MPCS 58020 2017 Homework 4

Assignment date: May 11, 2017

Due date: May 22, 2017

1. Create a 100 point discretely sampled Gaussian filter g_t by evaluating 100 equally spaced points of $g(x) = e^{-x^2}$ on the interval $-3 \leq x \leq 3$. Also, create an ARMA(2,1) process x_t with $\phi_1 = .5$, $\phi_2 = .1$, and $\theta = .5$. Using this signal x_t and filter g_t, do the following:

   - Calculate the discrete convolution $x * g$ using periodic boundary conditions with a hand-coded routine (no convolution libraries) that carries out the calculation in physical space. Do this for signals of length 1000, 10000, 100000, and 1000000. Time your results.
   - Repeat in Fourier space. Verify that your results are the same and compare performance. You may use an FFT package in the language of your choice.

2. Derive an expression for the Fourier Transform of the following functions, then plot the function and it's transform (pick a value for the constants in the plot). You may use symbolic integration software, but show the integral you set up. You may also use the function transforms listed in the notes together with properties of Forier Transform to derive the new fucntion transforms.

   (a) $f(t) = Ae^{-at^2}$ where a and A are nonzero real constants

   (b) $f(t) = sin(2\pi vt) + cos(2\pi vt)$ where v is a nonzero real constant

   (c) $\gamma(t) = \int_{-\infty}^{\infty} f(t')f(t+t')dt'$, for any (sufficiently well behaved) real valued function f.

   (d) $f(t) = \begin{cases} \frac{1}{a} & \text{for } -a/2 \leq t \leq a/2 \\ 0 & \text{otherwise} \end{cases}$ for real $a \neq 0$.

3. Create an instance x_t of an invertible ARMA(2,2) process with coefficients of your choice using $n = 10000$. Filter the process using a physical space filter defined as $g(t) = -2te^{-t^2/2}$ (using either convolution or FFT). Submit the following plots: a) x_t, b) filtered x_t, c) the filter in physical space, d) the filter in Fourier space. Explain what you observe in (b).

4. Derive an analytic expression for the autocovariance function for the ARMA(2,2) process defined in the previous problem. Compare a plot of this expression vs. the sample autocovariance from your sample signal x_t.

5. Shumway problem 4.2

6. Shumway problem 4.4

7. Shumway problem 4.8

8. Shumway Problem 4.9