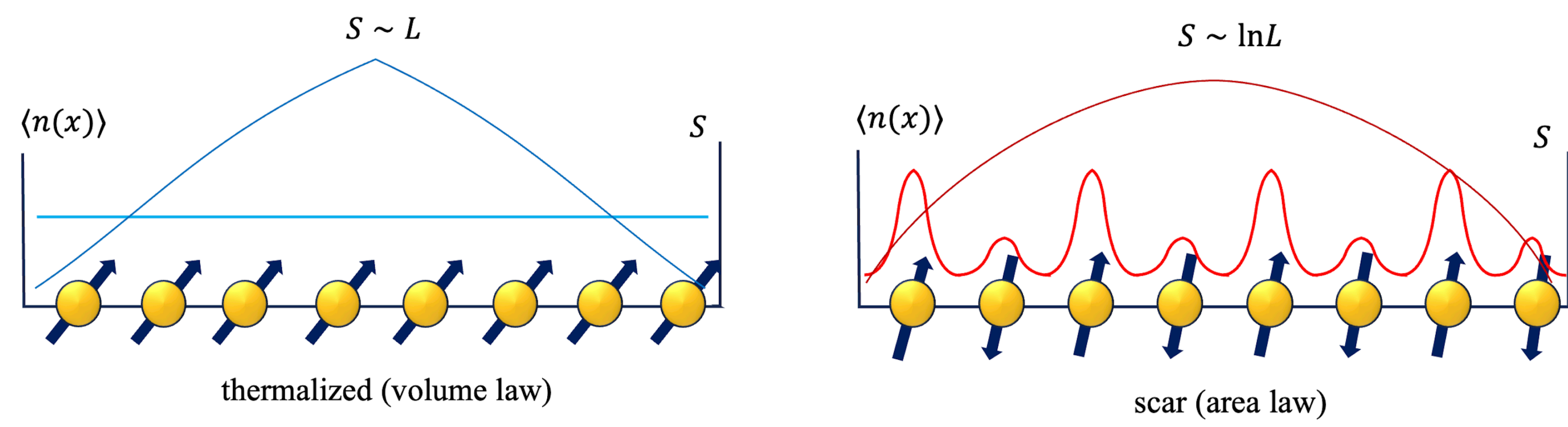


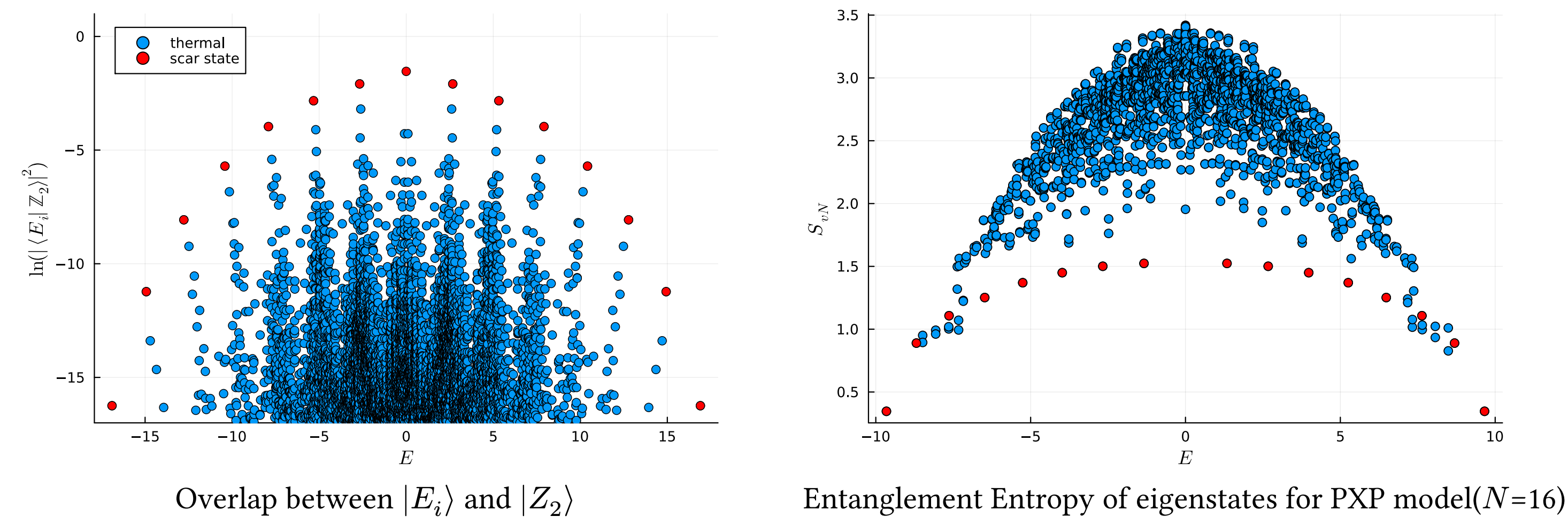
Thermalization and Entanglement



Quantum entanglement originates from the interaction between particles and typically spreads ballistically in a quantum many-body system. The thermodynamics can emerge from such isolated quantum system, where the entropy is related to the quantum entanglement, obeying a volume law scaling.

The violation of thermalization is recently observed in the near-term quantum simulations[1], where certain high energy quantum states, dubbed quantum many-body scars, can maintain low entangled and retain its memory forever. It offers new possibilities for quantum information and energy manipulation.

Quantum many-body scars and PXP model



Quantum many-body scars allow for coherent oscillations and long-lived dynamics in systems that would otherwise equilibrate quickly[2]. Via the Rydberg blockade, the Rydberg atoms array can be approximated by a “PXP” model[3] :

$$H_{\text{PXP}} = \sum_{i=1}^N P_{i-1} \sigma_i^x P_{i+1}$$

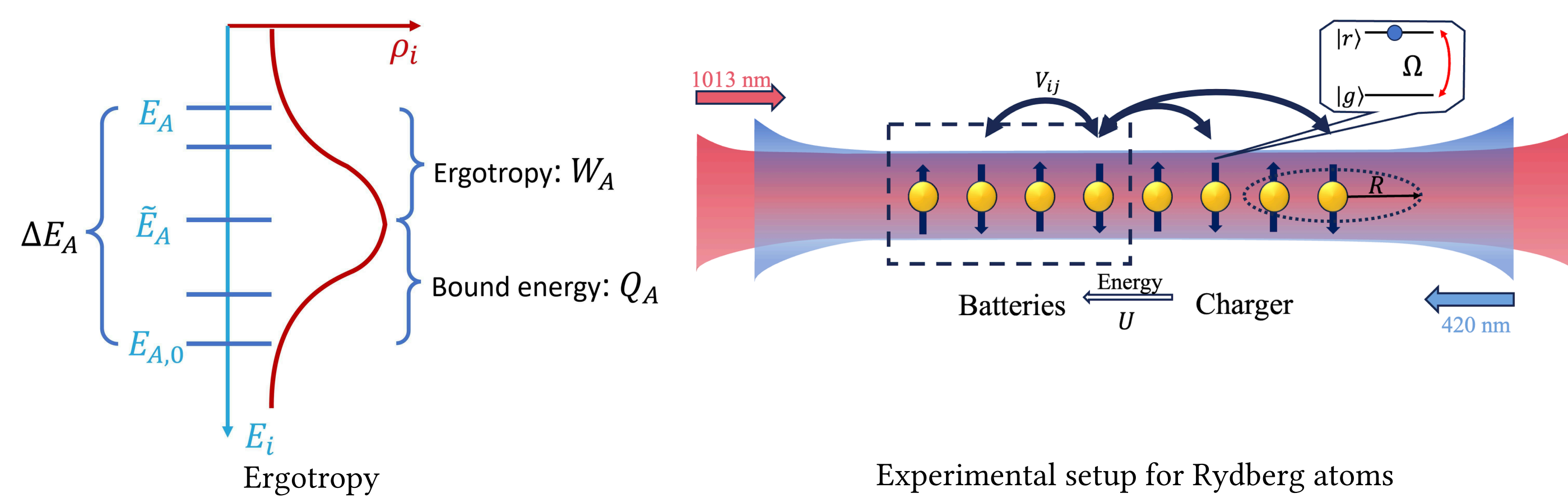
where $\sigma_i^x = |\odot\rangle\langle\bullet_i| + |\bullet_i\rangle\langle\odot_i|$ is the Pauli x matrix on site i , The projectors onto the ground state, $P_i = |\odot_i\rangle\langle\odot_i|$, constrain the dynamics by allowing an atom to flip its state only if both of its neighbors are in the ground state.

Quantum Battery and Ergotropy in spin chains

Quantum batteries are quantum devices that can store and supply energy [4], the performance of which to extract energy is determined by ergotropy.

Ergotropy is the maximal amount of work that can be extracted from a quantum state by applying unitary operations. The ergotropy of a mixed state ρ with respect to H is defined as:

$$W = \text{tr}(\rho H) - \min_U [\text{tr}(U \rho U^\dagger H)]$$

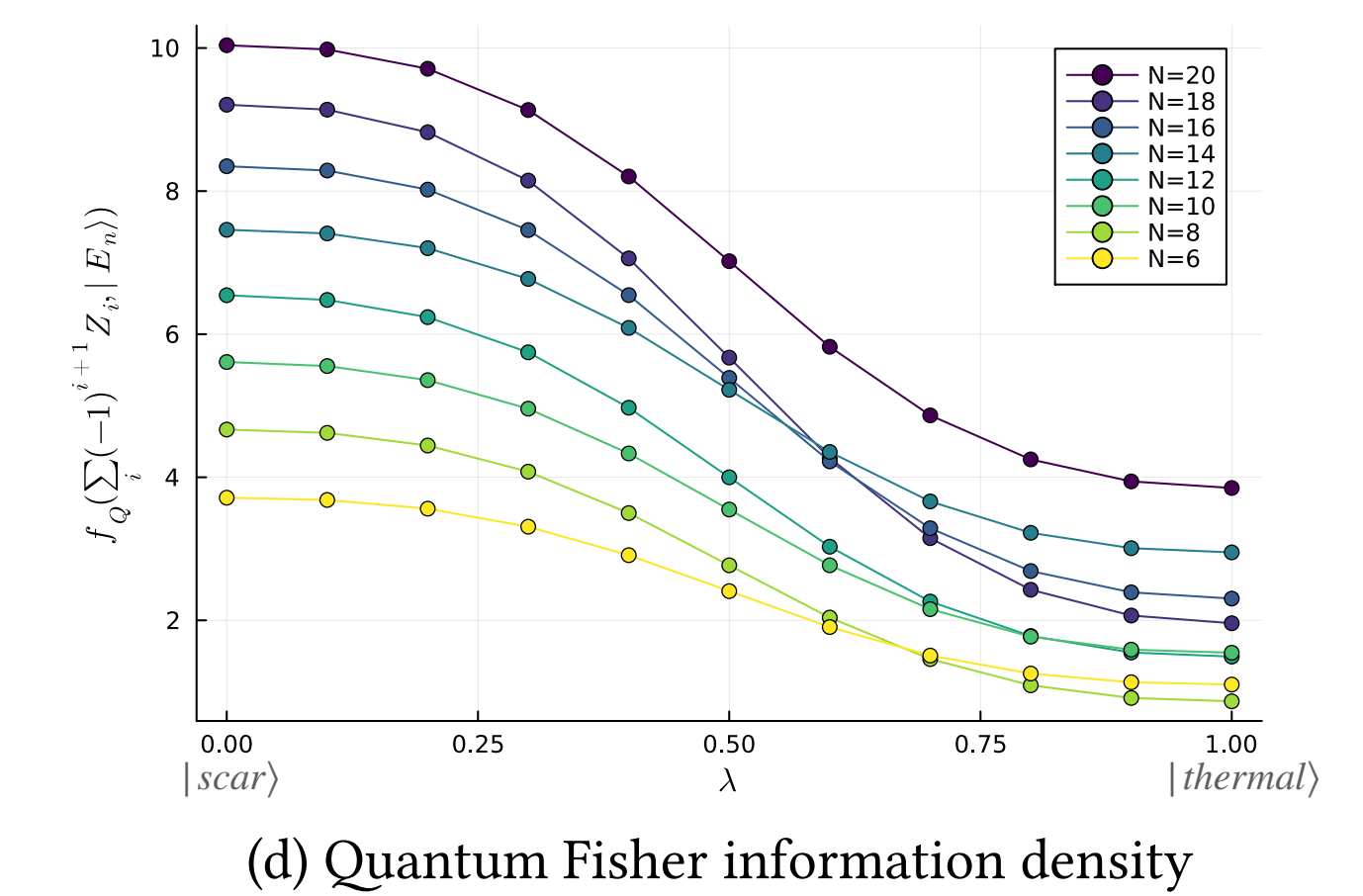
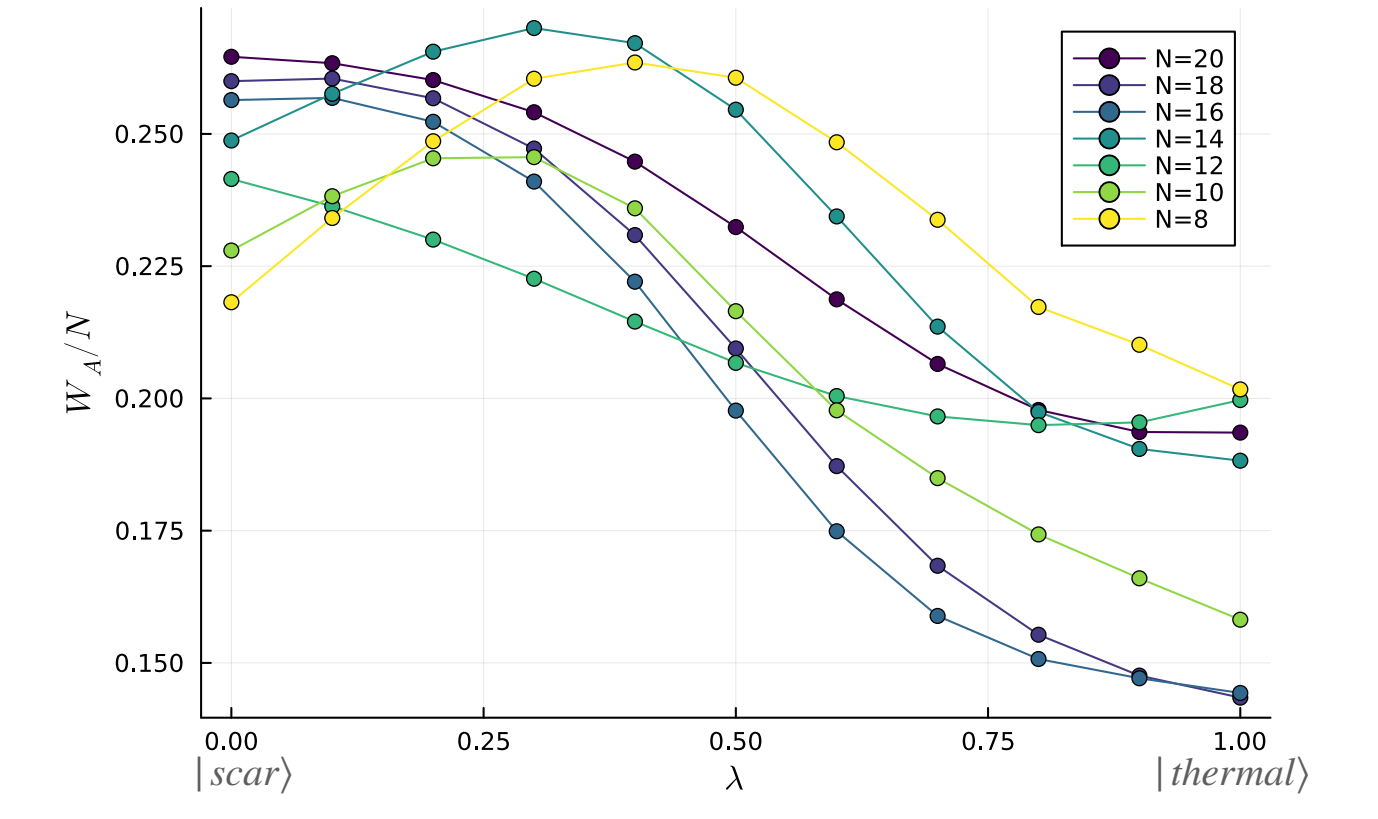
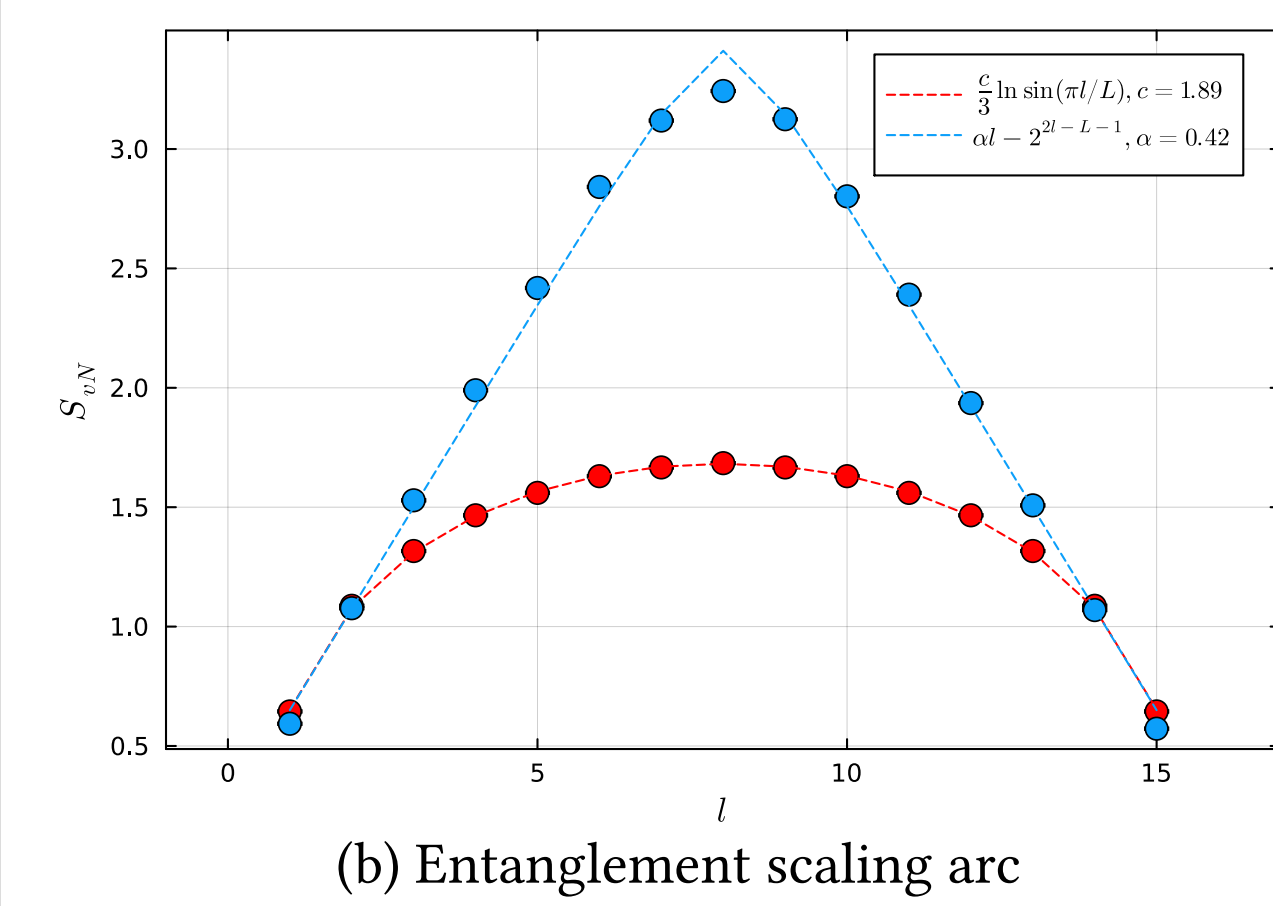
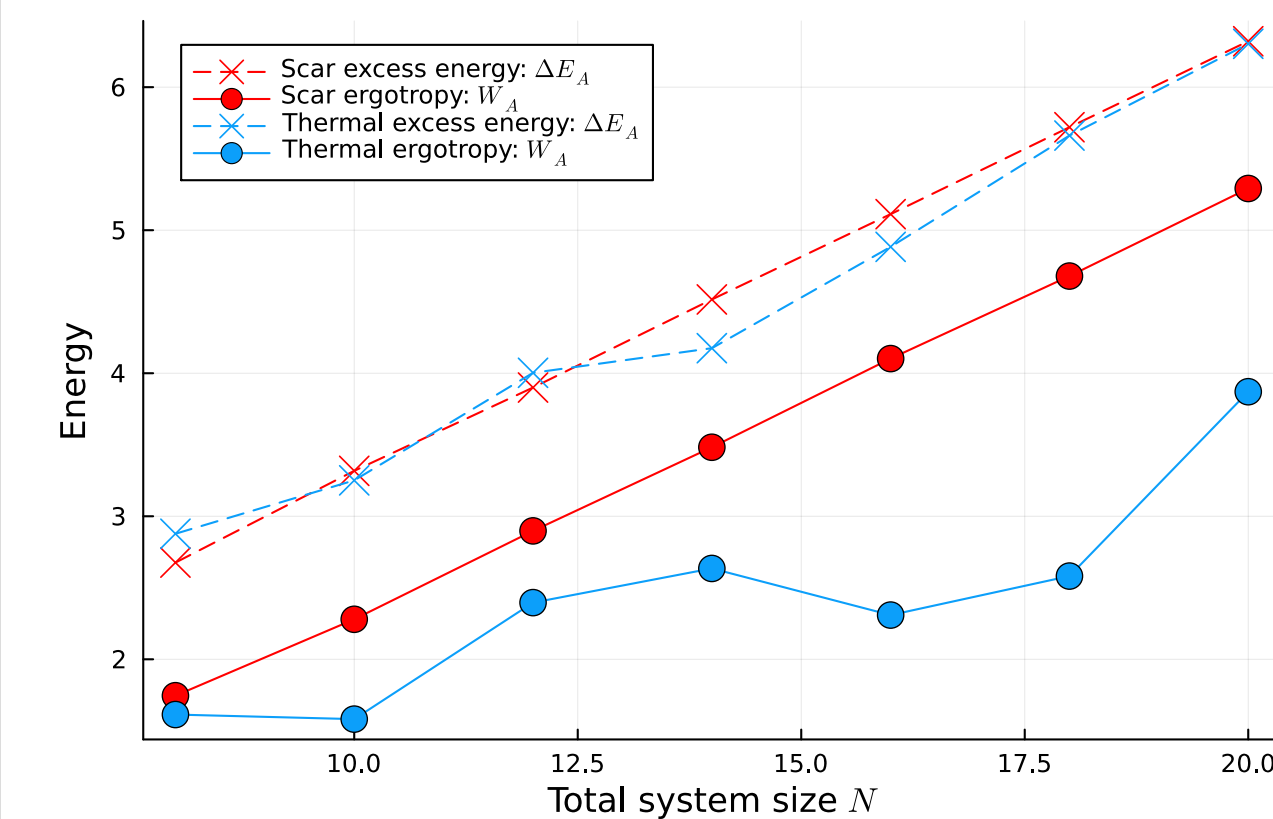


- $\Delta E_A = \text{tr}(\rho_A H_A) - \min(H_A)$, the energy charged to the battery.
- $Q_A = \min_U (\text{tr}(\rho_A U^\dagger H_A) - \min(H_A))$, the gap denotes the bound energy inaccessible by any unitary operations.

The right figure shows experimental setup for Rydberg atoms, individual ^{87}Rb atoms which are trapped using optical tweezers arranged into defect-free arrays. Coherent interactions V_{ij} between the atoms are enabled by exciting them to a Rydberg state with strength Ω .

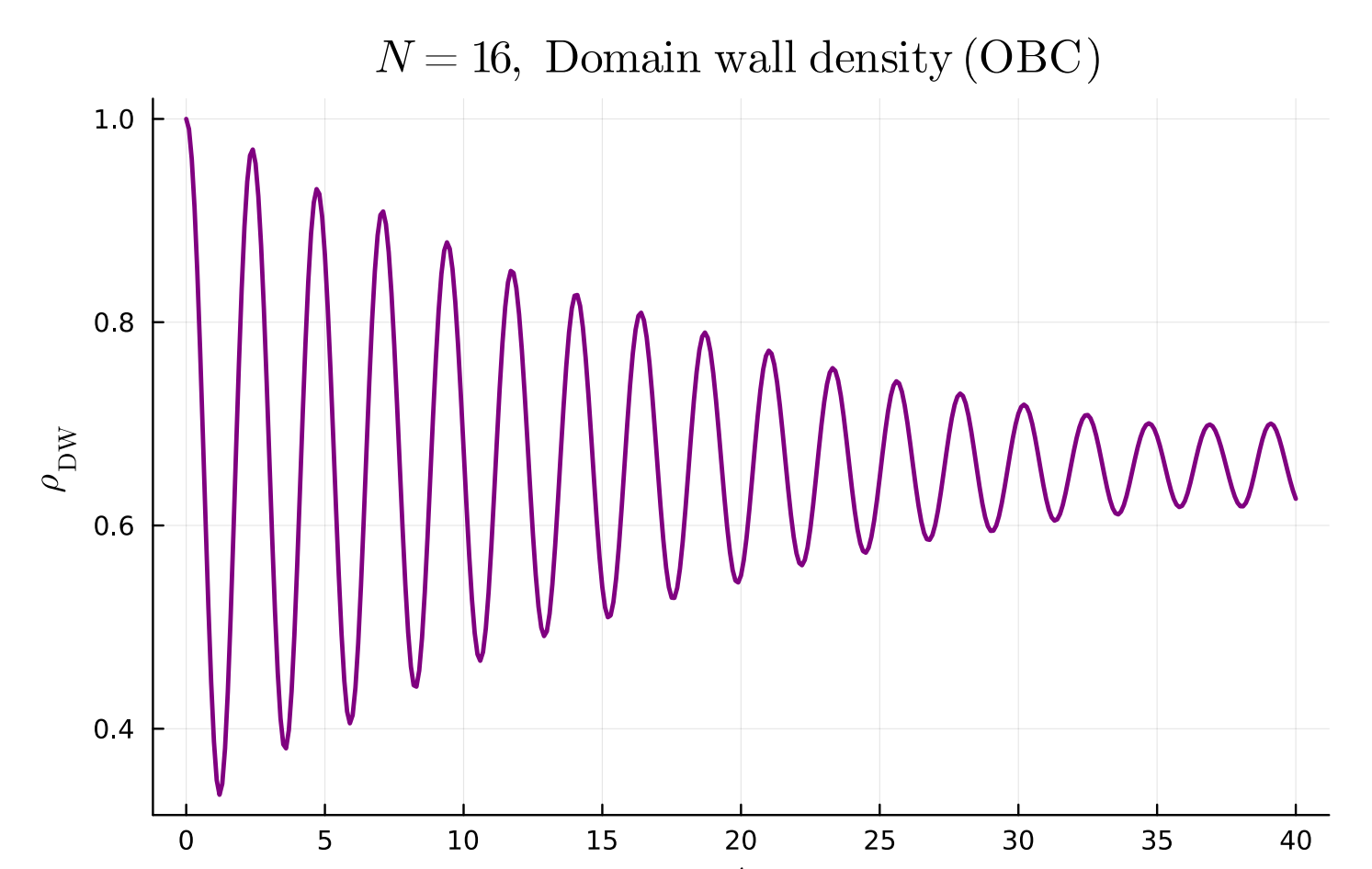
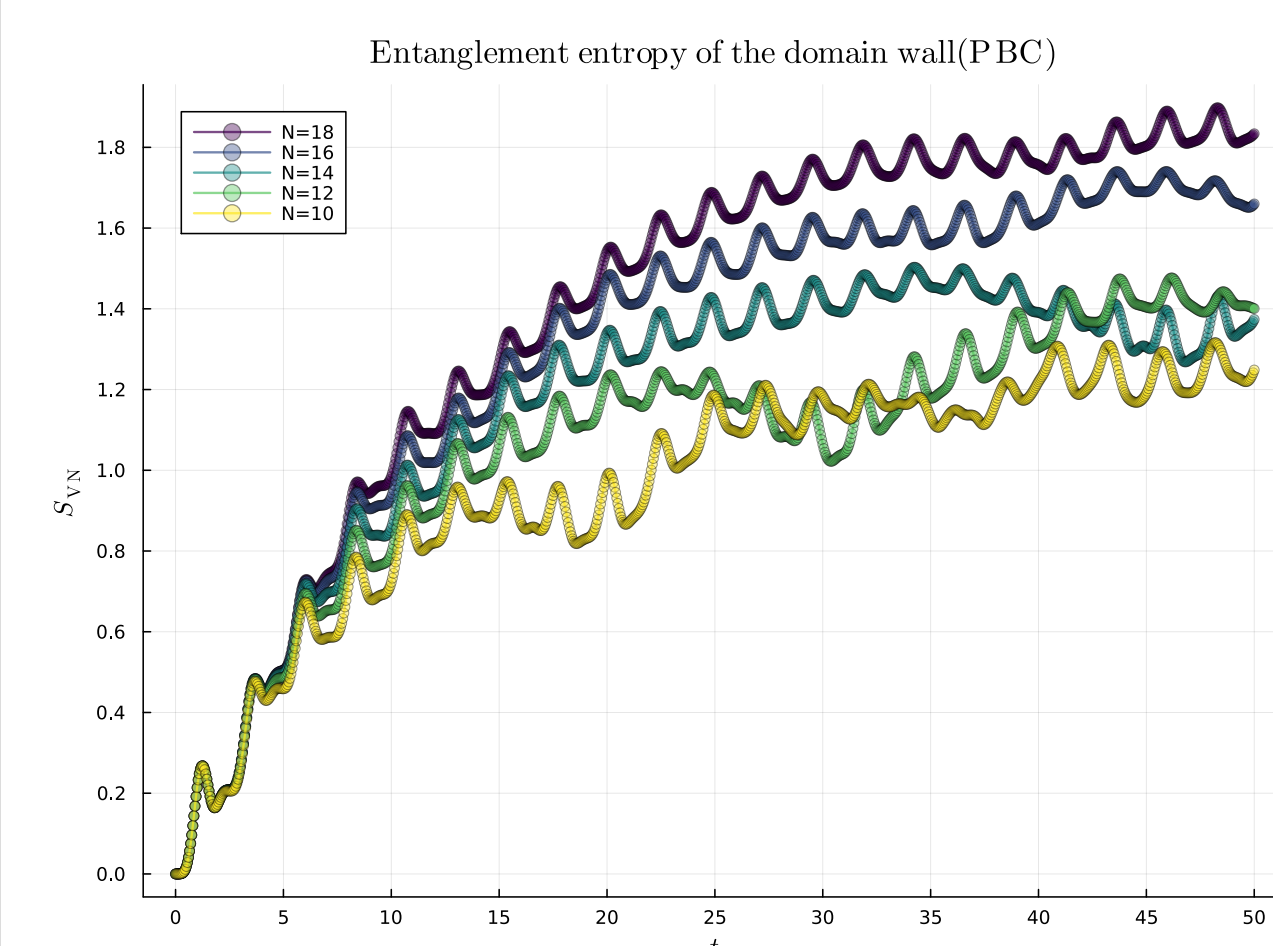
Numerical Results

Eigenstate

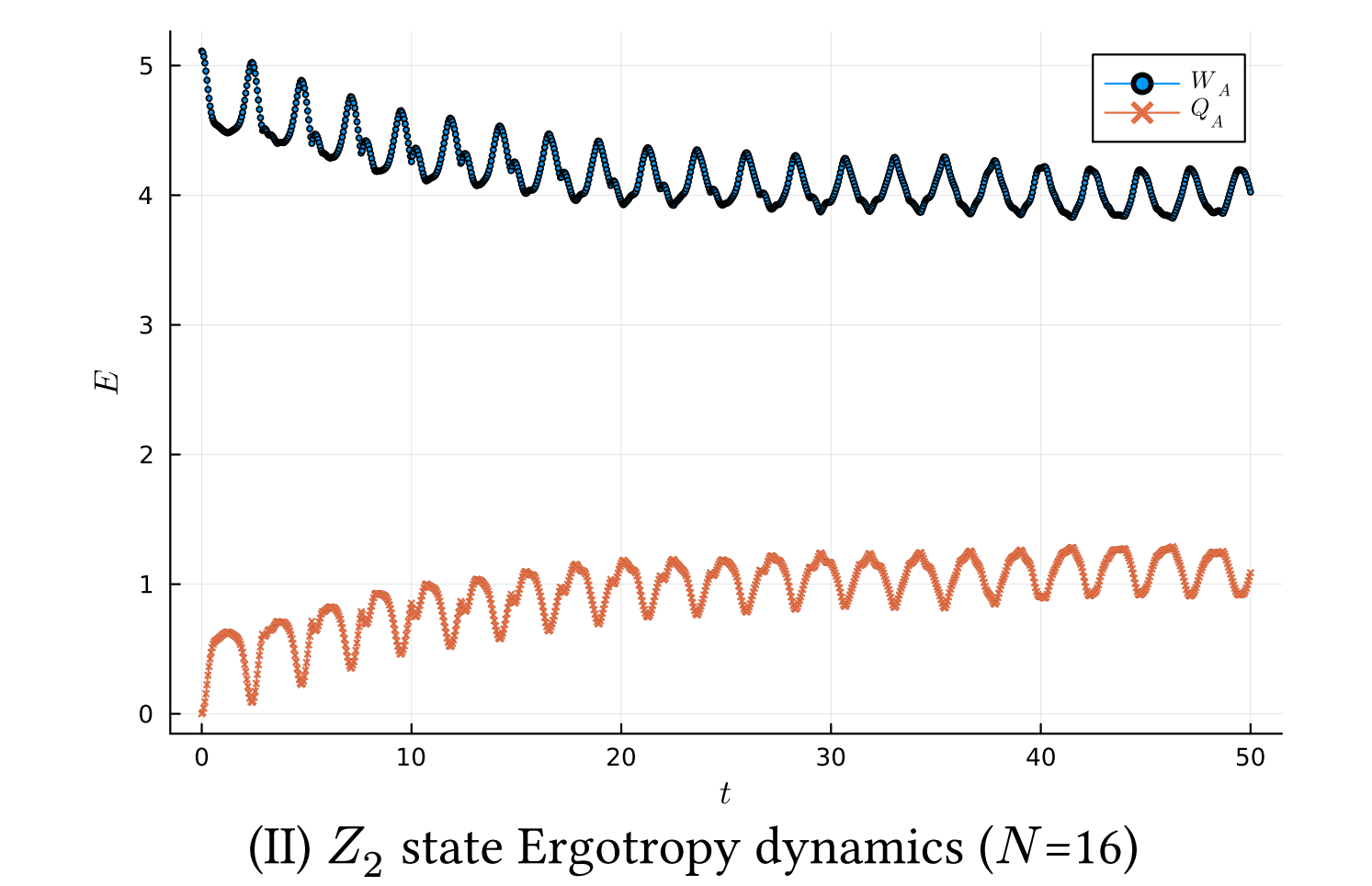
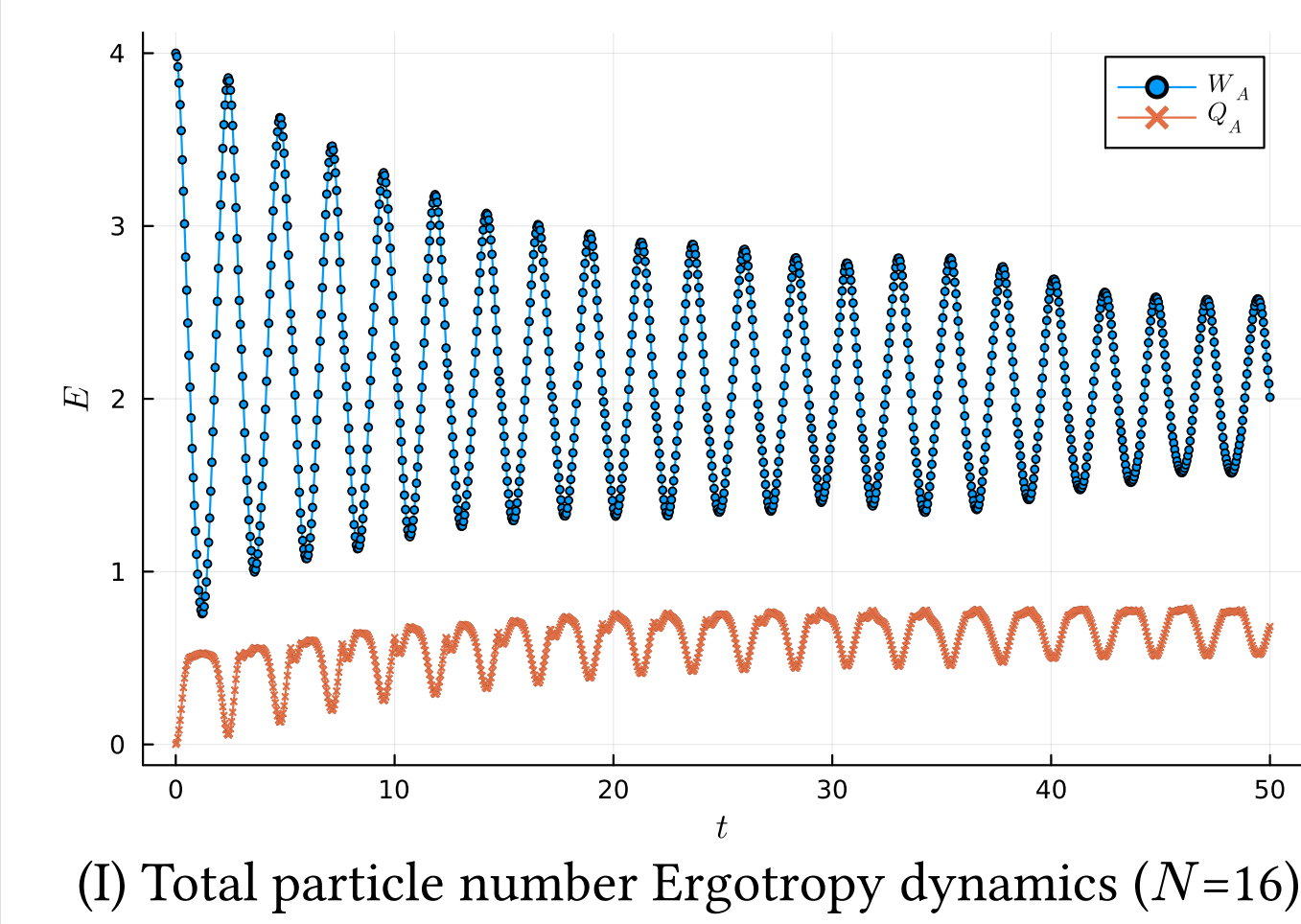


- (a) Scar v.s. thermal state: finite size scaling of their ergotropy and charged energies. The low bound energy gap of the scar state is due to its low entanglement entropy.
- (b) The entanglement entropy: scar obeys the critical area law; the thermal state obeys the volume law. The low complexity of the scar allows for a matrix product state (MPS) representation.
- (c) Superposition of scar and thermal states tunes the ergotropy density.
- (d) Superposition of scar and thermal states tunes the multipartite entanglement, witnessed by the quantum Fisher information.

Dynamics



The spread of entanglement entropy and the relaxation of the domain wall density: $\rho_{\text{DW}} = \sum_i \frac{1-Z_i}{2N}$ after a quantum quench.



The relaxation of the energy and the ergotropy density after the quench. The oscillations are attributed to the scar contributions.

- (I) The energy is defined by the non-interacting Hamiltonian: $H = \sum_j \frac{1-Z_j}{2N}$
- (II) The energy is defined by the PXP interacting Hamiltonian.
- More numerical results in preparation.

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