In [364]:
```
import warnings
warnings.filterwarnings('ignore')
```

CQF Exam One - 2021/3/14

NOTE TO GRADER:

Sorry had a hard time convert this report to PDF and failed, can only do HTML instead

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Question 1(a)

Theoretically, quarterly and monthly Sharpe Ration (SR) would be:

$SR_{quarterly}$ = $SR_{annually} * \frac{1}{\sqrt{4}} = \frac{R_a - R_a}{\sigma_a} * \frac{1}{2} = 0.265$

$SR_{monthly}$ = $SR_{annually} * \frac{1}{\sqrt{12}} = 0.153$

Question 1(b)

Annually, P(P&L < 0) = P(x < - $SR_{annually}$) = P(x < - 0.53)

Quarterly, P(P&L < 0) = P(x < - $SR_{quarterly}$) = P(x < - 0.265)

Monthly, P(P&L < 0) = P(x < - $SR_{monthly}$) = P(x < - 0.153)

Now $P(x < -0.53) < P(x < -0.265) < P(x < -0.153)$, as a quick example, taking standard normal distribution (0,1) from python's scipy CDF function:

P(P&L < 0) = P(x < - $SR_{annually}$) = P(x < - 0.53) = 29.81%

P(P&L < 0) = P(x < - $SR_{quarterly}$) = P(x < - 0.265) = 39.55%

P(P&L < 0) = P(x < - $SR_{monthly}$) = P(x < - 0.153) = 43.92%

which means probability of an annualized P&L is smaller than 0 is 29.81%, that of quarterly and monthly are 39.55% and 43.92%, and that illustrates

“Evaluating the P&L more frequently, means its appears more risky than it actually is.”

python code calculates CDFs of standard normal distribution for Question 1(b):

In [1]:
```
from scipy.stats import norm
print("P(x<-0.53) = {:.2%}".format(norm.cdf(-0.53)))
print("P(x<-0.265) = {:.2%}".format(norm.cdf(-0.265)))
print("P(x<-0.153) = {:.2%}".format(norm.cdf(-0.153)))
```

```
P(x<-0.53) = 29.81%
P(x<-0.265) = 39.55%
P(x<-0.153) = 43.92%
```

Question 2 (a)

Assuming there is no budget constraint (to be clear, my understanding of it is $1^T\omega$ does not have to be necessarily equal to 1. Lagrangian function with respect to ω and λ:

$$L(\omega, \lambda) = \frac{1}{2}\omega^T\Sigma\omega + \lambda[m - \gamma - (\mu - \gamma\mathbf{1})^T\omega]$$

and solve for the first order condition:

$$\frac{\delta L}{\delta \omega} = \omega^T\Sigma - \lambda(\mu - \gamma\mathbf{1})^T = 0$$
$$\frac{\delta L}{\delta \lambda} = m - \gamma - (\mu - \gamma\mathbf{1})^T\omega = 0$$

since $\frac{\delta^2 L}{\delta \omega^2} = \Sigma$,which is positive, we have reached the optimal weight vector:

$$\lambda = \frac{(m - \gamma)}{(\mu - \gamma\mathbf{1})^T\Sigma^{-1}(\mu - \gamma\mathbf{1})}(i)$$
$$\omega = \lambda(\mu - \gamma\mathbf{1})\Sigma^{-1}(ii)$$

that's essentially as far as we can go without putting some values into the correlation matrix

Question 2 (b)

The following function is based on formula (ii) and returns get optimal(ω), target portfolio return (based on result of $\omega^T * \mu$, not mirrowing input μ sort of like a validation) and portfolio std deviation:

In [68]:
```
import numpy as np

# define a portfolio optimize function based on analytical result
def analytical_portfolio_optimize(mu, sigma, corr, m, r) -> tuple:
    # one vector
    ones = np.ones((mu.shape[0],1))
    # covariance matrix
    covar = sigma.T.dot(corr).dot(sigma)
    covar_inverse = np.linalg.inv(covar)
    r_vector = r*ones
    # formula (i) from previous cell
    _lambda = (m-r)/((mu - r_vector).T.dot(covar_inverse).dot(mu - r_vector))
    omega = _lambda*(mu-r_vector).T.dot(covar_inverse)

    # return value is (portfolio weights, portfolio calculated return, portfol
io std deviation)
    return omega, (r + omega.dot(mu-r_vector))[0][0], (omega.dot(covar).dot(om
ega.T))[0][0]**0.5
```

define parameters and run target return value(m) = 5%

$\textbf{portfolio weights}$ are: 0.11306403, 0.30116639, -0.23623674, 0.20893623

$\textbf{portfolio\_sigma}$: 0.03774847334530612

$\textbf{portfolio\_target\_return}$ should be equal to $r + (\mu - \gamma\mathbf{1})\omega$, which is 0.05

In [66]:
```
# asset expected returns
mu = np.array([[.04],[.08],[.12],[.15]])
# asset std deviation
sigma = np.diag([.07,.12,.18,.26])
# correlation matrix
corr = np.array([[1.0,.2,.5,.3],[.2,1.0,.7,.4],[.5,.7,1.0,.9],[.3,.4,.9,1.0]])
# risk free rate
r = 0.03
# RUN m = 5%
weights, portfolio_target_return, portfolio_sigma = analytical_portfolio_optim
ize(mu=mu, sigma=sigma, corr=corr, m=0.05, r=r)
print(f"portfolio weights: {weights}")
print(f"portfolio risk(std deviation): {portfolio_sigma}")
print(f"calculated portfolio return: {portfolio_target_return}")
```

```
portfolio weights: [[ 0.11306403  0.30116639 -0.23623674  0.20893623]]
portfolio risk(std deviation): 0.03774847334530612
calculated portfolio return: 0.05
```

when m = 7.5 %, 10%, 12.5%:

In [65]:
```
for m in [0.075, 0.1,0.125]:
    weights, portfolio_target_return, portfolio_sigma = \
        analytical_portfolio_optimize(mu=mu, sigma=sigma, corr=corr, m=m, r=r)
    print(f"portfolio target return: {m}")
    print(f"portfolio weights: {weights}")
    print(f"portfolio risk(std deviation): {portfolio_sigma}")
    print(f"calculated portfolio return: {portfolio_target_return}\n")
```

```
portfolio target return: 0.075
portfolio weights: [[ 0.25439408  0.67762437 -0.53153267  0.47010651]]
portfolio risk(std deviation): 0.08493406502693876
calculated portfolio return: 0.07499999999999998

portfolio target return: 0.1
portfolio weights: [[ 0.39572412  1.05408235 -0.82682859  0.73127679]]
portfolio risk(std deviation): 0.13211965670857143
calculated portfolio return: 0.1

portfolio target return: 0.125
portfolio weights: [[ 0.53705416  1.43054033 -1.12212452  0.99244707]]
portfolio risk(std deviation): 0.17930524839020404
calculated portfolio return: 0.125
```

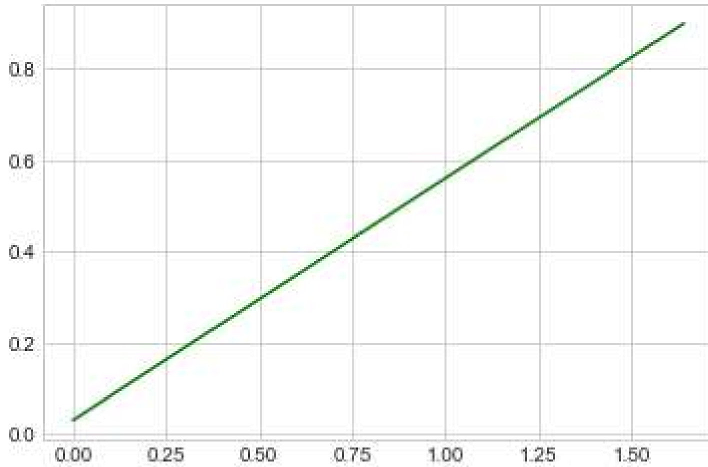
Plotting a Efficient Frontier

based on the constraint: $\gamma - (\mu - \gamma \mathbf{1})^T \omega = m$, and the fact that there is not budget constraints, this portfolio contains risky assets and risk free asset, so its shape should be a straight line, whose slope is the sharpe ratio.

In [148]:
```
from matplotlib.pyplot import plot, style

m_min = 0.03
m_max = 0.9
n = 1000
# asset expected returns
mu = np.array([[.04],[.08],[.12],[.15]])
# asset std deviation
sigma = np.diag([.07,.12,.18,.26])
# correlation matrix
corr = np.array([[1.0,.2,.5,.3],[.2,1.0,.7,.4],[.5,.7,1.0,.9],[.3,.4,.9,1.0]])

#to contain returns and sigmas from analytical portfolio
portfolio_returns, portfolio_sigmas = [], []

# m is expected return ranging from
for m in range(int(m_min*n), int(m_max*n), 1):
    m = m/n
    _, _return, _sigma = analytical_portfolio_optimize(mu=mu, sigma=sigma, cor
r=corr, m=m, r=r)
    portfolio_returns.append(_return)
    portfolio_sigmas.append(_sigma)
```

In [149]:
```
style.use('seaborn-whitegrid')
plot(portfolio_sigmas, portfolio_returns, color='green', linestyle='solid')
```

Out[149]: [<matplotlib.lines.Line2D at 0x2437be7bb48>]



Question 3

VaR sensitivities:

$$\frac{\delta VaR(\omega)}{\delta\omega_i} = \mu_i + Factor * \frac{(\Sigma\omega)_i}{\sqrt{\omega^T\Sigma\omega}}$$

$$\frac{\delta ES(\omega)}{\delta\omega_i} = \mu_i - \frac{\phi(Factor)}{1-c} * \frac{(\Sigma\omega)_i}{\sqrt{\omega^T\Sigma\omega}}$$

In [183]:
```
# portfolio asset, std deviation and weights
sigma = np.diag([.3,.2,.15])
corr = np.array([[1.0,.8,.5],[.8,1.0,.3],[.5,.3,1.0]])
omega = np.array([[0.5], [0.2], [0.3]])
mu = np.array([[.0],[.0],[.0]])
# confidence
c = 0.99
```

define inverse of normal distribution CDF and PDF as following:

In [186]:
```
from scipy.stats import norm
# factor(), pdf() takes argument such as 0.01, 0.05, 0.1
# factor() return a Z score and pdf() return a PDF
factor = lambda x: norm.ppf(1-x)
pdf = lambda x: norm.pdf(x)
```

given correlation and variance matrix we could calculate covariance matrix:

$$\Sigma = \sigma^T \cdot Corr \cdot \sigma$$

In [187]:
```
covar = sigma.T.dot(corr).dot(sigma)
print(f"Covariance matrix is\n {covar}")
```

```
Covariance matrix is
 [[0.09   0.048  0.0225]
 [0.048  0.04   0.009 ]
 [0.0225 0.009  0.0225]]
```

define portfolio std deviation (stddev) as $\sqrt{\omega^T \Sigma \omega}$, which can be calculated as:

In [188]:
```
stddev = (omega.T.dot(covar).dot(omega))**0.5
print(f"portfolio standard deviation is \n {stddev[0][0]}")
```

```
portfolio standard deviation is
 0.20869834690289238
```

So $\textbf{VaR sensitivities wrt each asset, } \frac{\delta VaR(\omega)}{\delta \omega_i} = \mu_i + Factor * \frac{(\Sigma\omega)_i}{\sqrt{\omega^T \Sigma \omega}}$ can be calculated as following:

In [194]:
```
mu + (factor(c)/stddev) * covar.dot(omega)
```

Out[194]:
```
array([[-0.68386475],
       [-0.38679881],
       [-0.2207094 ]])
```

So $\textbf{ES sensitivities wrt each asset, } \frac{\delta ES(\omega)}{\delta \omega_i} = \mu_i - \frac{\phi(Factor)}{1-c} * \frac{(\Sigma\omega)_i}{\sqrt{\omega^T \Sigma \omega}}$ can be calculated as following:

In [195]:
```
mu - (pdf(factor(c))/((1-c)*stddev)) * covar.dot(omega)
```

Out[195]:
```
array([[-0.78347958],
       [-0.44314167],
       [-0.25285893]])
```

Question 4 (a)

Notice: For Question 4, terms in bold are columns in the analysis dataset to help better understand it

Before we start, we need to define some global variable and functions to help us analye, read in raw data and calculate daily log return

$$\textbf{Daily Log Return} = Log(\frac{ClosingPrice_t}{ClosingPrice_{t-1}})$$

In [468]:
```
import pandas as pd
import numpy as np
from scipy.stats import norm

##############################################################################
###########
# HANDY FUNCTIONS
# factor() takes argument such as 0.99, 0.95, 0.90 and return a Z-score
factor = lambda x: norm.ppf(x)
# square_root() returns square_root
square_root = lambda x: np.sqrt(x)
# calculate daily return
log_return = lambda x: np.log(x[-1]/x[0])

##########b###################################################################
############

##############################################################################
###########
# GLOBAL INPUTS
HISTORICAL_VAR_ROLLING_DAYS = 21
FORWARD_VAR_ROLLING_DAYS = 10
DAILY_RETURN_ROLLING_DAYS = 2 # daily return
VAR_QUANTILE = 0.99
FACTOR = factor(VAR_QUANTILE)
##############################################################################
###########

# reading XLSX file, assuming the ftse 100 excel is in the same folder as this
notebook
data_21d = pd.read_excel("./Data_FTSE100- January 2021.xlsx", skiprows=1)
data_21d = data_21d.set_index('Date')
data_21d['Daily Log Return'] = data_21d['Closing Price'].rolling(window=DAILY_
RETURN_ROLLING_DAYS).apply(log_return)
```

21 historical day rolling standard deviation and forward 10 Day 99th VaR based on 21 historical day rolling daily standard deviation

$\textbf{21D Daily Deviation} = \frac{\sum(DailyReturn_i - \mu)}{N-1}$, where N = 21

$\textbf{10 Day VaR} = Factor * \Sigma_t * \sqrt{10}$

In [469]:
```
# 21 DAY DAILY Deviation and VaR
data_21d[f'{HISTORICAL_VAR_ROLLING_DAYS}D Daily Deviation'] = data_21d['Daily
 Log Return'].rolling(window=HISTORICAL_VAR_ROLLING_DAYS).std()
data_21d[f"{FORWARD_VAR_ROLLING_DAYS} Day VaR"] = data_21d[f'{HISTORICAL_VAR_R
OLLING_DAYS}D Daily Deviation']*-Factor*square_root(FORWARD_VAR_ROLLING_DAYS)
```

Calculate Breaches and Consecutive Breaches

we calculate 10 day log return by $\mathbf{10\ Day\ Log\ Return} = Log(\frac{ClosingPrice_t}{ClosingPrice_{t-10}})$

we shift existing 10 VaR column 10 days (10 rows) later by Python Pandas shift function to get comparable 10 VaR and 10 Day Log Return to same row for calculation (called $\mathbf{Shifted\ 10\ VaR}$ below).

$\mathbf{Breach}$ is a bool column that indicates if there is a breach or not

$\mathbf{Rolling\ Consecutive\ Breaches}$ column indicates number of consecutive breaches, which is calculated by $get\_consecutive\_breaches()$

and finally $\mathbf{Total\ Consecutive\ Breaches}$ is based on $\mathbf{Rolling\ Consecutive\ Breaches}$ and only takes the largest number of a row from $\mathbf{Rolling\ Consecutive\ Breaches}$ for example a row of rolling consecutive breaches like 1,2,3,4 would mean total consecutive breaches is 4

In [470]:

```
# function get consecutive breaches
def get_rolling_consecutive_breaches(breach: list) ->list:
    consecutive_breaches = []

    for i in range(len(breach)):
        if breach[i] == False:
            consecutive_breaches.append(0)
        elif breach[i] == True:
            consecutive_breaches.append( breach[i-1] +consecutive_breaches[-1
])
    return consecutive_breaches

# function get consecutive breaches based on rolling_consecutive_breaches
def get_total_consecutive_breaches(rolling_consecutive_breaches: list) ->list:
    total = 0
    for i in range(len(rolling_consecutive_breaches[:-1])):
        if rolling_consecutive_breaches[i+1] == 0:
            total = total + rolling_consecutive_breaches[i]
    return total

log_return = lambda x: np.log(x[-1]/x[0])
data_21d[f'{FORWARD_VAR_ROLLING_DAYS}D Log Return'] = data_21d['Closing Price'
].rolling(window=FORWARD_VAR_ROLLING_DAYS+1).apply(log_return)
data_21d[f'Shifted {FORWARD_VAR_ROLLING_DAYS} Day VaR'] = data_21d[f"{FORWARD_
VAR_ROLLING_DAYS} Day VaR"].shift(FORWARD_VAR_ROLLING_DAYS)
data_21d[f'{FORWARD_VAR_ROLLING_DAYS}D Breach'] = data_21d[f'{FORWARD_VAR_ROLL
ING_DAYS}D Log Return'] < data_21d[f'Shifted {FORWARD_VAR_ROLLING_DAYS} Day Va
R']
data_21d[f'{FORWARD_VAR_ROLLING_DAYS}D Rolling Consecutive Breaches'] = get_ro
lling_consecutive_breaches(data_21d[f'{FORWARD_VAR_ROLLING_DAYS}D Breach'].tol
ist())
```

21D: sample data of breach days

In [471]:
```
data_21d[ data_21d[f'{FORWARD_VAR_ROLLING_DAYS}D Breach'] ].head(10)
```

Out[471]:

Date	Closing Price	Daily Log Return	21D Daily Deviation	10 Day VaR	10D Log Return	Shifted 10 Day VaR	10D Breach	10D Rolling Consecutive Breaches
2010-01-28	5145.74	-0.013843	0.008309	-0.061123	-0.066252	-0.062370	True	0
2010-05-04	5411.11	-0.025936	0.011191	-0.082329	-0.056896	-0.046298	True	0
2010-05-05	5341.93	-0.012867	0.010954	-0.080584	-0.079455	-0.048439	True	1
2010-05-06	5260.99	-0.015268	0.011011	-0.081003	-0.084249	-0.051919	True	2
2010-05-07	5123.02	-0.026575	0.012011	-0.088357	-0.100621	-0.054720	True	3
2010-05-10	5387.42	0.050322	0.017085	-0.125685	-0.060540	-0.056555	True	4
2010-05-11	5334.21	-0.009926	0.016885	-0.124215	-0.075728	-0.057088	True	5
2010-07-05	4823.53	-0.003014	0.011506	-0.084645	-0.094033	-0.089368	True	0
2011-03-16	5598.23	-0.017187	0.008614	-0.063368	-0.055022	-0.051109	True	0
2011-08-04	5393.14	-0.034869	0.011582	-0.085205	-0.089806	-0.072342	True	0

21D: average of 21D Daily Deviation:

In [472]:
```
data_21d[f'{HISTORICAL_VAR_ROLLING_DAYS}D Daily Deviation'].mean()
```

Out[472]: 0.010266728665813643

21D: Percentage of VaR breaches

In [473]:
```
number_of_breaches = data_21d[data_21d[f'{FORWARD_VAR_ROLLING_DAYS}D Breach']]
.shape[0]
number_of_days = data_21d[~data_21d[f'Shifted {FORWARD_VAR_ROLLING_DAYS} Day V
aR'].isnull() ].shape[0]
breach_percentage_21 = number_of_breaches/number_of_days
print(f"Number of Breaches:{'{0:.4%}'.format(breach_percentage_21)}")
```

```
Number of Breaches:2.2495%
```

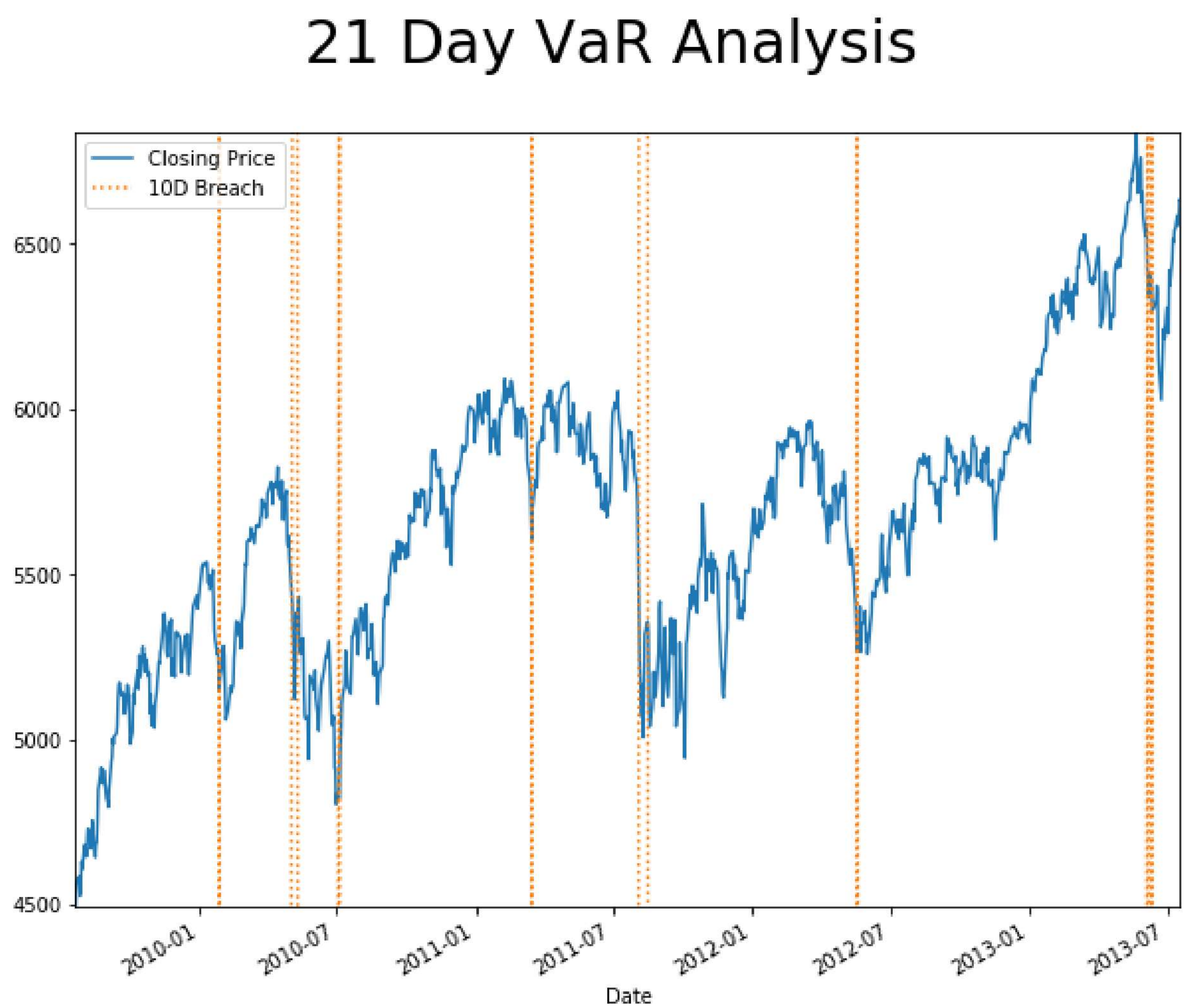
21D : Number of Consecutive Breaches days

In [474]:
```
rolling_consecutive_breaches = data_21d[f'{FORWARD_VAR_ROLLING_DAYS}D Rolling Consecutive Breaches'].tolist()
consecutive_breaches_21 = get_total_consecutive_breaches(rolling_consecutive_breaches)
print(f'Number of Consecutive Breaches: {consecutive_breaches_21} times')
```

```
Number of Consecutive Breaches: 14 times
```

21D: Plots

In [459]:
```
import matplotlib.pyplot as plt

max_price = data_21d['Closing Price'].max()
min_price = data_21d['Closing Price'].min()
fig, ax = plt.subplots()
fig.suptitle("21 Day VaR Analysis", fontsize=30)
graph_size = (10,8)

data_21d['Closing Price'].plot(figsize=graph_size, ylim = (min_price, max_pric
e) ,legend=True)
VaR_plot_21 = (data_21d[f'{FORWARD_VAR_ROLLING_DAYS}D Breach'].astype(int) *(m
ax_price)).plot(linestyle='dotted', figsize=graph_size,legend=True)
```



Question 4(b)

Without repeating articulating much of details, the following code, which repeats the above code used for 21 Day VaR analysis, calculates 99%/10day VaR with 42 historical day horizon by first defining HISTORICAL_VAR_ROLLING_DAYS as 42

In [520]:
```
#################################################################################
# GLOBAL INPUTS
HISTORICAL_VAR_ROLLING_DAYS = 42
FORWARD_VAR_ROLLING_DAYS = 10
DAILY_RETURN_ROLLING_DAYS = 2 # daily return
VAR_QUANTILE = 0.99
FACTOR = factor(VAR_QUANTILE)
#################################################################################

# reading XLSX file, assuming the ftse 100 excel is in the same folder as this notebook
data_42d = pd.read_excel("./Data_FTSE100- January 2021.xlsx", skiprows=1)
data_42d = data_42d.set_index('Date')
data_42d['Daily Log Return'] = data_42d['Closing Price'].rolling(window=DAILY_RETURN_ROLLING_DAYS).apply(log_return)

data_42d[f'{HISTORICAL_VAR_ROLLING_DAYS}D Daily Deviation'] = data_42d['Daily Log Return'].rolling(window=HISTORICAL_VAR_ROLLING_DAYS).std()
data_42d[f"{FORWARD_VAR_ROLLING_DAYS} Day VaR"] = data_42d[f'{HISTORICAL_VAR_ROLLING_DAYS}D Daily Deviation']*-Factor*square_root(FORWARD_VAR_ROLLING_DAYS)
data_42d[f'{FORWARD_VAR_ROLLING_DAYS}D Log Return'] = data_42d['Closing Price'].rolling(window=FORWARD_VAR_ROLLING_DAYS+1).apply(log_return)
data_42d[f'Shifted {FORWARD_VAR_ROLLING_DAYS} Day VaR'] = data_42d[f"{FORWARD_VAR_ROLLING_DAYS} Day VaR"].shift(FORWARD_VAR_ROLLING_DAYS)
data_42d[f'{FORWARD_VAR_ROLLING_DAYS}D Breach'] = data_42d[f'{FORWARD_VAR_ROLLING_DAYS}D Log Return'] < data_42d[f'Shifted {FORWARD_VAR_ROLLING_DAYS} Day VaR']
data_42d[f'{FORWARD_VAR_ROLLING_DAYS}D Rolling Consecutive Breaches'] = get_rolling_consecutive_breaches(data_42d[f'{FORWARD_VAR_ROLLING_DAYS}D Breach'].tolist())
```

Sample data

In [521]:
```
data_42d[data_42d[f'{FORWARD_VAR_ROLLING_DAYS}D Breach']].head(5)
```

Out[521]:

Date	Closing Price	Daily Log Return	42D Daily Deviation	10 Day VaR	10D Log Return	Shifted 10 Day VaR	10D Breach	10D Rolling Consecutive Breaches
2010-05-04	5411.11	-0.025936	0.008804	-0.064766	-0.056896	-0.050046	True	0
2010-05-05	5341.93	-0.012867	0.008888	-0.065382	-0.079455	-0.050578	True	1
2010-05-06	5260.99	-0.015268	0.009162	-0.067404	-0.084249	-0.051901	True	2
2010-05-07	5123.02	-0.026575	0.009687	-0.071263	-0.100621	-0.053352	True	3
2010-05-10	5387.42	0.050322	0.012619	-0.092832	-0.060540	-0.054214	True	4

42D: Average of 42D Daily Deviation:

In [522]:
```
data_42d[f'{HISTORICAL_VAR_ROLLING_DAYS}D Daily Deviation'].mean()
```

Out[522]: 0.010372089369491314

42D: Percentage of VaR breaches

In [523]:
```
number_of_breaches = data_42d[data_42d[f'{FORWARD_VAR_ROLLING_DAYS}D Breach']]
.shape[0]
number_of_days = data_42d[~data_42d[f'Shifted {FORWARD_VAR_ROLLING_DAYS} Day V
aR'].isnull() ].shape[0]
breach_percentage_42 = number_of_breaches/number_of_days
print(f"Number of Breaches:{'{0:.4%}'.format(breach_percentage_42)}")
```

```
Number of Breaches:2.1944%
```

42D : Number of Consecutive Breaches days

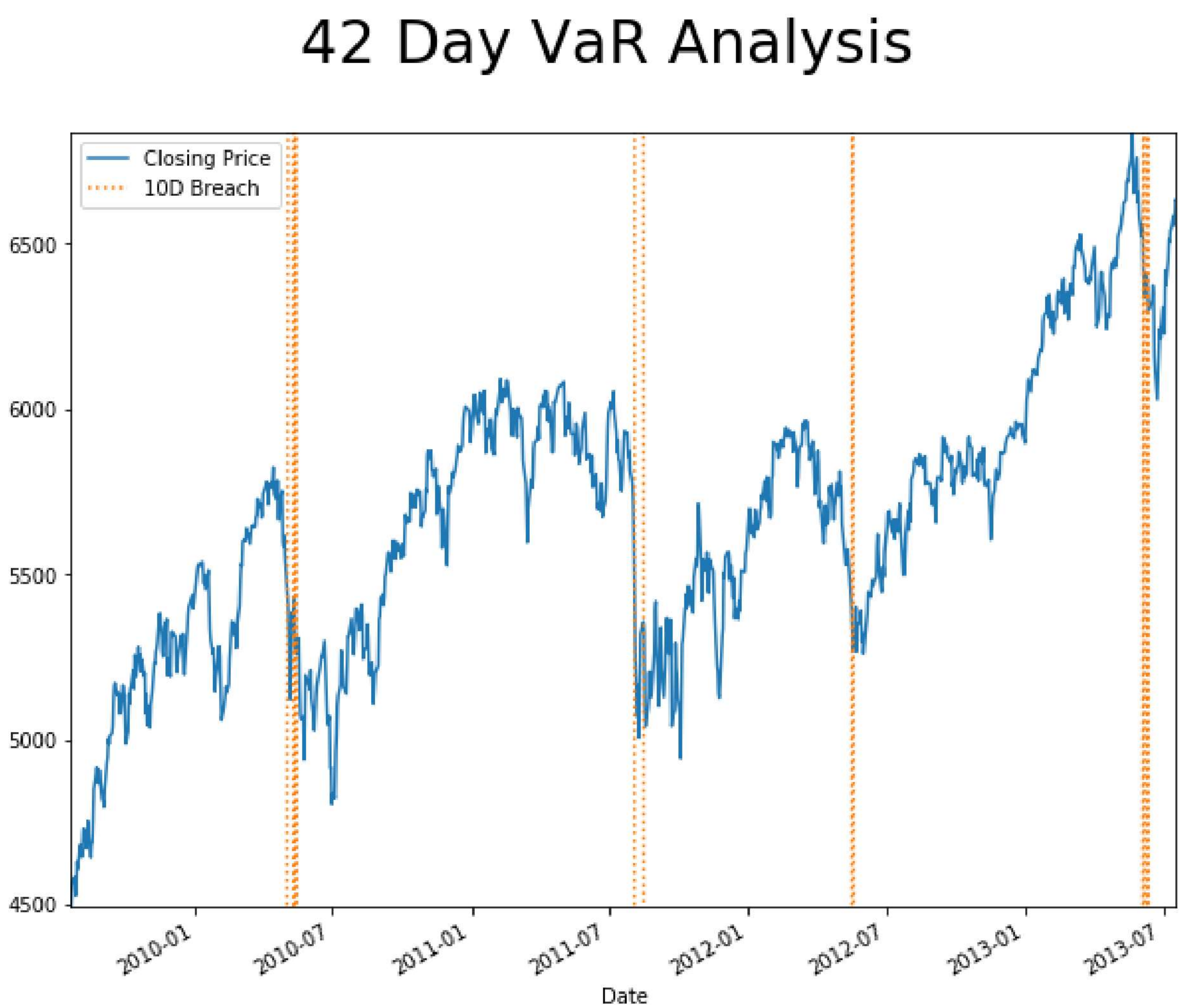
In [524]:
```
rolling_consecutive_breaches = data_42d[f'{FORWARD_VAR_ROLLING_DAYS}D Rolling
 Consecutive Breaches'].tolist()
consecutive_breaches_42 = get_total_consecutive_breaches(rolling_consecutive_b
reaches)
print(f'Number of Consecutive Breaches: {consecutive_breaches_42} times')
```

```
Number of Consecutive Breaches: 15 times
```

Plots - 21D and 42D altogether

In [525]:
```
import matplotlib.pyplot as plt

max_price = data_42d['Closing Price'].max()
min_price = data_42d['Closing Price'].min()
fig, ax = plt.subplots()
fig.suptitle("42 Day VaR Analysis", fontsize=30)
graph_size = (10,8)

data_42d['Closing Price'].plot(figsize=graph_size, ylim = (min_price, max_pric
e) ,legend=True)
(data_42d[f'{FORWARD_VAR_ROLLING_DAYS}D Breach'].astype(int) *(max_price)).plo
t(
    linestyle='dotted', figsize=graph_size,legend=True)
```

Out[525]:
```
<matplotlib.axes._subplots.AxesSubplot at 0x1a262f4b288>
```



Question 4 - Summary Table & Summary plots

In [533]:
```
pd.DataFrame([[breach_percentage_21, consecutive_breaches_21],[breach_percentage_42, consecutive_breaches_42]],
             columns = ['VaR Breach %', 'Consecutive Breaches'], index = ['21 Day', '42 Day'])
```

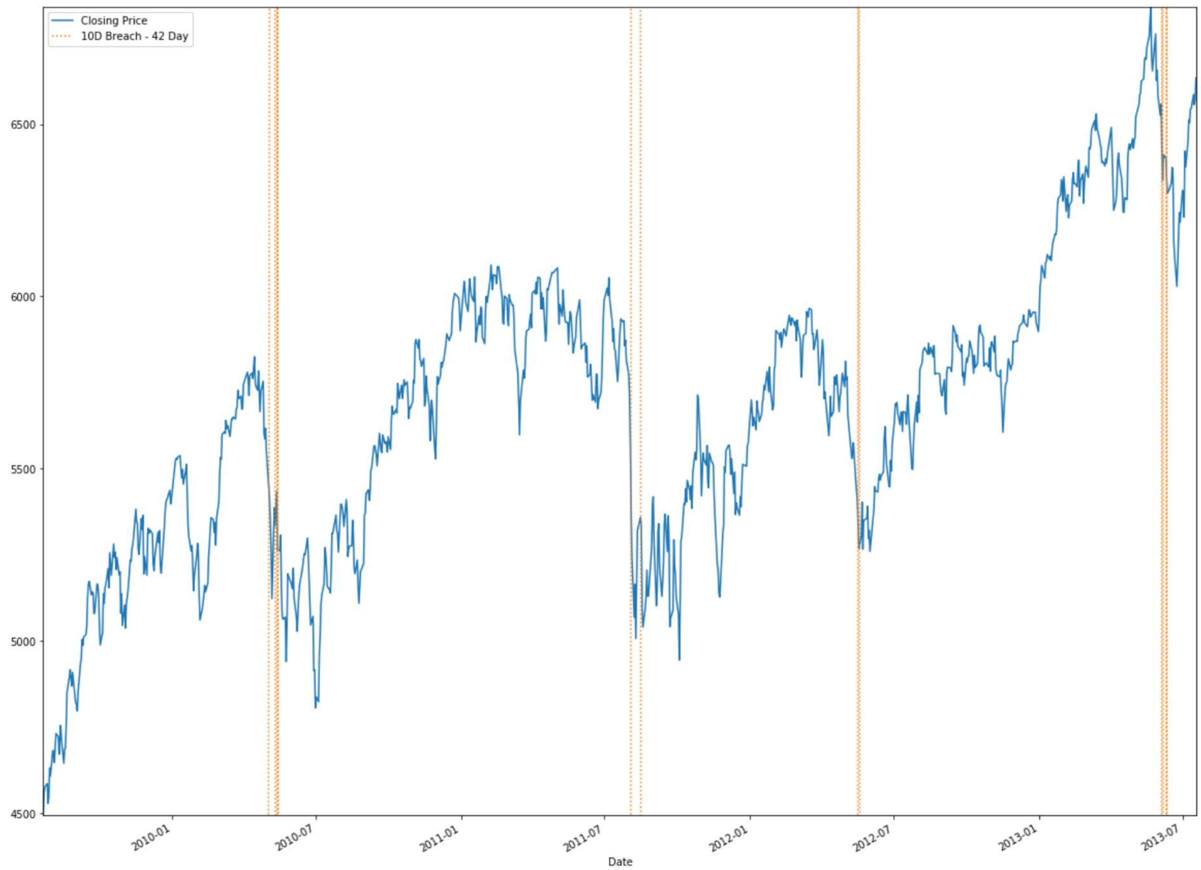
Out[533]:

	VaR Breach %	Consecutive Breaches
21 Day	0.022495	14
42 Day	0.021944	15

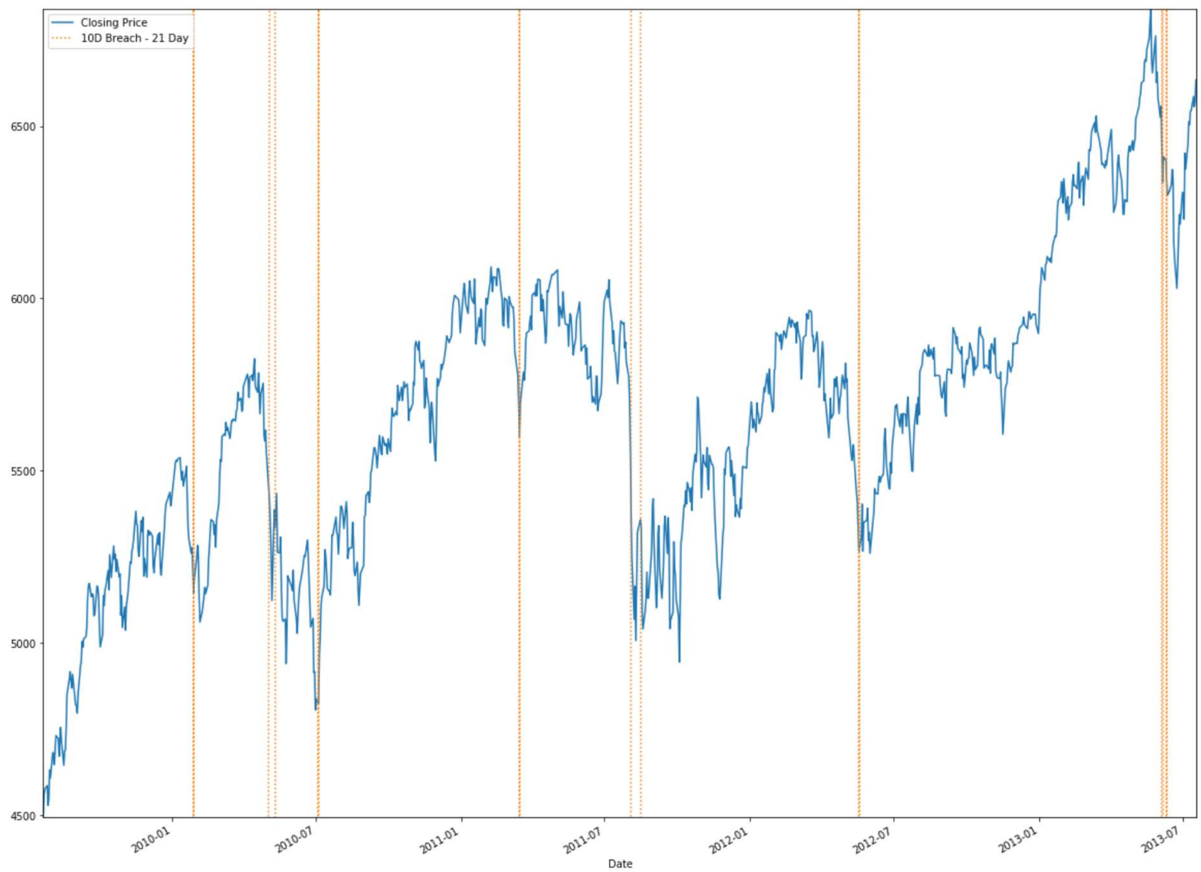
In [536]:
```
import matplotlib.pyplot as plt

max_price = data_42d['Closing Price'].max()
min_price = data_42d['Closing Price'].min()

fig, ax = plt.subplots()
fig.suptitle("42 Day Combined VaR Analysis", fontsize=30)
graph_size = (20,16)

data_42d = data_42d.rename({'10D Breach': '10D Breach - 42 Day'},axis=1)
data_42d['Closing Price'].plot(figsize=graph_size, ylim = (min_price, max_pric
e) ,legend=True)
(data_42d[f'{FORWARD_VAR_ROLLING_DAYS}D Breach - 42 Day'].astype(int) *(max_pr
ice)).plot(
    linestyle='dotted', figsize=graph_size,legend=True)

fig, ax = plt.subplots()
fig.suptitle("21 Day Combined VaR Analysis", fontsize=30)
data_21d['Closing Price'].plot(figsize=graph_size, ylim = (min_price, max_pric
e) ,legend=True)
data_21d = data_21d.rename({'10D Breach': '10D Breach - 21 Day'},axis=1)
(data_21d[f'{FORWARD_VAR_ROLLING_DAYS}D Breach - 21 Day'].astype(int) *(max_pr
ice)).plot(
    linestyle='dotted', figsize=graph_size,legend=True)
```

Out[536]: <matplotlib.axes._subplots.AxesSubplot at 0x1a269d6e548>

42 Day Combined VaR Analysis



21 Day Combined VaR Analysis



In [530]:
```
data_42d
```

Out[530]:

Date	Closing Price	Daily Log Return	42D Daily Deviation	10 Day VaR	10D Log Return	Shifted 10 Day VaR	10D Breach - 42 Day	10D Rolling Consecutive Breaches
2009-07-22	4493.73	NaN	NaN	NaN	NaN	NaN	False	0
2009-07-23	4559.80	0.014596	NaN	NaN	NaN	NaN	False	0
2009-07-24	4576.61	0.003680	NaN	NaN	NaN	NaN	False	0
2009-07-27	4586.13	0.002078	NaN	NaN	NaN	NaN	False	0
2009-07-28	4528.84	-0.012571	NaN	NaN	NaN	NaN	False	0
...	...	...	...	...	...	...	...	...
2013-07-15	6586.11	0.006271	0.011838	-0.087089	0.043179	-0.077647	False	0
2013-07-16	6556.35	-0.004529	0.011855	-0.087215	0.039259	-0.077525	False	0
2013-07-17	6571.93	0.002374	0.011830	-0.087026	0.053452	-0.078530	False	0
2013-07-18	6634.36	0.009455	0.011902	-0.087560	0.032584	-0.085435	False	0
2013-07-19	6630.67	-0.000556	0.011842	-0.087119	0.039240	-0.085503	False	0

1009 rows × 8 columns

Question 5

(a)

Section 32.5

Backtesting of the bank-wide risk model must be based on a VaR measure calibrated at a $\mathbf{99th\ percentile}$ confidence level.

(1) An exception or an outlier occurs when either the actual loss or the hypothetical loss of the bank-wide trading book registered in a day of the backtesting period exceeds the corresponding daily VaR measure given by the model. As per MAR99.8, exceptions for actual losses are counted separately from exceptions for hypothetical losses; the overall number of exceptions is the greater of these two amounts.

(2) In the event either the P&L or the daily VaR measure is not available or impossible to compute, it will count as an outlier

(b)

Section 32.19

If any given trading desk experiences either more than 12 exceptions at the 99th percentile or $\mathbf{30\ exceptions\ at\ the\ 97.5th\ percentile}$ in the most recent 12-month period, the capital requirement for all of the positions in the trading desk must be determined using the standardised approach.1

(c)

Section 32.34

The PLA requirements are based on two test metrics:

(1) $\mathbf{the\ Spearman\ correlation\ metric}$ to assess the correlation between RTPL and HPL; and

(2) $\mathbf{the\ Kolmogorov\text{-}Smirnov\ (KS)\ test\ metric}$ to assess similarity of the distributions of RTPL and HPL.

```
In [ ]:
```