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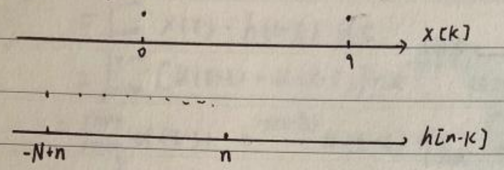
第二章 5 7 10 11 12 19 20 23 40 46 47

5. $x[n]=\begin{cases}1, & 0\le n\le 9\\ 0, & 其他\end{cases}$ $h[n]=\begin{cases}1, & 0\le n\le N\\ 0, & 其他\end{cases}$

$y[n]=x[n]*h[n]$

$=\sum_{k=-\infty}^{+\infty} x[k]\,h[n-k]$



$N\le 9$.

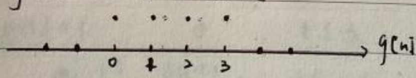
$-N+4\le 0 \Rightarrow N\ge 4$

$14-N\ge 9 \Rightarrow N\le 5$

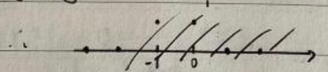
$\Rightarrow N=4$.

7. $y[n]=\sum_{k=-\infty}^{\infty} x[k]\,g[n-2k]$

$g[n]=u[n]-u[n-4]$



(a) $y[n]=g[n-2]$



$\therefore y[n]=u[n-2]-u[n-6]$

(b) $y[n]=u[n-4]-u[n-8]$.

(c) ∵在输入信号上产生一个时移在输出信号上没有产生相同的时移

∴不是线性时不变的

(d) $y[n]=\sum_{k=0}^{\infty} g[n-2k]$

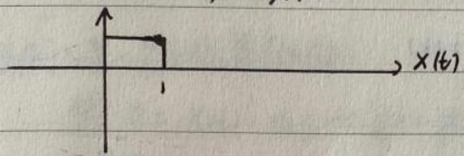
$=u[n]-u[n-4]+u[n-2]-u[n-6]+u[n-4]-u[n-8]+u[n-6]-u[n-10]$

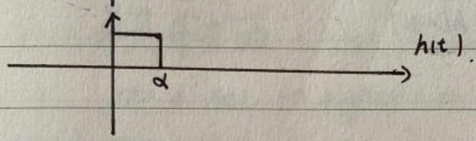
$=u[n]+u[n-2]$.

10、 $x(t) = \begin{cases} 1, & 0 \le t \le 1 \\ 0, & 其他 \end{cases}$

$h(t) = x(\frac{t}{\alpha})$

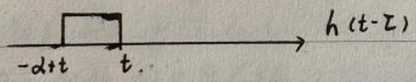
(a) $h(t) = \begin{cases} 1, & 0 \le t \le \alpha \\ 0, & 其他 \end{cases}$



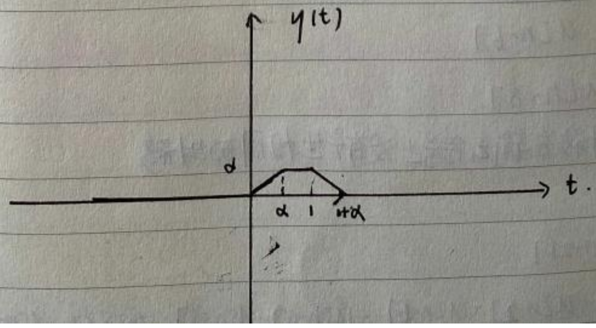


$y(t) = x(t) * h(t)$

$= \int_{-\infty}^{+\infty} x(\tau) \cdot h(t-\tau)\, d\tau$



$\therefore y(t) = \begin{cases} 0, & t < 0 \\ t, & 0 \le t < \alpha \\ \alpha, & \alpha \le t < 1 \\ -t+\alpha+1, & 1 \le t < \alpha+1 \\ 0, & t > \alpha+1 \end{cases}$



(b) 若 $\frac{dy(t)}{dt}$ 仅有3个不连续点

则 $\alpha = 1$.

11、 $x(t) = u(t-3) - u(t-5)$

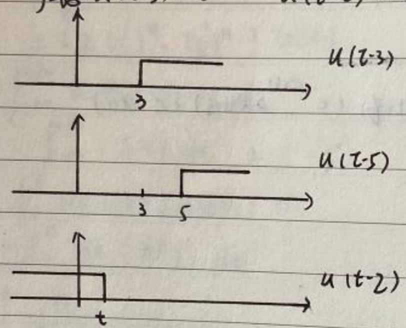
$h(t) = e^{-3t} u(t)$

(a) $y(t) = x(t) * h(t)$

$= \int_{-\infty}^{+\infty} x(\tau) \cdot h(t-\tau)\, d\tau$.

$= \int_{-\infty}^{+\infty} [u(\tau-3) - u(\tau-5)] \cdot e^{-3(t-\tau)} u(t-\tau)\, d\tau$

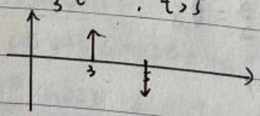
$= \int_{-\infty}^{+\infty} u(\tau-3) \cdot e^{-3(t-\tau)} u(t-\tau)\, d\tau - \int_{-\infty}^{+\infty} u(\tau-5) \cdot e^{-3(t-\tau)} u(t-\tau)\, d\tau$.



$\therefore y(t) = \begin{cases} 0, & t \le 3. \\ \int_3^t e^{-3(t-\tau)} d\tau, & 3 < t \le 5 \\ \int_3^t e^{-3(t-\tau)} d\tau - \int_5^t e^{-3(t-\tau)} d\tau, & t > 5 \end{cases}$

$\therefore y(t) = \begin{cases} 0, & t \le 3 \\ \frac{1}{3} - \frac{1}{3} e^{9-3t}, & 3 < t \le 5 \\ \frac{1}{3} e^{15-3t} - \frac{1}{3} e^{9-3t}, & t > 5 \end{cases}$

(b) $\frac{dx(t)}{dt}$



$g(t) = \int_{-\infty}^{+\infty} \frac{dx(\tau)}{d\tau} \cdot e^{-3(t-\tau)} u(t-\tau)\, d\tau$.

$g(t) = \begin{cases} 0, & t < 3 \\ e^{9-3t}, & 3 \le t < 5 \\ e^{9-3t} - e^{15-3t}, & t \ge 5 \end{cases}$ $\therefore g(t) = e^{-3(t-3)} u(t-3) - e^{-3(t-5)} u(t-5)$.

(c) 计算可知 $g(t) = \frac{dy(t)}{}$

12、$y(t) = e^{-t}u(t) * \sum_{k=-\infty}^{\infty} \delta(t-3k)$

证明 $y(t) = Ae^{-t}$

$y(t) = e^{-t}u(t) * \sum_{k=-\infty}^{\infty}\delta(t-3k)$

$\cancel{= \int_{-\infty}^{+\infty} e^{-\tau}u(\tau)\cdot\sum_{k=-\infty}^{\infty}\delta(t-\tau-3k)\,d\tau}$.

$= \sum_{k=-\infty}^{\infty} e^{-t}u(t) * \delta(t-3k)$

$= \sum_{k=-\infty}^{+\infty} e^{-(t-3k)}u(t-3k)$

$\cancel{= e^{-t}\sum_{k=-\infty}^{+\infty} e^{3k}u(t-3k)}$

$\therefore$ $0 \le t \le 3$ 时

$y(t) = \cdots + e^{-(t+9)}u(t+9) + e^{-(t+6)}u(t+6) + e^{-(t+3)}u(t+3) + e^{-t}u(t)$

$= e^{-t}(1 + e^{-3} + e^{-6} + \cdots)$

$= e^{-t}\dfrac{1}{1-e^{-3}}$

$\therefore A = \dfrac{1}{1-e^{-3}}$

19. $w[n] = \frac{1}{2}w[n-1] + x[n]$

$y[n] = \alpha y[n-1] + \beta w[n]$

(a) $w[n] = \frac{1}{\beta}y[n] - \frac{\alpha}{\beta}y[n-1]$.

$w[n-1] = \frac{1}{\beta}y[n-1] - \frac{\alpha}{\beta}y[n-2]$.

$w[n] - \frac{1}{2}w[n-1] = \frac{1}{\beta}y[n] - \frac{\alpha}{\beta}y[n-1] - \frac{1}{2\beta}y[n-1] + \frac{\alpha}{2\beta}y[n-2] = x[n]$.

$\therefore$ $\beta = 1$. $\alpha + \frac{1}{2} = \frac{3}{4}$ $\frac{\alpha}{2} = \frac{1}{8}$

$\Rightarrow \begin{cases} \alpha = \frac{1}{4} \\ \beta = 1. \end{cases}$

(b) S_1 : $w[n] = \frac{1}{2}w[n-1] + x[n]$

S_2 : $y[n] = \frac{1}{4}y[n-1] + w[n]$

$y[n] = -\frac{1}{8}y[n-2] + \frac{3}{4}y[n-1] + x[n]$.

$x[n]=\delta[n]$ 时

$w[n]=(\frac{1}{2})^n u[n]$.

$w[n]=\delta[n]$ 时

$y[n]=(\frac{1}{4})^n u[n]$.

∴ 单位~~冲激~~脉冲响应为

$h[n] = (\frac{1}{2})^n u[n] * (\frac{1}{4})^n u[n]$

$= \sum_{k=-\infty}^{+\infty} (\frac{1}{2})^k u[k] \cdot (\frac{1}{4})^{n-k} u[n-k]$

$= \sum_{k=0}^{n} (\frac{1}{2})^k \cdot (\frac{1}{4})^{n-k} \cdot u[n]$

$= [2(\frac{1}{2})^n - (\frac{1}{4})^n]\ u[n]$.

20. (a) $\int_{-\infty}^{+\infty} u_0(t) \cos(t)\, dt$

$= \int_{-\infty}^{+\infty} \delta(t)\, dt = 1$

(b) $\int_0^5 \sin(2\pi t)\, \delta(t+3)\, dt$.

$= \int_0^5 \sin(-6\pi)\, dt$

$= 0$.

(c) $\cancel{\int_{-5}^{5} u_1(1-\tau) \cos(2\pi\tau)\, d\tau}$.

$\cancel{x(t) = \cos(2\pi t)[u(t+5) - u(t-5)]}$

$\cancel{\frac{dx(t)}{dt} = \int_{-5}^{5} u_1(t-\tau) \cos(2\pi\tau)\, d\tau}$.

$\cancel{\text{原式} = \frac{dx(t)}{dt}\Big|_{t=1} = 0}$.

$x(t) = \int_{-\infty}^{+\infty} x(\tau)\, \delta(t-\tau)\, d\tau$

$= x(t) * \delta(t)$

$x'(t) = x(t) * u_1(t)$.

令 $x(t) = \cos 2\pi t\, [u(t+5) - u(t-5)]$

∴ $x'(t) = \int_{-5}^{5} \cos 2\pi\tau \cdot u_1(t-\tau)\, d\tau$

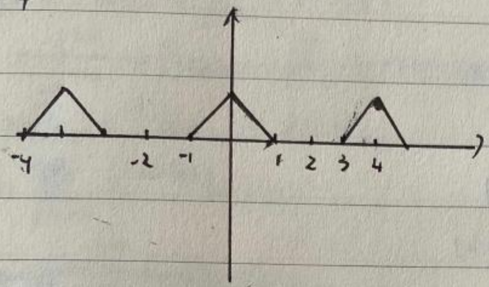
∵ $x'(1) = 0$

∴ 原式 $= 0$.

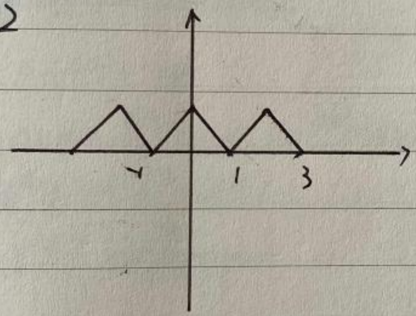
23. ~~$y = \int_{-\infty}^{+\infty} x(\tau)\, h(t-\tau)\, d\tau$~~

$$y(t) = \int_{-\infty}^{+\infty} x(\tau)\cdot h(t-\tau)\, d\tau$$

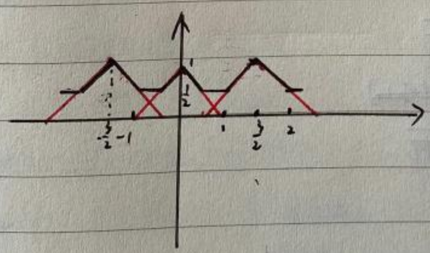
(a) $T = 4$



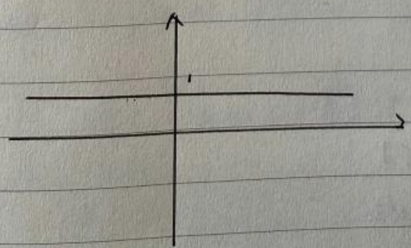
(b) $T=2$



(c) $T=\frac{3}{2}$



(d) $T=1$



40、$y(t)=\int_{-\infty}^{t} e^{-(t-\tau)}\, x(\tau-2)\, d\tau$.

~~(a) $y(t)=\int_{-\infty}^{t} e^{-(t-(\tau-2)-2)}\, x(\tau-2)\, d(\tau-2)$~~

~~令 $m=\tau-2$~~

~~$\therefore y(t)=\int_{-\infty}^{t-2} e^{-(t-2-m)}\, x(m)\, dm$~~

~~$y(t)=x(t)*h(t)$~~

~~$=\int_{-\infty}^{+\infty} x(t)\cdot h(t-\tau)\, d\tau$.~~

~~$\therefore$ 单位冲激响应为 $e^{-(t-2)}u(t-2)$~~

$y(t)=\int_{-\infty}^{t} e^{-(t-\tau)}\, x(\tau-2)\, d\tau$

令 $m=\tau-2$.

$\therefore y(t)=\int_{-\infty}^{t-2} e^{-(t-m-2)}\, x(m)\, dm$

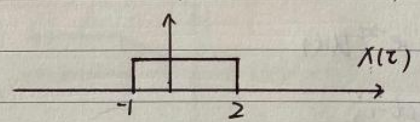
$=\int_{-\infty}^{t-2} e^{-(t-2-m)}\, x(m)\, dm$

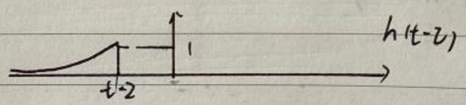
~~$x(m)*e^{-m}=\int_{-\infty}^{t-2} x(m)$~~

$x(t)*e^{-(t-2)}=\int_{-\infty}^{+\infty} x(\tau)\cdot e^{-(t-2-\tau)}\, d\tau$.

$\therefore h(t)=e^{-(t-2)}u(t-2)$

(b)





1° $t-2\le -1 \Rightarrow t\le 1$

$\Rightarrow y(t)=0$

2° $1<t\le 4$

$y(t)=\int_{-1}^{t-2} e^{-(t-2-\tau)}\, d\tau$

$=e^{-(t-2)}\cdot e^{\tau}\Big|_{-1}^{t-2}$

$=1-e^{-(t-1)}$

3° $t>4$

$y(t)=\int_{-1}^{2} e^{-(t-2-\tau)}\, d\tau$

$=e^{-(t-2)}\, e^{\tau}\Big|_{-1}^{2}$

$=e^{-(t-4)}-e^{-(t-1)}$

$\therefore y(t)=\begin{cases} 0, & t\le 1 \\ 1-e^{-(t-1)}, & 1<t\le 4 \\ e^{-(t-4)}-e^{-(t-1)} \end{cases}$

46、$x(t) = 2e^{-3t}u(t-1)$

$x(t) \to y(t)$.

$\frac{dx(t)}{dt} \to -3y(t) + e^{-2t}u(t)$

$y(t) = x(t) * h(t)$

$g(t) = \frac{dx(t)}{dt} * h(t)$.

由11题得

$g(t) = \frac{dy(t)}{dt}$

$\therefore -3y(t) + e^{-2t}u(t) = \frac{dy(t)}{dt}$.

$\frac{dx(t)}{dt} = -6e^{-3t}u(t-1) + 2e^{-3t}\delta(t-1)$

$\therefore \frac{dx(t)}{dt} = -6e^{-3t}u(t-1) + 2e^{-3}\delta(t-1)$

$\therefore \frac{dx(t)}{dt} = -3x(t) + 2e^{-3}\delta(t-1)$.

$\because \frac{dx(t)}{dt} \to -3x(t) + \cancel{2e^{-3}h(t-1)}\ e^{-2t}u(t)$

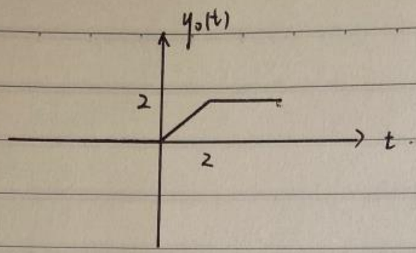
$\frac{dx(t)}{dt} \to -3x(t) + 2e^{-3}h(t-1)$.

$\therefore 2e^{-3}h(t-1) = e^{-2t}u(t)$

$\therefore h(t) = \frac{1}{2}e^{3} \cdot e^{-2(t+1)}u(t+1)$

$= \frac{1}{2}e^{-2t+1}u(t+1)$.

47, (a)



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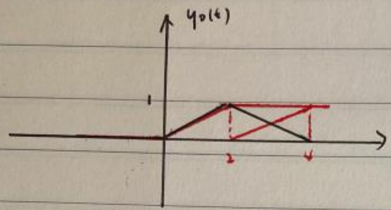
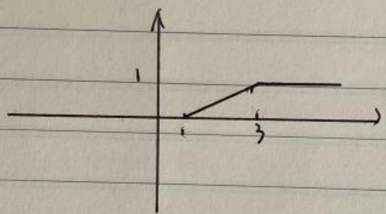
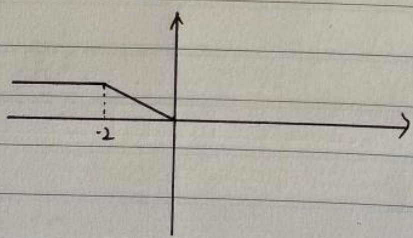


(c)



(e)



(f)