

# Frame

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当独立出压强分量时，原先的 $\psi$ 演化就表示成了

$$\frac{\partial \psi}{\partial t} = F\psi + ip\psi + i\frac{\hbar}{2}\nabla^2\psi$$

此时的 $F = aj + bk = (a + bi)j = fj$ 只有两个分量，我们可以进一步推导复数形式的演化表达式。令 $\psi = \psi_1 + \psi_2 = (\phi_1 + \phi_2 i) + (\phi_3 + \phi_4 i)j$ ，于是

$$\begin{aligned} F\psi &= (-a\phi_3 - b\phi_4) + (a\phi_4 - b\phi_3)i + (a\phi_1 + b\phi_2)j + (b\phi_1 - a\phi_2)k \\ &= [(-a\phi_3 - b\phi_4) + (a\phi_4 - b\phi_3)i] + [(a\phi_1 + b\phi_2) + (b\phi_1 - a\phi_2)i]j \\ &= -f\bar{\psi}_2 + f\bar{\psi}_1 j \end{aligned}$$

于是就有

$$\begin{cases} \frac{\partial \psi_1}{\partial t} = -f\bar{\psi}_2 + ip\psi_1 + i\frac{\hbar}{2}\nabla^2\psi_1 \\ \frac{\partial \psi_2}{\partial t} = f\bar{\psi}_1 + ip\psi_2 + i\frac{\hbar}{2}\nabla^2\psi_2 \end{cases}$$