Error Theories & Data Processing (English)

误差理论与数据处理（英文）

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Chapter1: Preface

Outlines:

- The importance of error studying.
- The objectives of this course.
- The basic concepts of error, including the definitions, the sources and the classifications of error.
- The meaning of Accuracy.
- Significant digit and data computation.

The importance of error studying

Science starts from measuring. No measuring, no fine science.

—Mendeleev 门捷列夫

If you can measure something of your interest and describe it with quantity, you may understand it; If you cannot do that, your understanding is little and shallow.

—Kelvin 开尔文

Information technologies include measuring technology, computer technology and communication technology, and measuring technology is the key and foundation.

—钱学森

The objectives of this course

- Understand the properties of error and investigate the sources of error, so as to reduce or remove them;
- Properly handle measuring and data processing, and correctly analyze the results of measurement, in order to obtain outcomes as close to true values as possible;
- Correctly arrange the experiments, design or select instruments and apply measuring methods, thus construct the optimal system to obtain the best result at the lowest cost.

Basic concepts of error

Definition and expression of error
误差的定义及表示法:

Error=measurement value(测量值) – true value(真值)

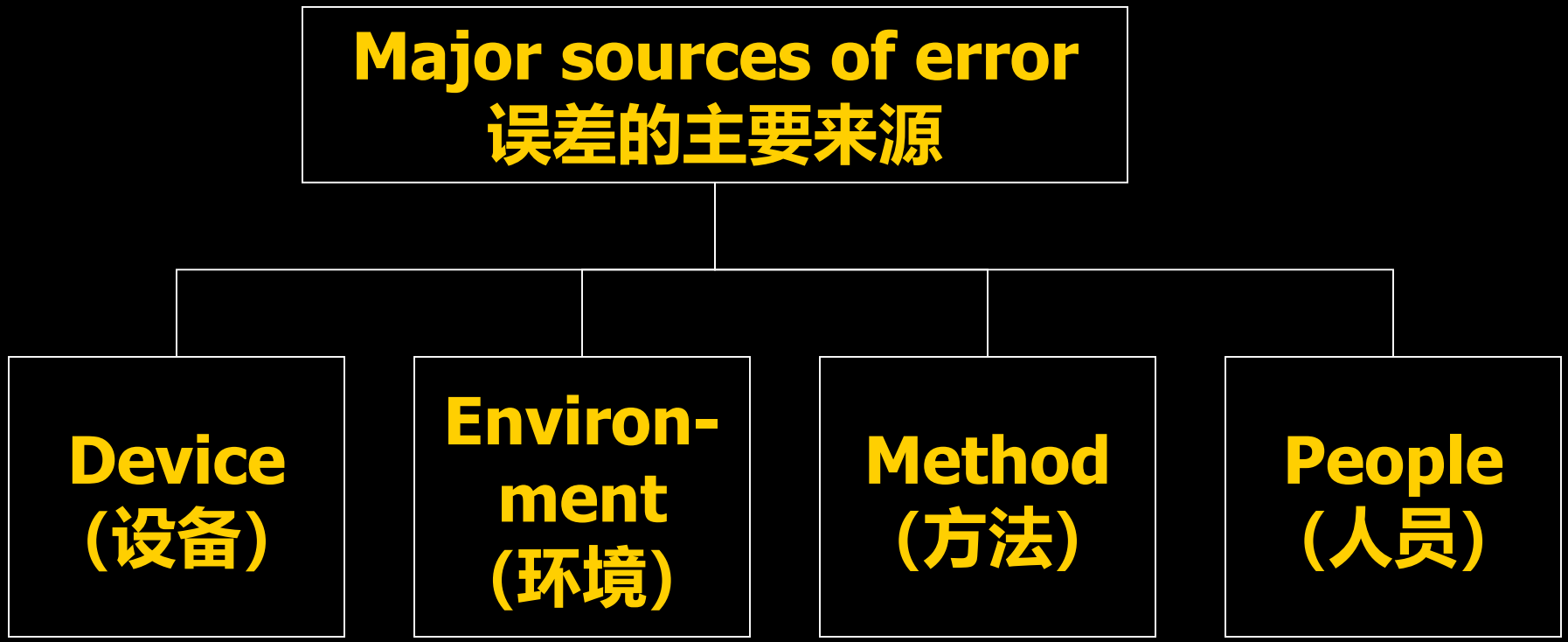
$$\Delta L = L - L_0$$

True value L_0 is the real, exact value of the measured object (practically unknown).

- Theoretical true value理论真值: theoretically defined or deduced.
- Conventional true value约定真值: universally admitted value of standard (e.g. international prototype of the kilogram).
- Relative true value相对真值 (actual value实际值): measured value by instrument with higher accuracy.

The sources of error (误差来源)

The sources of error are everywhere, and almost every step of the measuring procedure brings in error.



Error caused by devices(测量设备误差)

- Error from standard gauge 标准量具误差

  Normally such error is required to be within 1/3 to 1/10 of overall error.

- Error from measuring instrument 测量仪器误差
  - ✓ The principle of instrument is imperfect ;
  - ✓ Major parts of the instrument have manufacturing and assembly error;
  - ✓ Calibration error; Graduation error;
  - ✓ Reading resolution of the instrument is limited;
  - ✓ Digital instrument has quantization error (rounding error) ;
  - ✓ Wear and tear of the instrument will cause error.

- Error from accessories of the instrument 仪器的附件误差

Error caused by environment (环境误差)

The actual parameters实际参数 of environment may be different from the required condition, including temperature, humidity, air pressure, vibration, velocity, gravity, electro-magnetic interference, etc.

- For electric measurements, the environment error mostly comes from temperature, power supply voltage, and electro-magnetic interference;
- In laser measurements, the air temperature, humidity, dust and pressure will affect the wave length of the laser, and thus cause error.

Error caused by method (方法误差)

This is also called principle error 理论误差

- The method we use in measuring may be imperfect

- The method we use in measuring work may have approximation in calculation

  For example, when using perimeter to measure the diameter of a circle we calculate:

  $d=s/\pi$, $\pi=3.1415926535\ldots\ldots\approx3.14$

Error cause by people （人员误差）

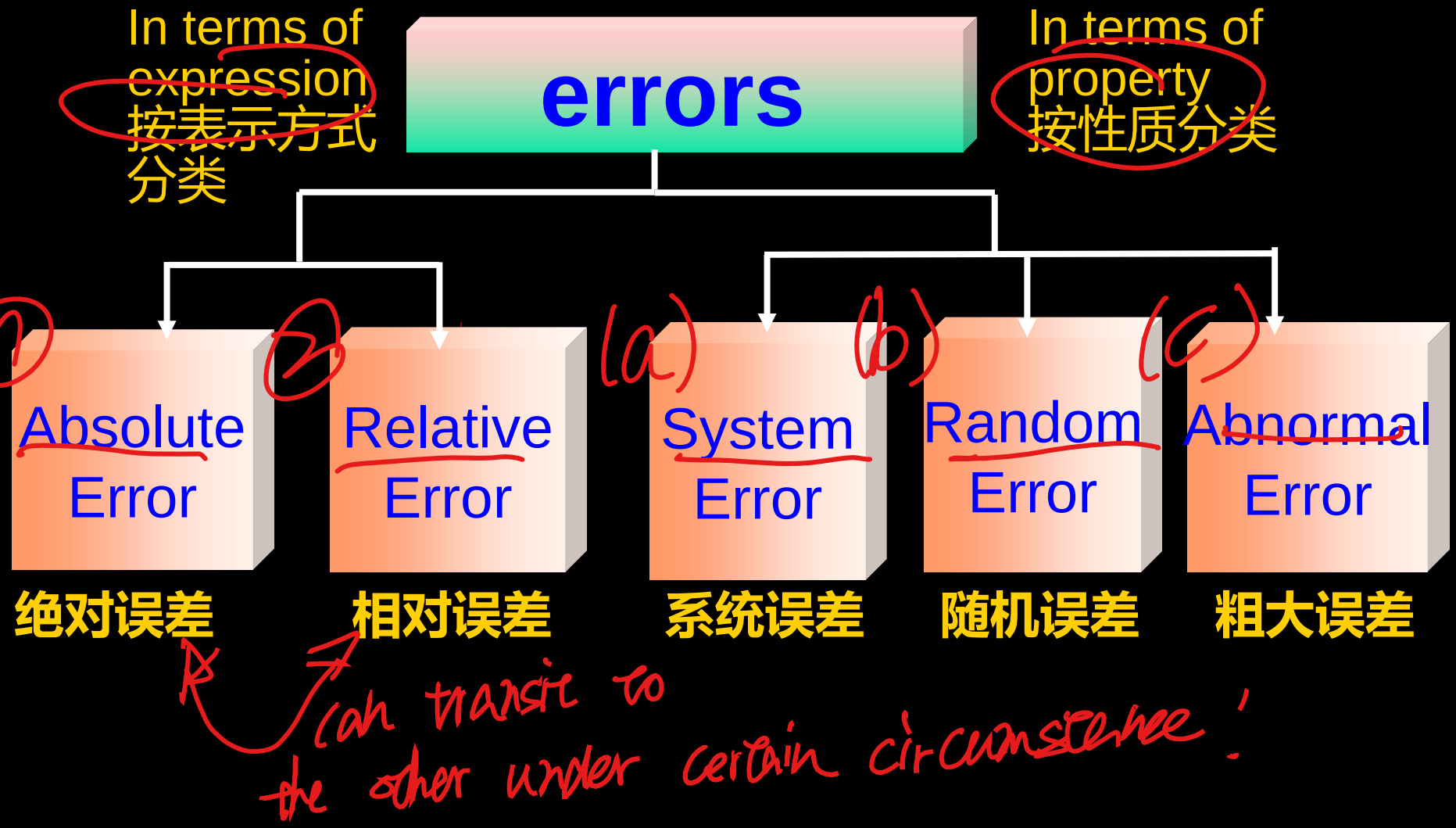
People are different in everyway, such as habit, strength, capability, mental status,…, etc

People make mistakes

How to reduce error caused by people

- Fully understand the principle of measuring, possible sources of error and the property of the device.
- Master the procedures of the measuring.
- Do the measuring carefully.
- Properly handle the measurement data.

The classifications of error 误差分类



Absolute Error

Absolute Error=measurement value – true value

$$\Delta L = L - L_0$$

The properties of Absolute Error :

- It could be positive or negative;
- It has the same unit as measurement value;
- It is used to evaluate the accuracy of measurements of the same object.

Correction（修正值）

In order to remove system error, the quantity added to the measurement result is called correction.

True value ≈ Measurement value + Correction

Correction = True value – Measurement value

$L_0 = L + C$ $\quad C = L_0 - L = -\Delta L$

Relative Error

Relative Error: $r = \frac{\Delta L}{L_0} \approx \frac{\Delta L}{L}$

The properties of Relative Error :

- It could be positive or negative;
- It does not have any unit, normally expressed in "%" ;
- It is used to evaluate the accuracy of measurements of different objects.

Fiducial Error of instrument
仪器的引用误差

Definition:

$$r_m = \frac{\Delta x_m}{x_m}$$

The maximum indication error within the range **量程内最大示值误差**

Range (maximum reading) of the instrument **仪器全量程**

Fiducial Error of instrument **仪器(最大)引用误差**

- Fiducial Error is a type of relative error;
- It uses specific value on the denominator, i.e. the full range of the instrument, so it is also called Fiducial Relative Error or Error over the range 量程误差;
- It reflexes the accuracy of corresponding instrument.

System Error

Errors that maintain the same sign and absolute value in multiple measurements under the same condition, or errors that change with certain pattern when the measuring condition changes.

System errors are classified as follows:

- Constant system error 恒定系统误差
- Variable system error 可变系统误差
  - Linear system error 线性系统误差
  - Periodic system error 周期系统误差
  - Complex system error 复杂系统误差

In engineering practices, under different conditions, system errors are treated as known system errors(已定系统误差) or unknown system errors(未定系统误差).

Random Error

- Under the same condition and with multiple measurements, the absolute value and the sign of the error change randomly without predictable pattern.
- Random disturbances are the causes of random error.
- Such randomness follows statistic rules with large number of repeated measurements and can be cancelled out by taking average.

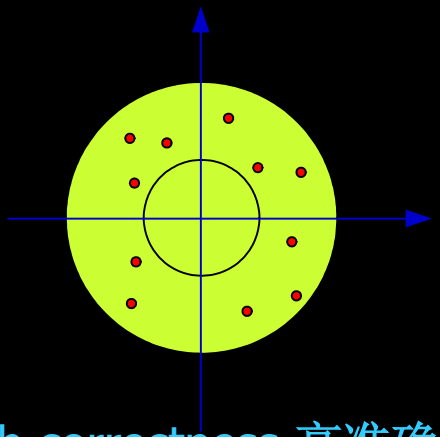
Abnormal Error

Errors that exceed the expectation under given condition, and they are apparently some mistakes of measuring.

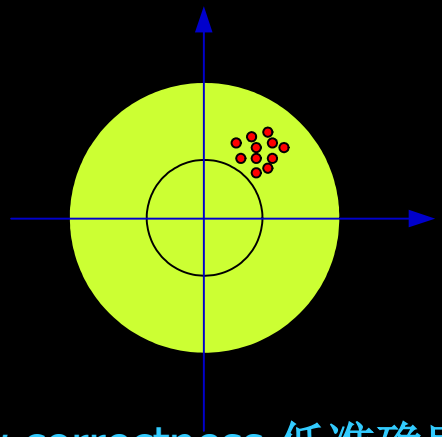
Causes could be abnormal change of measuring condition, failure of instrument, mistake of people…

Measurement with abnormal error carries wrong information, and it needs to be identified and removed from consideration.

The meaning of accuracy 精确度的含义



High correctness 高准确度
Low precision 低精密度



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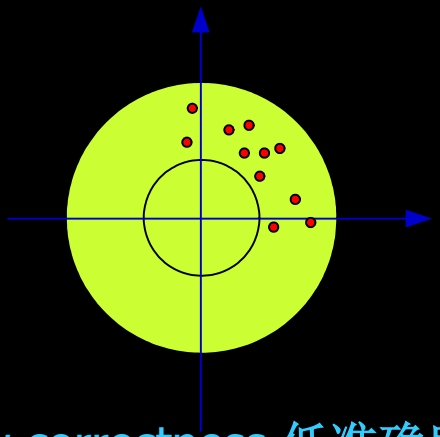

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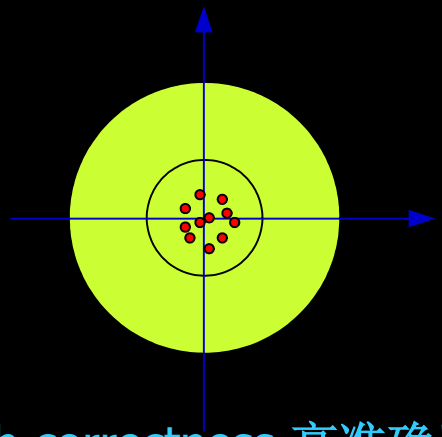

High correctness 高准确度
High precision 高精密度

Definition of accuracy(精度的定义)

Accuracy reflexes how close between the measurement and the true value. Its level can be represented by the quantity of error. The smaller is the error, the higher is the accuracy.

Accuracy is the joint index of correctness and precision, i.e. Accuracy = Correctness + Precision

Correctness indicates the influence of system error, precision indicates the influence of random error, and accuracy is the joint influence of both system and random errors.

Significant Digit & Data Computation
有效数字与数据运算

Significant Digit :

For any approximate number, the left most non-zero digit is called the first significant digit. From the first significant digit to the last digit, all digits, zero or non-zero, are called significant digits.

047.200150

Rules of significant digit in measurement :

For ordinary measurements, the last digit is not reliable, the second digit to the last is reliable.

For important measurements, we can use an extra digit for the data and error, and in such case, the third digit to the last is reliable, the second digit to the last is not reliable, and the last digit is only for reference.

examples & expression 示例和数字表达方式

35.6 Vs 0.0356 0.0027 ≠0.00270

2400 ??? $2.4x10^3$ or $2.40x10^3$ or $2.400x10^3$

Standard expression:

$ax10^n$; $1 \le a < 10$

$L=20.531 \pm 0.01$

$L=20.53 \pm 0.01=(2.053 \pm 0.001)x10^1$

For important measurements:

$L=15.214 \pm 0.042$

Rounding-off method 舍入规则

If the part to be dropped off is greater than half unit of the last digit of the remaining part, the last digit adds 1;

If the part to be dropped off is less than half unit of the last digit of the remaining part, the last digit doesn't change;

If the part to be dropped off is equal to half unit of the last digit of the remaining part, the last digit is adjusted to even number.

Examples of rounding-off operation

Original data:	Rounded off data:
3.14159	3.142
2.71729	2.717
4.51050	4.510
3.21550	3.216
6.378501	6.379
7.691499	7.691
5.43460	5.435

Rules of data computation 数据运算规则

In arithmetic computation of approximate numbers, for the result to be more accurate, all computing data may have an extra digit after their significant digits, which is called the safety digit.

Rules of data computation数据运算规则

For summation and subtraction, the data with the fewest number of decimal digits should be the reference, and other data may keep an extra decimal digit, but the final result should have the same number of decimal digits as the reference;

在加减运算时，各运算数据以小数位数最少的数据位数为准，其余各数据可多保留一位小数，但最后结果应保留最少的小数位数；

For multiplication and division, the computing data with the fewest number of significant digits should be the reference, and other data may keep an extra significant digit, but the final result should have the fewest number of significant digits as the reference;

在乘除运算时，各运算数据以有效位数最少的数据为准，其余各数据可以多保留一位有效位数，但最后的结果应象参考数据一样保留最少的有效位数；

For computation of square and square root, the computing numbers are treated the same way as in multiplication and division;

在平方和开方运算时，按照乘除运算处理；

Rules of data computation 数据运算规则

For logarithm arithmetic, in order to avoid losing accuracy, we should use n-digit or (n+1)-digit logarithmic table if the approximate number has n significant digits;

在对数运算时，**n**位有效数字的数据应该使用**n**位或者**(n+1)**的对数表，以免损失精度；

For computation of trigonometric function, the number of significant digits of the function should increase as error of the angle reduces;

在三角函数运算中，所取函数值的位数应该随角度误差的减少而增多，其对应关系如下表所示：

Error of the angle	10″	1″	0.1″	0.01″
Digits of the function	5	6	7	8