# Notations

September 28, 2025

- RIME in delay coordinates:

$$V_{\boldsymbol{b}}(\nu) = \sum_a \lambda_a(\nu) \frac{\gamma(\nu)}{\Omega_p(\nu)} \int \mathrm{d}^2\boldsymbol{u}\, \tilde{A}(\boldsymbol{u_b} - \boldsymbol{u}, \nu)\, T(\boldsymbol{u}, \eta_a) \tag{1}$$

  where

$$\lambda_a(\nu) = \Delta\eta \exp\left(i2\pi\eta_a\nu\right) \tag{2}$$

- Covariance and response matrix:

$$\langle V^A_{\boldsymbol{b}}(\nu_i) V^{B\,*}_{\boldsymbol{b}}(\nu_j)\rangle = \sum_\alpha \bar{P}(\boldsymbol{u_b}, \eta_\alpha)\, Q^{\alpha,AB}_{ij} + N^{AB}_{ij}$$

  where

$$\begin{aligned} Q^{\alpha,AB}_{ij} &= Q^{\alpha,\mathrm{alt}}_{ij} R^{AB}_{ij}\, \mathbf{B}_{ij}, \\ N^{AB}_{ij} &= \tilde{N}^{AB}_{ij} R^{AB}_{ij} \end{aligned} \tag{3}$$

  where we have defined

  - The quadratic term of Fourier transform in the $\alpha$-th delay bin:

$$Q^{\alpha,\mathrm{alt}}_{ij} \equiv \sum_{a\in\{\alpha\}} \lambda_a(\nu_i)\lambda^*_a(\nu_j) \tag{4}$$

    If no LST binning, it represents a quadratic Fourier term:

$$Q^{a,\mathrm{alt}}_{ij} \equiv \lambda_a(\nu_i)\lambda^*_a(\nu_j) \tag{5}$$

  - The operations on the frequency axis (quadratic form of arbitrary weights $\gamma$, flagging $F$)

$$R^{AB}_{ij} = \tilde{R}_{A,i}\tilde{R}^*_{B,j} \qquad\qquad \tilde{R}_{A,i} = F_A(\nu_i)\gamma_A(\nu_i) \tag{6}$$

– Residual transformation (beam effects) in observer-coordinate modes:

$$
\begin{aligned}
\mathbf{B}_{ij} &\equiv \frac{1}{\Omega_p(\nu_i)\Omega_p(\nu_j)} \int \mathrm{d}^2\boldsymbol{u}\, \tilde{A}(\boldsymbol{u_b} - \boldsymbol{u}, \nu_i)\tilde{A}(\boldsymbol{u_b} - \boldsymbol{u}, \nu_j) \\
&= \frac{1}{\Omega_p(\nu_i)\Omega_p(\nu_j)} \int \mathrm{d}^2\boldsymbol{\theta}\, A(\boldsymbol{\theta}, \nu_i)A(\boldsymbol{\theta}, \nu_j)
\end{aligned}
\tag{7}
$$

where $\Omega_p$ is the beam normalisation.

- Quadratic estimate (represented using linear algebra in the frequency vector space):

$$
q_\alpha \propto \mathrm{Tr}\left(E^\alpha\, x_A x_B^\dagger\right) \tag{8}
$$

$$
\begin{aligned}
\langle q_\alpha \rangle &\propto \sum_\beta p_\beta\, \mathrm{Tr}\left(E^\alpha\, Q^{\beta,AB}\right) + \mathrm{Tr}\left(E^\alpha\, N^{AB}\right). \\
&\equiv \sum_\beta W_\beta^{AB}\, p_\beta + b_\alpha^{AB}.
\end{aligned}
\tag{9}
$$

where

$$
W_\beta^{AB} = \mathrm{Tr}\left(E^\alpha\, Q^{\beta,AB}\right), \qquad b_\alpha^{AB} = \mathrm{Tr}\left(E^\alpha\, N^{AB}\right).
$$

And the norm is the sum of $W_\beta^{AB}$ over $\beta$.