Unwrapping Customer Delight: Using Frequentist and Bayesian Regression Models to Optimize Surprise Gift Strategies

The Estée Lauder Companies
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Introduction & Problem Statement

In today's fast-paced, competitive, and customer-centric market, creating personalized customer experiences is no longer a luxury but a necessity for retaining loyalty, driving sales, and fostering long-term relationships. The beauty industry exemplifies this dynamic, where customers often form deep emotional connections with brands and expect tailored, memorable interactions that cater to their unique preferences. As competition intensifies, it becomes crucial for companies to move beyond traditional engagement strategies and embrace innovative approaches that resonate deeply with their customers. One such strategy is the use of surprise gift campaigns—unexpected gestures such as complimentary product samples, limited-edition items, or exclusive rewards. These gifts leverage the element of surprise to delight customers, creating positive experiences that can enhance brand affinity and potentially drive increased spending in the future.

The significance of such campaigns lies not only in their ability to boost short-term sales but also in their potential to solidify customer relationships, foster goodwill, and differentiate a brand in a crowded marketplace. However, determining the true effectiveness of these campaigns presents a challenge. Do they genuinely drive higher spending and enhance customer loyalty, or are they merely an added cost with limited tangible impact? This project, in collaboration with The Estée Lauder Companies Inc., seeks to answer these critical questions by utilizing advanced statistical and machine learning techniques to evaluate and quantify the effects of surprise gift strategies on customer spending.

Our analysis centers on developing a Bayesian Regression Discontinuity Design (RDD), a causal inference technique that estimates the impact of receiving a gift by comparing outcomes just above and below a predetermined threshold. By leveraging NumPyro, a probabilistic programming library in Python, the team aims to construct a robust framework that not only measures the causal effects of surprise gifts but also accounts for inherent uncertainty in the analysis. A key focus of this study is understanding the return on investment (ROI) of these campaigns, a critical metric for businesses aiming to balance customer engagement strategies with profitability. ROI quantifies the financial return relative to the cost of the campaign, providing actionable insights that inform strategic decisions about the scalability and sustainability of surprise gifting initiatives.

The Estée Lauder Companies Inc. provided a dataset simulating customer spending behaviors, reflecting real-world dynamics influenced by such campaigns. This dataset includes variables such as spending in 2021 and 2022, capturing a longitudinal perspective on customer behavior. To analyze this data, we will employ Bayesian RDD methodologies augmented with Markov Chain Monte Carlo (MCMC) sampling techniques. These tools allow us to generate not just point estimates of the treatment effect but also credible intervals, offering a comprehensive view of the potential variability and reliability of the findings.

By combining cutting-edge machine learning tools with rigorous statistical modeling, this project aims to deliver actionable insights into the effectiveness of surprise gift campaigns. These insights will empower The Estée Lauder Companies to refine their customer engagement strategies, optimize resource allocation, and confidently scale initiatives that drive both customer satisfaction and long-term financial growth.

Exploratory Data Analysis (EDA)

In preparation for developing a Bayesian Regression Discontinuity Design (RDD) model to assess the impact of surprise gift campaigns on customer spending, we conducted an extensive exploratory data analysis (EDA). This step was critical to uncover insights into customer behavior, identify patterns, and understand key trends within the dataset. Below, we dive into the primary findings from our EDA and how they inform the subsequent modeling process.

1. **Descriptive Statistics and Spending Trends**
   The dataset revealed a clear upward trend in customer spending from 2021 to 2022. The mean spending increase between the two years was approximately $14.84, indicating a general rise in customer expenditure. Further examination of percentiles supported this conclusion, with the 25th percentile of spending increasing from $28.62 in 2021 to $41.49 in 2022. This trend held true across the 50th and 75th percentiles as well as the maximum spending observed. Through boxplot visualizations in Figure 1 below, it became evident that the distribution of spending was more clustered around the $50-$100 range in 2022, while in 2021, spending was more spread out and clustered below $50. This shift suggests that customers, on average, spent more in 2022, which could be influenced by factors such as increased brand engagement, product launches, or the impact of surprise gift campaigns.

2. **High Spenders and Change in Spending**
   We focused on identifying "high spenders" – defined as customers whose spending increased by more than $10 between 2021 and 2022. Most of these high spenders exhibited an average increase of $15 in their 2022 spending compared to the previous year, as shown in Figure 2. This was a significant finding, as it underscored the potential for positive customer behavior changes that surprise gift campaigns might influence.

3. **Clustering Analysis**
   Using a K-Nearest Neighbors (KNN) model, we categorized customers into clusters based on their spending habits. The most significant volume of clusters fell below the $100 threshold, with diminishing volumes beyond the $200 mark. This clustering allowed us to identify distinct customer segments and recognize that a large proportion of customers spent modest amounts, while only a few exhibited high spending behaviors. By understanding these clusters, we were able to tailor our subsequent analysis to focus on the key customer segments that might be most influenced by surprise gifts.

4. **Threshold Analysis of Spending Changes**
   We conducted a threshold analysis to assess how spending behavior differed between the two years. When the threshold was set to $0, we identified 18,508 customers who spent in both 2021 and 2022. However, when the threshold was increased to $500 or more, the number of high spenders dropped to zero, indicating that the majority of customers did not experience drastic changes in spending over this range. The analysis revealed that most customers had a change in spending of $0, and the smallest group of customers saw a difference of $30 in

spending between the two years. This insight will be critical for the Bayesian RDD model, as it highlights the relative stability in customer spending across the years while also pinpointing smaller groups who might demonstrate more significant behavioral changes.

5. **Binary**

   In order to analyze the impact of the free gift treatment on customer spending behavior, we added a binary column to the data indicating whether or not each customer received the treatment. This gift threshold was set at $80, meaning that treatment T is defined as a step function (boolean) at this value. This allowed us to distinguish between customers who received a gift (1) and those who did not (0) as part of our EDA and for our initial frequentist model.

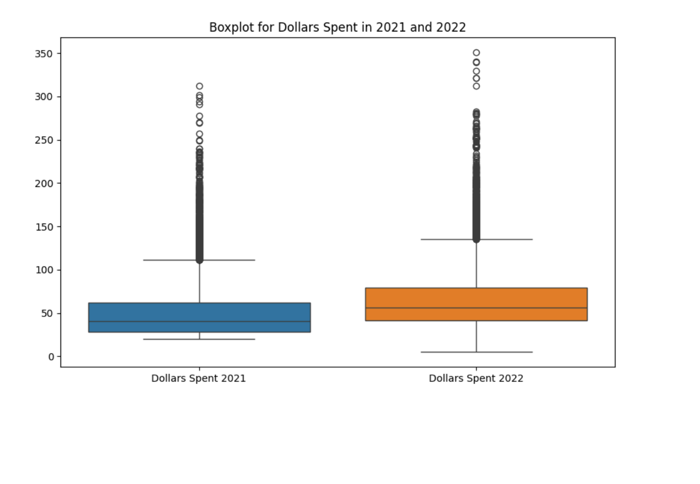

Figure 1: Boxplot for Dollars Spent in 2021 and 2022

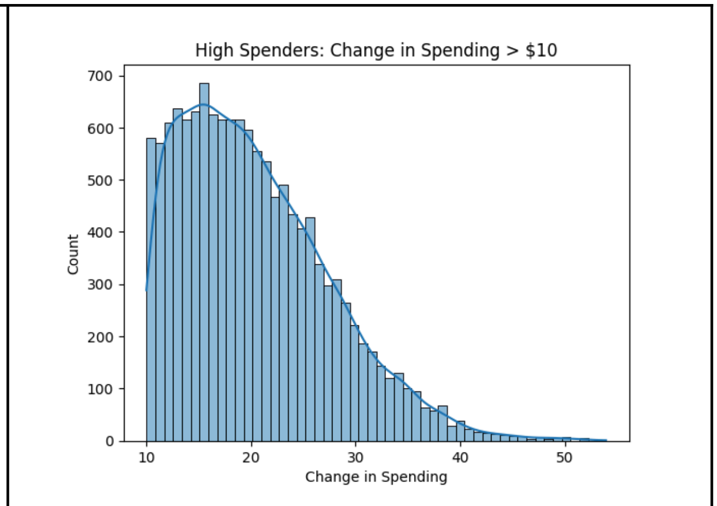

Figure 2: Change in Spending for High Spenders

Figure 3: Clustering Spending Patterns

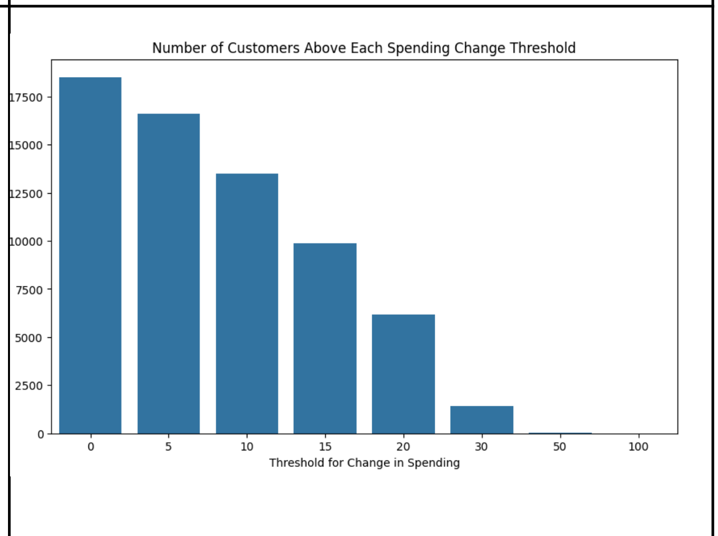

Figure 4: Spending Thresholds

The EDA provided critical insights into customer behavior, spending trends, and segmentation. These findings laid the groundwork for implementing the Bayesian RDD model, which will measure the exact causal impact of surprise gift campaigns on customer spending. Key takeaways include the identification of a general upward trend in spending, distinct customer clusters, and the presence of a high-spending subgroup likely influenced by the campaigns. The application of machine learning techniques, such as clustering and threshold analysis, revealed significant behavioral patterns that will guide the subsequent modeling process. These insights not only enhance our understanding of customer dynamics but also ensure a data-driven approach to evaluating the return on investment (ROI) of surprise gift strategies. This comprehensive EDA equips us to proceed with confidence in building robust models that reflect the underlying data patterns and inform actionable business strategies.

Bandwidth & Analysis

A Regression Discontinuity Design (RDD) is a method for estimating causal effects by leveraging a predetermined cutoff point on a continuous assignment variable to determine treatment eligibility. By comparing observations just above and below this threshold, RDD aims to isolate the treatment's impact from other factors. A regression line in RDD represents the relationship between the assignment variable and the outcome on either side of a cutoff point, with bandwidth selection determining the range of data used near this cutoff. The discontinuity at the cutoff indicates an abrupt change in the outcome, reflecting the treatment effect attributable to the treatment group. A carefully chosen bandwidth ensures that the treatment and control groups near the cutoff are comparable, minimizing bias while maintaining enough data points for a reliable analysis.

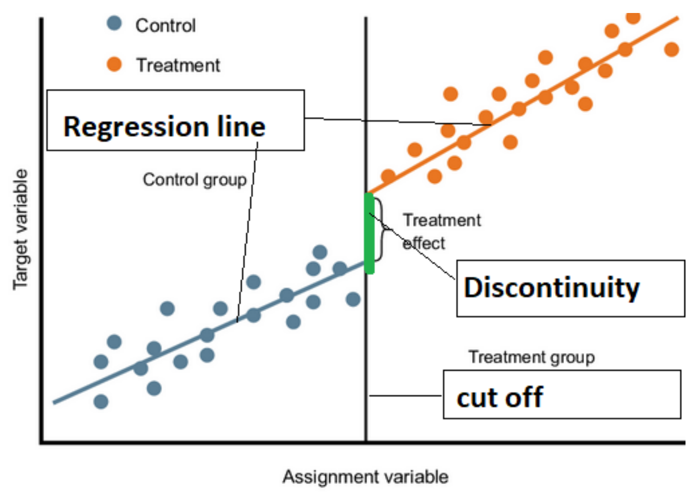

Figure 5: Example of Regression Discontinuity Design. Source: Muhammad Asad Kamran, *Regression Discontinuity in Causal Inference: An Introduction*, February 26, 2023. Retrieved from LinkedIn.

1. **Treatment Effect and Confidence Intervals**
   In Figure 7, a slight bell-shaped curve between $25 and $45 can be observed, indicating potential bias as the treatment effect fluctuates noticeably. Beyond this range in Figure 6, the treatment effect stabilizes, and the confidence intervals narrow, indicating reduced bias and variance. The optimal bandwidth of $55 is chosen to minimize bias while maintaining stable treatment effect estimates and tight confidence intervals, ensuring reliable causal inference.

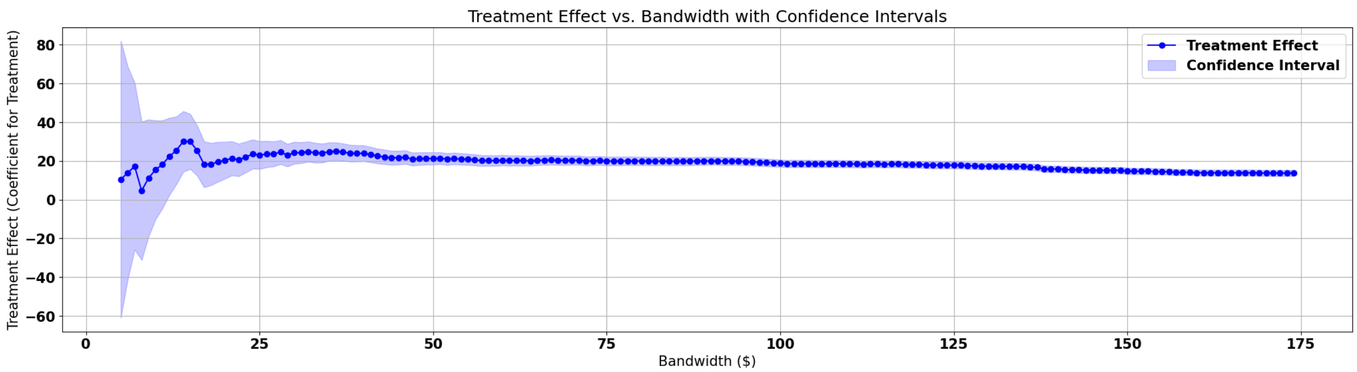

Figure 6: Treatment Effect vs Bandwidth

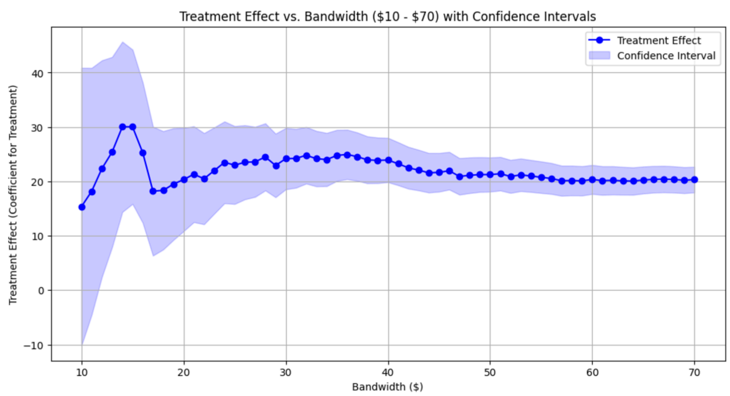

Figure 7: Treatment Effect vs Bandwidth ($10 – $70)

2. **P-value of Levene test**

   In Figure 8, before the bandwidth of $55, the p-values exhibit significant fluctuations, especially between 0 and 25, indicating an inconsistent balance between the treatment and control groups due to the inclusion of fewer observations. Between $25 and $45, the p-values begin to stabilize but still show variability. These fluctuations suggest that smaller bandwidths may lead to less reliable estimates, as fewer data points near the cutoff are included in the analysis. This supports selecting $55 as the optimal bandwidth, where the p-values stabilize further, ensuring balanced comparisons and more robust treatment effect estimates.

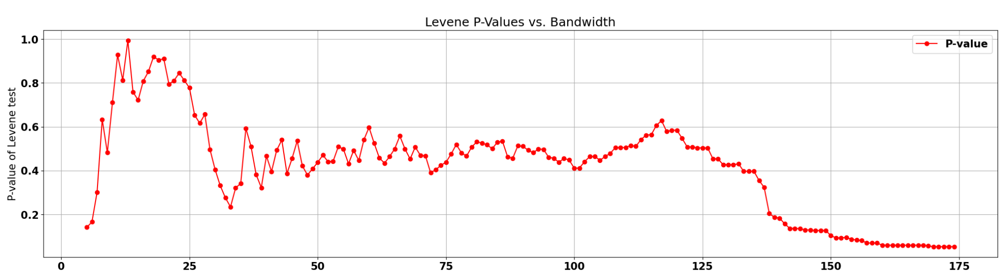

Figure 8: Levene P-Values by Bandwidth

3. **Data Size vs. Bandwidth**
   The plot in Figure 9 shows a clear upward trend, indicating that as the bandwidth increases, the data size increases. This relationship is significant because it suggests that broader bandwidths allow for the inclusion of more data points. A bandwidth that is too narrow may result in a small sample size, leading to high variance and unreliable estimates. On the other hand, a bandwidth that is too wide may introduce bias by incorporating irrelevant data. At bandwidth $55, the data size is large enough to ensure that outliers or noise do not overly influence the treatment effect estimates.

4. **Rate of Change of Treatment Effect vs. Bandwidth**
   At a bandwidth of $55, the rate of change in the treatment effect stabilizes near zero in Figure 10, indicating that the treatment effect no longer fluctuates significantly as the bandwidth increases. This stability suggests that the treatment effect estimates are consistent at this bandwidth. $55 balances between including enough observations for statistical power and maintaining the comparability of groups near the cutoff.

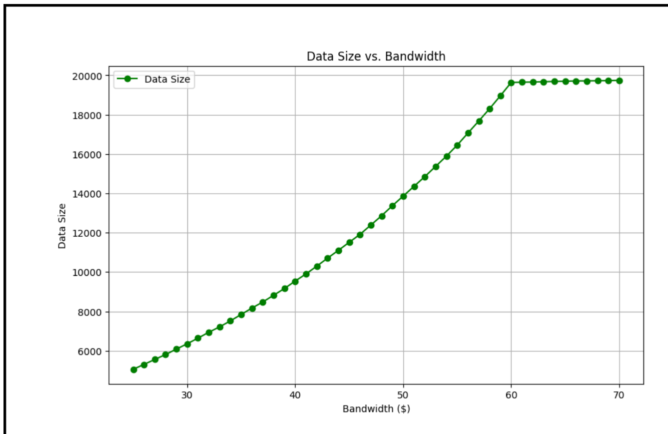

Figure 9: Data Size by Bandwidth

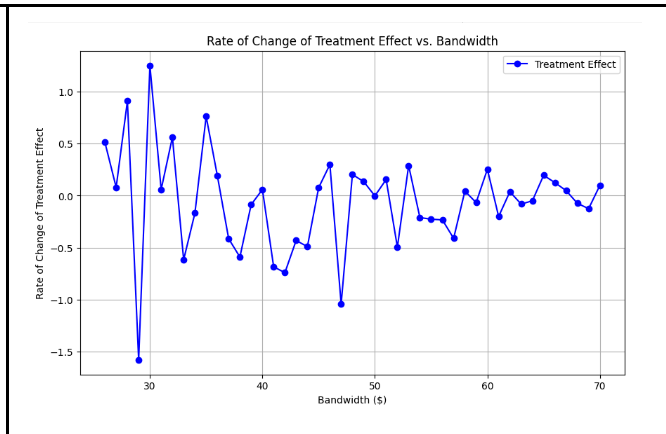

Figure 10: ROC of Treatment Effect by Bandwidth

Bandwidth Choice:
The analysis of the treatment effect, confidence intervals, rate of change, Levene's p-values, and data size collectively point to $55 as the optimal bandwidth. At $55, there is a balance between bias and variance. This bandwidth ensures that the estimates are robust and not overly influenced by outliers or noise while maintaining a substantial sample size.

Frequentist Model & Analysis

Before completing our Bayesian model and analysis, we began with a frequentist model and analysis utilizing basic regression discontinuity analysis. This is because we want to estimate the effect of a treatment where the treatment assignment is based on a cutoff in a continuous variable and we have previously determined the cut-off of whether or not the customer receives a gift to be greater than or equal to $80 spent in 2021.

As this report investigates the potential effect of receiving a surprise gift on future customer spending, we began with directly addressing our research question and formulating a null and alternative hypothesis.

Specifically, the research question asks whether receiving a surprise gift has a causal effect on future customer spending and seeks to quantify the magnitude of this effect. The null hypothesis posits that receiving a surprise gift does not significantly impact future customer spending, while the alternative hypothesis suggests that receiving a gift has a significant positive or negative impact.

The dependent variable for this analysis is customer spending in 2022, following the receipt (or non-receipt) of a surprise gift. The independent variables include customer spending in 2021, prior to receiving the gift, and a binary indicator variable representing treatment status. This treatment variable equals 1 if the customer received a gift and 0 otherwise.

Model Specification

To explore these relationships, we applied an Ordinary Least Squares (OLS) regression model. This model includes terms for the main effects of 2021 expenditure and treatment status, as well as an interaction term to examine how the relationship between 2021 and 2022 expenditures differs based on treatment.

The general model specification is as follows:

$$y_{2022} = \beta_0 + \beta_1 \cdot x_{2021} + \beta_T \cdot T + \beta_2 \cdot (x_{2021} \cdot T) + \epsilon$$

where:

- y_{2022} is the expenditure in 2022 (dependent variable).
- x_{2021} is the expenditure in 2021 (independent variable).
- T is the treatment indicator, a binary variable that takes the value 1 if the customer received a gift (i.e., spent more than $80 in 2021) and 0 otherwise.
- β_T measures the main effect of treatment.
- β_2 captures the interaction effect between 2021 expenditure and treatment.

which we derived from this initial standard model specification:

$y = \beta_0 + \beta_T T + \beta_x x + \beta_{int} xT + \epsilon$. (Ordinary Least Squares - statsmodels 0.15.0 (+8). (n.d.).)

Interaction Term

The interaction term $\beta_2 \cdot (x_{2021} \cdot T)$ is included to investigate whether the relationship between 2021 and 2022 expenditures changes depending on whether the customer received a gift. A significant interaction term would imply that the spending behavior in 2022 is influenced differently for those who received the gift compared to those who did not.

Results

P-Values

The p-values for each coefficient indicate the statistical significance of the corresponding effect. Low p-values (typically below 0.05) suggest that the observed effect is unlikely to have occurred by random chance. The main effect of 2021 expenditure is expected to be significant, as past spending behavior is typically a strong predictor of future behavior. The significance of the treatment effect will determine whether receiving the gift had a measurable impact on 2022 spending. Finally, a significant interaction term would suggest that treatment status alters the relationship between 2021 and 2022 expenditures in a meaningful way.

Regression Coefficients

The key coefficients from the OLS regression model provide important insights into the relationship between past and future spending and the impact of receiving a gift. The coefficient β_1, which represents the effect of 2021 expenditure on 2022 expenditure, indicates how spending behavior carries over from one year to the next. A positive β_1 suggests that higher spending in 2021 predicts higher spending in 2022, independent of whether the customer received a gift.

The main treatment effect, represented by β_T, captures the influence of receiving the gift in 2021 on 2022 expenditure. A positive β_T implies that receiving the gift increased spending in 2022, whereas a negative value would indicate a decrease in spending as a result of the gift.

The interaction effect, β_2, examines whether the relationship between 2021 and 2022 expenditure is affected by treatment status. A significant interaction term suggests that receiving the gift modified the typical spending pattern, either amplifying or reducing the effect of 2021 expenditure on 2022 outcomes.

With these results, coefficients and values in mind, the model provides strong evidence that past expenditure and receiving a gift influence future spending.

Model OLS Results ($55 Bandwidth)

```
                            OLS Regression Results
==============================================================================
Dep. Variable:     Dollars Spent 2022   R-squared:                       0.875
Model:                            OLS   Adj. R-squared:                  0.875
Method:                 Least Squares   F-statistic:                 3.833e+04
Date:                Tue, 19 Nov 2024   Prob (F-statistic):               0.00
Time:                        05:32:03   Log-Likelihood:                -61122.
No. Observations:               16451   AIC:                         1.223e+05
Df Residuals:                   16447   BIC:                         1.223e+05
Df Model:                           3
Covariance Type:            nonrobust
======================================================================================
                         coef    std err          t      P>|t|      [0.025      0.975]
--------------------------------------------------------------------------------------
const                 10.0695      0.267     37.708      0.000       9.546      10.593
Dollars Spent 2021     1.0975      0.006    192.891      0.000       1.086       1.109
Treatment             20.7680      1.476     14.070      0.000      17.875      23.661
Interaction           -0.1993      0.016    -12.837      0.000      -0.230      -0.169
==============================================================================
Omnibus:                        1.132   Durbin-Watson:                   2.007
Prob(Omnibus):                  0.568   Jarque-Bera (JB):                1.144
Skew:                           0.000   Prob(JB):                        0.564
Kurtosis:                       2.959   Cond. No.                     1.21e+03
==============================================================================
```

Figure 11: OLS Regression Results

Overall Model

The overall model demonstrates a strong explanatory power, with an R-squared value of 0.875, indicating that 87.5% of the variation in dollars spent in 2022 can be explained by the predictors: 2021 spending, treatment, the interaction term, and the constant. This high value suggests that the model captures a significant portion of the variance in spending behavior, leaving only 12.8% unexplained. The analysis was conducted on 16,451 observations, representing approximately 82.26% of the total dataset used within the specified bandwidth.

Coefficients

The coefficients further illuminate the model's insights. The constant (intercept) coefficient of 10.0695 represents the predicted average 2022 dollar spending for customers who spent $0 in 2021 and did not receive the treatment. The 2021 spending coefficient of 1.0975 indicates that customers are expected to spend 110% of their 2021 spending in 2022, representing a 10% increase, highlighting a strong positive relationship between past and future spending. The treatment coefficient of 20.7680 shows that customers who received the surprise gift spent, on average, $20.76 more in 2022 compared to those who did not receive the gift. The interaction coefficient of -0.1993 suggests diminishing returns, implying that as a customer's spending in 2021 increases, the impact of the surprise gift on 2022 spending decreases slightly.

The statistical significance of all these effects is confirmed by a P-value of 0.000, indicating meaningful impacts on 2022 spending.

Residual Analysis

Residual analysis supports the model's reliability. The Omnibus value of 1.132, combined with a Prob(Omnibus) of 0.568, suggests that the residuals are consistent with a normal distribution, satisfying the assumption of normality. Additionally, the skew value of 0.000 confirms that residuals are symmetrically distributed around zero.

OLS Takeaways

In summary, the model's R-squared value indicates a strong fit at 0.875, demonstrating its ability to explain most of the variance in 2022 spending. The negative interaction term suggests that the effect of the surprise gift diminishes as prior spending increases, so as 2021 spending increases, the additional effect treatment slightly diminishes. Furthermore, the residual analysis affirms the model's statistical robustness, making it a reliable tool for interpreting the effects of past spending and treatment on future customer behavior.

Bayesian Model & Analysis

While the Frequentist model provides an initial benchmark for the effect of surprise gifts on customer purchases, the Bayesian adds depth by accounting for uncertainty and incorporating prior knowledge, making it ideal for nuanced, real-world applications. Bayesian Inferencing treats probability as a degree of belief, thus incorporating prior knowledge, whereas the frequentist method interprets probability as the frequency of an event in repeated trials.

We'll translate the frequentist interaction model into a Bayesian framework using Numpyro. The model will be:

$$y_{2022} \sim N(\mu, \sigma)$$

$$\mu = \alpha + \beta_1 x_{2021} + \beta_2 T + \beta_3 (x_{2021} T)$$

where α is the intercept, β_1 represents the effect of 2021 expenditure, β_T is the main effect of the treatment, β_2 is the interaction term for treatment, and σ represents the noise term, assumed to follow a Half-Cauchy distribution, as is typical in Bayesian modeling. As a reminder, x_{2021} represents the expenditure in 2021, which serves as an independent variable to predict the expenditure in 2022. μ is the expected value of the 2022 expenditure, given the model parameters and the input data. (Bayesian Regression Using NumPyro — NumPyro documentation.)

Utilizing the previously determined bandwidth of 55, we filtered our dataset, reducing the total number of data points from 20,000 to 16,451. This indicates a retention rate of 82.25%. We proceeded with these filtered data points for further analysis.

Markov Chain Monte Carlo (MCMC) sampling was used to estimate the posterior distributions of these parameters, with trace plots and posterior distributions analyzed for convergence and credible intervals.

Three models with different priors (Uniform, Normal, and Half-Cauchy) were implemented to analyze the treatment effect (β_2), which represents the impact of receiving a gift on spending in 2022. These were performed via NUTS (N-U-Turn-Sampler), and the models used the following parameters:

- μ: Expected value of 2022 spending
- α: Intercept term, capturing baseline spending.
- β_1: Coefficient for 2021 spending, assessing its influence on 2022 spending.
- β_2: Treatment effect of receiving a gift on spending in 2022.
- β_3: Interaction term between 2021 spending and treatment, indicating any modification in the spending relationship.
- σ: Standard deviation, capturing data variability around the mean.

Uniform Priors Model: Model Specification

Priors:

- $\alpha \sim \text{Uniform}(-10, 10)$: Intercept term has a uniform prior across a wide range, implying no specific initial belief about the baseline spending level.

β_1 ~ Uniform(-10, 10): Coefficient for the effect of 2021 spending is assumed to be uniformly distributed, indicating an open range for the relationship between past and future spending.

β_2 ~ Uniform(-10, 10): Treatment effect (impact of receiving a gift) has a wide, uninformative uniform prior, allowing it to vary freely across possible values.

β_3 ~ Uniform(-10, 10): Interaction term for spending and treatment also has a uniform prior, implying no prior belief about how treatment might modify the effect of past spending.

σ ~ Uniform(0, 10): Standard deviation parameter has a uniform prior, accommodating a broad range of possible variability.

Uniform priors are uninformative, meaning they don't impose any prior beliefs about the parameter values. This allows the data alone to drive the estimates, which is helpful when there's no strong prior knowledge or when an exploratory analysis is desired. As shown in Figure 12, the broad range from -10 to 10 allows the parameters to take on a wide set of possible values, giving flexibility in identifying any effects. This model is particularly useful as a baseline. If results from the Uniform Priors Model align with other models, it suggests that the findings are not overly dependent on specific prior assumptions.

Normal Priors Model: Model Specification

Priors:

α ~ Normal(0, 5): Assumes the baseline spending (intercept) is centered around 0 with moderate spread, reflecting reasonable uncertainty.

β_1 ~ Normal(0, 5): Coefficient for the effect of 2021 spending is assumed to be normally distributed with a central tendency near zero, allowing moderate variation.

β_2 ~ Normal(0, 5): Treatment effect (impact of receiving a gift) also has a prior centered at zero with moderate spread, suggesting we initially expect no strong effect but allow flexibility.

β_3 ~ Normal(0, 5): Interaction term has a normal prior, indicating we assume any modification of past spending by treatment is likely modest but could vary.

σ ~ Exponential(1): Standard deviation parameter has an Exponential prior, which constrains it to be positive but favors smaller values, reflecting an expectation of limited variability.

Normal priors provide a more informative but flexible structure than uniform priors. Centering priors around 0 reflects a conservative stance, assuming no effect unless the data strongly suggests otherwise. As shown in Figure 13, the standard deviation of 5 allows for reasonable deviations from zero, meaning we do not overly constrain the estimates while still offering some structure. This model is useful when there's an expectation that effects are not extreme. It offers a balance between complete unconstrained (as in the uniform priors) and stronger prior beliefs. The Normal Priors Model is appropriate for refining estimates and observing the effect in a less unconstrained context.

Half-Cauchy Priors Model: Model Specification

Priors:

α ~ Normal(0, 5): Same as in the Normal Priors Model, centering the intercept around 0 with moderate spread.

β_1 ~ Normal(0, 5): Coefficient for the effect of 2021 spending is also centered around 0, suggesting we expect a modest influence of past spending on future spending.

β_2 ~ Normal(0, 5): Treatment effect has a normal prior, allowing it to vary around zero but favoring modest effects.

β_3 ~ Normal(0, 5): Interaction term has the same prior as in the Normal Priors Model, indicating moderate flexibility.

σ ~ Half-Cauchy(5): The standard deviation has a Half-Cauchy prior, which is more informative and better suited for handling variance in Bayesian models by allowing heavier tails and thus accommodating occasional extreme values.

The Half-Cauchy prior on sigma is beneficial in cases where there might be occasional outliers or larger-than-expected variance in the data. As seen in Figure 14, this makes the model more robust to data variability without inflating the overall uncertainty excessively. By using normal priors for most parameters but a Half-Cauchy for sigma, the model strikes a balance between incorporating moderate prior beliefs and allowing for flexibility in the data's variability. The Half-Cauchy Priors Model is particularly useful in real-world data scenarios where occasional extreme values may arise. It's valuable for estimating the treatment effect in a way that remains robust to potential outliers in spending data.

Each model offers a different level of prior informativeness:

- Uniform Priors: Provide a completely uninformative baseline to see if the data alone can support a strong effect.
- Normal Priors: Offer moderate constraints and are helpful in scenarios where we expect the treatment effect to exist but are cautious about extreme values.
- Half-Cauchy Priors: Useful for handling variability robustly, allowing the model to be less sensitive to potential outliers.

Using these models together helps confirm the stability of the treatment effect (β_2) across different assumptions, showing that the estimated effect of the surprise gift on future spending is robust and not driven by specific prior choices.

Posterior Distributions for β_2 (Treatment Effect)

1. Uniform Priors Model:
    a. Mean of β_2: Approximately 20.75, indicating a positive effect of receiving a gift on future spending.
    b. 94% HDI (Highest Density Interval): Ranges from around 18.54 to 23.34, which shows a credible interval.

c. The Uniform Priors Model gives a slight overestimate of the treatment effect with a slightly higher mean compared to other models, reflecting the benchmark nature of the uniform prior.

2. Normal Priors Model:
    a. Mean of β_2: Approximately 19.11, which is higher than the Uniform Priors Model.
    b. 94% HDI: Ranges from 16.90 to 21.60, showing a credible interval that is centered higher than in the Uniform model.
    c. The Normal Priors Model suggests a stronger positive treatment effect than the Uniform Priors Model, likely because it allows more probability mass around the central values. The effect is robust, with the HDI comfortably within the positive range, implying that receiving a gift significantly increases spending.
3. Half-Cauchy Priors Model:
    a. Mean of β_2: Approximately 19.11, closely aligning with the Normal Priors Model.
    b. 94% HDI: Spans from 16.77 to 21.45, similar to the Normal Priors Model, which suggests a comparable impact estimate.
    c. The Half-Cauchy Priors Model is robust to variability, as indicated by the slightly wider shape of the posterior. It reinforces the results from the Normal Priors Model, confirming a positive and stable treatment effect on future spending.

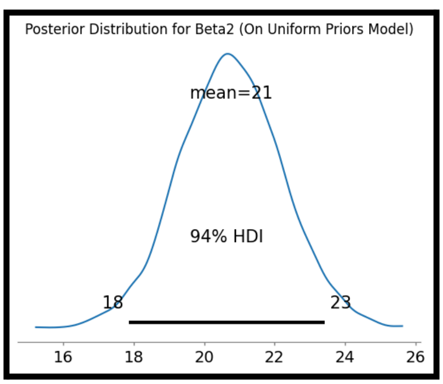

Figure 12: Uniform Priors Model

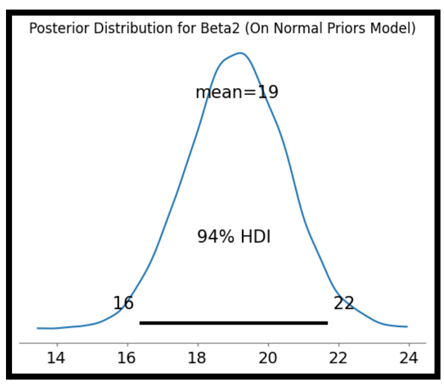

Figure 13: Normal Priors Model

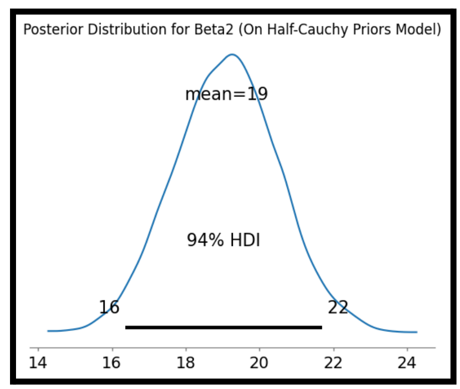

Figure 14: Half-Cauchy Priors Model

Trace Plots for Convergence

The trace plots for each model, as portrayed below, indicate stable sampling paths across all iterations, suggesting that the MCMC chains have converged. The lack of significant drift or periodic trends in the trace plots for all parameters (α, β_1, β_2, β_3, and σ) implies that the posterior estimates are reliable and that the model has reached a steady state.

Convergence across models (Uniform, Normal, and Half-Cauchy) further indicates that the posterior distributions are consistent and robust, showing no sensitivity to initialization or random fluctuations.

1. Uniform Priors Model

In Figure 15, the trace plots for each parameter are relatively stable with minimal fluctuations, indicating that the MCMC process has likely converged. The lack of trends or periodic patterns in the trace plots further supports the reliability of these estimates under the Uniform Priors Model.

2. Normal Priors Model
   In Figure 16, the trace plots for the Normal Priors Model also show stable sampling paths for all parameters, with no significant trends or oscillations. This stability confirms convergence, indicating that the model reached reliable posterior estimates under normal prior assumptions.
3. Half-Cauchy Priors Model
   Similar to the other models, the trace plots in Figure 17 display stable sampling paths, with no noticeable trends or random walk patterns. The Half-Cauchy Priors Model thus confirms convergence and reliability in the parameter estimates, particularly under the Half-Cauchy assumption for sigma.

The Bayesian analysis shows a significant positive impact of receiving a gift on future spending, with β_2 consistently positive across all models. The Half-Cauchy and Normal priors suggest a higher treatment effect magnitude compared to the Uniform priors, implying that surprise gifts may drive customer spending substantially. The use of different priors demonstrates that while the magnitude of the effect may vary slightly, the positive direction and significance of the treatment effect hold, supporting the reliability of the analysis. This analysis provides evidence that the gift treatment likely increases customer spending, which could justify the continuation or expansion of such surprise gift campaigns.

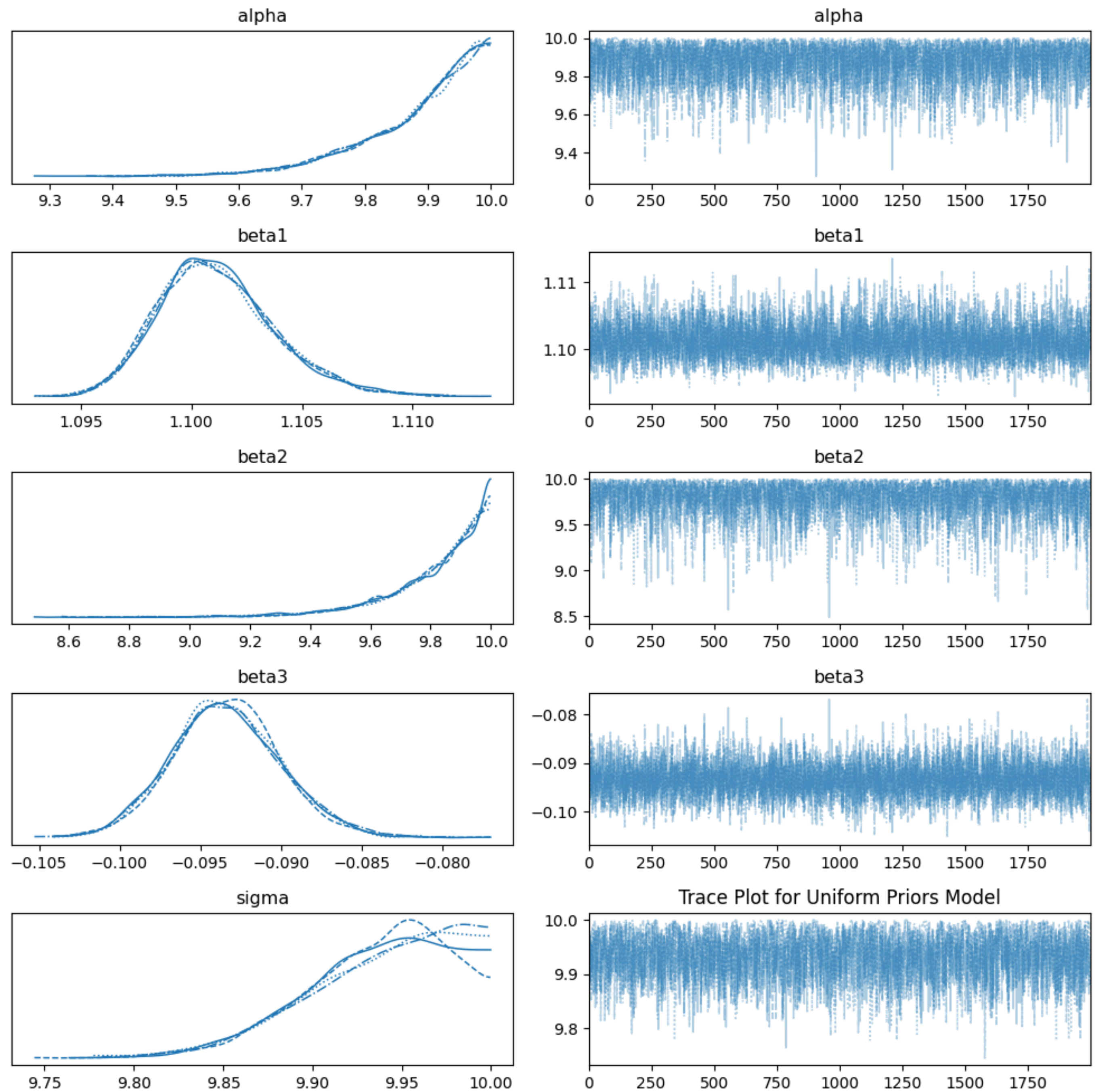

Figure 15: Trace Plot for Uniform Priors Model

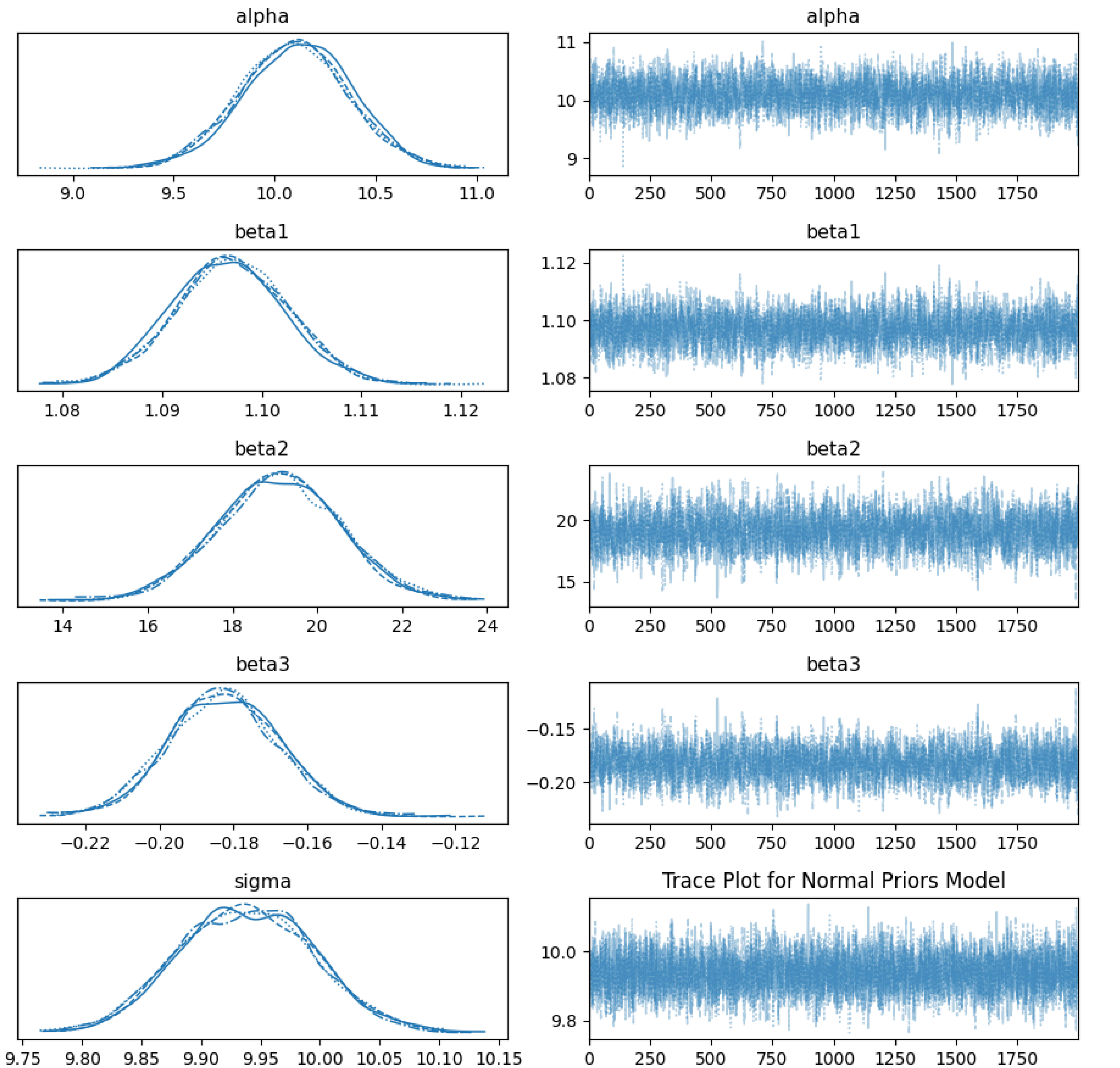

Figure 16: Trace Plot for Normal Priors Model

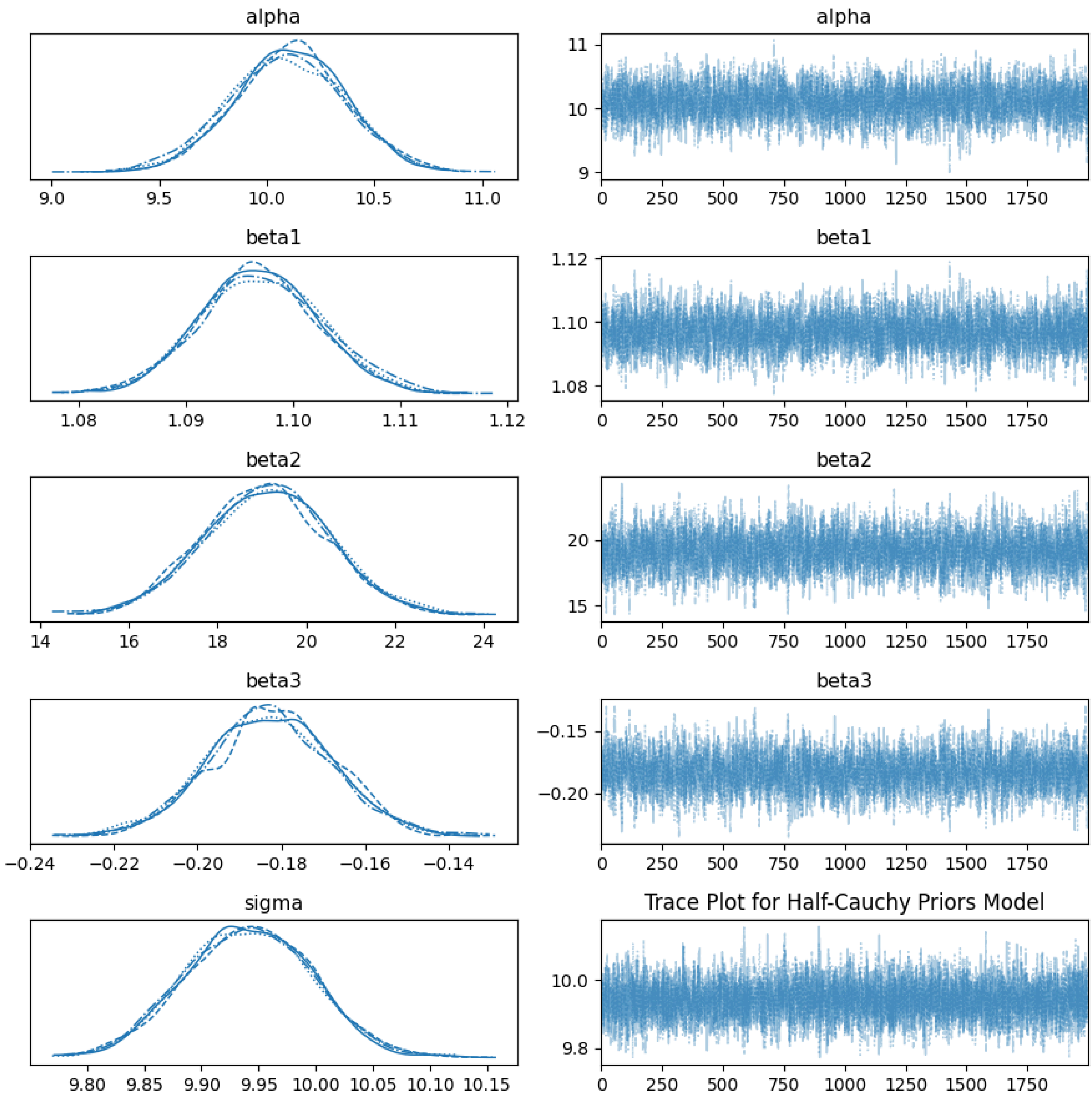

Figure 17: Trace Plot for Half-Cauchy Priors Model

Comparison Between Frequentist & Bayesian Models

Model Structure and Interpretation

1. Frequentist Model (OLS):
    a. The OLS model estimates the treatment effect (β_2) on customer spending in 2022.
    b. Treatment Coefficient: 20.7680, indicating that receiving a gift is associated with an average increase of about $20.77 in 2022 spending compared to those who did not receive a gift.
    c. The frequentist model provides a single point estimate and assumes that this estimate holds across all observations, without accounting for prior beliefs or uncertainty.
2. Bayesian Models:
    a. The Bayesian models provide a probabilistic interpretation of the treatment effect (β_2), allowing for uncertainty and prior beliefs to influence the estimates.
    b. Treatment Effect (β_2):
        - Uniform Priors Model: Mean effect of 20.75, indicating a more conservative estimate of the gift's impact.
        - Normal Priors Model: Mean effect of 19.11, suggesting a moderate positive impact.
        - Half-Cauchy Priors Model: Mean effect of 19.11, similar to the Normal Priors Model, reinforcing the positive effect.
    c. Bayesian models offer a range (credible intervals) for the treatment effect, which reflects uncertainty and allows for more nuanced conclusions than the single point estimate in the frequentist model.

Goodness of Fit

1. Frequentist Model (OLS):
    a. R-squared: 0.875, meaning the model explains 87.5% of the variance in spending. This value suggests that 12.8% of the variation remains unexplained, indicating a strong fit, as the model captures a significant portion of the variance in spending behavior.
    b. Residual Analysis: The residual graphs have ideal properties for a regression model. Constant variance and normally distributed residuals ensure that the model captures the key relationships in the data and that its predictions are accurate.
2. Bayesian Models:
    a. Bayesian models do not produce an R-squared but instead provide posterior distributions that offer insights into model fit through credible intervals and trace plots.
    b. Trace Plots: The stable trace plots across models suggest that the MCMC process reached steady-state values, indicating a well-fitting model.
    c. Credible Intervals: The Bayesian models' credible intervals for β_2 provide a range of likely treatment effects, which can be more informative than a single point estimate in capturing the effect's variability.

Handling of Uncertainty

1. Frequentist Model (OLS):
   a. The OLS model provides a fixed treatment effect with a confidence interval but does not directly account for prior beliefs or allow for probabilistic interpretations of the parameters. The model assumes that the treatment effect is the same for all individuals, with no flexibility for incorporating uncertainty beyond the confidence interval.
2. Bayesian Models:
   a. The Bayesian approach naturally incorporates uncertainty by providing posterior distributions for each parameter. Each model's credible interval (94% HDI) for β_2 offers a probabilistic interpretation, showing the range within which the treatment effect likely falls, accounting for uncertainty in the estimates.
   b. Bayesian models accommodate prior beliefs, with Normal and Half-Cauchy priors yielding slightly higher estimates for β_2 compared to the Uniform Priors Model. This flexibility allows for more realistic modeling, particularly when prior knowledge or assumptions are relevant.

Treatment Effect (Comparison of β_2 and Treatment Coefficient)

1. Frequentist Model (OLS):
   a. The OLS model estimated the treatment effect at 20.7680, which is significantly higher than the Bayesian estimates. The high effect estimate in the OLS model could be inflated due to unmodeled variability or skewness in the residuals, as indicated by the residual analysis.
2. Bayesian Models:
   a. The Bayesian models provide more conservative estimates for the treatment effect (`beta2`), with means ranging from 20.75 (Uniform Priors) to 19.1 (Normal and Half-Cauchy Priors).
   b. This discrepancy suggests that the frequentist model may overestimate the treatment effect, while Bayesian models offer a more cautious and potentially realistic range by incorporating uncertainty and prior information.

Conclusion

Frequentist Model Limitations:

The OLS model provides a straightforward and interpretable estimate but may overstate the effect of the gift on spending due to lack of flexibility to account for prior knowledge or uncertainty. The single point estimate and deviation from normal residuals highlight limitations in capturing the full complexity of the data.

Bayesian Model Advantages:

The Bayesian models present a more robust framework for estimating the treatment effect, with consistent results across different prior assumptions. The credible intervals and trace plots confirm that the positive effect of the treatment is reliable, but the Bayesian estimates remain lower than the frequentist estimate, suggesting a more cautious view of the treatment's impact. Bayesian methods provide greater flexibility in handling uncertainty and prior beliefs, resulting in a more nuanced and potentially accurate analysis of the treatment effect.

In summary, while the frequentist model offers a higher treatment effect estimate, the Bayesian models provide a more conservative and probabilistic view of the impact, which might better reflect the true influence of receiving a gift on future spending. This comparison demonstrates the value of Bayesian analysis in producing reliable estimates when there is uncertainty or variability in the data.

Regression Discontinuity Design Plots

Frequentist RDD Plots

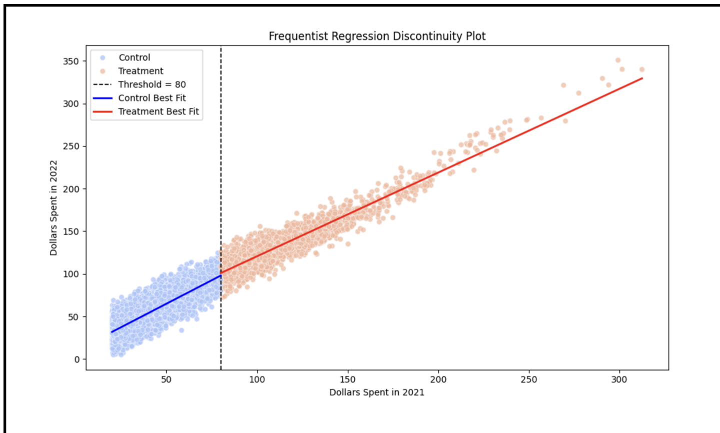

Figure 18: Frequentist Regression Discontinuity Plot

Figure 19: Frequentist Regression Discontinuity Plot (with Bandwidth)

Bayesian RDD Plots

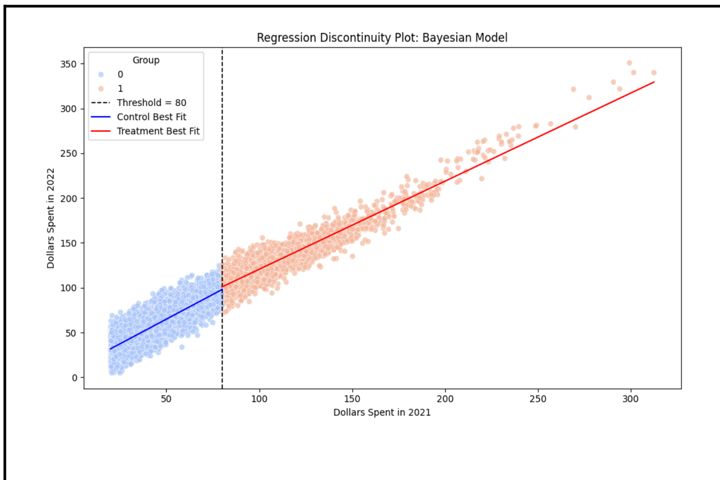

Figure 19: Bayesian Regression Discontinuity Plot

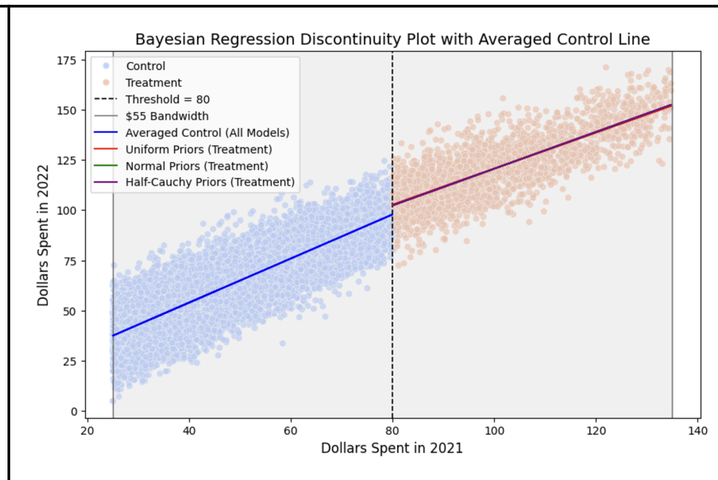

Figure 20: Bayesian Regression Discontinuity Plot (with Bandwidth)

RDD Plot Analysis

The Frequentist and Bayesian RDD plots illustrate the $80 gift threshold cutoff, accompanied by two best-fit lines representing the control and treatment groups. Both models reveal a noticeable positive jump at the cutoff point, signifying the treatment effect. This jump is particularly pronounced in the zoomed-in plots focusing on the bandwidth, where the separation between the control and treatment lines is clearly visible.

A consistent pattern emerges across the models: the slope of the treatment group is less steep than that of the control group, indicating a diminishing treatment effect with higher prior spending. This aligns with our earlier observations from the OLS analysis, which identified a negative interaction term, suggesting that the impact of the gift decreases as past spending increases.

In the Bayesian RDD plot, three prior structures - Uniform, Normal, and Half-Cauchy - are represented. However, the Normal Priors line is obscured by the Half-Cauchy line due to their overlap. The treatment effect sizes derived from the plots correspond to our model estimates: $20.77 for the Frequentist model, $20.75 for the Bayesian model with Uniform Priors, and $19.11 for both the Normal and Half-Cauchy Priors. These values explain the observed positive jump at the cutoff.

These visual and numerical findings validate our prior analyses, reinforcing the robustness of the models and their alignment with the expected treatment effects. The consistent behavior across models and priors underscores the reliability of our conclusions regarding the impact of surprise gifts on customer spending.

Conclusion

This analysis establishes that both the Frequentist and Bayesian models effectively demonstrate the positive impact of surprise gift campaigns on customer spending, with each approach yielding comparable treatment effect estimates. The Frequentist model calculates a treatment effect of $20.77, reflecting a significant increase in customer spending among those who received a surprise gift. Meanwhile, the Bayesian model offers a slightly lower but still substantial treatment effect of $19.11. Although both methods align closely in their findings, the Bayesian approach emerges as the more robust and reliable option due to its ability to incorporate uncertainty and prior knowledge, making it better suited for guiding strategic business decisions.

From a financial perspective, the ROI analysis highlights the economic viability of these campaigns. With a $10 gift valuation, the Bayesian treatment effect translates to a mean ROI of 91.1%, signifying that the revenue generated from the campaign nearly doubles the investment cost. While the Frequentist model suggests an even higher ROI, its single-point estimate does not provide the same level of probabilistic insight as the Bayesian model. The Bayesian framework's credible intervals enable a nuanced understanding of the treatment effect's variability, offering more realistic and actionable guidance, especially in contexts where decision-making involves risk and uncertainty.

While the Frequentist model offers a straightforward and interpretable estimate, its lack of flexibility in handling variability or integrating prior knowledge limits its utility in complex, real-world scenarios. In contrast, the Bayesian model accounts for inherent uncertainty, making its insights more adaptable and robust. This flexibility is particularly critical for The Estée Lauder Companies, where customer behavior is influenced by diverse and dynamic factors.

Given these findings, we recommend that The Estée Lauder Companies prioritize the Bayesian framework for evaluating and refining future surprise gift campaigns. The treatment effect estimate of $19.11, combined with an ROI of 91.1%, underscores the substantial value these campaigns deliver. Furthermore, we advocate for continued investment in surprise gift strategies, with a focus on optimizing targeting criteria to maximize returns and deepen customer engagement. The ability to leverage Bayesian insights ensures that the company can confidently scale these initiatives while mitigating risks associated with overestimating their impact.

In conclusion, while both models affirm the effectiveness of surprise gift campaigns, the Bayesian approach provides a more detailed, reliable, and practical basis for strategic decision-making. By continuing to implement and refine these campaigns, The Estée Lauder Companies can achieve meaningful increases in customer spending, strengthen loyalty, and secure long-term financial growth.

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