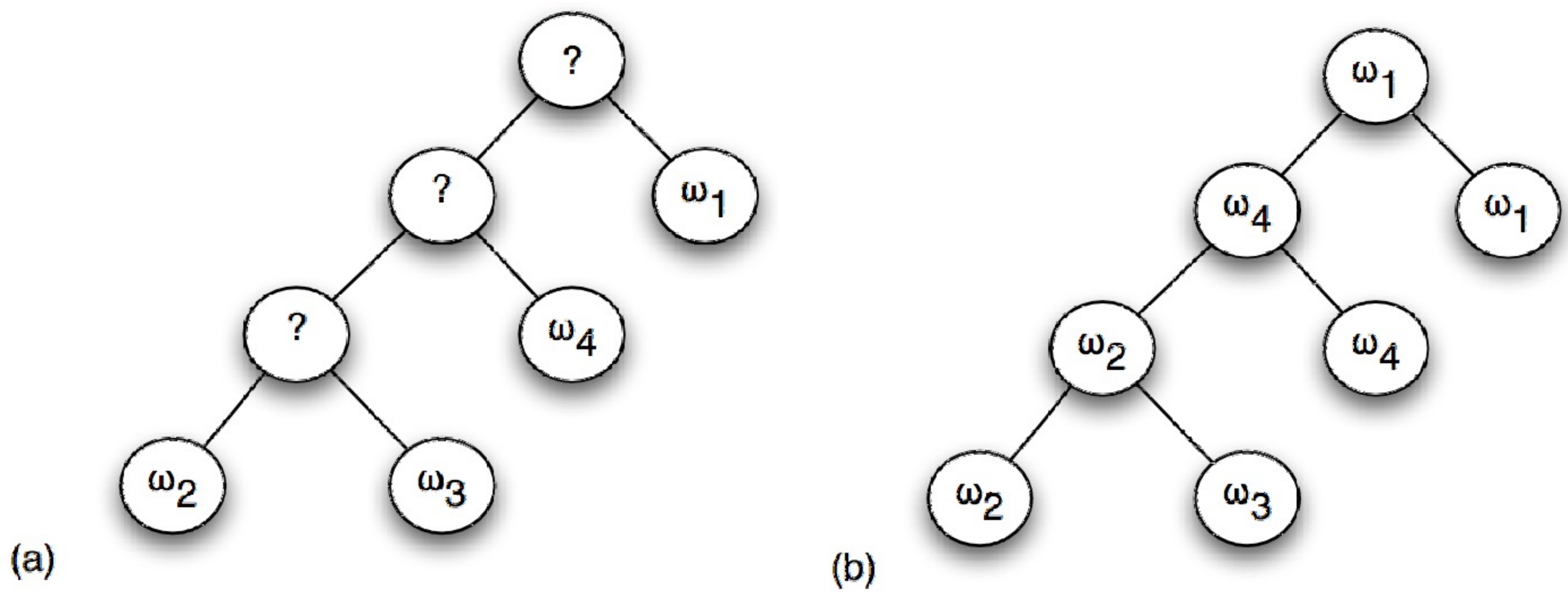
  $\omega_1 \succ_1 \omega_2 \succ_1 \omega_3$

  $\omega_3 \succ_2 \omega_1 \succ_2 \omega_2$

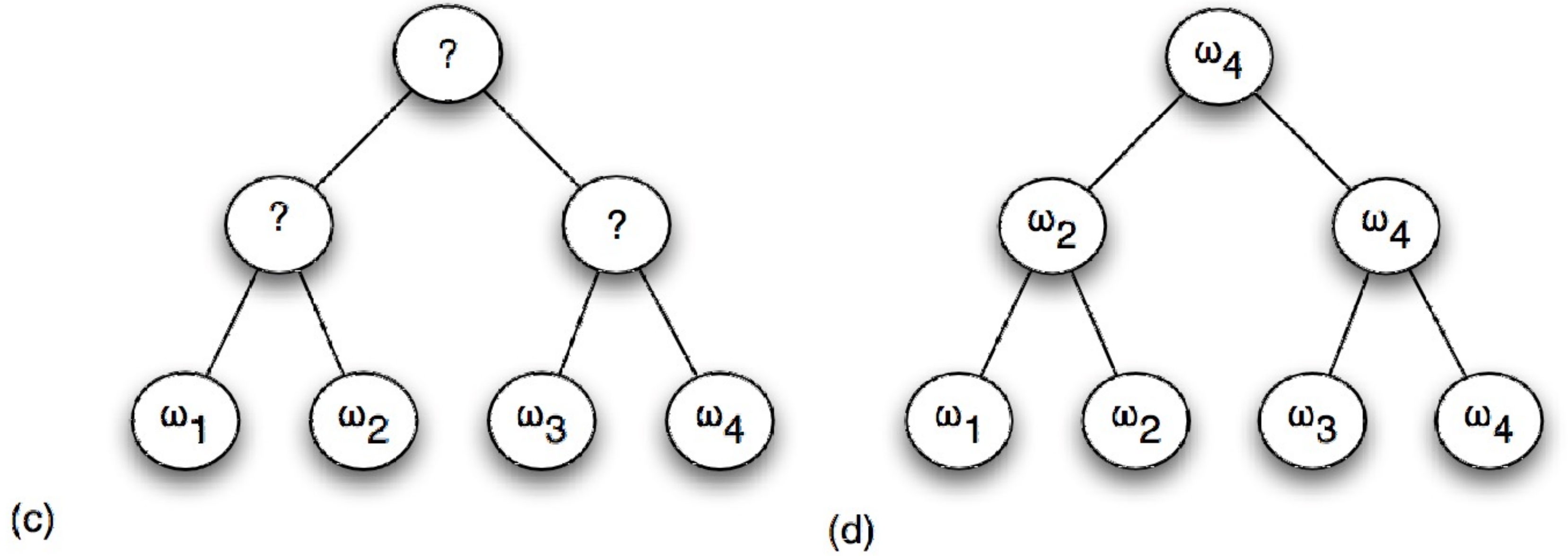
  $\omega_2 \succ_3 \omega_3 \succ_3 \omega_1$
- For every possible candidate, there is another candidate that is preferred by a majority of voters!
- This is *Condorcet's paradox:* there are situations *in* which, *no matter which outcome we choose, a* majority *of voters will be unhappy with the outcome chosen.*

Sequential Majority Elections

- A variant of plurality, in which players play in a series of rounds: either a *linear sequence or a tree (knockout* tournament).



Sequential Majority Elections



Linear Sequential Pairwise Elections

- Here, we pick an ordering of the outcomes – the *agenda – which determines who plays against who.*
- For example, if the agenda is:

$$\omega_2 , \omega_3 , \omega_4 , \omega_1$$

- then the first election is between ω_2 and ω_3, and the winner goes on to an election with ω_4, and the winner of this election goes in an election with ω_1.

Anomalies with Sequential Pairwise Elections

- Suppose:

  33 voters have preferences

  $$\omega_1 \succ \omega_2 \succ \omega_3$$

  33 voters have preferences

  $$\omega_3 \succ \omega_1 \succ \omega_2$$

  33 voters have preferences

  $$\omega_2 \succ \omega_3 \succ \omega_1$$

- Then *for every candidate, we can fix an agenda for that candidate to win in a sequential pairwise election!*

Majority Graphs

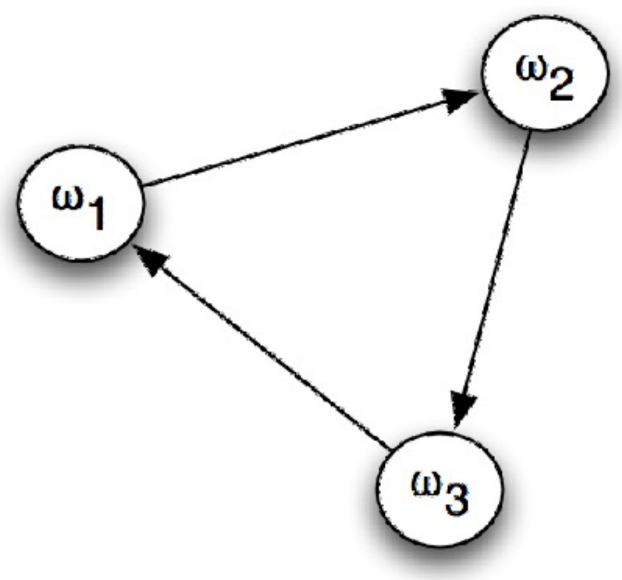
- This idea is easiest to illustrate by using a *majority graph.*
- A directed graph with:

  vertices = candidates

  an edge (*i, j) if i would beat j is a simple majority* election.
- A *compact representation of voter preferences.*

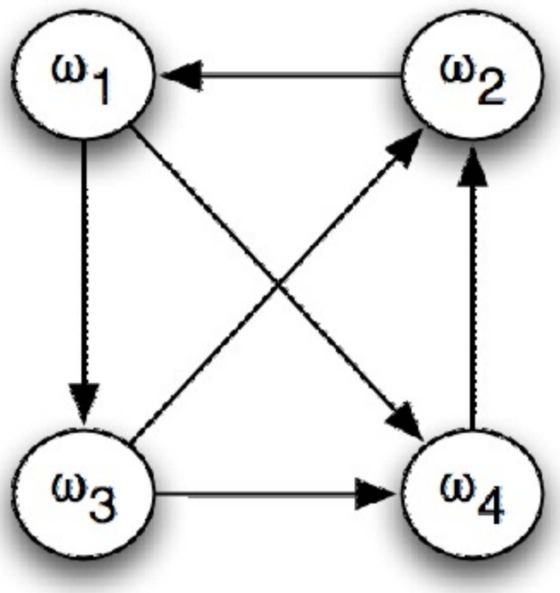
Majority Graph for the Previous Example

- with agenda (ω_3 , ω_2 , ω_1), ω_1 wins
- with agenda (ω_1 , ω_3 , ω_2), ω_2 wins
- with agenda (ω_1 , ω_2 , ω_3), ω_3 wins



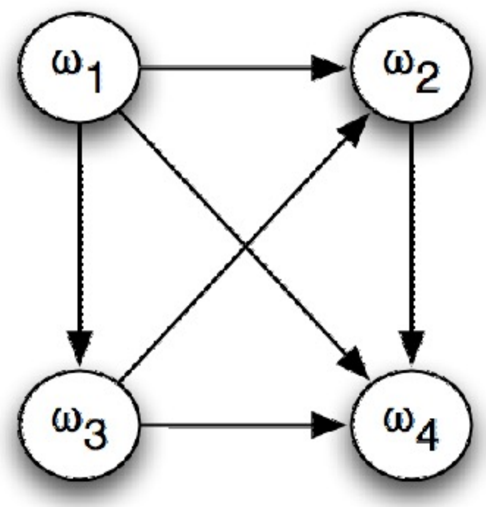
Another Majority Graph

- Give agendas for each candidate to win with the following majority graph.



Condorcet Winners

- A *Condorcet winner is a candidate that would beat every other candidate in a pairwise election.*
- Here, ω_1 is a Condorcet winner.



Voting Procedures: Borda Count

- One reason plurality has so many anomalies is that it *ignores most of a voter's preference orders: it only* looks at the *top ranked candidate.*
- The *Borda count takes whole preference order into* account.
- For each candidate, we have a variable, counting the strength of opinion in favour of this candidate.
- If ω_i appears first in a preference order, then we increment the count for ω_i by k-1; we then increment the count for the next outcome in the preference order by k-2, …, until the final candidate in the preference order has its total incremented by 0.
- After we have done this for all voters, then the totals give the ranking.

Desirable Properties of Voting Procedures

- Can we classify the properties we want of a “good” voting procedure?
- Two key properties:
- *The Pareto property*;
  - *If everybody prefers ω_i over ω_j, then ω_i should be ranked over ω_j in the social outcome.*
- *Independence of Irrelevant Alternatives (IIA)*.
  - *Whether ω_i is ranked above ω_j in the social outcome should depend only on the relative orderings of ω_i and ω_j in voters profiles.*

Arrow’s Theorem

- *For elections with more than 2 candidates, the only voting procedure satisfying the Pareto condition and IIA is a dictatorship, in which the social outcome is in fact simply selected by one of the voters.*
- This is a *negative result: there are fundamental limits to* democratic decision making!

Strategic Manipulation

- We already saw that sometimes, voters can benefit by *strategically misrepresenting their preferences*, *i.e.*, lying – tactical voting.
- Are there any voting methods which are *non-manipulable, in the sense that voters can never* benefit from misrepresenting preferences?

The Gibbard-Satterthwaite Theorem

- The answer is given by the Gibbard-Satterthwaite theorem:

  The only non-manipulable voting method satisfying the Pareto property for elections with more than 2 candidates is a dictatorship.

- In other words, every "realistic" voting method is prey to strategic manipulation . . .

Computationally Complexity to the Rescue!

- Gibbard-Satterthwaite only tells us that manipulation is possible *in principle.*

  It does not give any indication of *how to misrepresent* preferences.
- Bartholdi, Tovey, and Trick showed that there are elections that are prone to manipulation in principle, but where manipulation was *computationally complex.*
- “Single Transferable Vote” is NP-hard to manipulate!