

1. The answers are below:

```
> fib = function(n) {  
+   s = numeric(n)  
+  
+   if (n <= 1) s[n] = 0  
+   else {  
+     s[1:(n - 1)] = fib(n - 1)  
+     if (n == 2) s[n] = 1  
+     else s[n] = s[n - 1] + s[n - 2]  
+   }  
+  
+   s  
+ }  
>  
> fib(1)  
[1] 0  
> fib(2)  
[1] 0 1  
> fib(3)  
[1] 0 1 1  
> fib(10)  
[1] 0 1 1 2 3 5 8 13 21 34
```

2. The answers are below:

```
(a) > clusters.medians = function(x, c) {  
+   lenc = length(c)  
+  
+   d = outer(c, x, function(cj, xi) abs(xi - cj))  
+   d.minnum = apply(d, 2, which.min)  
+  
+   con = outer(1:lenc, d.minnum, function(num, minnum) num == minnum)  
+  
+   xv = unlist(apply(con, 1, function(t) median(x[t])))  
+   xv  
+ }  
>  
> find.clusters.medians = function(x, c) {  
+   ctmp1 = c  
+   repeat {  
+     ctmp2 = clusters.medians(x, ctmp1)  
+     if (all(abs(ctmp1 - ctmp2) < 1e-07)) break  
+     else ctmp1 = ctmp2  
+   }
```

```

+   }
+   ctmp1
+ }
>
> x = faithful$eruptions
> find.clusters.medians(x, c(2,4))
[1] 1.9830 4.3415
(b) > find.clusters.medians(x, c(2,3,4))
[1] 1.9830 3.9665 4.5330
(c) > find.clusters.medians(x, c(2,3,4,5))
[1] 1.967 3.600 4.150 4.600

```

3. The answers are below:

```

> sign.matrix = function(x) outer(x, x, function(x1, x2) sign(x1 - x2))
>
> conc = function(x, y) {
+   conc.mtx = sign.matrix(x)
+   conc.mty = sign.matrix(y)
+   conc.z = conc.mtx + conc.mty
+   c = length(which(conc.z < 0 | conc.z > 0))
+   n = length(x)
+   c / (n * (n - 1))
+ }
>
> conc(x = 1:5, y = c(3, 1, 4, 5, 2))
[1] 0.6
>
> set.seed(782)
> x = round(rnorm(1000))
> y = x + round(rnorm(1000))
> conc(x, y)
[1] 0.8518939

```

4. The answers are below:

```

(a) > nba.df = read.csv("https://raw.githubusercontent.com/
zzdxzhangzhi/assignments/master/782/NBA2016-2017.csv",
+ stringsAsFactors = FALSE)
> names(nba.df) = c("team1", "team2", "wins")
> head(nba.df)

```

	team1	team2	wins
1	Atlanta Hawks	Boston Celtics	2
2	Atlanta Hawks	Brooklyn Nets	2
3	Atlanta Hawks	Charlotte Hornets	1
4	Atlanta Hawks	Chicago Bulls	3
5	Atlanta Hawks	Cleveland Cavaliers	3

```

6 Atlanta Hawks      Dallas Mavericks      2
>
> nba.names = nba.df$team1[seq(1, 870, length = 30)]
> nba.names
[1] "Atlanta Hawks"      "Boston Celtics"      "Brooklyn Nets"
    "Charlotte Hornets"
[5] "Chicago Bulls"      "Cleveland Cavaliers"  "Dallas Mavericks"
    "Denver Nuggets"
[9] "Detroit Pistons"    "Golden State Warriors" "Houston Rockets"
    "Indiana Pacers"
[13] "Los Angeles Clippers" "Los Angeles Lakers"   "Memphis Grizzlies"
    "Miami Heat"
[17] "Milwaukee Bucks"    "Minnesota Timberwolves" "New Orleans Pelicans"
    "New York Knicks"
[21] "Oklahoma City Thunder" "Orlando Magic"        "Philadelphia 76ers"
    "Phoenix Suns"
[25] "Portland Trail Blazers" "Sacramento Kings"     "San Antonio Spurs"
    "Toronto Raptors"
[29] "Utah Jazz"          "Washington Wizards"
> log.likelihood.r = function(r, times, s) {
+   rn = s - sum(r)
+   rr = c(r, rn)
+
+   if (all(rr > 0)) {
+     mtx = outer(rr, rr, function(ri, rj) log(ri / (ri + rj)))
+     rankv = c(t(mtx)[which(row(mtx) != col(mtx))])
+     sum(times * rankv)
+   } else {
+     -Inf
+   }
+ }
>
> s = 1000
> Q = function(r) {
+   -log.likelihood.r(r, nba.df$wins, s)
+ }
>
> count = length(nba.names)
> result = optim(seq(1, 29, length = 29), Q, method = "BFGS",
+               control = list(maxit = 200))
> result
$par
[1] 28.920344 49.262769 8.487427 20.865839 26.562360
45.009903 18.735556 26.484090 21.994617
[10] 127.851303 59.221865 27.412854 47.775175 12.710449
32.153364 26.968452 28.090105 17.226476

```

```
[19] 19.848702 15.942769 38.873109 14.320608 13.473253
11.341333 28.116503 17.982849 83.968346
[28] 43.789158 47.019261
```

```
$value
[1] 761.4917
```

```
$counts
function gradient
      147      143
```

```
$convergence
[1] 0
```

```
$message
NULL
```

```
>
> ratio = 100 / result$par[which.max(result$par)]
> r.value = result$par * ratio
> rr.value = c(r.value, (s - sum(result$par)) * ratio)
> rr.value
[1] 22.620297 38.531300 6.638514 16.320396 20.775979
35.204884 14.654177 20.714760 17.203279
[10] 100.000000 46.320893 21.441200 37.367766 9.941587
25.149031 21.093608 21.970918 13.473837
[19] 15.524833 12.469774 30.404938 11.200987 10.538221
8.870722 21.991566 14.065441 65.676566
[28] 34.250068 36.776521 30.966569
>
> rank.table = data.frame(nba.names, rr.value, stringsAsFactors = FALSE)
> ordered.rank = rank.table[order(rank.table$rr.value, decreasing = TRUE),]
> colnames(ordered.rank) = c("name", "rank")
> rownames(ordered.rank) = 1:30
> ordered.rank
```

	name	rank
1	Golden State Warriors	100.000000
2	San Antonio Spurs	65.676566
3	Houston Rockets	46.320893
4	Boston Celtics	38.531300
5	Los Angeles Clippers	37.367766
6	Utah Jazz	36.776521
7	Cleveland Cavaliers	35.204884
8	Toronto Raptors	34.250068
9	Washington Wizards	30.966569
10	Oklahoma City Thunder	30.404938

```

11      Memphis Grizzlies  25.149031
12      Atlanta Hawks     22.620297
13 Portland Trail Blazers  21.991566
14      Milwaukee Bucks   21.970918
15      Indiana Pacers     21.441200
16      Miami Heat         21.093608
17      Chicago Bulls      20.775979
18      Denver Nuggets     20.714760
19      Detroit Pistons    17.203279
20      Charlotte Hornets  16.320396
21      New Orleans Pelicans 15.524833
22      Dallas Mavericks   14.654177
23      Sacramento Kings   14.065441
24 Minnesota Timberwolves  13.473837
25      New York Knicks    12.469774
26      Orlando Magic      11.200987
27      Philadelphia 76ers  10.538221
28      Los Angeles Lakers  9.941587
29      Phoenix Suns       8.870722
30      Brooklyn Nets     6.638514
>

```

(b)

```

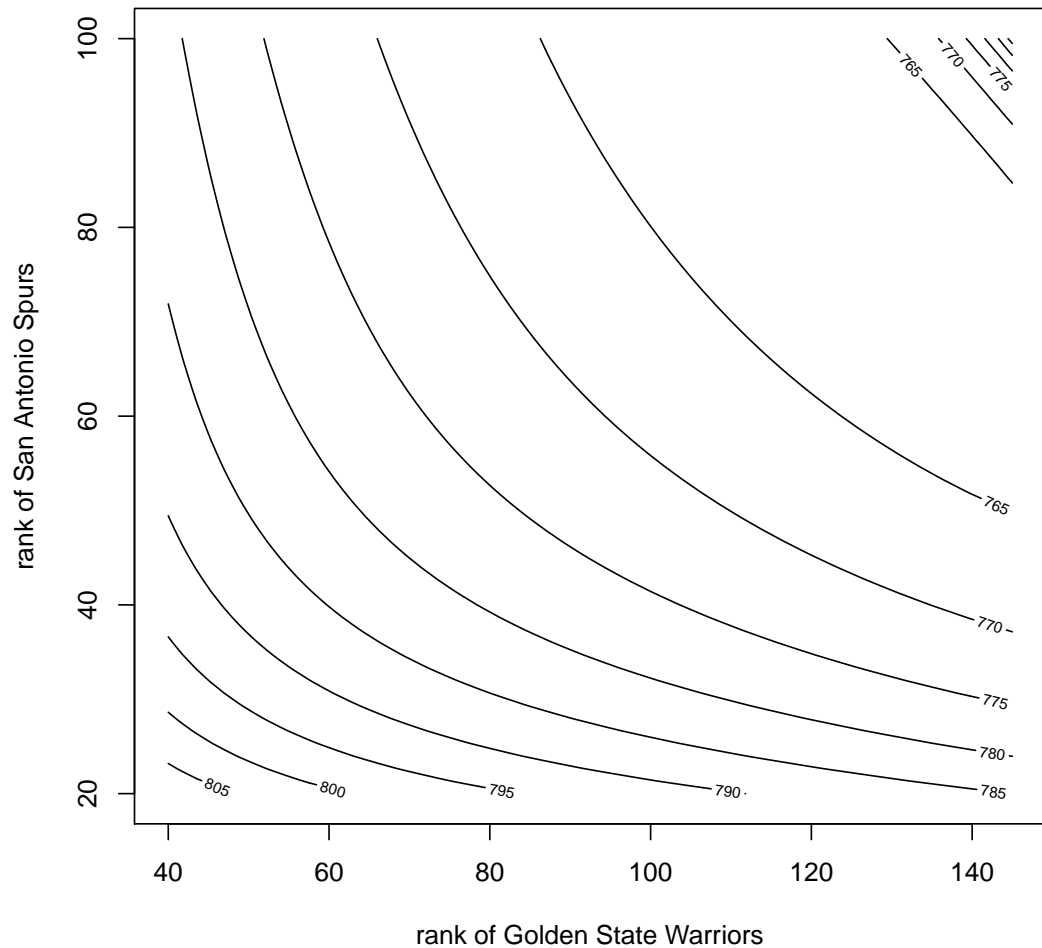
(c) > ranks = c(result$par, s - sum(result$par))
> ranks.sort = sort(ranks, decreasing = TRUE)
> first2 = c(which(round(ranks) == round(ranks.sort[1])),
+             which(round(ranks) == round(ranks.sort[2]))))
> first2
[1] 10 27
>
> Q2 = function(r1, r2) {
+   m = max(length(r1), length(r2))
+   if (length(r1) < m)
+     r1 = rep(r1, length = m)
+   if (length(r2) < m)
+     r2 = rep(r2, length = m)
+
+   ans = numeric(m)
+   for (i in 1:m) {
+     ranks[first2] = c(r1[i], r2[i])
+     ans[i] = -log.likelihood.r(ranks[-length(ranks)], nba.df$wins, s)
+   }
+   ans
+ }
>
> r1 = seq(40, 145, length = 61)

```

```

> r2 = seq(20, 100, length = 61)
> z = outer(r1, r2, Q2)
> contour(r1, r2, z,
+         xlab = paste("rank of", nba.names[first2[1]]),
+         ylab = paste("rank of", nba.names[first2[2]]))
>

```



```

> r1 = seq(40, 145, length = 1001)
> r2 = seq(20, 100, length = 6)
> z = outer(r1, r2, Q2)
> par(mfrow = c(2, 3))
> ran = range(z)
> for(j in 1:length(r2)) {
+   plot(r1, z[, j], ylim = ran, type = "l",
+        main = paste("rank of",
+                      nba.names[first2[2]],

```

```

+         "=",
+         r2[j]),
+       xlab = paste("rank of", nba.names[first2[1]]),
+       ylab = "Q")
+ }
>

```

