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第9章 2, 5, 7, 8, 9, 13, 21 (a, b, i, j), 22 (a, b, c), 28, 31, 32, 33, 34, 35.

2. (a) $X(s) = \int_{-\infty}^{+\infty} x(t) e^{-st} dt$

$$= \int_1^{+\infty} e^{-(s+5)t} dt$$

$$= -\frac{1}{s+5} e^{-(s+5)t} \Big|_1^{+\infty}$$

$$= \frac{e^{-(s+5)}}{s+5}, \text{ Re}(s) > -5$$

(b) $G(s) = \int_{-\infty}^{-t_0} A e^{-st} e^{-5t} dt$

$$= \frac{A}{s+5} e^{-(s+5)t} \Big|_{-\infty}^{-t_0}$$

$$\therefore A = 1, t_0 = -1$$

收敛域为 $\text{Re}(s) < -5$

5. (a) 在 s 平面有极点 $s = -2$

在无限远有一个极点

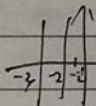
(b) 在 s 平面无极点

在无限远点有一个极点

(c) 在 s 平面有一个极点 $s = 1$

在无限远点无极点

7.



$$s^2 + s + 1 = 0$$

$$s^2 + s + \frac{1}{4} = -\frac{3}{4}$$

$$s = -\frac{1}{2} \pm \frac{\sqrt{3}}{2}j$$

有 4 个信号

8. $g(t) = x(t) e^{2t}$

$$G(s) = X(s-2)$$

$$x(t) = g(t) e^{-2t}$$

$\therefore G(s)$ 有两极点 -1 和 1

$g(t)$ 收敛

$\therefore g(j\omega)$ 收敛

\therefore 在 $X(s)$ 的收敛域内 0 在收敛域内

$\therefore x(t)$ 是右边信号

$\therefore x(t)$ 是双边信号

$\therefore x(t)$ 是双边信号

9. $X(s) = \frac{2(s+2)}{(s+3)(s+4)}$

$= 2\left(\frac{2}{s+4} - \frac{1}{s+3}\right)$

$\text{Re}\{s\} > -3 \therefore X(t)$ 是右边信号.

$\therefore X(t) = 4e^{-4t}u(t) - 2e^{-3t}u(t)$

13. $g(t)$ 有两极点 1 和 -1 . $\lim_{s \rightarrow 0} s = 0$.

$g(t) = X(t) + \alpha X(-t)$

$X(t) = \beta e^{-t}u(t)$

$X(-t) = \beta e^t u(-t)$

$\therefore g(t) = \beta e^{-t}u(t) + \alpha \beta e^t u(-t)$

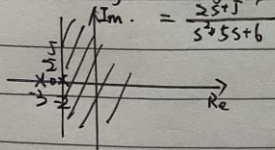
$G(s) = \frac{1}{2} \frac{1}{s+1} + \frac{1}{2} \frac{1}{s-1}$

$e^{-t}u(t) \rightarrow \text{Laplace transform } \frac{1}{s+1} \quad \text{Re}\{s\} > -1$

$e^t u(-t) \rightarrow \text{Laplace transform } -\frac{1}{s-1} \quad \text{Re}\{s\} < 1$

$\therefore \alpha = -1, \beta = \frac{1}{2}$

21. (a) $X(s) = \frac{1}{s+2} + \frac{1}{s+3} \quad \text{Re}\{s\} > -2$



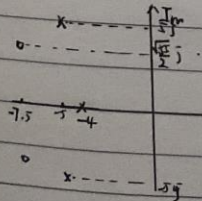
(b) $X(t) = e^{-4t}u(t) + e^{-st}(\sin St)u(t)$

由欧拉公式得

$x(t) = e^{-4t}u(t) + \frac{1}{2j} (e^{-st+jSt} - e^{-st-jSt})u(t)$

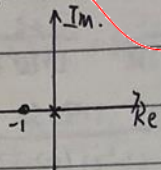
$\therefore X(s) = \frac{1}{s+4} + \frac{1}{2j} \left(\frac{1}{s+jS} - \frac{1}{s-jS} \right)$

$= \frac{1}{s+4} + \frac{5}{(s+4)^2 + 25} \quad \text{Re}\{s\} > -4$



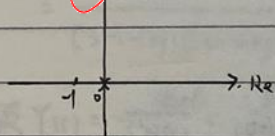
(i). $x(t) = \delta(t) + u(t)$

$X(s) = 1 + \frac{1}{s} \quad \text{Re}\{s\} > 0$



(j). $x(t) = \delta(3t) + u(3t)$

$X(s) = 1 + \frac{1}{3} \times \frac{3}{s} = 1 + \frac{1}{s} \quad \text{Re}\{s\} > 0$



22. (a) $\therefore \sin 3t u(t) \xrightarrow{L} \frac{3}{s^2 + 9} \quad \text{Re}\{s\} > 0$

$\therefore \frac{1}{3} \sin 3t u(t) \xrightarrow{L} \frac{1}{s^2 + 9} \quad \text{Re}\{s\} > 0$

$\therefore x(t) = \frac{1}{3} \sin 3t u(t)$

(b). $\therefore \cos 3t u(t) \xrightarrow{L} \frac{s}{s^2 + 9} \quad \text{Re}\{s\} > 0$

$\therefore \cos -3t u(t) \xrightarrow{L} -\frac{s}{s^2 + 9} \quad \text{Re}\{s\} < 0$

$\therefore x(t) = -\cos 3t u(t)$

(c). ~~由~~ 由 s 域平稳性得

$x(t) = -e^{-t} \cos 3t u(t)$

28. (a) $\text{Re}\{s\} < -2$

$-2 < \text{Re}\{s\} < -1$

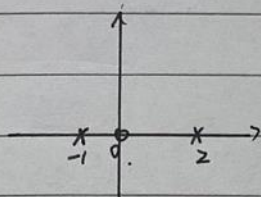
$-1 < \text{Re}\{s\} < 1$

~~$\text{Re}\{s\} > 1$~~

~~$\text{Re}\{s\} > 2$~~

(b). 对于 $\text{Re}\{s\} > 1$, 有系统是因果的, 其他是非因果的。
对于 $\text{Re}\{s\} < 1$, 有系统是稳定的, 其他是不稳定的。

31. (a) $H(s) = \frac{1}{s^2 - s - 2}$



(b) (i). \therefore 系统是稳定的

\therefore 收敛域包括 $j\omega$ 轴.

\therefore 收敛域为 $-1 < \text{Re}\{s\} < 2$.

$$H(s) = \frac{1}{3} \frac{1}{s-2} - \frac{1}{3} \frac{1}{s+1}$$

$$\therefore h(t) = -\frac{1}{3} e^{-t} u(t) + \frac{1}{3} e^{2t} u(-t)$$

(ii) \therefore 系统是因果的

\therefore 收敛域为 $\text{Re}\{s\} > 2$.

$$\therefore h(t) = -\frac{1}{3} e^{-t} u(t) + \frac{1}{3} e^{2t} u(t)$$

(iii) \therefore 系统既不稳定又不因果

\therefore 收敛域为 $\text{Re}\{s\} < -1$.

$$\therefore h(t) = \frac{1}{3} e^{-t} u(-t) - \frac{1}{3} e^{2t} u(-t)$$

32. $x(t) = e^{2t}$ 时 $y(t) = \frac{1}{6} e^{2t}$

$$\therefore H(2) = \frac{1}{6}$$

$$sH(s) + 2H(s) = \frac{1}{s+4} + \frac{b}{s} \quad \text{Re}\{s\} > 0$$

$$4H(2) = \frac{2}{3} = \frac{1}{6} + \frac{b}{2}$$

$$\Rightarrow b = 1$$

$$\therefore H(s) = \frac{2}{s(s+4)} \quad \text{Re}\{s\} > 0$$

33.

$$33. x(t) = e^{-t} u(t) + e^t u(-t)$$

$$\therefore X(s) = \frac{1}{s+1} - \frac{1}{s-1}, \quad -1 < \operatorname{Re}\{s\} < 1$$

$$\therefore Y(s) = \frac{s+1}{s^2+2s+2} \cdot \left(\frac{1}{s+1} - \frac{1}{s-1} \right)$$

$$= \frac{-2(s+1)}{(s^2+2s+2)(s^2-1)}$$

$$= \frac{-2}{(s^2+2s+2)(s-1)}$$

$$= \frac{-2}{[s-(-1+j)] [s-(-1-j)] (s-1)}, \quad -1 < \operatorname{Re}\{s\} < 1$$

$$\text{设 } Y(s) = \frac{As+B}{s^2+2s+2} + \frac{C}{s-1}$$

$$\therefore As^2 - As + Bs - B + Cs^2 + 2Cs + 2C = -2$$

$$\Rightarrow C = -\frac{2}{5}, \quad A = \frac{2}{5}, \quad B = \frac{6}{5}$$

$$\therefore Y(s) = \frac{2}{5} \cdot \frac{s+3}{s^2+2s+2} - \frac{2}{5} \frac{1}{s-1}$$

$$= \frac{2}{5} \frac{s+1}{(s+1)^2+1} + \frac{4}{5} \frac{1}{(s+1)^2+1} - \frac{2}{5} \frac{1}{s-1}, \quad -1 < \operatorname{Re}\{s\} < 1$$

$$\therefore y(t) = \frac{2}{5} e^{-t} \cos t u(t) + \frac{4}{5} e^{-t} \sin t u(t) + \frac{2}{5} e^t u(-t)$$

34. $H(s)$ 因果稳定.

\therefore 收敛域包含 $j\omega$ 轴且为右半信号.

$$\frac{1}{s} H(s) \xrightarrow{s \rightarrow \infty} 0$$

$$\text{输入为 } u(t) \text{ 时 } X(s) = \frac{1}{s}, \quad \operatorname{Re}\{s\} > 0$$

$$\therefore \frac{1}{s} H(s) \text{ 绝对可积.}$$

$$\therefore H(0) = 0$$

$$\text{输入为 } tu(t) \text{ 时 } X(s) = \frac{1}{s^2}, \quad \operatorname{Re}\{s\} > 0$$

$$\therefore \frac{1}{s^2} H(s) \text{ 不绝对可积.}$$

\therefore 零阶 不限 = 次项.

$$\text{设 } Y(s) = s^2 H(s) + 2s H(s) + 2H(s)$$

$$\therefore H(s) = \frac{Y(s)}{s^2+2s+2}$$

$\therefore Y(s)$ 是有限持续期且绝对可积

$\therefore Y(s)$ 的收敛域是整个 s 平面

~~$\therefore H(s) = 0.2$~~

~~$\therefore H(s) = 1$~~

$\therefore Y(s)$ 没有极点 只有零点

$\therefore H(s)$ 有一个零点且 $H(1) = 0.2$

$\therefore H(s) = \frac{s}{s^2 + 2s + 2}, \operatorname{Re}\{s\} > -1$

35. (a) $\frac{Y(s)}{X(s)} = \frac{s^2 - s - 6}{s^2 + 2s + 1}$

\therefore 微分方程为 $y''(t) + 2y'(t) + y(t) = x''(t) - x'(t) - 6x(t)$

(b) \therefore 该系统是因果的

极点为 $\operatorname{Re}\{s\} = -1 < 0$

\therefore 该系统是稳定的

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