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第三章 1 13 15 34 35 43

1、$T=8$. $a_1=a_{-1}=2$ $a_3=4j$ $a_{-3}=-4j$

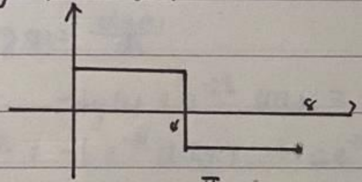
$T=8 \Rightarrow \omega_0=\frac{2\pi}{T}=\frac{\pi}{4}$

$x(t)=2e^{j\frac{\pi}{4}t}+2e^{-j\frac{\pi}{4}t}+4je^{j\frac{3\pi}{4}t}-4je^{-j\frac{3}{4}\pi t}$

$=4\cos(\frac{\pi}{4}t)-8\sin\frac{3}{4}\pi t$

$=4\cos\frac{\pi}{4}t+8\cos(\frac{3}{4}\pi t+\frac{\pi}{2})$

13、$H(j\omega)=\frac{\sin 4\omega}{\omega}$.



$T=8 \Rightarrow \omega_0=\frac{\pi}{4}$

设 $x(t)$ 的傅里叶级数形式为 $x(t)=\sum_{k=-\infty}^{+\infty}a_k e^{jk\frac{\pi}{4}t}$

$\therefore H(jk\omega)=\frac{\sin k\pi}{k\omega}$ 恒为0

$\because$ 频率响应恒为0

$\therefore$ 输出 $y(t)=0$.

15、$H(j\omega)=\begin{cases}1, & |\omega|\le 100\\ 0, & |\omega|>100\end{cases}$

$T=\frac{\pi}{6}$ $\omega_0=\frac{2\pi}{\frac{\pi}{6}}=12$

设 $x(t)=\sum_{k=-\infty}^{+\infty}a_k e^{jk\cdot 12t}$

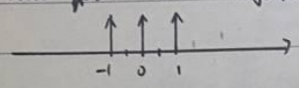
$a_k\cdot H(jk\omega)=a_k$

$\therefore |k\omega|>100 \Rightarrow |k|\ge 9$

$\therefore |k|\ge 9$ 时 $a_k=0$.

4、$h(t)=e^{-4|t|}$

(a) $x(t)=\sum_{k=-\infty}^{+\infty}\delta(t-n)$ 周期为1



$a_k=\frac{1}{T}\int_T x(t)e^{-jk\omega_0 t}dt$

$=\int_{-\frac{1}{2}}^{\frac{1}{2}}\delta(t)e^{-jk\omega_0 t}dt = 1$

∴所有傅里叶级数系数都为1

$H(j\omega)=\int_{-\infty}^{+\infty}h(t)e^{-j\omega t}dt$

$=\int_0^{+\infty}e^{-4t}e^{-j\omega t}dt+\int_{-\infty}^{0}e^{4t}e^{-j\omega t}$

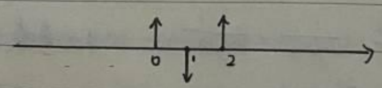
$=\frac{1}{-4-j\omega}e^{(-4-j\omega)t}\Big|_0^{+\infty}+\frac{1}{4-j\omega}e^{(4-j\omega)t}\Big|_{-\infty}^{0}$

$=\frac{1}{4+j\omega}+\frac{1}{4-j\omega}=\cancel{\frac{2j\omega}{16+\omega^2}}$

$\cancel{\therefore b_k=\frac{2j\omega}{16+\omega^2}}$

$\therefore y(t)=\sum_{k=-\infty}^{+\infty}\frac{2jk\omega}{16+k^2\omega^2}e^{jk\omega t}=\sum_{k=-\infty}^{+\infty}\left(\frac{1}{4-j2\pi k}+\frac{1}{4+j2\pi k}\right)e^{jk2\pi t}$

(b): $\cancel{a_k=1}$



$\cancel{a_k=\frac{1}{2}\int_{-\frac{1}{2}}^{\frac{3}{2}}\delta(t)e^{-jk\omega_0 t}}$ $T=2$, $\omega_0=\frac{2\pi}{2}=\pi$

$a_k=\frac{1}{2}\int_{-\frac{1}{2}}^{\frac{3}{2}}(\delta(t)-\delta(t-1))e^{-jk\omega_0 t}dt$

$=\frac{1}{2}-\frac{1}{2}\cos k\pi$

$\therefore a_k=\begin{cases}0, & k是偶数\\ 1, & k是奇数\end{cases}$

$\therefore b_k=\begin{cases}0, & k是偶数\\ \frac{1}{4+j\omega}+\frac{1}{4-j\omega}, & k是奇数\end{cases}$ $\therefore y(t)=\sum_{k=-\infty}^{+\infty}\left(\frac{1}{4-jk\pi}+\frac{1}{4+jk\pi}\right)e^{jk\pi t}$ （k是奇数）.

(c) $T=1$, $\omega_0 = 2\pi$.

$a_k = \int_{-\frac{1}{4}}^{\frac{1}{4}} e^{-jk2\pi t} dt$.

$= -\frac{1}{jk2\pi} e^{-jk2\pi t} \Big|_{-\frac{1}{4}}^{\frac{1}{4}}$

$= -\frac{1}{jk2\pi} (e^{-jk\frac{\pi}{2}} - e^{jk\frac{\pi}{2}})$

$= -\frac{1}{jk2\pi} (\cos k\frac{\pi}{2} - j\sin k\frac{\pi}{2} - \cos k\frac{\pi}{2} - j\sin k\frac{\pi}{2})$

$= \frac{\sin\frac{\pi}{2}k}{\pi k}$

$a_0 = \frac{1}{T}\int_T x(t)\, dt = \frac{1}{2}$

k为偶数时 $a_k = 0$

k为奇数时，$a_k = \frac{\sin\frac{\pi}{2}k}{\pi k}$

$\therefore k=0$ 时 $b_k = \frac{1}{4}$

k为偶数时，$b_k = 0$

k为奇数时，$b_k = \frac{\sin\frac{\pi}{2}k}{\pi k} (\frac{1}{4+j2\pi k} + \frac{1}{4-j2\pi k})$

$\therefore y(t) = \frac{1}{4} + \sum_{k=-\infty}^{\infty} \frac{\sin\frac{\pi}{2}k}{\pi k} (\frac{1}{4+j2\pi k} + \frac{1}{4-j2\pi k})$ （k为奇数）.

35、 $\omega_0 = \frac{2\pi}{T} = 14$

$H(j\omega) = \begin{cases} 1, & |\omega| \geq 250 \\ 0, & 其他 \end{cases}$

$a_k H(jk\omega) = a_k$

$\therefore |k\omega| < 250$ 时 $k = 0$

$\Rightarrow |k| \leq 17$ 时 $a_k = 0$.

3. (a) (i) 证明：$x(t+\frac{T}{2}) = \sum_{k=-\infty}^{+\infty} e^{j\pi k} a_k e^{jk\frac{2\pi}{T}(t\ )}$

$$= \sum_{k=-\infty}^{+\infty} a_k e^{jk\frac{2\pi}{T}t} e^{j(\ k)\pi} = \sum_{k=-\infty}^{+\infty} a_k e^{jk\frac{2\pi}{T}t} \cos k\pi$$

$$x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk\frac{2\pi}{T}t}$$

k为偶数时，显然 $x(t) = -x(t+\frac{T}{2})$

k为奇数时，$\cos k\pi = -1$

$$\therefore x(t) = -x(t+\frac{T}{2}).$$

(ii) 证明：$x(t+\frac{T}{2}) = \sum_{k=-\infty}^{+\infty} a_k \cos k\pi\, e^{jk\frac{2\pi}{T}t}$

$$x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk\frac{2\pi}{T}t}$$

$$\therefore x(t) + x(t+\frac{T}{2}) = \sum_{k=-\infty}^{+\infty} (1+\cos k\pi)\, a_k e^{jk\frac{2\pi}{T}t} = 0.$$

$\therefore (1+\cos k\pi)\ a_k = 0$

1° k为奇数，$1+\cos k\pi = 0$

2° k为偶数，$1+\cos k\pi$ 恒不为0.

$\therefore$ k为偶数时 a_k 恒为0

$\therefore$ 满足式 P3.43.2，则它是奇谐的.

(b) $T=2$，$\omega_0 = \pi$

~~$a_k = \frac{1}{2}\int_2 x(t) e^{-jk\pi t} dt$~~

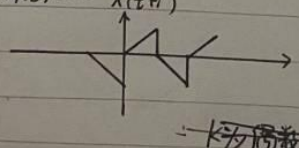
~~$= \frac{1}{2}\int_2 x(t)(\cos k\pi t - j\sin k\pi t)\, dt$~~

~~$a_2 = 0$~~

~~$\frac{1}{2}\int_0^1 t\, e^{-j2\pi t} dt + \frac{1}{2}\int_1^2 x(t)\, e^{-j2\pi t} dt = 0.$~~ 35.

$x(t) = -x(t+1)$

$\therefore$



$\therefore$ ~~k为偶数时，$a_k = 0$~~

~~k为奇数时~~.

$$a_k=\frac{1}{2}\int_2 x(t)e^{-jk\omega_0 t}dt$$

$$=\frac{1}{2}\int_2 x(t)e^{-jk\pi t}dt$$

$$=\frac{1}{2}\int_0^1 te^{-jk\pi t}dt+\frac{1}{2}\int_1^2(1-t)e^{-jk\pi t}dt$$

$\cancel{令 -jk\pi=m}$

$\cancel{\therefore a_k=\frac{1}{2}\int_0^1 te^{mt}dt+\frac{1}{2}\int_1^2(1-t)e^{mt}dt}$

$\cancel{=\frac{1}{2}(\frac{t}{m}-\frac{1}{m^2})e^{mt}\Big|_0^1+\frac{1}{2}(\frac{1}{m}-\frac{t}{m}+\frac{1}{m^2})e^{mt}\Big|_1^2}$

$\cancel{=\frac{1}{2}[\frac{1}{m}e^{m}-\frac{1}{m^2}e^{m}+\frac{1}{m^2}+\frac{1}{m}e^{2m}-\frac{2}{m}e^{2m}+\frac{1}{m^2}e^{2m}-\frac{1}{m}e^{m}+\frac{1}{m}e^{m}-\frac{1}{m^2}e^{m}]}$

$\because x(t)$是一个奇谐周期信号

$\therefore x(t)=-x(t+\frac{T}{2})$

$x(t+\frac{T}{2})=-x(t)$

$$a_k=\frac{1}{T}\int_T x(t)e^{-jk\omega_0 t}dt$$

$$=\frac{1}{T}\int_0^{\frac{T}{2}}x(t)e^{-jk\omega_0 t}dt+\frac{1}{T}\int_{\frac{T}{2}}^{T}x(t)e^{-jk\omega_0 t}dt$$

$$=\frac{1}{T}\int_0^{\frac{T}{2}}x(t)e^{-jk\omega_0 t}dt+\frac{1}{T}\int_0^{\frac{T}{2}}x(m+\frac{T}{2})e^{-jk\pi}e^{-jk\omega_0 m}dm$$

$$=\frac{1}{T}\int_0^{\frac{T}{2}}[x(t)+x(t+\frac{T}{2})e^{-jk\pi}]e^{-jk\omega_0 t}dt$$

$T=2\quad \omega_0=\pi$

$$\therefore a_k=\frac{1}{2}\int_0^1[x(t)+x(t+\frac{T}{2})e^{-jk\pi})e^{-jk\pi t}dt.$$

$$=\frac{1-e^{-jk\pi}}{2}(\frac{jk\pi+1}{k^2\pi^2}e^{-jk\pi}+\frac{1}{k^2\pi^2})$$

$$=\frac{1-\cos k\pi}{2}(\frac{-jk\pi-1}{-k^2\pi^2}\cos k\pi-\frac{1}{k^2\pi^2})$$

k为偶数时 $a_k=0$

k为奇数时 $a_k=\frac{1}{jk\pi}+\frac{2}{k^2\pi^2}$

(c) 不可能　由(a)(i)得 $x(t+\frac{T}{2})=\sum_{k=-\infty}^{+\infty}a_k e^{jk\frac{2\pi}{T}t}\cos k\pi$

若是偶谐的 k为偶数时 $x(t)=x(t+\frac{T}{2})$ $\therefore\frac{T}{2}$为其基波周期.

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(d) (1) $X(t)=\sum_{k=-\infty}^{+\infty} a_k e^{jk\frac{2\pi}{T}t}$

$X(t+T)=\sum_{k=-\infty}^{+\infty} a_k\cdot e^{jk2\pi}\, e^{jk\frac{2\pi}{T}t}$

若a_1，a_{-1}为非零.

~~若$a_1\neq 0$~~ $X(t)=a_0+a_1 e^{j\frac{2\pi}{T}t}+a_{-1}e^{j-\frac{2\pi}{T}t}+\cdots$

$X(t+t_0)=a_0+a_1 e^{j\frac{2\pi}{T}(t+t_0)}+a_{-1}e^{j-\frac{2\pi}{T}(t+t_0)}+\cdots$

令$X(t)=X(t+t_0)$

$e^{j\frac{2\pi}{T}t_0}=1$ 的最小t_0值为周期.

$\therefore t_0=T$.

(2) $a_k e^{jk\frac{2\pi}{T}(t+t_0)}$ 周期$\frac{T}{k}$

$a_l e^{jl\frac{2\pi}{T}(t+t_0)}$ 周期$\frac{T}{l}$

$\because$ k和l无公共因子

$\therefore$ 基波周期为T.