Zorah Zafari
Macro C- PSET 3
May 25, 2024

Sticky Wage Model

- assuming that nominal wages are sticky for one period

$W_1 = W_0$

Short-run Equilibrium

$Y_1 = A_1 N_1$

$W_1 = W_0$

$\frac{W_1}{P_1} = A_1$

$Y_1 = C_1$

$\frac{M_1}{P_1} = \xi^{1/v}\left(1 - \frac{1}{Q_1}\right)^{1/v} C_1^{\gamma/v}$

$1 = \beta E_1 \left\{ Q_1 \frac{P_1}{P_2} \frac{C_2^{-\gamma}}{C_1^{-\gamma}} \right\}$

long-run Equilibrium

$Y_t = A_t N_t$

$\frac{W_t}{P_t} = A_t$

$\frac{W_t}{P_t} = \frac{\chi N_t^{\varphi}}{C_t^{-\gamma}}$

$Y_t = C_t$

$\frac{M_t}{P_t} = \xi^{1/v}\left(1 - \frac{1}{Q_t}\right)^{1/v} C_t^{\gamma/v}$

$1 = \beta E_t \left\{ Q_t \frac{P_t}{P_{t+1}} \frac{C_{t+1}^{-\gamma}}{C_t^{-\gamma}} \right\}$

a) Are the firms on their labor curve? Explain.

Yes, with sticky wages, the firms will be on their labor demand curve. They can adjust their prices and output. Firms will hire as many workers as needed to produce the output demand at price P.

b) Are households on their labor supply curve? Explain.

No, with sticky wages, households cant optimally adjust their labor supply. They are constrained by $W_1 = W_0$ which means $\frac{W_1}{P_1} \neq \frac{\chi N_1^{\varphi}}{C_1^{-\gamma}}$

c) How does labor market clear?

labor market clears when $Y_1 = A_1 N_1$, $W_1 = W_0$ and $\frac{W_1}{P_1} = A_1$. firms choose price such that $P_1 = \frac{W_1}{A_1} = \frac{W_0}{A_1}$

at time $t = 1$, households supply labor and consume. In case of sticky wages the ratio of consumption and labor is no longer equal to $\frac{W_1}{P_1}$, now its equal to $\frac{W_0}{P_1}$

$Y_1 = C_1$

$A_1 N_1 = C_1$

$A_1 = \frac{C_1}{N_1} \rightarrow \frac{W_1}{P_1} \neq \frac{C_1}{N_1}$ $\quad \frac{W_0}{P_1} = \frac{C_1}{N_1}$

$\frac{W_t}{P_t} = \frac{\chi N_t^{\varphi}}{C_t^{-\gamma}} \longrightarrow W_1 = W_0$

labor supply becomes sticky wages

d Solve for long-run steady state.

all exogenous variables for $t \geq 2$: $A_t = A$, $M_t = M$ are constant.
to solve for steady state for $C_2 = C$ and $P_2 = P$ and plug into short run solutions.

$$C = Y = \left[\frac{1}{\chi} A^{1+\varphi}\right]^{\frac{1}{\gamma+\varphi}} \qquad \frac{M}{P} = \zeta^{1/\nu}(1-\beta)^{-1/\nu} Y^{\gamma/\nu}$$

$Y_t = A_t N_t$ $\quad N_t = \frac{Y_t}{A_t} = \frac{C_t}{A_t}$

$W_t/P_t = A_t$

$W_t/P_t = \chi N_t^{\varphi} C_t^{\gamma}$

$Y_t = C_t$

$M_t/P_t = \zeta^{1/\nu}(1 - 1/Q_t)^{-1/\nu} C_t^{\gamma/\nu}$

$1 = \beta E_t \left\{ Q_t \frac{P_t}{P_{t+1}} \frac{C_{t+1}^{-\gamma}}{C_t^{-\gamma}} \right\}$

$$A_t = \chi N_t^{\varphi} C_t^{\gamma} \rightarrow \chi \left[\frac{C_t}{A_t}\right]^{\varphi} C_t^{\gamma}$$

$$A_t A_t^{\varphi} = \chi C_t^{\varphi} C_t^{\gamma}$$

$$A_t^{1+\varphi} = \chi C_t^{\varphi+\gamma} \rightarrow C = \left[\frac{1}{\chi} A_t^{1+\varphi}\right]^{\frac{1}{\gamma+\varphi}}$$

e Does the Classic Dichotomy hold in the long-Run? Explain.

yes, any change in M causes proportional change in P, leaving Y, C, N unchanged.

f Solve for output and money market equilibrium in the short-run.

$Y_1 = A_1 N_1$

$W_1 = W_0$

$\frac{W_1}{P_1} = A_1 \qquad P_1 = \frac{W}{A}$

$Y_1 = C_1$

$\frac{M_1}{P_1} = \zeta^{1/\nu}(1 - \frac{1}{Q_1})^{1/\nu} C_1^{\gamma/\nu}$

$1 = \beta E_1 \left\{ Q_1 \frac{P_1}{P_2} \frac{C_2^{-\gamma}}{C_1^{-\gamma}} \right\}$

$\left(\frac{1}{Q_1 \beta} \frac{P_2}{P_1}\right)^{\frac{1}{\gamma}} C_1 = C_2$

$$\frac{W_0}{P_1} = \chi N_1^{\varphi} Y_1^{\gamma} \qquad \frac{M_1}{P_1} = \zeta^{1/\nu}\left(1 - \frac{1}{Q_1}\right)^{-1/\nu} Y_1^{\gamma/\nu}$$

$$P_1 \to \frac{W_0}{A_1}$$

$$1 = \beta \left\{ Q \frac{P_0}{P} \frac{Y^{-\gamma}}{Y_1^{-\gamma}} \right\}$$

$$C_1 = Y_1 = \left\{ \frac{1}{\beta Q_1} \frac{P}{P_0} \right\}^{\frac{1}{\gamma}} Y$$

$$= \left\{ \frac{1}{\beta Q_1} \frac{P_1}{P} \right\}^{\frac{1}{\gamma}} Y = \left\{ \frac{1}{\beta Q} \frac{W_0}{A_1 P} \right\}^{\frac{1}{\gamma}} Y$$

g Does the Classic Dichotomy hold in the short-run?

NO, from the Euler Equation, changes in the nominal interest rate Q_1 have a direct effect on output and consumption $Y_1 = C_1$

For a given level of Y_1 can manipulate nominal interest rate by changing the money supply M_1.

$\Rightarrow$ Classic Dichotomy doesn't hold in short run.

h Explain intuitively how an increase in the money supply affects output in the short-run.

Since wages are sticky — an increase in money supply will cause households to consume more which will drive up output since firms wont increase prices (they're already on their labor demand curve).
any reaction to consumption/output comes from interest Rates (from euler we have that the level of output $Y_1 = C_1$ is pinned down by Q_1). so if $M\uparrow$ then $Q\uparrow$

i How does productivity affect output? explain intuitively.

Changes in productivity have no effect on real output — it does not affect real interest rate and therefore doesnt affect consumption demand. even with sticky wages, firms will adjust prices if productivity goes up but output should remain unchanged.

j Derive the labor wedge. is it procyclical or counter-cyclical?

labor wedge : the distortion (implicit tax) in the labor market

$$(1-\tau_t^N) = \frac{MRS_t}{MPL_t} = \frac{\chi N_t^{\varphi} C_t^{\gamma}}{(1-\alpha) Y_t / N_t}$$

Here its counter-cyclical.

$$\tau_t^N = 1 - \frac{MRS}{MPL} \Rightarrow MPL > MRS \text{ in Recession} : N \text{ is low, } w \text{ is high}$$

labor wedge at $t=1$ is : $(1-\tau_1^N) = \frac{MRS}{MPL} = \chi A_1^{\gamma-1} N_1^{\gamma+\varphi}$

from Johannes tex file :

$$\frac{\chi N_t^{\varphi}}{C_t^{-\gamma}} = \frac{(1-\alpha) A_t N_t^{-\alpha}}{(1+\mu_t)}$$

We closed the model by combining labor supply and labor demand. but now HH are no longer on labor supply curve. The endogenous markup makes the equation hold. In the data, the labor wedge moves endogenously with business cycle.

k What moments of the data would you use to discriminate between the predictions of the sticky price and sticky wage model

I would look at the how prices, output, and labor adjust with changes in productivity
($\partial P/\partial A$, $\partial Y/\partial A$, $\partial N/\partial A$)
I would also look at how wages adjust with changes in money supply ($\partial W/\partial M$)

In the data, it would also be important to look at moments of inflation, real wages, labor, output consumption, real interest rates and productivity. I would start by looking at standard deviations.