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3. 解：(a) $\sin(2\pi t+\frac{\pi}{4})$

设 $x(t)=\sin(2\pi t+\frac{\pi}{4})$

由欧拉关系 $e^{j(2\pi t+\frac{\pi}{4})}=\cos(2\pi t+\frac{\pi}{4})+j\sin(2\pi t+\frac{\pi}{4})$

$e^{-j(2\pi t+\frac{\pi}{4})}=\cos(2\pi t+\frac{\pi}{4})-j\sin(2\pi t+\frac{\pi}{4})$

$\therefore \sin(2\pi t+\frac{\pi}{4})=\frac{1}{2j}(e^{j(2\pi t+\frac{\pi}{4})}-e^{-j(2\pi t+\frac{\pi}{4})})$

$=\frac{e^{j\frac{\pi}{4}}}{2j}e^{j2\pi t}-\frac{e^{j-\frac{\pi}{4}}}{2j}e^{-j2\pi t}.$

$\therefore X(j\omega)=\frac{\pi}{j}e^{j\frac{\pi}{4}}\delta(\omega-2\pi)-\frac{\pi}{j}e^{-j\frac{\pi}{4}}\delta(\omega+2\pi).$

(b) $1+\cos(6\pi t+\frac{\pi}{8})$

设 $y(t)=1+\cos(6\pi t+\frac{\pi}{8})$

~~由分析~~ ~~公式~~ $\cancel{a_k=\frac{1}{T}\int_T x(t)}$ 综合 $x(t)=\sum_{k=-\infty}^{+\infty}a_k e^{jk\omega_0 t}$

$\Rightarrow a_0=1$

由欧拉关系：$e^{j(6\pi t+\frac{\pi}{8})}=\cos(6\pi t+\frac{\pi}{8})+j\sin(6\pi t+\frac{\pi}{8})$

$e^{-j(6\pi t+\frac{\pi}{8})}=\cos(6\pi t+\frac{\pi}{8})-j\sin(6\pi t+\frac{\pi}{8})$

$\Rightarrow \cos(6\pi t+\frac{\pi}{8})=\frac{1}{2}e^{j(6\pi t+\frac{\pi}{8})}+\frac{1}{2}e^{-j(6\pi t+\frac{\pi}{8})}$

$\therefore Y(j\omega)=2\pi\delta(\omega)+\pi e^{j\frac{\pi}{8}}\delta(\omega-6\pi)+\pi e^{-j\frac{\pi}{8}}\delta(\omega+6\pi).$

4(a) 解：$x(t)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}X(j\omega)e^{j\omega t}d\omega.$

$=\frac{1}{2\pi}\int_{-\infty}^{+\infty}2\pi\delta(\omega)e^{j\omega t}d\omega+\frac{1}{2\pi}\int_{-\infty}^{+\infty}\pi\delta(\omega-4\pi)e^{j\omega t}d\omega+\frac{1}{2\pi}\int_{-\infty}^{+\infty}\pi\delta(\omega+4\pi)e^{j\omega t}d\omega$

$=1+\frac{1}{2}e^{j4\pi t}+\frac{1}{2}e^{-j4\pi t}$

$=1+\cos 4\pi t$

10 解(a) $x(t)=t\left(\frac{\sin t}{\pi t}\right)^2$

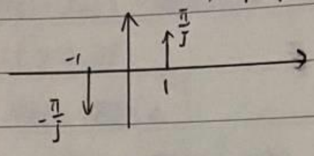
$= t\,\frac{\sin t}{\pi t}\cdot\frac{\sin t}{\pi t}$

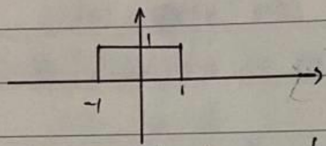
$= \frac{1}{\pi}\sin t\cdot\frac{\sin t}{\pi t}$

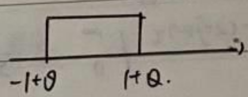
令 $x_1(t)=\sin t \qquad x_2(t)=\frac{\sin t}{\pi t}$

则 $X_1(j\omega)=\frac{\pi}{j}[\delta(\omega-1)-\delta(\omega+1)]$

$X_2(j\omega)=\begin{cases}1, & |\omega|<1\\ 0, & |\omega|>1\end{cases}$

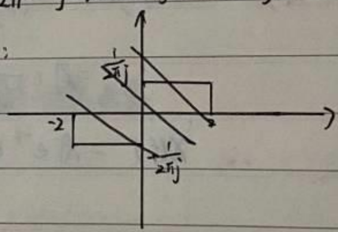


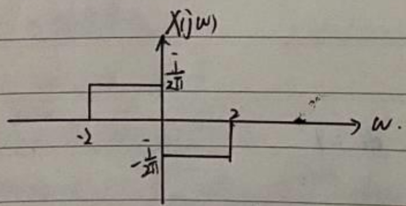




由乘积性质得 $X(j\omega)=\frac{1}{2\pi^2}\int_{-\infty}^{+\infty}X_1(j\theta)X_2(j(\omega-\theta))\,d\theta$

∴ Fourier Transform:





(b) 由帕斯瓦尔定理 $\int_{-\infty}^{+\infty}|x(t)|^2dt=\frac{1}{2\pi}\int_{-\infty}^{+\infty}|X(j\omega)|^2d\omega$

$\therefore A=\frac{1}{2\pi}\int_{-2}^{0}-\frac{1}{4\pi^2}\,d\omega+\frac{1}{2\pi}\int_{0}^{2}-\frac{1}{4\pi^2}\,d\omega$

$=\frac{1}{2\pi^3}$

11.解: $y(t)=x(t)*h(t)$,　$g(t)=x(3t)*h(3t)$

由卷积性质得 $Y(j\omega)=X(j\omega)\cdot H(j\omega)$

$$G(j\omega)=\frac{1}{3}X(\frac{j\omega}{3})\cdot\frac{1}{3}H(\frac{j\omega}{3})$$

$$=\frac{1}{9}X(\frac{j\omega}{3})H(\frac{j\omega}{3})$$

$$Y(\frac{j\omega}{3})=X(\frac{j\omega}{3})H(\frac{j\omega}{3})$$

$\therefore G(j\omega)=\frac{1}{9}Y(\frac{j\omega}{3})$

$\therefore g(t)=\frac{1}{3}y(3t)$.

$\therefore A=\frac{1}{3}$　$B=3$.

14.解: $\int_{-\infty}^{+\infty}Ae^{-2t}u(t)e^{-j\omega t}dt$

$=\int_{0}^{+\infty}Ae^{-(2+j\omega)t}dt$

$=-A\frac{1}{2+j\omega}e^{-(2+j\omega)t}\Big|_{0}^{+\infty}=\frac{A}{2+j\omega}$.

$\therefore \frac{A}{2+j\omega}=(1+j\omega)X(j\omega)\Rightarrow X(j\omega)=\frac{A}{(1+j\omega)(2+j\omega)}$

$$=A(\frac{1}{1+j\omega}-\frac{1}{2+j\omega})$$

$\therefore x(t)=Ae^{-t}u(t)-Ae^{-2t}u(t)$

由帕斯瓦尔定理

$\int_{-\infty}^{+\infty}|x(t)|^2dt=1$

$\therefore \int_{0}^{\infty}(A^2e^{-2t}+A^2e^{-4t}-2A^2e^{-3t})dt=1$.

$\frac{1}{2}+\frac{1}{4}-\frac{2}{3}=\frac{1}{A^2}$

$\Rightarrow A^2=12$

$\because x(t)$ 为非负

$\therefore x(t)=2\sqrt{3}(e^{-t}-e^{-2t})u(t)$

15、解：$\frac{1}{2\pi}\int_{-\infty}^{+\infty} Re\{X(jw)\} e^{jwt} dw = |t| e^{-|t|}$

~~$\because t<0$ 时 $X(t)=0$.~~

~~$\therefore \frac{1}{2\pi}\int_{0}^{\infty} Re\{X(jw)\} e^{jwt} dw = t e^{-t}$.~~

$\because X(t)$ 为实值信号

$\therefore \frac{1}{2\pi} \quad Ev\{X(t)\} = |t| e^{-|t|}$

$Ev\{X(t)\} = 2\pi |t| e^{-|t|}$

$\therefore X(t)+X(-t) = 4\pi |t| e^{-|t|}$

$\therefore t>0$ 时

$X(t) = 4\pi t e^{-t}$

$\therefore X(t) = 4\pi t e^{-t} u(t)$

24.(a) (1) $Re\{X(jw)\}=0 \Rightarrow X(t)$ 是实奇信号.

$\therefore$ a. d.

(2) $Im\{X(jw)\}=0 \Rightarrow X(t)$ 是实偶信号

$\therefore$ e, f

(3) 存在实数 α, 使 $e^{jw\alpha}X(jw)$ 为实函数

时移 $-\alpha$.

$\because$ 0 也为实数

$\therefore$ a, b, e, f

(4) $\int_{-\infty}^{+\infty} X(jw)\, dw = 0$.

~~$X(0)$~~ $X(t) = \int_{-\infty}^{+\infty} X(jw) e^{jwt} dw$

$X(0) = \int_{-\infty}^{+\infty} X(jw)\, dw = 0$

$\therefore$ a, b, c, d, f.

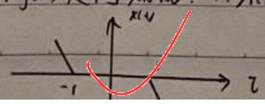
(5) $\int_{-\infty}^{+\infty} w X(jw)\, dw = 0$

$\int_{-\infty}^{+\infty} jw X(jw)\, dw = 0 \Rightarrow \left.\frac{dx(t)}{dt}\right|_{t=0} = 0$.

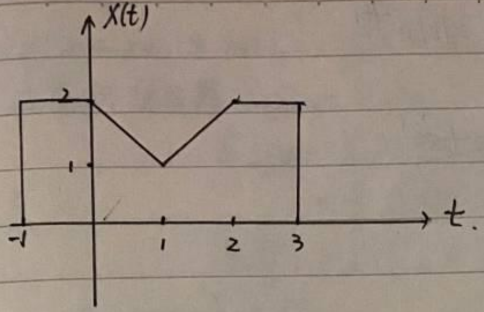
$\therefore$ b, c, e, f

(6) $X(jw)$ 是周期的. $\therefore X(t)$ 是周期的. $\therefore$ a.

(b)



25.



解: $X(j\omega)=\int_{-\infty}^{+\infty} x(t)e^{-j\omega t}dt$.

(a) ∵ $x(t+1)$是实偶信号

且$x(t+1)$的fourier transform $e^{j\omega}X(j\omega)$ 的幅角为0.

∴ $\measuredangle X(j\omega) = -\omega$.

(b) $X(j0)=\int_{-\infty}^{+\infty} x(t)\,dt = 2\times4-1=7$.

(c) $x(t)=\frac{1}{2\pi}\int_{-\infty}^{+\infty} X(j\omega)e^{j\omega t}d\omega$

∴ $x(0)=2=\frac{1}{2\pi}\int_{-\infty}^{+\infty} X(j\omega)d\omega$

∴ $\int_{-\infty}^{+\infty} X(j\omega)d\omega = 4\pi$.

(d) $\int_{-\infty}^{+\infty} X(j\omega)\frac{2\sin\omega}{\omega}e^{j2\omega}d\omega$.

由卷积性质得 $x(t)*y(t) \xleftrightarrow{F} X(j\omega)Y(j\omega)$

对于 $y(t)=\begin{cases}1, & |t|<1\\0, & |t|>1\end{cases}$. $Y(j\omega)=\frac{2\sin\omega}{\omega}$.

∴ $x(t)*y(t)=\frac{1}{2\pi}\int_{-\infty}^{+\infty} X(j\omega)\frac{2\sin\omega}{\omega}e^{j\omega t}d\omega$

令 $h(t)=x(t)*y(t)=\int_{-\infty}^{+\infty} x(\tau)y(t-\tau)d\tau$.

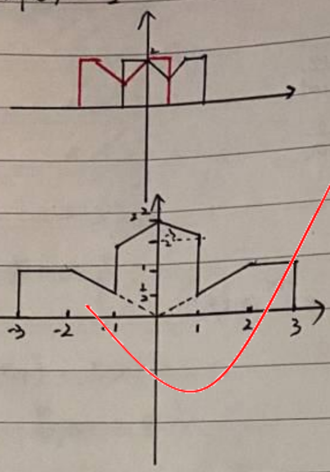
∴ $h(2)=2\times2-0.5=3.5$

∴ $\int_{-\infty}^{+\infty} X(j\omega)\frac{2\sin\omega}{\omega}e^{j2\omega}=7\pi$

(e) $\int_{-\infty}^{+\infty} |X(j\omega)|^2 d\omega = 2\pi\int_{-\infty}^{+\infty} |x(t)|^2 dt$

$=\frac{76}{3}\pi$

(f) 1. 即為 $\frac{x(t)+x(-t)}{2}$



32. 解：(a) $h(t)=\frac{\sin 4(t-1)}{\pi(t-1)}$ $\quad x(t)=\cos(6t+\frac{\pi}{2})$

$\therefore y(t)=x(t) * h(t)$

$\therefore Y(j\omega)=X(j\omega)H(j\omega)$

$$=e^{-j\omega}\cdot\begin{cases}1, & |\omega|<4\\ 0, & |\omega|>4\end{cases}\cdot e^{j\omega\frac{\pi}{12}}\,\pi[\delta(\omega-6)+\delta(\omega+6)]$$

$$=e^{j\omega(\frac{\pi}{12}-1)}\cdot 0$$

$$=0$$

$\therefore y(t)=0$

(b) $x(t)=\sum_{k=0}^{\infty}(\frac{1}{2})^k\sin 3kt$.

$$H(j\omega)=\begin{cases}e^{-j\omega}, & |\omega|<4\\ 0, & |\omega|>4\end{cases}$$

$\therefore Y(j\omega)=e^{-j\omega}\cdot\frac{\pi}{j}[\frac{1}{2}\delta(\omega-3)-\frac{1}{2}\delta(\omega+3)]$

$\cancel{\therefore y(t)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}e^{-j\omega}\cdot\frac{\pi}{j}[\delta(\omega-3)-\delta(\omega+3)]\cdot e^{j\omega t}\,d\omega}$

$\cancel{=\frac{1}{2j}\int_{-\infty}^{+\infty}[\delta(\omega-3)-\delta(\omega+3)]e^{j\omega(t-1)}\,d\omega}$

$\cancel{=\frac{1}{2j}e^{j(3t-3)}-\frac{1}{2j}e^{-j(3t-3)}}$ $\quad\therefore y(t)=\frac{1}{2}\sin(3t-3)$

35、(a) $H(j\omega)$ 的模为 $\frac{\sqrt{a^2+\omega^2}}{\sqrt{a^2+\omega^2}}=1$

$$H(j\omega)=\frac{a^2-\omega^2-2a\omega j}{a^2+\omega^2}$$

$$\therefore \angle H(j\omega) = -\arctan\frac{2a\omega}{a^2-\omega^2}$$

$$= -2\arctan\frac{\omega}{a}$$

$$H(j\omega)=\frac{a-j\omega}{a+j\omega} = \cancel{1-\frac{2\omega j}{a+j\omega}}$$

$$= -1 + \frac{2a}{a+j\omega}$$

∴单位冲激响应 $h(t) = -\delta(t) + 2a e^{-at}u(t)$

(b) $a=1$ 时 $H(j\omega) = \frac{1-j\omega}{1+j\omega}$

$$h(t) = -\delta(t) + 2e^{-t}u(t)$$

输入 $x(t) = \cos(\frac{1}{\sqrt{3}}t) + \cos t + \cos(\sqrt{3}t)$

$|H(j\omega)|=1$ $\quad \angle H(j\omega) = -\arctan\frac{2\omega}{1-\omega^2}$

$$\therefore y(t) = \cos(\frac{1}{\sqrt{3}}t-\frac{\pi}{3}) + \cos(t-\frac{\pi}{2}) + \cos(\sqrt{3}t-\frac{2\pi}{3})$$

36、(a) $x(t)=e^{-t}u(t)+e^{-3t}u(t)$

$$X(j\omega)=\frac{1}{1+j\omega}+\frac{1}{3+j\omega}$$

$$Y(j\omega)=\frac{2}{1+j\omega}-\frac{2}{4+j\omega}$$

$$\therefore H(j\omega) = 2\,\frac{\frac{1}{1+j\omega}-\frac{1}{4+j\omega}}{\frac{1}{1+j\omega}+\frac{1}{3+j\omega}} = 3\times\frac{3+j\omega}{(2+j\omega)(4+j\omega)}$$

(b) $h(t) = \frac{3}{2}(e^{-2t}+e^{-4t})u(t)$.

(c) $H(j\omega) = \frac{9+3j\omega}{(j\omega)^2+6(j\omega)+8}$

$$\therefore \frac{d^2y(t)}{dt^2} + 6\frac{dy(t)}{dt} + 8y(t) = 9x(t) + 3\frac{dx(t)}{dt}.$$

37. (a) $\cancel{X(j\omega) = \int_{-\infty}^{+\infty} x(t) e^{-j\omega t} dt}$

$\cancel{= \int_{-1}^{0} (1+t) e^{-j\omega t} dt + \int_{0}^{1} (1-t) e^{-j\omega t} dt.}$

$\cancel{= \int_{-1}^{0} t e^{-j\omega t} dt - \int_{0}^{1} t e^{-j\omega t} dt + \int_{-1}^{0} e^{-j\omega t} dt + \int_{0}^{1} e^{-j\omega t} dt}$

$\cancel{= -\frac{1}{j\omega}(t+\frac{1}{j\omega}) e^{-j\omega t}\Big|_{-1}^{0} + \frac{1}{j\omega}(t+\frac{1}{j\omega}) e^{-j\omega t}\Big|_{0}^{1} - \frac{1}{j\omega} e^{-j\omega t}\Big|_{-1}^{0} - \frac{1}{j\omega} e^{-j\omega t}\Big|_{0}^{1}}$

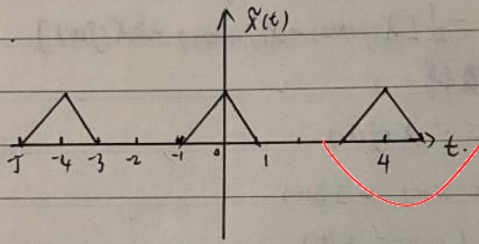
$\cancel{= -\frac{2}{(j\omega)^2} - \frac{2}{j\omega}}.$

令 $x_1(t) = \begin{cases} 1, & |\omega| < \frac{1}{2} \\ 0, & |\omega| > \frac{1}{2} \end{cases}$

$\because x(t) = x_1(t) * x_1(t)$

$\therefore X(j\omega) = 4 \frac{\sin^2 \frac{1}{2}\omega}{\omega^2}$

(b).



(c) $\cancel{\tilde{X}(j\omega)} = \sum_{k=-\infty}^{+\infty} \delta(t-4k)$ 的 fourier transform 为 $\frac{\pi}{2} \sum_{k=-\infty}^{+\infty} \delta(\omega - \frac{\pi}{2}k)$

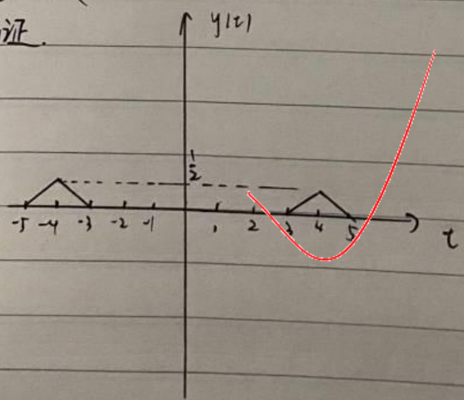
$\therefore \tilde{X}(j\omega) = X(j\omega) \cdot \frac{\pi}{2} \sum_{k=-\infty}^{+\infty} \delta(\omega - \frac{\pi}{2}k) = \frac{\pi}{2} \sum_{k=-\infty}^{+\infty} X(j\frac{\pi}{2}k)$

$\tilde{X}(j\omega) = G(j\omega) \cdot \frac{\pi}{2} \sum_{k=-\infty}^{+\infty} \delta(\omega - \frac{\pi}{2}k) = \frac{\pi}{2} \sum_{k=-\infty}^{+\infty} G(j\frac{\pi}{2}k).$

$\therefore X(j\frac{\pi}{2}k) = G(j\frac{\pi}{2}k)$ 时 可相等

$\cancel{X(j\frac{\pi}{2}k) =}$

(d) 由(c)已证.



43. $g(t) = x(t)\cos^2 t * \frac{\sin t}{\pi t}$

设 $s(t) = \cos^2 t \cancel{* \frac{\sin t}{\pi t}}$

假设存在线性时不变系统 S，单位冲激响应为 $h(t)$

则有 $g(t) = x(t) * h(t)$

$\cancel{g(t) = x(t) \cdot s(t)}$

$\cancel{G(j\omega) = X(j\omega) * S(j\omega)}$

$S(j\omega) = \frac{1}{2}\pi[\delta(\omega-2) + 2\delta(\omega) + \delta(\omega+2)] \cdot \cancel{\begin{cases} 1, & |\omega|<1 \\ 0, & \text{otherwise} \end{cases}}$

$= \cancel{\pi\delta(\omega)}$

$\cancel{\therefore G(j\omega) = X(j\omega) * \pi\delta(\omega)}$

$\cancel{\therefore g(t) = x(t) \cdot \frac{1}{2}}$

$x(t)s(t) \overset{\mathcal{F}}{\longleftrightarrow} \frac{1}{2\pi} X(j\omega) * S(j\omega)$

$= \frac{1}{4}[X(j(\omega-2)) + X(j(\omega+2)) + 2X(j\omega)]$

经滤波得

$G(j\omega) = \frac{1}{2} X(j\omega)$

$\therefore g(t) = x(t) * \frac{1}{2}\delta(t)$

$\therefore h(t) = \frac{1}{2}\delta(t)$

$\therefore$ 存在一个线性时不变系统 S，有 $x(t) \xrightarrow{S} g(t)$

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