1. The answers are below:

```
> fib = function(n) {
+    s = numeric(n)
+
+    if (n <= 1) s[n] = 0
+    else {
+      s[1:(n - 1)] = fib(n - 1)
+      if (n == 2) s[n] = 1
+      else s[n] = s[n - 1] + s[n - 2]
+    }
+
+    s
+ }
>
> fib(1)
[1] 0
> fib(2)
[1] 0 1
> fib(3)
[1] 0 1 1
> fib(10)
 [1]  0  1  1  2  3  5  8 13 21 34
```

2. The answers are below:

   (a)
```
> clusters.medians = function(x, c) {
+    lenc = length(c)
+
+    d = outer(c, x, function(cj, xi) abs(xi - cj))
+    d.minnum = apply(d, 2, which.min)
+
+    con = outer(1:lenc, d.minnum, function(num, minnum) num == minnum)
+
+    xv = unlist(apply(con, 1, function(t) median(x[t])))
+    xv
+ }
>
> find.clusters.medians = function(x, c) {
+    ctmp1 = c
+    repeat {
+      ctmp2 = clusters.medians(x, ctmp1)
+      if (all(abs(ctmp1 - ctmp2) < 1e-07)) break
+      else ctmp1 = ctmp2
```

```
+   }
+   ctmp1
+ }
>
> x = faithful$eruptions
> find.clusters.medians(x, c(2,4))
[1] 1.9830 4.3415
```

(b)
```
> find.clusters.medians(x, c(2,3,4))
[1] 1.9830 3.9665 4.5330
```

(c)
```
> find.clusters.medians(x, c(2,3,4,5))
[1] 1.967 3.600 4.150 4.600
```

3. The answers are below:

```
> sign.matrix = function(x) outer(x, x, function(x1, x2) sign(x1 - x2))
>
> conc = function(x, y) {
+   conc.mtx = sign.matrix(x)
+   conc.mty = sign.matrix(y)
+   conc.z = conc.mtx + conc.mty
+   c = length(which(conc.z < 0 | conc.z > 0))
+   n = length(x)
+   c / (n * (n - 1))
+ }
>
> conc(x = 1:5, y = c(3, 1, 4, 5, 2))
[1] 0.6
>
> set.seed(782)
> x = round(rnorm(1000))
> y = x + round(rnorm(1000))
> conc(x, y)
[1] 0.8518939
```

4. The answers are below:

(a)
```
> nba.df = read.csv("https://raw.githubusercontent.com/
zzdxzhangzhi/assignments/master/782/NBA2016-2017.csv",
+ stringsAsFactors = FALSE)
> names(nba.df) = c("team1", "team2", "wins")
> head(nba.df)
          team1              team2 wins
1 Atlanta Hawks       Boston Celtics    2
2 Atlanta Hawks        Brooklyn Nets    2
3 Atlanta Hawks    Charlotte Hornets    1
4 Atlanta Hawks        Chicago Bulls    3
5 Atlanta Hawks Cleveland Cavaliers    3
```

2

```
6 Atlanta Hawks    Dallas Mavericks    2
>
> nba.names = nba.df$team1[seq(1, 870, length = 30)]
> nba.names
 [1] "Atlanta Hawks"         "Boston Celtics"        "Brooklyn Nets"
         "Charlotte Hornets"
 [5] "Chicago Bulls"         "Cleveland Cavaliers"   "Dallas Mavericks"
       "Denver Nuggets"
 [9] "Detroit Pistons"       "Golden State Warriors" "Houston Rockets"
         "Indiana Pacers"
[13] "Los Angeles Clippers"  "Los Angeles Lakers"    "Memphis Grizzlies"
       "Miami Heat"
[17] "Milwaukee Bucks"       "Minnesota Timberwolves" "New Orleans Pelicans"
    "New York Knicks"
[21] "Oklahoma City Thunder" "Orlando Magic"         "Philadelphia 76ers"
      "Phoenix Suns"
[25] "Portland Trail Blazers" "Sacramento Kings"      "San Antonio Spurs"
       "Toronto Raptors"
[29] "Utah Jazz"             "Washington Wizards"
> log.likelihood.r = function(r, times, s) {
+   rn = s - sum(r)
+   rr = c(r, rn)
+
+   if (all(rr > 0)) {
+     mtx = outer(rr, rr, function(ri, rj) log(ri / (ri + rj)))
+     rankv = c(t(mtx)[which(row(mtx) != col(mtx))])
+     sum(times * rankv)
+   } else {
+     -Inf
+   }
+ }
>
> s = 1000
> Q = function(r) {
+   -log.likelihood.r(r, nba.df$wins, s)
+ }
>
> count = length(nba.names)
> result = optim(seq(1, 29, length = 29), Q, method = "BFGS",
+                control = list(maxit = 200))
> result
$par
 [1]  28.920344  49.262769   8.487427  20.865839  26.562360
45.009903  18.735556  26.484090  21.994617
[10] 127.851303  59.221865  27.412854  47.775175  12.710449
32.153364  26.968452  28.090105  17.226476
```

```
[19]   19.848702   15.942769   38.873109   14.320608   13.473253
11.341333   28.116503   17.982849   83.968346
[28]   43.789158   47.019261


$value
[1] 761.4917

$counts
function gradient
     147       143

$convergence
[1] 0

$message
NULL


>
> ratio = 100 / result$par[which.max(result$par)]
> r.value = result$par * ratio
> rr.value = c (r.value, (s - sum(result$par)) * ratio)
> rr.value
 [1]   22.620297   38.531300    6.638514   16.320396   20.775979
35.204884   14.654177   20.714760   17.203279
[10] 100.000000   46.320893   21.441200   37.367766    9.941587
25.149031   21.093608   21.970918   13.473837
[19]   15.524833   12.469774   30.404938   11.200987   10.538221
8.870722   21.991566   14.065441   65.676566
[28]   34.250068   36.776521   30.966569
>
> rank.table = data.frame(nba.names, rr.value, stringsAsFactors = FALSE)
> ordered.rank = rank.table[order(rank.table$rr.value, decreasing = TRUE),]
> colnames(ordered.rank) = c("name", "rank")
> rownames(ordered.rank) = 1:30
> ordered.rank
                      name       rank
1    Golden State Warriors 100.000000
2        San Antonio Spurs  65.676566
3          Houston Rockets  46.320893
4           Boston Celtics  38.531300
5     Los Angeles Clippers  37.367766
6                Utah Jazz  36.776521
7      Cleveland Cavaliers  35.204884
8          Toronto Raptors  34.250068
9       Washington Wizards  30.966569
10  Oklahoma City Thunder  30.404938
```

```
11        Memphis Grizzlies  25.149031
12          Atlanta Hawks  22.620297
13 Portland Trail Blazers  21.991566
14        Milwaukee Bucks  21.970918
15         Indiana Pacers  21.441200
16             Miami Heat  21.093608
17          Chicago Bulls  20.775979
18         Denver Nuggets  20.714760
19        Detroit Pistons  17.203279
20      Charlotte Hornets  16.320396
21   New Orleans Pelicans  15.524833
22       Dallas Mavericks  14.654177
23       Sacramento Kings  14.065441
24 Minnesota Timberwolves  13.473837
25        New York Knicks  12.469774
26          Orlando Magic  11.200987
27      Philadelphia 76ers  10.538221
28      Los Angeles Lakers   9.941587
29            Phoenix Suns   8.870722
30           Brooklyn Nets   6.638514
>
```

(b)
```
> log.likelihood.r.deriv = function(r, i, times1, times2) {
+   rlen = length(r)
+
+   if (all(r > 0)) {
+     mtx1 = outer(r[i], r[-i], function(ri, rj) rj / ri * (ri + rj))
+     mtx2 = outer(r[i], r[-i], function(ri, rj) 1 / (ri + rj))
+     deriv1 = c(t(mtx1), 1 / r[i]) * times1[(rlen * (i - 1) + 1) : (rlen * i)]
+     deriv2 = c(t(mtx2), 1 / (s - sum(r))) * times2[(rlen * (i - 1) + 1) : (rlen *
+
+     deriv.val = sum(deriv1) - sum(deriv2)
+     deriv.val
+   } else {
+     -Inf
+   }
+ }
>
> Q.derivs = function(r) {
+   order.team2 = order(nba.df$team2)
+   wins.team2 = nba.df$wins[order.team2]
+   rlen = length(r)
+   gradients = numeric(rlen)
+   for (i in 1:rlen) {
+     gradients[i] = log.likelihood.r.deriv(r, i, nba.df$wins, wins.team2)
+   }
+
```

```
+    gradients
+ }
>
> result.deriv = optim(seq(1, 29, length = 29), Q, gr = Q.derivs,
+                         method = "BFGS")
> result.deriv
$par
 [1]  1  2  3  4  5  6  7  8  9 10 11 12 13 14 15 16 17 18 19 20
21 22 23 24 25 26 27 28 29

$value
[1] 1109.275

$counts
function gradient
      29        1

$convergence
[1] 0

$message
NULL

>
```
(c)
```
> ranks = c(result$par, s - sum(result$par))
> ranks.sort = sort(ranks, decreasing = TRUE)
> first2 = c(which(round(ranks) == round(ranks.sort[1])),
+            which(round(ranks) == round(ranks.sort[2])))
> first2
[1] 10 27
>
> Q2 = function(r1, r2) {
+    m = max(length(r1), length(r2))
+    if (length(r1) < m)
+      r1 = rep(r1, length = m)
+    if (length(r2) < m)
+      r2 = rep(r1, length = m)
+
+    ans = numeric(m)
+    for (i in 1:m) {
+      ranks[first2] = c(r1[i], r2[i])
+      ans[i] = -log.likelihood.r(ranks[-length(ranks)], nba.df$wins, s)
+    }
+
+    ans
+ }
```

6

```
>
> r1 = seq(40, 145, length = 61)
> r2 = seq(20, 100, length = 61)
> z = outer(r1, r2, Q2)
> contour(r1, r2, z,
+         xlab = paste("rank of", nba.names[first2[1]]),
+         ylab = paste("rank of", nba.names[first2[2]]))
>
```
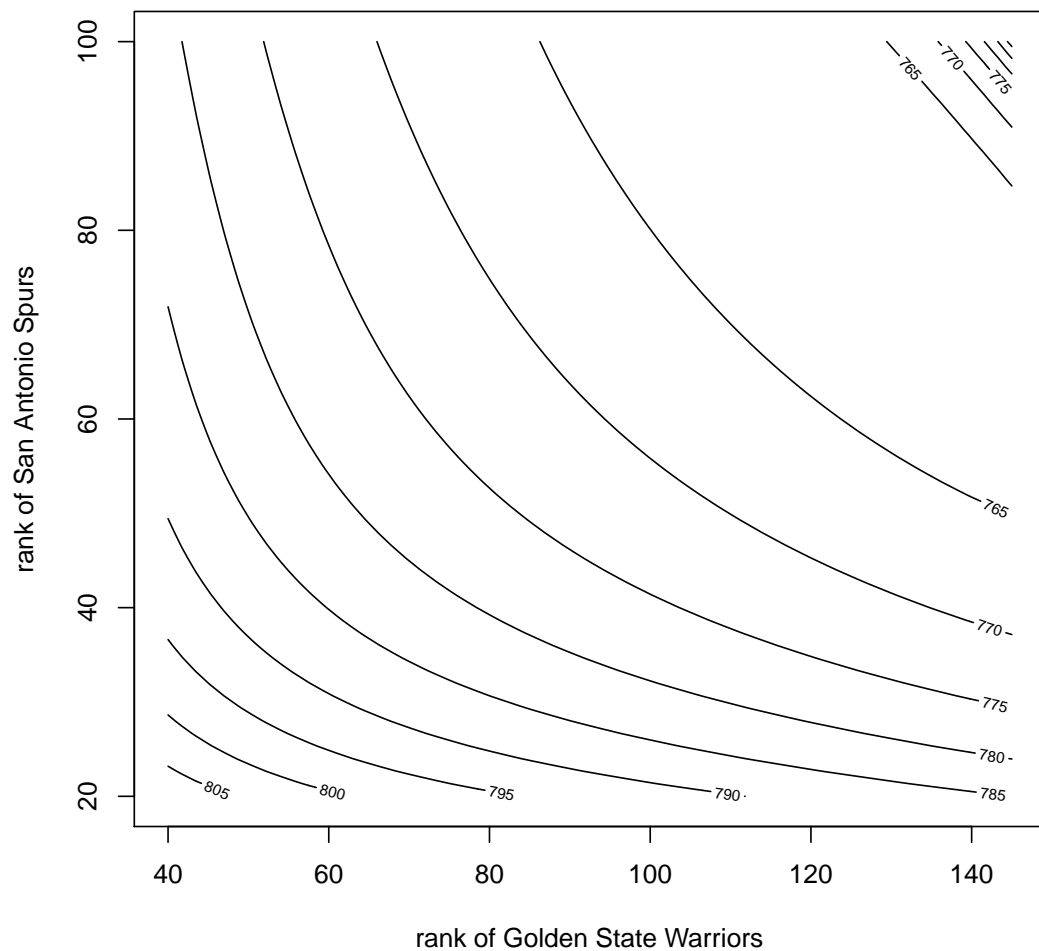


```
> r1 = seq(40, 145, length = 1001)
> r2 = seq(20, 100, length = 6)
> z = outer(r1, r2, Q2)
> par(mfrow = c(2, 3))
> ran = range(z)
> for(j in 1:length(r2)) {
+   plot(r1, z[, j], ylim = ran, type = "l",
```
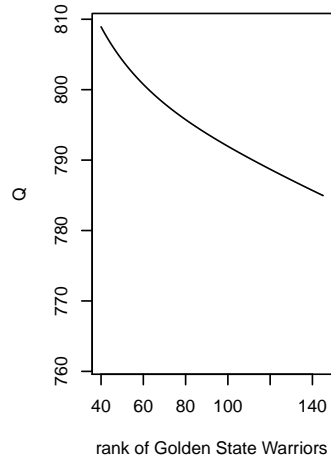
```
+        main = paste("rank of",
+                     nba.names[first2[2]],
+                     "=",
+                     r2[j]),
+        xlab = paste("rank of", nba.names[first2[1]]),
+        ylab = "Q")
+ }
>
```
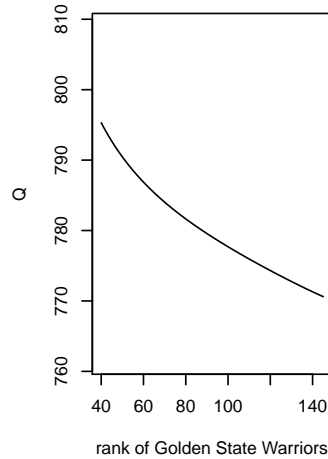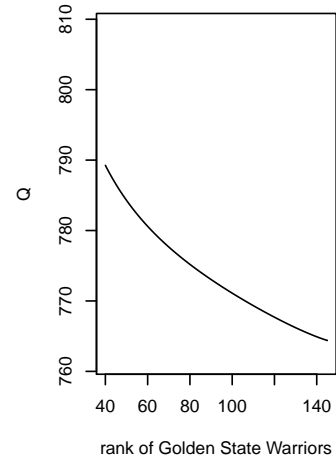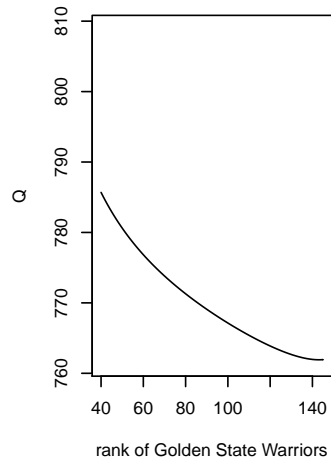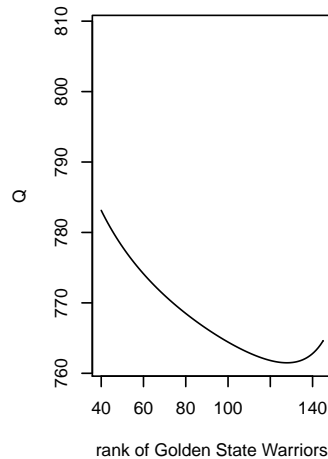
rank of San Antonio Spurs = 20

rank of Golden State Warriors

rank of San Antonio Spurs = 36

rank of Golden State Warriors

rank of San Antonio Spurs = 52

rank of Golden State Warriors

rank of San Antonio Spurs = 68

rank of Golden State Warriors

rank of San Antonio Spurs = 84

rank of Golden State Warriors

rank of San Antonio Spurs = 100

rank of Golden State Warriors