

MPCS 58020 2017: Homework 1

Due date: Monday April 3, 2017 5:30pm (before class)

Solve the following problems and show your work. Email your solutions to Bryce Allen <bdallen@uchicago.edu> with "MPCS 58020: HW1" in the subject line. Scanned handwritten work is fine.

1. An airplane can safely fly if at least half of its engines are functional. If each engine independently functions with probability p , for what values of p is a three-engine plane safer than a five-engine plane?
2. Show that De Morgan's laws are true for sets (using either a formal proof or an informal explanation with diagrams):
 - (a) $(A \cup B)^c = A^c \cap B^c$
 - (b) $(A \cap B)^c = A^c \cup B^c$
3. An insurance company writes a policy to the effect that an amount of money A must be paid if some event E occurs within a year. If the company estimates that E will occur within a year with probability p , what should it charge the customer so that its expected profit will be 10 percent of A ?
4. Two design teams, one named C and the other N, are asked to separately design a new product within a month. From experience we know that:
 - (a) The probability that team C is successful is $2/3$.
 - (b) The probability that team N is successful is $1/2$.
 - (c) The probability that at least one team is successful is $3/4$.

Assuming that exactly one successful design is produced, what is the probability that it was designed by team N?

5. Show that

$$A \cup B \cup C = A \cup (A^c \cap B) \cup (A^c \cap B^c \cap C)$$

6. Pat and Taylor have a date at a given time, and each will arrive at the meeting place with a delay between 0 and 1 hour, with all pairs of delays being equally likely. The first to arrive will wait for 15 minutes and will leave if the other has not yet arrived. What is the probability that they will meet?