Technical Report: Monte Carlo pi Estimation (Task 4)

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Course: IE 221 Probability | Fall 2025-2026 4

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1. **Introduction**
   The objective of this task is to experimentally verify the Strong Law of Large Numbers (SLLN) using the Monte Carlo method to estimate the value of pi.

- The Monte Carlo method is a direct application of SLLN, where random sampling is used to obtain numerical results for deterministic problems.

2. Theoretical Background

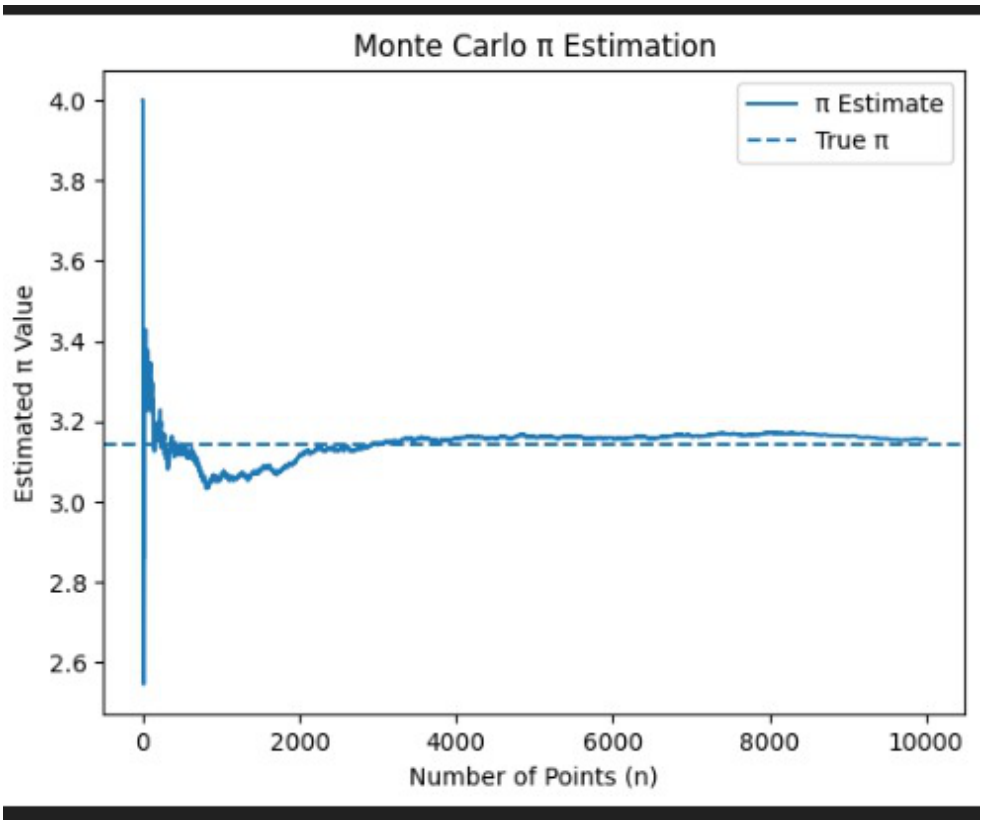
- According to the Strong Law of Large Numbers (SLLN), the arithmetic mean of independent and identically distributed random variables converges almost surely to their expected value as n approaches infinity.
- In this estimation, random points (x, y) are dropped into a unit square [0,1] x 0,1].
- The ratio of points falling inside a unit quarter circle $x\text{^}2 + y\text{^}2 \leq 1$ to the total number of points converges to pi/4.
- Multiplying this proportion by 4 gives the estimate for pi.

3. Methodology

- **Data Generation:** Successive observations X_1, X_2, ..., X_n and Y_1, Y_2, ..., Y_n were generated from a standard uniform distribution U[0,1].
- **Condition Check:** For each point, the condition x^2 + y^2 ≤ 1 was tested to determine if it resides within the quarter circle.
- **Estimation Formula:** The estimation was calculated as: Pi = 4 X {Number of points inside circle}) / ({Total number of points}).
- **Simulation Size:** A sufficiently large n = 10,000 was used to ensure convergence.

4. Results and Analysis

- The generated data was saved to results/pi_estimation_data.csv.
- The pi estimate vs. number of points (n)" graph shows high volatility in the early stages of the simulation.
- As n increases, the estimate settles and converges toward the true value of pi approximation 3.1415.



- The visual evidence reflects the "almost sure convergence" property of SLLN, as the sample paths stay near the expected value.

5. Conclusion

- The simulation successfully demonstrates that as the sample size increases, the experimental results approach the theoretical value, confirming the boundaries of the SLLN.