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第9章 2, 5, 7, 8, 9, 13, 21 (a, b, i, j), 22 (a, b, c), 28, 31, 32, 33, 34, 35.

$$\begin{aligned} 2. (a) X(s) &= \int_{-\infty}^{+\infty} x(t) e^{-st} dt \\ &= \int_1^{+\infty} e^{-(s+5)t} dt \\ &= -\frac{1}{s+5} e^{-(s+5)t} \Big|_1^{+\infty} \\ &= \frac{e^{-(s+5)}}{s+5}, \quad \text{Re}(s) > -5. \end{aligned}$$

$$\begin{aligned} (b) G(s) &= \int_{-\infty}^{-t_0} A e^{-st} e^{-st} dt \\ &= \frac{A}{s+5} e^{-(s+5)t} \Big|_{t=t_0}^{-\infty} \\ \therefore A &= 1, t_0 = -1. \end{aligned}$$

收敛域为 $\text{Re}(s) < -5$.

5. (a) 在 s 平面有极点 $s = -2$.

在无限远有一个极点

(b) 在 s 平面无极点

在无限远有一个极点

(c) 在 s 平面有一个极点 $s = 1$

在无限远无极点

$$\begin{aligned} 7. \quad & \begin{array}{|c|c|c|} \hline & 1 & \\ \hline -2 & -1 & \\ \hline \end{array} & \begin{aligned} s^2 + s + 1 &= 0 \\ s^2 + s + \frac{1}{4} &= -\frac{3}{4} \\ s &= -\frac{1}{2} \pm \frac{\sqrt{3}}{2}j \end{aligned} \end{aligned}$$

有 4 个信号.

$$\begin{aligned} 8. \quad g(t) &= x(t) e^{2t} & G(s) &= X(s-2) \\ x(t) &= g(t) e^{-2t} & \therefore G(s) &\text{有两极点 } -1 \text{ 和 } 1. \\ g(t) &\text{收敛} & \therefore G(j\omega) &\text{收敛} \\ \therefore x(t) &\text{收敛域为 } 0 & \text{在收敛域内} \\ \therefore x(t) &\text{是右边信号} & \therefore g(t) &\text{是双边信号} \\ & & \therefore x(t) &\text{是双边信号} \end{aligned}$$

9. $X(s) = \frac{2(s+2)}{(s+3)(s+4)}$

$= 2 \left(\frac{2}{s+4} - \frac{1}{s+3} \right)$

$\text{Re}\{s\} > -3 \therefore X(t)$ 是右信号.

$\therefore X(t) = 4e^{-4t} u(t) - 2e^{-3t} u(t)$

13. $g(t)$ 有两极点 1 和 -1 . $\therefore \lim_{s \rightarrow 0} s = 0$.

$g(t) = X(t) + \alpha X(-t)$

$X(t) = \beta e^{-t} u(t)$

$X(-t) = \beta e^t u(-t)$

$\therefore g(t) = \beta e^{-t} u(t) + \alpha \beta e^t u(-t)$

$G(s) = \frac{1}{2} \frac{1}{s+1} + \frac{1}{2} \frac{1}{s-1}$

$e^{-t} u(t) \rightsquigarrow \text{Laplace transform } \frac{1}{s+1} \quad \text{Re}\{s\} > -1$

$e^t u(-t) \rightsquigarrow \text{Laplace transform } -\frac{1}{s-1} \quad \text{Re}\{s\} < 1$

$\therefore \alpha = -1, \beta = \frac{1}{2}$

21. (a) $X(s) = \frac{1}{s+2} + \frac{1}{s+3} \quad \text{Re}\{s\} > -2$

$\text{Im.} = \frac{2s+5}{s^2+s+6}$

(b) $X(t) = e^{-4t} u(t) + e^{-st} (\sin St) u(t)$

由欧拉公式得

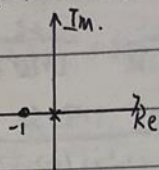
$x(t) = e^{-4t} u(t) + \frac{1}{2j} (e^{-(s-jS)t} - e^{-(s+jS)t}) u(t)$

$\therefore X(s) = \frac{1}{s+4} + \frac{1}{2j} \left(\frac{1}{s-jS} - \frac{1}{s+jS} \right)$

$= \frac{1}{s+4} + \frac{S}{(s+4)^2 + 25} \quad \text{Re}\{s\} > -4$

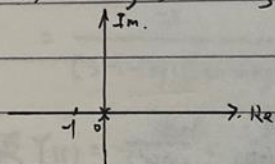
(i). $x(t) = \delta(t) + u(t)$

$X(s) = 1 + \frac{1}{s} \quad \text{Re}\{s\} > 0$



(j). $x(t) = \delta(3t) + u(3t)$

$X(s) = 1 + \frac{1}{3} \times \frac{3}{s} = 1 + \frac{1}{s} \quad \text{Re}\{s\} > 0$



22. (a) $\therefore \sin 3t u(t) \xrightarrow{L} \frac{3}{s^2 + 9} \quad \text{Re}\{s\} > 0$

$\therefore \frac{1}{3} \sin 3t u(t) \xrightarrow{L} \frac{1}{s^2 + 9} \quad \text{Re}\{s\} > 0$

$\therefore x(t) = \frac{1}{3} \sin 3t u(t)$

(b). $\therefore \cos 3t u(t) \xrightarrow{L} \frac{s}{s^2 + 9} \quad \text{Re}\{s\} > 0$

$\therefore \cos -3t u(t) \xrightarrow{L} -\frac{s}{s^2 + 9} \quad \text{Re}\{s\} < 0$

$\therefore x(t) = -\cos 3t u(t)$

(c). ~~由~~ 由 s 域平稳性质得

$x(t) = -e^{-t} \cos 3t u(1-t)$

28. (a) $\text{Re}\{s\} < -2$

$-2 < \text{Re}\{s\} < -1$

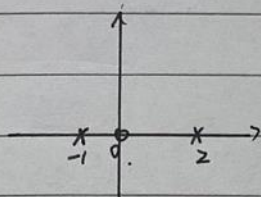
$-1 < \text{Re}\{s\} < 1$

~~$\text{Re}\{s\} > 1$~~

~~$\text{Re}\{s\} > 2$~~

(b). 对于 $\text{Re}\{s\} > 1$, 有系统是因果的, 其他是非因果的.
对于 $\text{Re}\{s\} < 1$, 有系统是稳定的, 其他是不稳定的.

31. (a) $H(s) = \frac{1}{s^2 - s - 2}$



(b) (i). \therefore 系统是稳定的

\therefore 收敛域包括 $j\omega$ 轴.

\therefore 收敛域为 $-1 < \text{Re}\{s\} < 2$.

$$H(s) = \frac{1}{3} \frac{1}{s-2} - \frac{1}{3} \frac{1}{s+1}$$

$$\therefore h(t) = -\frac{1}{3} e^{-t} u(t) + \frac{1}{3} e^{2t} u(t)$$

(ii) \therefore 系统是因果的

\therefore 收敛域为 $\text{Re}\{s\} > 2$.

$$\therefore h(t) = -\frac{1}{3} e^{-t} u(t) + \frac{1}{3} e^{2t} u(t)$$

(iii) \therefore 系统既不稳定又不因果

\therefore 收敛域为 $\text{Re}\{s\} < -1$.

$$\therefore h(t) = \frac{1}{3} e^{-t} u(-t) - \frac{1}{3} e^{2t} u(-t)$$

32. $x(t) = e^{2t}$ 时 $y(t) = \frac{1}{b} e^{2t}$.

$$\therefore H(2) = \frac{1}{b}$$

$$sH(s) + 2H(s) = \frac{1}{s+4} + \frac{b}{s} \quad \text{Re}\{s\} > 0$$

$$4H(2) = \frac{2}{3} = \frac{1}{b} + \frac{b}{2}$$

$$\Rightarrow b = 1$$

$$\therefore H(s) = \frac{2}{s(s+4)} \quad \text{Re}\{s\} > 0$$

②

$$33. x(t) = e^{-t} u(t) + e^t u(-t)$$

$$\therefore X(s) = \frac{1}{s+1} - \frac{1}{s-1}, \quad -1 < \operatorname{Re}\{s\} < 1$$

$$\therefore Y(s) = \frac{s+1}{s^2+2s+2} \cdot \left(\frac{1}{s+1} - \frac{1}{s-1} \right)$$

$$= \frac{-2(s+1)}{(s^2+2s+2)(s^2-1)}$$

$$= \frac{-2}{(s^2+2s+2)(s-1)}$$

$$= \frac{-2}{[s-(1+j)] [s-(-1-j)] (s-1)}, \quad -1 < \operatorname{Re}\{s\} < 1$$

$$\text{设 } Y(s) = \frac{As+B}{s^2+2s+2} + \frac{C}{s-1}$$

$$\therefore As^2 - As + Bs - B + Cs^2 + 2Cs + 2C = -2$$

$$\Rightarrow C = -\frac{2}{5}, \quad A = \frac{2}{5}, \quad B = \frac{6}{5}$$

$$\therefore Y(s) = \frac{2}{5} \cdot \frac{s+3}{s^2+2s+2} - \frac{2}{5} \frac{1}{s-1}$$

$$= \frac{2}{5} \frac{s+1}{(s+1)^2+1} + \frac{4}{5} \frac{1}{(s+1)^2+1} - \frac{2}{5} \frac{1}{s-1}, \quad -1 < \operatorname{Re}\{s\} < 1$$

$$\therefore y(t) = \frac{2}{5} e^{-t} \cos t u(t) + \frac{4}{5} e^{-t} \sin t u(t) + \frac{2}{5} e^t u(-t)$$

34. $H(s)$ 因果稳定

\therefore 收敛域包含 $j\omega$ 轴且为右半信号

$$\frac{1}{s} H(s) \xrightarrow{s \rightarrow \infty} 0$$

$$\text{输入为 } u(t) \text{ 时 } X(s) = \frac{1}{s}, \quad \operatorname{Re}\{s\} > 0$$

$$\therefore \frac{1}{s} H(s) \text{ 绝对可积}$$

$$\therefore H(0) = 0$$

$$\text{输入为 } tu(t) \text{ 时 } X(s) = \frac{1}{s^2}, \quad \operatorname{Re}\{s\} > 0$$

$$\therefore \frac{1}{s^2} H(s) \text{ 不绝对可积}$$

\therefore 零极点不限 = 次项

$$\text{设 } Y(s) = s^2 H(s) + 2s H(s) + 2H(s)$$

$$\therefore H(s) = \frac{Y(s)}{s^2+2s+2}$$

$\therefore Y(s)$ 是有限持续期且绝对可积

$\therefore Y(s)$ 的收敛域是整个 s 平面

~~$\Rightarrow H(s) = 0.2$~~

~~$\Rightarrow H(s) = 1$~~

$\therefore Y(s)$ 没有极点 只有零点

$\therefore H(s)$ 有一个零点且 $H(1) = 0.2$

$\therefore H(s) = \frac{s}{s^2 + 2s + 2} \quad \operatorname{Re}\{s\} > -1$

25. (a) $\frac{Y(s)}{X(s)} = \frac{s^2 - s - 6}{s^2 + 2s + 1}$

\therefore 微分方程为 $y''(t) + 2y'(t) + y(t) = x''(t) - x'(t) - 6x(t)$

(b) \therefore 该系统是因果的

极点为 $\operatorname{Re}\{s\} = -1 < 0$

\therefore 该系统是稳定的