CQF Exam One

January 2021 Cohort

Instructions

Answers to all questions **are required**. Requested mathematical and all computational workings must be provided. Portal, upload and logistical questions to Orinta.Juknaite@fitchlearning.com. Clarifying questions are welcome to Richard.Diamond@fitchlearning.com. Tutor is unable to re-explain calculation or confirm correct numerical answers/Basel section numbers. Please make a good use of lecture material.

- Please name ONE PDF and ONE ZIP file to start with your own LASTNAME_.
- Submit report in ONE PDF including all graphs. If your report is a Python notebook or Excel with handwritten inserts – print as ONE PDF cutting unnecessary output. If you submit multiple pdf or image files, absence of one report that provides workings and answers to all exam tasks will result in a request to resubmit or marks deduction.
- All code/Excel to be uploaded as ONE ZIP FILE.

Computation in Python/R encouraged. However for Exam One, all tasks are possible to solve in Excel using *MMULT(), MINV()* for portfolio computation and *IF()* for VaR backtesting.

Marking Scheme: Q1 10% **Q2** 28% **Q3** 20% **Q4** 27% **Q5** 15%

Optimal Portfolio Allocation

Question 1. Assume a trading strategy reports Annualised Sharpe Ratio of 0.53, as commonly done.

(a) How would you scale Sharpe Ratio with time? Compute Quarterly SR and Monthly SR estimates.

(b) "Evaluating the P&L more frequently, means its appears more risky than it actually is." Illustrate this by computing Quarterly and Monthly Loss Probability. Formula below has x Normal Random variable, therefore the probability is taken from Normal CDF.

$$\Pr(\text{P\&L} < 0) = \Pr(x < -\text{SR}).$$

Question 2. For this question, consider an investment universe composed of the following risky assets:

Assets	$\boldsymbol{\mu}$	$\boldsymbol{\sigma}$
A	0.04	0.07
B	0.08	0.12
C	0.12	0.18
D	0.15	0.26

$$\boldsymbol{Corr} = \begin{pmatrix} 1 & 0.2 & 0.5 & 0.3 \\ 0.2 & 1 & 0.7 & 0.4 \\ 0.5 & 0.7 & 1 & 0.9 \\ 0.3 & 0.4 & 0.9 & 1 \end{pmatrix}$$

Consider optimization for a target return m, with the net of allocations invested (borrowed) in the risk-free asset (if interested what happened with the budget constraint please see exercises)

$$\underset{\boldsymbol{w}}{\operatorname{argmin}} \frac{1}{2}\boldsymbol{w^T \Sigma w}, \qquad \text{s.t. } r + (\boldsymbol{\mu} - r\mathbf{1})^T \mathbf{w} = m$$

(a) Formulate the Lagrangian function and its partial derivatives. Derive the analytical solution for optimal allocations $\boldsymbol{w}^*$. Provide handwritten or typeset mathematical working.

(b) For the risk-free rate of 3% and a range of target return values $m = 5\%, 7.5\%, 10\%, 12.5\%$ compute optimal allocations $\boldsymbol{w}^*$ (each portfolio), portfolio risk $\sigma_\Pi = \sqrt{\boldsymbol{w}^T \boldsymbol{\Sigma} \boldsymbol{w}}$, and plot the Efficient Frontier. **Note:** it is your task to determine the nature of μ_Π and shape of the frontier.

Question 3. For a given allocated portfolio below, compute VaR and ES sensitivities *wrt* each asset i.

Asset	μ	σ	w
1	0	0.30	50%
2	0	0.20	20%
3	0	0.15	30%

$$\boldsymbol{Corr} = \begin{pmatrix} 1 & 0.8 & 0.5 \\ 0.8 & 1 & 0.3 \\ 0.5 & 0.3 & 1 \end{pmatrix}$$

$$\frac{\partial \text{VaR}(w)}{\partial w_i} = \mu_i + \text{Factor} \times \frac{(\boldsymbol{\Sigma w})_i}{\sqrt{\boldsymbol{w}^T \boldsymbol{\Sigma w}}} \quad \text{and} \quad \frac{\partial \text{ES}(w)}{\partial w_i} = \mu_i - \frac{\phi(\text{Factor})}{1-c} \times \frac{(\boldsymbol{\Sigma w})_i}{\sqrt{\boldsymbol{w}^T \boldsymbol{\Sigma w}}}$$

where $()_i$ refers to i-th element, confidence $c = 99\%$, $1 - c = 0.01$, and Factor $= \Phi^{-1}(1-c)$ is *inverse Normal cdf*, while $\phi()$ is simply Normal *pdf*. Signs in formulae are for the left tail – lower percentile of 0.01. Tutor will not confirm Factor sign/matrix computation/output.

Regulation and Techniques

Question 4. As a market risk analyst, each day you calculate VaR from the available prior data. Then, you wait ten days to compare your prediction value VaR_{t-10} to the realised return and check if the prediction about the worst loss was breached. You are given a dataset with *Closing Prices* (FTSE 100).

- Implement VaR backtesting by computing 99%/10day Value at Risk using the rolling window of 21 returns to compute σ. The rolling window technique will give you the time series of $\text{VaR}_{10D,t}$.

$$\text{VaR}_{10D,t} = \text{Factor} \times \sigma_t \times \sqrt{10}$$

3.1 (a) Report the percentage of VaR breaches and (b) number of consecutive breaches. (c) Provide a plot which clearly identifies breaches. Further Instructions given below, use as needed.

3.2 Repeat backtesting for the sample of 42 returns to compute σ. Provide outputs (a), (b), (c) in one Table, together with 3.1.

Question 5. The Basel Framework can be referred to in interactive form: `https://www.bis.org/basel_framework/index.htm`. To evaluate how good the bank's modelling of risk factors is, we refer to section MAR32 approach to backtesting. Provide exact answers together with one most relevant section number.

(a) On which risk measure the firmwide risk model must be based? At which confidence level?

(b) How many exceptions are allowed in a trading desk-level backtesting at 97.5th percentile before it will be forced to exercise the standardised approach?

(c) What are two key metrics that satisfy Profit Loss Attribution (PLA) test requirement?

END OF EXAM (Version A4)

Further Instructions

Please make good use of lecture exercises and problem-solving sessions. The tutor is unable to confirm numerical answers and methods of calculation/spreadsheets.
To compute the 99%/10day Value at Risk for an investment in the market index on the rolling basis. We drop the expected return (mean) from the VaR formula

$$\text{VaR}_{10D,t} = \text{Factor} \times \sigma_t \times \sqrt{10}$$

- Practical VaR calculation drops μ_{10D} for two reasons. First, 21-day sample average return (or alike) is not a robust quantity. Second, for a diversified portfolio/market index the quantity is negligible.
- Appropriate Factor value to be used (Standard Normal Percentile), the tutor will not confirm the numerical value. It is also your task to identify the eligible number of observations for which VaR is available and can be backtested: N_{obs} will not be confirmed.
- Compute a column of rolling standard deviation over log-returns for observations $1-21, 2-22, \ldots$. Compute VaR for each day t, after the initial period. This is your worst loss prediction.
- **Regardless** of how many observations there are in a sample (10, 21, 100, etc.), variance is *an average of squared daily differences* $\frac{\sum(r_t-\mu)^2}{(N-1)}$ and so, timescale remains 'daily'.
- VaR is fixed at time t and compared to the return realised from t to $t+10$. A breach occurs when that forward realised 10-day return $\ln(S_{t+10}/S_t)$ is below the VaR_t quantity.

  $r_{10D,t+10} < \text{VaR}_{10D,t}$ means breach, given both numbers are negative.

  In Excel, you will have a column for VaR_t series, a column of $r_{10D,t+10}$ series, and indicator column $\{0,1\}$ for a breach using $IF()$ function.
- To obtain the conditional probability of breach $N_{conseq}/N_{breaches}$, identify consecutive breaches. For example, the sequence $1,1,1$ means two consecutive breaches occurred.

Extra Tasks

These tasks are not part of the exam and not graded. However, they provide paths for advanced modelling and your own further exploration.

Backtesting For comparison to naive sample std dev, backtesting can be done with EWMA on variance, with the same rolling window of 21 or 42 observations. EWMA estimated for each next period as follows:

$$\sigma^2_{t+1\,|\,t} = \lambda\,\sigma^2_{t\,|\,t-1} + (1-\lambda)\,r_t^2$$

advised $\lambda = 0.72$ is smaller than suggested by RiskMetrics but minimises out of sample forecasting errors.

The full GJR-GARCH model requires estimation of $(\omega, \alpha, \gamma, \beta)$ first (please see ARCH Lecture and its spreadsheet). GARCH model itself is biased towards the long-run average variance $\bar{\sigma}^2$ and not as responsive to the recent return information r_t^2. Therefore, we prefer EWMA.

Build Q-Q plots for 1D and 10D returns, and conclude if Normally distributed returns was a reasonable assumption. Log-returns being Normally distributed is the main assumption of Analytical VaR.
Without this assumption holding, the Normal Factor is not applicable.

As to the issue of independence of breaches in VaR, the applicable statistical test is for Christoffersen's exceedances independence.