MPCS 55001 Algorithm - HOMEWORK 1

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QUESTION 1

Let p denote the probability of each engine runs fine.

P(airplane with 3 engines fly safely) = $P^3 + \binom{1}{3} * p^2 * (1-p)$

P(airplane with 5 engines fly safely) = $P^5 + \binom{1}{5} * p^4 * (1-p) + \binom{2}{5} * p^3 * (1-p)^2$

we need to find the value of p such that P(airplane with 3 engines fly safely) >= P(airplane with 5 engines fly safely)

The value of p can be found by the following python script:

```
for i in range(0,1000,1):

    p=0.001*i

    print p**3 + 3*p**2*(1-p),p**5+5*p**4*(1-p)+10*p**3*(1-p)**2,p**3
+ 3*p**2*(1-p)¡p**5+5*p**4*(1-p)+10*p**3*(1-p)**2
```

So when $0 < p < 0.5$, P(airplane with 3 engines fly safely) > P(airplane with 5 engines fly safely)

So when $0.5 < p < 1$, P(airplane with 3 engines fly safely) < P(airplane with 5 engines fly safely)

QUESTION 2

Please look for a Venn diagram within this PDF file. Sorry I don't know how to adjust the position of it.

QUESTION 3

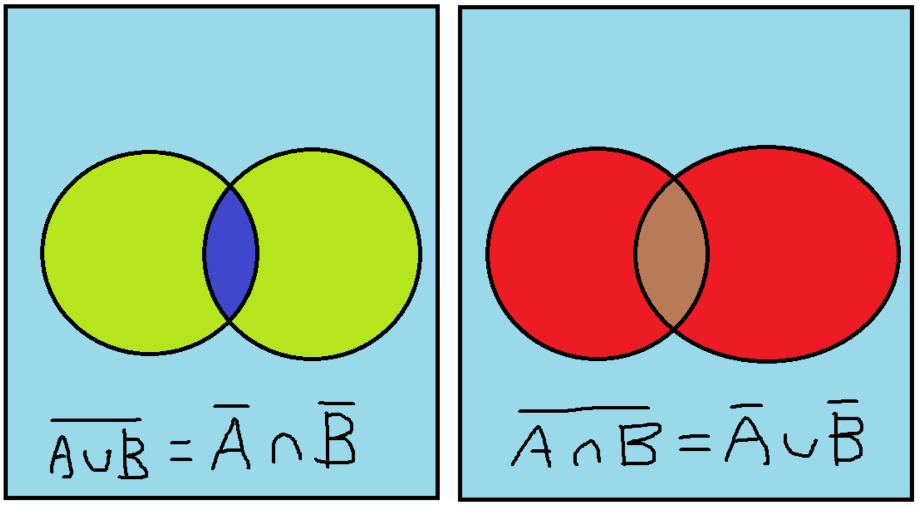

Figure 1: Venn Diagram for QUESTION 2

Let X denote the money it charges.

The expected amount of money paid = pA

So X - pA = 0.1A

So X = pA + 0.1*A

QUESTION 4

Let C denote the probability for C to succeed
Let N denote the probability for N to succeed

there are 4 items in the sample space: CN, C^cN, CN^c, N^cC^c

So according to the assumptions. we have $P(CN) = \frac{5}{12}$, $P(C^cN) = \frac{1}{12}$, $P(CN^c) = \frac{3}{12}$, $P(C^cN^c) = \frac{3}{12}$

So the probability that there is exactly one team succeed = $\frac{3}{12}$

So the probability that there is exactly one team succeed and the team is N = $\frac{1}{12}$

Assuming that exactly one successful design is produced, the probability that it was designed by team N = $\frac{1}{3}$

QUESTION 5

According to distributive law, $A \cup (A^c \cap B) = (A \cup A^c) \cap (A \cup B) = A \cup B$

$(A \cup B) \cup (A^c \cap B^c \cap C) = (A \cup B \cup A^c) \cap (A \cup B \cup B^c) \cap (A \cup B \cup C)$, in which $(A \cup B \cup A^c)$ and $(A \cup B \cup B^c)$ always evaluate to TRUE, so $(A \cup B) \cup (A^c \cap B^c \cap C) = (A \cup B \cup C)$.

Accordingly, $A \cup (A^c \cap B) \cup (A^c \cap B^c \cap C) = (A \cup B \cup C)$

QUESTION 6

The distribution of arrival time can be modeled by uniform distribution each, as the figure shows(at the bottom of this PDF), the green shadow area is that they will meet, with the total sample spaces as the square area, green plus blue. So the probability that they will meet = $\frac{green}{green \cap blue} = \frac{60*60}{60*60-45*45*0.5-45*45*0.5} = \frac{3600-2025}{3600} = 0.4375$

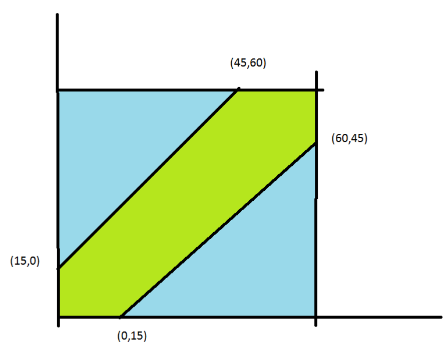

Figure 2: QUESTION 6