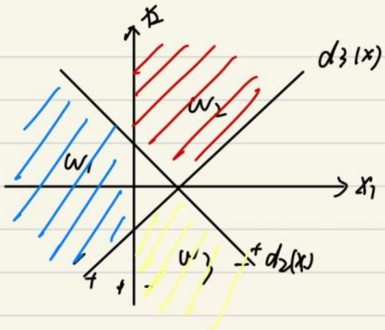
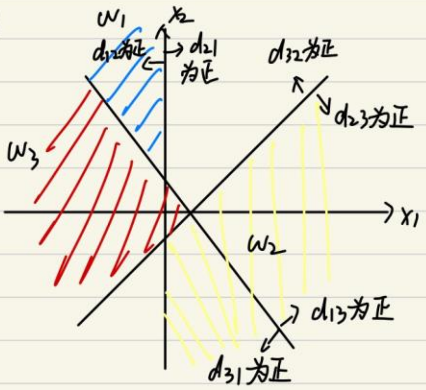
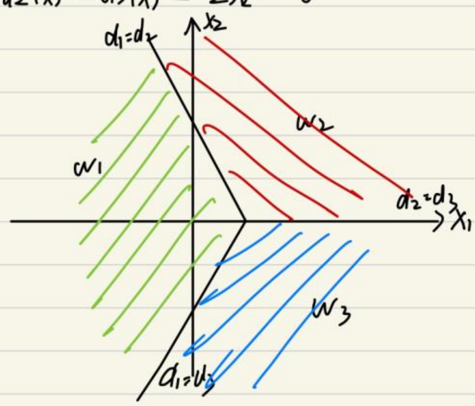
1. 解 (a) 判别界面 $\begin{cases} d_1(x) = -x_1 = 0 \\ d_2(x) = x_1 + x_2 - 1 = 0 \\ d_3(x) = x_1 - x_2 - 1 = 0 \end{cases}$



(b)



(c) $\begin{cases} d_1(x) - d_2(x) = -2x_1 - x_2 + 1 = 0 \\ d_1(x) - d_3(x) = -2x_1 + x_2 + 1 = 0 \\ d_2(x) - d_3(x) = 2x_2 = 0 \end{cases}$



2、解：类内总离差阵 $S_w = S_1 + S_2 = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

$\therefore S_w^{-1} = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix}$

$\vec{u} = S_w^{-1} (\vec{m}_1 - \vec{m}_2) = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$

$\therefore y_t = \dfrac{u' m_1 + u' m_2}{2} = -1$

3、$\vec{m}_1 = mean(w_1) = (0, \frac{2}{3}, \frac{2}{3})^T$

$\vec{m}_2 = mean(w_2) = (1, \frac{2}{3}, \frac{2}{3})^T$

$S_{w1} = \sum_j (\vec{x}_j' - \vec{m}_1)(\vec{x}_j' - \vec{m}_1)' = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{2}{3} & -\frac{1}{3} \\ 0 & -\frac{1}{3} & \frac{2}{3} \end{bmatrix}$

$S_{w2} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{2}{3} & -\frac{1}{3} \\ 0 & -\frac{2}{3} & \frac{2}{3} \end{bmatrix}$

$S_w = S_{w1} + S_{w2} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{4}{3} & -\frac{2}{3} \\ 0 & -1 & \frac{4}{3} \end{bmatrix}$

4、解：判别界面：$y = 3x_1 + 2x_2 + 6 = 0$

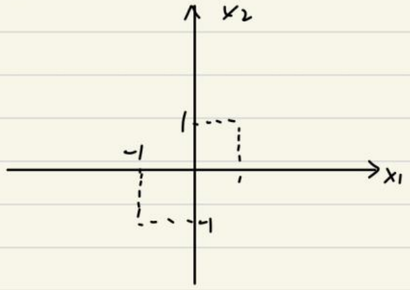
$\therefore$ 原点到判别界面的距离为 $d = \frac{6}{13}\sqrt{13}$

原点在 $y > 0$ 的一侧

5、代价函数 $\min_{w,b} \max_{\alpha} \{ \frac{1}{2} w^T w + \sum_{n=1}^{N} \alpha_n [1 - y_n (w^T x_n + b)] \}$

先优化 w 和 b，再优化 α 比原始问题更容易，优化 w 和 b 是通过求一次导实现的，显然比优化 α 容易。可以优先解决容易的部分

6、



两类样本线性不可分

$(1,1)^T$ → $(1,1,1,1)$

$(-1,-1)^T$ $\xrightarrow{\varphi(x)}$ $(1,-1,-1,1)$ 线性可分

$(0,-1)^T$ → $(1,0,-1,0)$

$(-1,0)^T$ → $(1,-1,0,0)$