

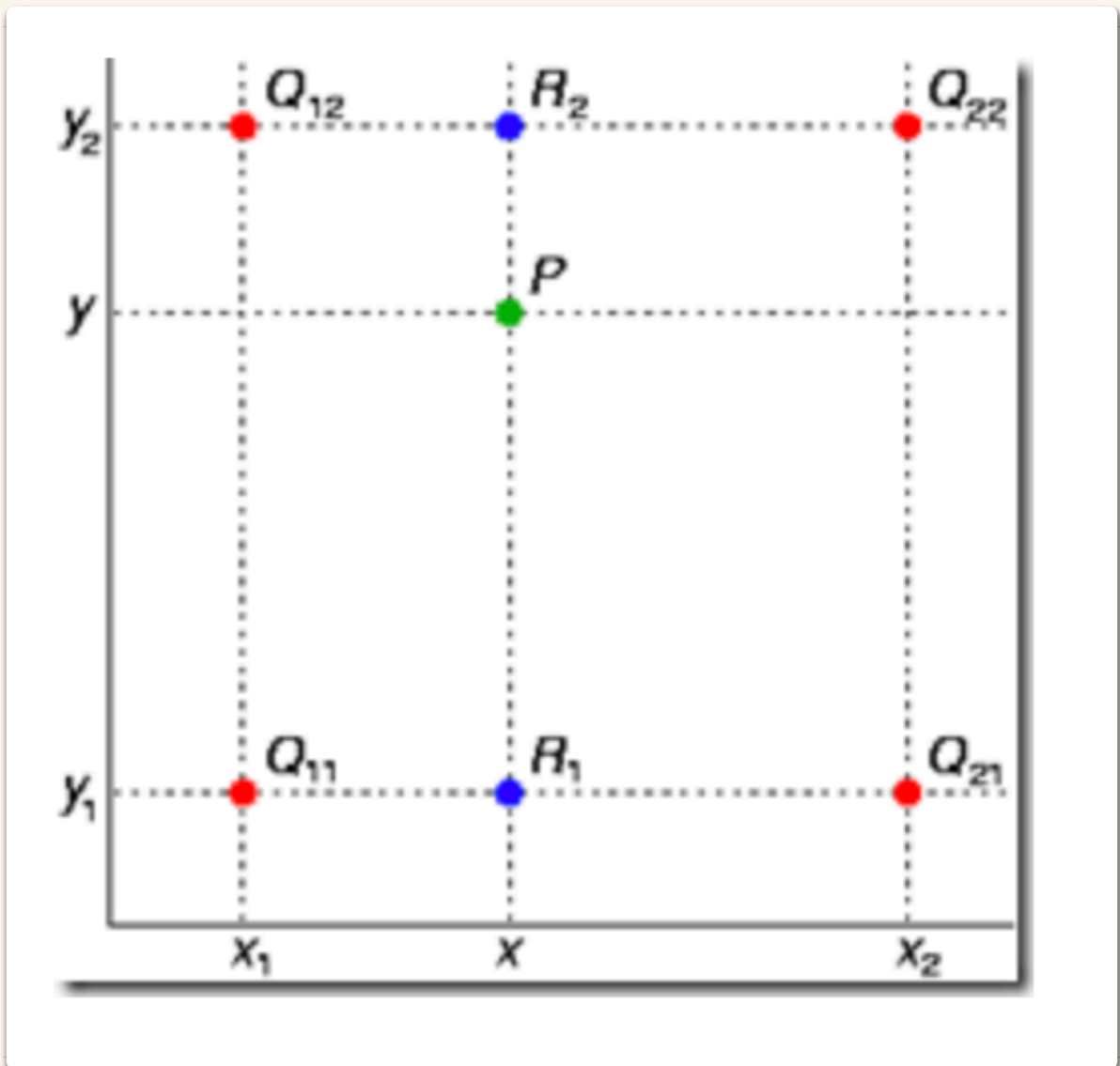
# Bilinear Interpolation

## Linear Interpolation

- 已知两点A B，要在AB中插入一点C，即在AB的连线上插入。可套用一次线性公式：

## 双线性

- 已知 $Q_{11}$ ,  $Q_{12}$ ,  $Q_{21}$ ,  $Q_{22}$ ，要在其中插入一点P。得到P点，需要两次线性插值，即双线性插值



1. 首先求出插值R1, R2

$$f(R_1) \approx \frac{x_2 - x}{x_2 - x_1} f(Q_{11}) + \frac{x - x_1}{x_2 - x_1} f(Q_{21}) \quad \text{where } R_1 = (x, y_1),$$

$$f(R_2) \approx \frac{x_2 - x}{x_2 - x_1} f(Q_{12}) + \frac{x - x_1}{x_2 - x_1} f(Q_{22}) \quad \text{where } R_2 = (x, y_2).$$

2. 由插值R1, R2求出插值P

$$f(P) \approx \frac{y_2 - y}{y_2 - y_1} f(R_1) + \frac{y - y_1}{y_2 - y_1} f(R_2).$$

3. 式子扩展化简即为

$$\begin{aligned} f(x, y) \approx & \frac{f(Q_{11})}{(x_2 - x_1)(y_2 - y_1)} (x_2 - x)(y_2 - y) + \frac{f(Q_{21})}{(x_2 - x_1)(y_2 - y_1)} (x - x_1)(y_2 - y) \\ & + \frac{f(Q_{12})}{(x_2 - x_1)(y_2 - y_1)} (x_2 - x)(y - y_1) + \frac{f(Q_{22})}{(x_2 - x_1)(y_2 - y_1)} (x - x_1)(y - y_1). \end{aligned}$$

4. 若选择一个坐标系使得Q点为(0, 0), (0, 1), (1, 0), (1, 1), 那么插值公式可以化简为矩阵形式