第二章作业　　周子涵　20180112180114

$1.^{(1)} X_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$　$X_2 = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$　$X_3 = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$　$X_4 = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$　$X_5 = \begin{bmatrix} 6 \\ 6 \end{bmatrix}$　$X_6 = \begin{bmatrix} 8 \\ 6 \end{bmatrix}$　$X_7 = \begin{bmatrix} 6 \\ 8 \end{bmatrix}$　$X_8 = \begin{bmatrix} 8 \\ 2 \end{bmatrix}$

$\|X_1 - Z_1\| = 0 < \|X_1 - Z_2\|$

$\|X_2 - Z_1\| = 2 < \|X_2 - Z_2\| = \sqrt{2}$

$\|X_3 - Z_1\| = 2 < \|X_3 - Z_2\| = \sqrt{2}$

$\|X_4 - Z_1\| = 2\sqrt{2} \leqslant \|X_4 - Z_2\| = 4\sqrt{2}.$

$\|X_5 - Z_1\| > \|X_5 - Z_2\| = 0$

$\|X_6 - Z_1\| = 10 > \|X_6 - Z_2\| = 2$

$\|X_7 - Z_1\| = 10 > \|X_7 - Z_2\| = 2$

$\|X_8 - Z_1\| = 8\sqrt{2} > \|X_8 - Z_2\| = 2\sqrt{2}.$

∴第一类 $\{X_1, X_2, X_3, X_4\}$　$Z_1 = (1,1)^T$

第二类 $\{X_5, X_6, X_7, X_8\}$　$Z_2 = (7,7)^T$

$\|X_1 - Z_1\| = \sqrt{2} < \|X_1 - Z_2\| = 6\sqrt{2}$

$\|X_2 - Z_1\| = \sqrt{2} < \|X_2 - Z_2\| = \sqrt{74}$

$\|X_3 - Z_1\| = \sqrt{2} < \|X_3 - Z_2\| = \sqrt{74}$

$\|X_4 - Z_1\| = \sqrt{2} < \|X_4 - Z_2\| = 3\sqrt{2}$

$\|X_5 - Z_1\| = 3\sqrt{2} > \|X_5 - Z_2\| = \sqrt{2}$

$\|X_6 - Z_1\| = \sqrt{74} > \|X_6 - Z_2\| = \sqrt{2}$

$\|X_7 - Z_1\| = \sqrt{74} > \|X_7 - Z_2\| = \sqrt{2}$

$\|X_8 - Z_1\| = \sqrt{2} > \|X_8 - Z_2\| = \sqrt{2}.$

∴第二次聚类结果与第一次一致

(2) $\|X_1 - Z_1\| = 2 < \|X_1 - Z_2\| = 2\sqrt{2}$

$\|X_2 - Z_1\| < \|X_2 - Z_2\|$

$\|X_3 - Z_1\| = 2\sqrt{2} > \|X_3 - Z_2\| = \sqrt{2}$

$\|X_4 - Z_1\| > \|X_4 - Z_2\|$

$\|X_5 - Z_1\| = 6\sqrt{2} > \|X_5 - Z_2\| = 4\sqrt{2}$

$\|X_6 - Z_1\| = 10 > \|X_6 - Z_2\|$

$\|X_7 - Z_1\| = 10 < \|X_7 - Z_2\|$

$\|X_8 - Z_1\| = 8\sqrt{2} > \|X_8 - Z_2\|$

∴第一类：$\{X_1, X_2\}$　$Z_1 = (1, 0)^T$

第二类：$\{X_3, \cdots, X_8\}$　$Z_2 = (5, \frac{16}{3})^T$

第二次聚类：

第一类：$\{X_1, \cdots, X_4\}$　$Z_1 = (1, 1)^T$

第二类：$\{X_5, \cdots, X_8\}$　$Z_2 = (7, 7)^T$

原始点、选取过近会增加运算.

2、

	x_1	x_2	x_3	x_4	x_5
x_1	0	$\sqrt{3}$	$\sqrt{15}$	$\sqrt{6}$	$\sqrt{11}$
x_2		0	$\sqrt{6}$	$\sqrt{5}$	$\sqrt{7}$
x_3			0	$\sqrt{13}$	$\sqrt{6}$
x_4				0	$\sqrt{7}$
x_5					0

$\sqrt{6}$

$\sqrt{5}$

$\sqrt{3}$
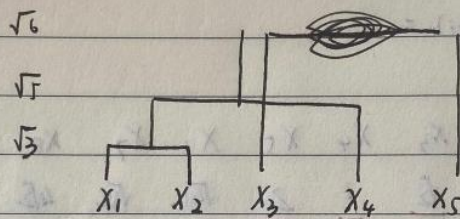
x_1 x_2 x_3 x_4 x_5

3、关系

	x_1	x_2	x_3	x_4	x_5
x_1					
x_2	0				
x_3	0	$\frac{1}{2}$			
x_4	0	0	0		
x_5	$\frac{1}{2}$	0	0	0	

x_1 x_2 x_3 x_4 x_5

4. $X_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $X_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $X_3 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ $X_4 = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$ $X_5 = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$

$X_6 = \begin{bmatrix} -4 \\ 5 \end{bmatrix}$ $X_7 = \begin{bmatrix} 3 \\ -5 \end{bmatrix}$ $X_8 = \begin{bmatrix} 4 \\ -4 \end{bmatrix}$ $X_9 = \begin{bmatrix} 3 \\ -4 \end{bmatrix}$ $X_{10} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$

对于第1个样本 $X_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\| X_1 - X_1 \| = 0 < 3 \Rightarrow \chi(0-3) = 1$

$\| X_1 - X_2 \| = 1 < 3 \Rightarrow \chi(1-3) = 1$

$\| X_1 - X_3 \| = \sqrt{2} < 3 \Rightarrow \chi(\sqrt{2}-3) = 1$

$\| X_1 - X_4 \| = 1 < 3 \Rightarrow \chi(1-3) = 1$

$\| X_1 - X_5 \| = 2 < 3 \Rightarrow \chi(2-3) = 1$

$X_6 - X_{10}$ 则 $\chi(X_i) = 0$.

$\Rightarrow \rho_1 = 5$

	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	
5 X_1	0	1	$\sqrt{2}$	1	2	$\sqrt{41}$	$\sqrt{34}$	$4\sqrt{2}$	5	$\sqrt{41}$	5
5 X_2	1	0	$\sqrt{5}$	$\sqrt{2}$	1	$5\sqrt{2}$	$\sqrt{41}$	$\sqrt{41}$	$4\sqrt{2}$	$3\sqrt{2}$	$4\sqrt{2}$
4 X_3	$\sqrt{5}$	$\sqrt{5}$	0	1	$\sqrt{10}$	5	$2\sqrt{5}$	$3\sqrt{2}$	$\sqrt{13}$	$3\sqrt{5}$	$3\sqrt{2}$
5 X_4	1	$\sqrt{2}$	1	0	$\sqrt{5}$	$4\sqrt{2}$	5	5	$3\sqrt{2}$	$\sqrt{52}$	5
4 X_5	2	1	$\sqrt{10}$	$\sqrt{5}$	0	$\sqrt{61}$	$5\sqrt{2}$	$4\sqrt{5}$	$\sqrt{41}$	$\sqrt{61}$	$\sqrt{41}$
4 X_6	$\sqrt{41}$	$5\sqrt{2}$	5	$4\sqrt{2}$	$\sqrt{61}$	0	1	1	$2\sqrt{2}$	10	$4\sqrt{2}$
4 X_7	$\sqrt{34}$	$\sqrt{41}$	$2\sqrt{5}$	5	$3\sqrt{2}$	1	0	$2\sqrt{2}$	1	$\sqrt{101}$	$2\sqrt{5}$
4 X_8	$4\sqrt{2}$	$\sqrt{41}$	$3\sqrt{2}$	5	$4\sqrt{5}$	1	$2\sqrt{2}$	0	1	9	$3\sqrt{2}$
4 X_9	5	$4\sqrt{2}$	$\sqrt{13}$	$3\sqrt{2}$	$\sqrt{41}$	$2\sqrt{2}$	1	1	0	$\sqrt{52}$	$3\sqrt{2}$
1 X_{10}	$\sqrt{41}$	$5\sqrt{2}$	$3\sqrt{5}$	$\sqrt{52}$	$\sqrt{61}$	10	$\sqrt{101}$	9	$\sqrt{82}$	0	$\sqrt{41}$

$\therefore \rho_2 = 5$ $\rho_3 = 4$ $\rho_4 = 5$ $\rho_5 = 4$ $\rho_6 = 4$ $\rho_7 = 4$ $\rho_8 = 4$ $\rho_9 = 4$ $\rho_{10} = 1$

$\Rightarrow \delta_1 = 5$ $\delta_2 = 4\sqrt{2}$ $\delta_3 = 1$ $\delta_4 = 5$ $\delta_5 = \sqrt{41}$ $\delta_6 = 4\sqrt{2}$ $\delta_7 = 2\sqrt{5}$

$\delta_8 = 3\sqrt{2}$ $\delta_9 = 3\sqrt{2}$ $\delta_{10} = \sqrt{41}$

$\Rightarrow \gamma = \{ 25, 20\sqrt{2}, 12\sqrt{5}, 25, 4\sqrt{41}, 16\sqrt{2}, 8\sqrt{5}, 12\sqrt{2}, 12\sqrt{2}, \sqrt{41} \}$