IE 221 PROBABILITY - TERM PROJECT (TW3)

Technical Report: Task 2 - Strong Law of Large Numbers (SLLN)

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1. Introduction

The objective of this task is to experimentally verify the **Strong Law of Large Numbers (SLLN)**. The SLLN is a fundamental theorem in probability that describes the result of performing the same experiment a large number of times. It states that the sample mean of i.i.d. variables converges almost surely to the theoretical expected value (mean) as the number of trials increases

2. Theoretical Background

For a sequence of independent and identically distributed (i.i.d.) random variables X1,X2 ,…,Xn with a finite expected value μ, the SLLN is expressed as:

$$P(\lim_{n\to\infty} \frac{S_n}{n} = \mu) = 1$$

In this project, we utilize the **Standard Uniform Distribution** U[0,1], where the theoretical mean is μ=0.5. According to the SLLN, the cumulative mean of our samples should eventually settle at 0.5 and remain there for almost all sample paths.

3. Methodology & Implementation

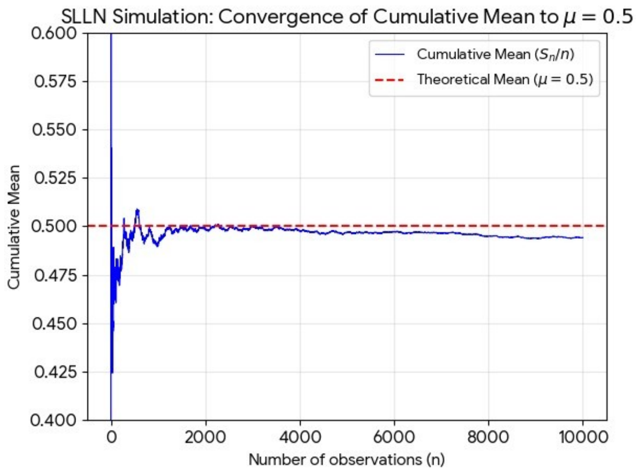
The simulation was developed using **Java** to ensure high performance with large datasets.

- **Sample Size:** We used a sufficiently large sample size of **n=10,000** observations to observe the convergence clearly.
- **Process:** The code generates random values from U[0,1] using the java.util.Random class. For each step i, it calculates the cumulative sum and divides it by the current number of observations to find the cumulative mean (Si=i1 ∑j=1iXj).
- **Output:** The resulting data points (n, cumulative mean, and theoretical mean) are exported to results/slln_data.csv for visualization.

4. Visual Analysis and Findings

The simulation produces a "Cumulative mean vs. number of observations (n)" graph.

- **Early Volatility:** At the beginning of the simulation (small values of n), the cumulative mean shows high volatility, oscillating significantly above and below the 0.5 line.
- **Convergence:** As n increases toward 10,000, the fluctuations decrease in magnitude. The sample path stabilizes and converges almost exactly to the theoretical reference line of **μ=0.5**.
- **Interpretation:** This visual stabilization reflects the "almost sure convergence" property. Even though the individual random variables vary between 0 and 1, their collective average becomes highly predictable as the sample size grows.



5. Conclusion

The SLLN simulation successfully demonstrates that empirical averages become more stable and accurate as more data is collected. The convergence observed in our Java implementation confirms that for the standard uniform distribution, the sample mean is a consistent estimator of the true population mean μ=0.5.