Matrix Elimination Calculation Process

The matrix $\boldsymbol{A_2}$ is shown in equation(A2).

$$\boldsymbol{x}=\begin{bmatrix} \delta\theta_{VSM} & \delta\omega_{VSM} & v_{0,d} & v_{0,q} & i_{cv,d} & \cdots \\ \cdots & i_{cv,q} & \gamma_d & \gamma_q & i_{o,d} & \cdots \\ \cdots & i_{o,q} & \varphi_d & \varphi_q & v_{PLL,d} & \cdots \\ \cdots & v_{PLL,q} & \varepsilon_{PLL} & \xi_d & \xi_q & \cdots \\ \cdots & q_m & \delta\theta_{PLL} & & & \end{bmatrix} \tag{A1}$$

Non-zero elements in the original state space matrix are represented by table elements in Figure A1. Move the state variable power angle $\delta\theta_{VSM}$ and angular velocity $\delta\omega_{VSM}$ to the first two rows, and extend the rows of the remaining state variables to obtain a new matrix form $\tilde{\boldsymbol{A}}_1$. The new order of state variables is shown in equation (A1).

Matrix transformation principle:: $Z_{(1,2)}$ is the $X_{(14,18)}$ element in the original matrix; $Z_{(2,2)}$ is the $X_{(18,18)}$ element in the original matrix; $Z_{(5,2)}$ is the $X_{(3,18)}$ element in the original matrix; $Z_{(6,2)}$ is the $X_{(4,18)}$ element in the original matrix; $Z_{(7,2)}$ is the $X_{(5,18)}$ element in the original matrix; $Z_{(8,2)}$ is the $X_{(6,18)}$ element in the original matrix; $Z_{(9,1)}$ is the $X_{(7,14)}$ element in the original matrix; $Z_{(10,1)}$ is the $X_{(8,14)}$ element in the original matrix; $Z_{(13,1)}$ is the $X_{(11,14)}$ element in the original matrix; $Z_{(14,1)}$ is the $X_{(12,14)}$ element in the original matrix; $Z_{(16,2)}$ is the $X_{(15,18)}$ element in the original matrix; $Z_{(17,2)}$ is the $X_{(16,18)}$ element in the original matrix; $X_{(2,3)}$ is the $X_{(18,1)}$ element in the original matrix; $X_{(2,4)}$ is the $X_{(18,2)}$ element in the original matrix; $X_{(2,9)}$ is the $X_{(18,7)}$ element in the original matrix; $X_{(2,10)}$ is the $X_{(18,8)}$ element in the original matrix; $X_{(2,14)}$ is the $X_{(18,12)}$ element in the original matrix; $X_{(2,15)}$ is the $X_{(18,13)}$ element in the original matrix. The remaining elements move to the next row in orde.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1		Z																	
2		Z	X	X					X	X				X	X				
3				X	X				X										
4			X			X				X									
5		Z	X	X	X		X		X	X	X					X		X	
6		Z	X	X		X		X	X	X		X					X		
7		Z	X	X	X				X	X						X		X	
8		Z	X	X		X			X	X							X		
9	Z		X						X	X									
10	Z			X					X	X									
11			X								X								
12				X								X							
13	Z		X	X									X						X
14	Z		X	X										X					X
15														X					
16		Z	X						X	X								X	
17		Z		X					X	X									
18			X	X					X	X								X	
19														X	X				

Fig.A1 Matrix form of $\tilde{A}_1$

$$A_2 = \begin{bmatrix} 0 & a_{12}\cdot\Delta\omega & 0 & \dots & 0 & \dots & 0 & s\Delta\theta \\ a_{21}\cdot\Delta\theta & a_{22}\cdot\Delta\omega & a_{23}\cdot\Delta Z_1 & \dots & 0 & \dots & a_{2n}\cdot\Delta Z_{n-2} & s\Delta\omega - a_{[2,(i+2)]}\cdot\Delta Z_i \\ a_{31}\cdot\Delta\theta & a_{32}\cdot\Delta\omega & a_{33}\cdot\Delta Z_1 & \dots & 0 & \dots & a_{3n}\cdot\Delta Z_{n-2} & s\Delta Z_1 - a_{[3,(i+2)]}\cdot\Delta Z_i \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ a_{[(i+2),1]}\cdot\Delta\theta & a_{[(i+2),2]}\cdot\Delta\omega & a_{[(i+2),3]}\cdot\Delta Z_1 & \dots & 0 & \dots & a_{[(i+2),n]}\cdot\Delta Z_{n-2} & s\Delta Z_i - a_{[(i+2),(i+2)]}\cdot\Delta Z_i \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ a_{n1}\cdot\Delta\theta & a_{n2}\cdot\Delta\omega & a_{n3}\cdot\Delta Z_1 & \dots & 0 & \dots & a_{nn}\cdot\Delta Z_{n-2} & s\Delta Z_{n-2} - a_{[n,(i+2)]}\cdot\Delta Z_i \end{bmatrix} \tag{A2}$$

Expression of parameters in matrix analysis of phase locked loop are:

$$\begin{cases} v_{pll,q1} = \dfrac{Z_{(14,1)}}{s - X_{(14,14)} - \dfrac{X_{(14,19)}}{s}P_1} \\ P_1 = X_{(19,14)} + X_{(19,15)}\dfrac{X_{(15,14)}}{s} \\ v_{pll,q2} = \dfrac{X_{(14,3)}}{s - X_{(14,14)} - \dfrac{X_{(14,19)}}{s}P_1} \\ v_{pll,q3} = \dfrac{X_{(14,4)}}{s - X_{(14,14)} - \dfrac{X_{(14,19)}}{s}P_1} \end{cases} \tag{A3}$$

$$\begin{cases} \varepsilon_{pll1} = \dfrac{Z_{(14,1)}}{s\left\{s - X_{(14,14)} - \dfrac{X_{(14,19)}}{s}P_1\right\}} \\ \varepsilon_{pll2} = \dfrac{X_{(14,3)}}{s\left\{s - X_{(14,14)} - \dfrac{X_{(14,19)}}{s}P_1\right\}} \\ \varepsilon_{pll3} = \dfrac{X_{(14,4)}}{s\left\{s - X_{(14,14)} - \dfrac{X_{(14,19)}}{s}P_1\right\}} \end{cases} \tag{A4}$$

The parameters of the phase locked loop on the electromechanical oscillation circuit are:

$$\begin{cases} PLL_1 = X_{(2,15)}\varepsilon_{pll1} + X_{(2,14)}v_{pll,q1} \\ PLL_2 = X_{(2,15)}\varepsilon_{pll2} + X_{(2,14)}v_{pll,q2} \\ PLL_3 = X_{(2,15)}\varepsilon_{pll3} + X_{(2,14)}v_{pll,q3} \end{cases} \tag{A5}$$

Now the 13th, 14th, 15th, and 19th rows and columns of the phase locked loop in the matrix can be eliminated to obtain a matrix $\tilde{A}_2$ of size 15×15.

Matrix analysis of residual variables:

First, eliminate intermediate variables γ_d, γ_q, ξ_d, ξ_q, φ_d, φ_q and q_m in the $\tilde{A}_2$ matrix.

1）Eliminate variable q_m

Move the element in the 18th column of the matrix $\tilde{A}_2$ to the right side of the equation, multiply the elements in the 18th row by $1/\left(s - X_{(18,18)}\right)$, then multiply this row by $X_{(5,18)}$, $X_{(7,18)}$, and $X_{(16,18)}$, and add them to the 5th, 7th, and 16th rows, respectively. At this time, the elements marked as the 18th rows and columns in the matrix $\tilde{A}_2$ are eliminated, and the matrix becomes $\tilde{A}_3$ of size 14×14.

2）Eliminate variables φ_d and φ_q

Shift the column elements marked as 11th and 12th in matrix $\tilde{A}_3$, then the expressions of φ_d and φ_q will be got. Multiply the 11th row by $\dfrac{1}{s - X_{(11,11)}}$, then multiply the 12th row by $\dfrac{1}{s - X_{(12,12)}}$. Afterwards, multiply the 11th row by $X_{(5,11)}$ and add it to the 5th row, then multiply the 12th row by $X_{(6,12)}$ and add it to the 6th row. At this time, the elements marked as the 11th and 12th rows and columns in the matrix $\tilde{A}_3$ are eliminated, and the matrix becomes $\tilde{A}_4$ of size 12×12.

3）Eliminate variables ξ_d and ξ_q

Shift the column elements marked as 16th and 17th in matrix $\tilde{A}_4$, then the expressions of ξ_d and ξ_q will be got. Multiply the 16th row by $\dfrac{1}{s - X_{(16,16)}}$, then multiply the 17th row by $\dfrac{1}{s - X_{(17,17)}}$, and then multiply the 16th row by $X_{(5,16)}$, and add it to the 5th row. Multiply the 16th row by $X_{(7,16)}$ to add to the 7th row, then multiply the 17th row by $X_{(6,17)}$ to add to the 6th row, and multiply the 17th row by $X_{(8,17)}$ to add to the 8th row. At this time, the elements marked as the 16th and 17th rows and columns in the matrix $\tilde{A}_4$ are eliminated, and the matrix becomes $\tilde{A}_5$ of size 10×10.

4）Eliminate variables γ_d and γ_q

Shift the column elements marked as 7th and 8th in matrix $\tilde{\boldsymbol{A}}_5$. Multiply the 7th row by $\frac{1}{s-X_{(7,7)}}$, then multiply the 8th row by $\frac{1}{s-X_{(8,8)}}$. Next, multiply the 7th row by $X_{(5,7)}$ to add to the 5th row and multiply the 8th row by $X_{(6,8)}$ add to the 6th row. At this time, the elements marked as the 7th and 8th rows and columns in the matrix $\tilde{\boldsymbol{A}}_5$ are eliminated, and the matrix becomes $\tilde{\boldsymbol{A}}_6$ of size 8×8.

The above complex coefficients are expressed by variables D_1-D_6 and Q_1-Q_6.

	1 θ	2 ω	3 $v_{o,d}$	4 $v_{o,q}$	5 $i_{cv,d}$	6 $i_{cv,q}$	9 $i_{o,d}$	10 $i_{o,q}$
1		Z						
2		Z	X	X			X	X
3				X	X		X	
4			X			X		X
5		D_1	D_2	D_3	D_4		D_5	D_6
6		Q_1	Q_2	Q_3		Q_4	Q_5	Q_6
9	Z		X				X	X
10	Z			X			X	X

Fig.A2 Matrix form of $\tilde{\boldsymbol{A}}_6$

The specific values of each parameter are:

$$D_1=\frac{X_{(5,16)}Z_{(16,2)}}{s}+Z_{(5,2)}+Z'_{72} \tag{A6}$$

$$D_2=\left[X_{(16,3)}+\frac{X_{(16,18)}X_{(18,3)}}{s-X_{(18,18)}}\right]\frac{X_{(5,16)}}{s}+ X_{(5,3)}+X_{(5,18)}\frac{X_{(18,3)}}{s-X_{(18,18)}}+ X_{(5,11)}\frac{X_{(11,3)}}{s-X_{(11,11)}}+X'_{73} \tag{A7}$$

$$D_3=\frac{X_{(5,16)}}{s}\left[X_{(16,4)}+\frac{X_{(16,18)}X_{(18,4)}}{s-X_{(18,18)}}\right]+ X_{(5,4)}+X_{(5,18)}\frac{X_{(18,4)}}{s-X_{(18,18)}}+X'_{74} \tag{A8}$$

$$D_4=X_{(5,5)}+\frac{X_{(5,7)}X_{(7,5)}}{s} \tag{A9}$$

$$D_5=\frac{X_{(5,16)}}{s}\left[X_{(16,9)}+\frac{X_{(16,18)}X_{(18,9)}}{s-X_{(18,18)}}\right]+ X_{(5,9)}+X_{(5,18)}\frac{X_{(18,9)}}{s-X_{(18,18)}}+X'_{79} \tag{A10}$$

$$D_6=\frac{X_{(5,16)}}{s}\left[X_{(16,10)}+\frac{X_{(16,18)}X_{(18,10)}}{s-X_{(18,18)}}\right]+ X_{(5,10)}+\frac{X_{(5,18)}X_{(18,10)}}{s-X_{(18,18)}}+X'_{710} \tag{A11}$$

$$Q_1=Z'_{82}+Z_{(6,2)}+\frac{X_{(6,17)}Z_{(17,2)}}{s} \tag{A12}$$

$$Q_2=\frac{X_{(6,8)}X_{(8,3)}}{s}+X_{(6,3)} \tag{A13}$$

$$Q_3=X'_{84}+X_{(6,4)}+X_{(6,12)}\frac{X_{(12,4)}}{s-X_{(12,12)}}+ \frac{X_{(6,17)}X_{(17,4)}}{s} \tag{A14}$$

$$Q_4=\frac{X_{(6,8)}X_{(8,6)}}{s}+X_{(6,6)} \tag{A15}$$

$$Q_5=X'_{89}+X_{(6,9)}+\frac{X_{(6,17)}X_{(17,9)}}{s} \tag{A16}$$

$$Q_6=X'_{810}+X_{(6,10)}+\frac{X_{(6,17)}X_{(17,10)}}{s} \tag{A17}$$

The expression of the intermediate variable is:

$$Z'_{72}=\frac{X_{(5,7)}}{s}\left[\frac{X_{(7,16)}Z_{(16,2)}}{s}+Z_{(7,2)}\right] \tag{A18}$$

$$X'_{73}=\frac{X_{(5,7)}}{s}\times \left\{\frac{X_{(7,16)}}{s}\left[X_{(16,3)}+\frac{X_{(16,18)}X_{(18,3)}}{s-X_{(18,18)}}\right]+ X_{(7,3)}+\frac{X_{(7,18)}X_{(18,3)}}{s-X_{(18,18)}}\right\} \tag{A19}$$

$$X'_{74}=\frac{X_{(5,7)}}{s}\times \left\{\frac{X_{(7,16)}}{s}\left[X_{(16,4)}+\frac{X_{(16,18)}X_{(18,4)}}{s-X_{(18,18)}}\right]+ X_{(7,4)}+\frac{X_{(7,18)}X_{(18,4)}}{s-X_{(18,18)}}\right\} \tag{A20}$$

$$X'_{79}=\frac{X_{(5,7)}}{s}\times \left\{\frac{X_{(7,16)}}{s}\left[X_{(16,9)}+\frac{X_{(16,18)}X_{(18,9)}}{s-X_{(18,18)}}\right]+ X_{(7,9)}+\frac{X_{(7,18)}X_{(18,9)}}{s-X_{(18,18)}}\right\} \tag{A21}$$

$$X'_{710}=\frac{X_{(5,7)}}{s}\times \left\{\frac{X_{(7,16)}}{s}\left[X_{(16,10)}+\frac{X_{(16,18)}X_{(18,10)}}{s-X_{(18,18)}}\right]+ X_{(7,10)}+\frac{X_{(7,18)}X_{(18,10)}}{s-X_{(18,18)}}\right\} \tag{A22}$$

$$Z'_{82}=\frac{X_{(6,8)}}{s}\left[Z_{(8,2)}+\frac{X_{(8,17)}Z_{(17,2)}}{s}\right] \tag{A23}$$

$$X'_{84}=\frac{X_{(6,8)}}{s}\left[X_{(8,4)}+\frac{X_{(8,17)}X_{(17,4)}}{s}\right] \tag{A24}$$

$$X'_{89}=\frac{X_{(6,8)}}{s}\left[X_{(8,9)}+\frac{X_{(8,17)}X_{(17,9)}}{s}\right] \tag{A25}$$

$$X'_{810}=\frac{X_{(6,8)}}{s}\left[X_{(8,10)}+\frac{X_{(8,17)}X_{(17,10)}}{s}\right] \tag{A26}$$

At this point, the intermediate variables have been eliminated, and the remaining variables will be eliminated in the next step.

5）Eliminate variables $i_{cv,d}$ and $i_{cv,q}$

The following elimination is based on matrix $\tilde{\boldsymbol{A}}_6$, First, shift the 5th and 6th columns of this matrix, then multiply the 5th row by $\frac{1}{s-D_4}$, and multiply the 6th row by $\frac{1}{s-Q_4}$. Next, multiply the 5th row by $X_{(3,5)}$ to add to the 3rd row. Multiply the 6th row by $X_{(4,6)}$ and add to the 4th row. At this time, the elements marked as the 5th and 6th rows and columns in the matrix $\tilde{\boldsymbol{A}}_6$ are eliminated, and the matrix becomes $\tilde{\boldsymbol{A}}_7$ of size 6×6.

	1 θ	2 ω	3 $v_{o,d}$	4 $v_{o,q}$	9 $i_{o,d}$	10 $i_{o,q}$
1		Z				
2		Z	X	X	X	X
3		Y_1	Y_2	Y_3	Y_4	Y_5
4		Y_6	Y_7	Y_8	Y_9	Y_{10}
9	Z		X		X	X
10	Z			X	X	X

Fig.A3 Matrix form of $\tilde{\boldsymbol{A}}_7$

Where the matrix elements are:

$$Y_1=\frac{X_{(3,5)}D_1}{s-D_4} \tag{A27}$$

$$Y_2=\frac{X_{(3,5)}D_2}{s-D_4} \tag{A28}$$

$$Y_3=\frac{X_{(3,5)}D_3}{s-D_4}+X_{(3,4)} \tag{A29}$$

$$Y_4=\frac{X_{(3,5)}}{s-D_4}D_5+X_{(3,9)} \tag{A30}$$

$$Y_5=\frac{X_{(3,5)}D_6}{s-D_4} \tag{A31}$$

$$Y_6=\frac{X_{(4,6)}}{s-Q_4}Q_1 \tag{A32}$$

$$Y_7=\frac{X_{(4,6)}}{s-Q_4}Q_2+X_{(4,3)} \tag{A33}$$

$$Y_8=\frac{X_{(4,6)}}{s-Q_4}Q_3 \tag{A34}$$

$$Y_9=\frac{X_{(4,6)}}{s-Q_4}Q_5 \tag{A35}$$

$$Y_{10}=\frac{X_{(4,6)}}{s-Q_4}Q_6+X_{(4,10)} \tag{A36}$$

6）Eliminate the variables $i_{o,d}$ and $i_{o,q}$

Shift the 9th and 10th columns of this matrix $\tilde{\boldsymbol{A}}_7$. First, multiply the 9th row by $\frac{1}{s-X_{9,9}}$, then multiply the 9th row by $X_{(2,9)}$ and add it to 2nd row. Next, multiply the 9th row by Y_4 and add it to 3rd row, then multiply the 9th row by Y_9 and add it to the 4th row. Last, multiply the 9th row by $X_{(10,9)}$ and add it to the 10th row. At this time, the elements marked as the 9th and 10th rows and columns in the matrix $\tilde{\boldsymbol{A}}_7$ are eliminated, and the matrix becomes $\tilde{\boldsymbol{A}}_8$ of size 4×4 .

	1 θ	2 ω	3 $v_{o,d}$	4 $v_{o,q}$
1		Z		
2	E_{21}	Z	X'_{23}	X'_{24}
3	E_{31}	Y_1	Y'_2	Y'_3
4	E_{41}	Y_6	Y'_7	Y'_8

Fig.A4 Matrix form of $\tilde{\boldsymbol{A}}_8$

Where the matrix elements are:

$$X'_{23}=\frac{X_{(10,9)}X_{(9,3)}X_{(2,10)}}{s-\frac{X_{(10,9)}X_{(9,10)}}{s-X_{(9,9)}}-X_{(10,10)}}\times \frac{1}{s-X_{(9,9)}}+\frac{X_{(9,3)}X_{(2,9)}}{s-X_{(9,9)}}+X_{(2,3)} \tag{A37}$$

$$X'_{24}=\frac{X_{(10,4)}X_{(2,10)}}{s-\frac{X_{(10,9)}X_{(9,10)}}{s-X_{(9,9)}}-X_{(10,10)}}+X_{(2,4)} \tag{A38}$$

$$Y_2^{'} = \frac{X_{(10,9)}X_{(9,3)}Y_5^{'}}{s - \frac{X_{(10,9)}X_{(9,10)}}{s - X_{(9,9)}} - X_{(10,10)}} \times \frac{1}{s - X_{(9,9)}} + \frac{X_{(9,3)}Y_4}{s - X_{(9,9)}} + Y_2 \tag{A39}$$

$$Y_3^{'} = \frac{X_{(10,4)}A_5^{'}}{s - \frac{X_{(10,9)}X_{(9,10)}}{s - X_{(9,9)}} - X_{(10,10)}} + Y_3 \tag{A40}$$

$$Y_7^{'} = \frac{X_{(10,9)}X_{(9,3)}Y_{10}^{'}}{s - \frac{X_{(10,9)}X_{(9,10)}}{s - X_{(9,9)}} - X_{(10,10)}} \times \frac{1}{s - X_{(9,9)}} + \frac{X_{(9,3)}Y_9}{s - X_{(9,9)}} + Y_7 \tag{A41}$$

$$Y_8^{'} = \frac{A_{10}^{'}X_{(10,4)}}{s - \frac{X_{(10,9)}X_{(9,10)}}{s - X_{(9,9)}} - X_{(10,10)}} + Y_8 \tag{A42}$$

$$E_{21} = \frac{X_{(2,10)}\left[\frac{X_{(10,9)}Z_{(9,1)}}{s - X_{(9,9)}} + Z_{(10,1)}\right]}{s - \frac{X_{(10,9)}X_{(9,10)}}{s - X_{(9,9)}} - X_{(10,10)}} + \frac{Z_{(9,1)}X_{(2,9)}}{s - X_{(9,9)}} \tag{A43}$$

$$E_{31} = \frac{\left[\frac{X_{(10,9)}Z_{(9,1)}}{s - X_{(9,9)}} + Z_{(10,1)}\right]A_5^{'}}{s - \frac{X_{(10,9)}X_{(9,10)}}{s - X_{(9,9)}} - X_{(10,10)}} + \frac{Z_{(9,1)}Y_4}{s - X_{(9,9)}} \tag{A44}$$

$$E_{41} = \frac{\left(\frac{X_{(10,9)}Z_{(9,1)}}{s - X_{(9,9)}} + Z_{(10,1)}\right)A_{10}^{'}}{s - \frac{X_{(10,9)}X_{(9,10)}}{s - X_{(9,9)}} - X_{(10,10)}} + \frac{Z_{(9,1)}Y_9}{s - X_{(9,9)}} \tag{A45}$$

The influence of phase locked loop is considered in the above matrix. At this time, parameters E_{21}, $X_{23}^{'}$ and $X_{24}^{'}$ in the above matrix should be modified.

7) Eliminate variables $v_{o,d}$ and $v_{o,q}$

First, shift the 4th column of the matrix $\tilde{\boldsymbol{A}}_8$, then multiply the 4th row by $\frac{1}{s - Y_{8p}}$ and X_{24p} and then add it to the 2nd row. Next, multiply the 4th row by Y_{3p} and add it to the 3rd row. Last, the 3rd column is shifted, and multiply this column by the corresponding coefficient, then add it to the 2nd row, then matrix $\tilde{\boldsymbol{A}}_9$ (size of 2×2) is obtained. The effect of the non-electromechanical oscillation circuit of the VSG on the electromechanical oscillation is obtained by the following equation:

$$F = F_\theta + F_\omega \tag{A46}$$

Where:

$$\begin{cases} F_\theta = \dfrac{\left(\frac{E_{41}}{s - Y_8^{'}}Y_3^{'} + E_{31}\right)\left(\frac{Y_7^{'}}{s - Y_8^{'}}X_{24}^{'} + X_{23}^{'}\right)}{s - \frac{Y_7^{'}}{s - Y_8^{'}}Y_3^{'} - Y_2^{'}} + \dfrac{E_{41}}{s - Y_8^{'}}X_{24}^{'} + E_{21} \\ F_\omega = \dfrac{\left(\frac{Y_6}{s - Y_8^{'}}Y_3^{'} + A1\right)\left(\frac{Y_7^{'}}{s - Y_8^{'}}X_{24}^{'} + X_{23}^{'}\right)}{s - \frac{Y_7^{'}}{s - Y_8^{'}}Y_3^{'} - Y_2^{'}} + \dfrac{Y_6}{s - Y_8^{'}}X_{24}^{'} \end{cases} \tag{A47}$$