Inertial Measurement Units I

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Stanford University

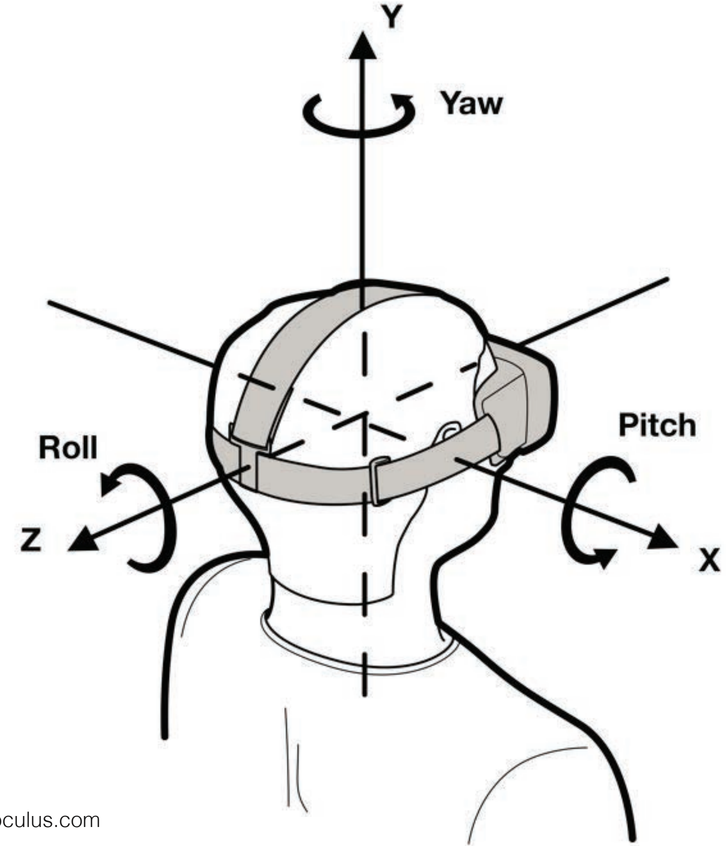
EE 267 Virtual Reality

Lecture 9

stanford.edu/class/ee267/

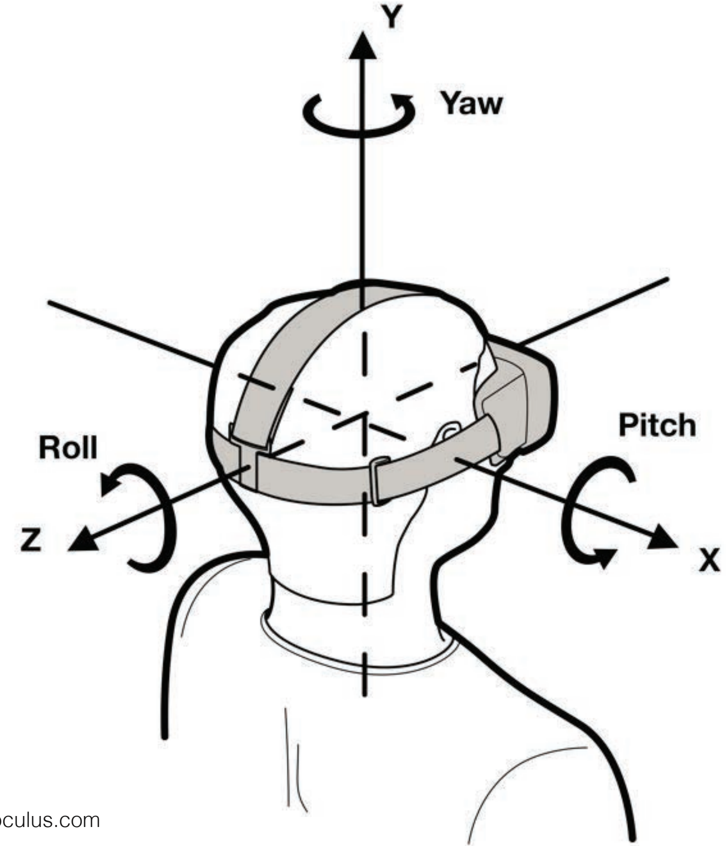
Lecture Overview

- coordinate systems (world, body/sensor, inertial, transforms)
- overview of inertial sensors: gyroscopes, accelerometers, and magnetometers
- gyro integration aka *dead reckoning*
- orientation tracking in *flatland*
- pitch & roll from accelerometer
- overview of VRduino



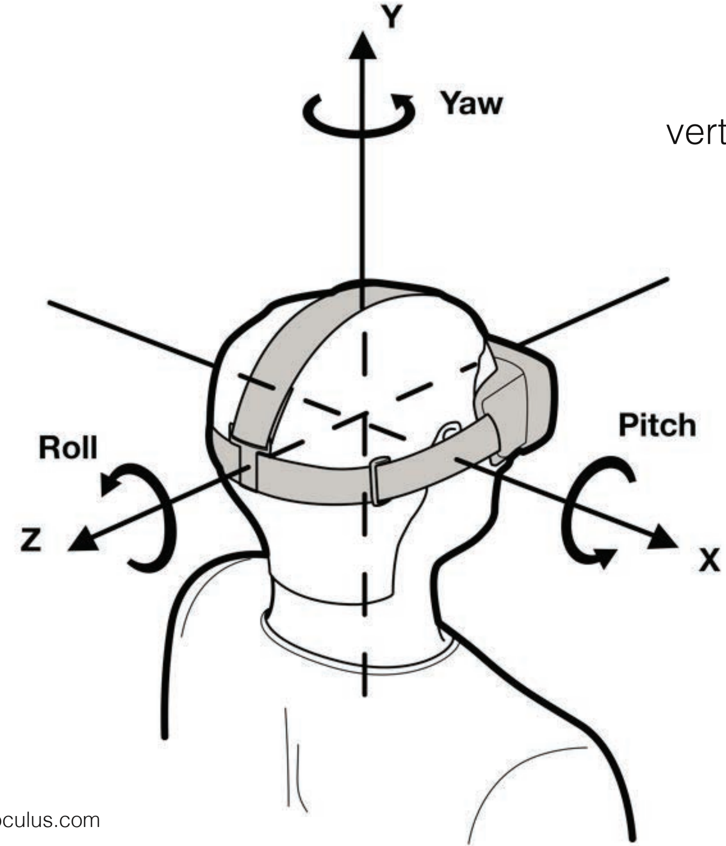
oculus.com

- primary goal: track orientation of head or other device
- orientation is the rotation of device w.r.t. world/earth or *inertial* frame
- rotations are represented by Euler angles (yaw, pitch, roll) or quaternions



oculus.com

- orientation tracked with IMU models relative rotation of sensor/body frame in world/ inertial coordinates
- example: person on the left looks up → pitch=90° or rotation around x-axis by 90°
- similarly, the world rotates around the sensor frame by -90° (inverse rotation)



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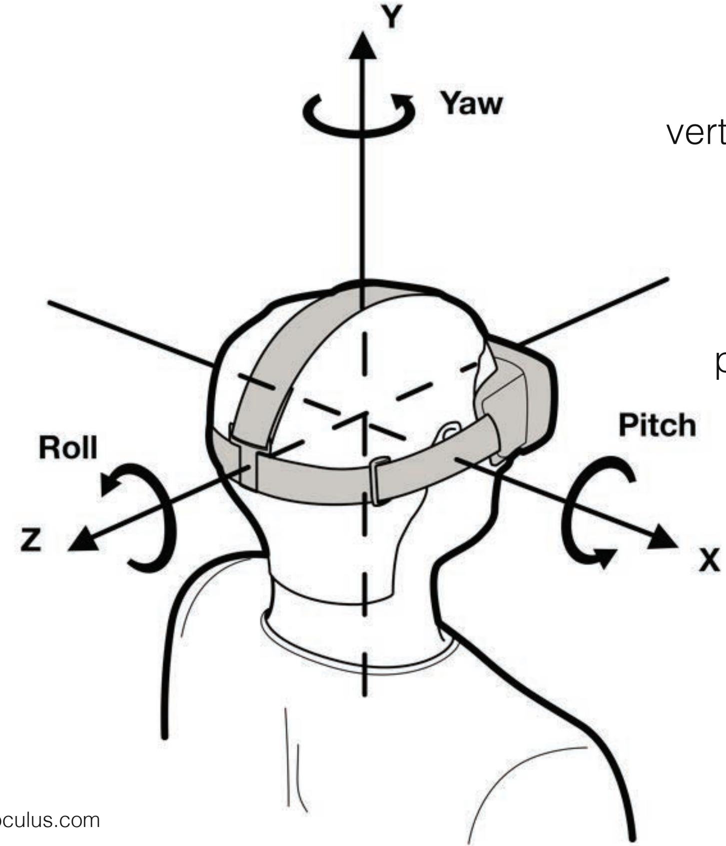
from lecture 2:

vertex in clip space

vertex

$$v_{clip} = M_{proj} \cdot M_{view} \cdot M_{model} \cdot v$$



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from lecture 2:

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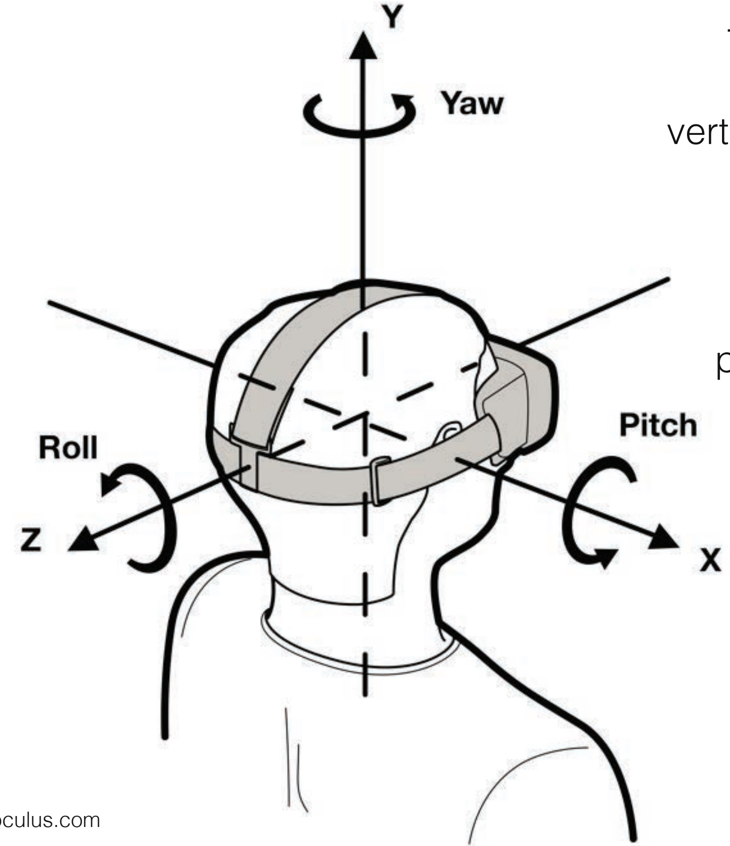
$$v_{clip} = M_{proj} \cdot M_{view} \cdot M_{model} \cdot v$$

projection matrix

view matrix

model matrix



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projection matrix

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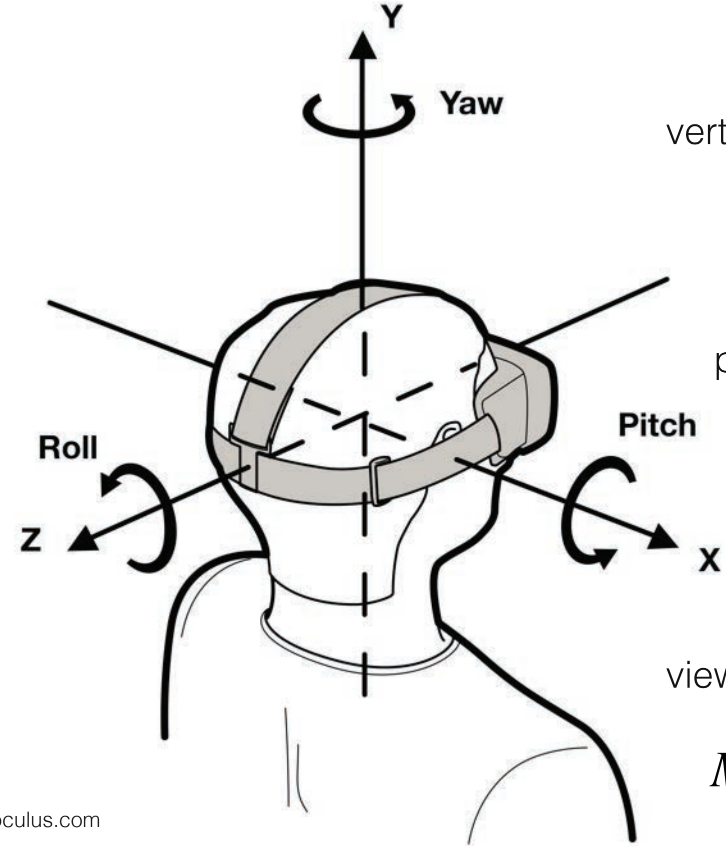
rotation

translation

$$M_{view} = R \cdot T(-eye)$$



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from lecture 2:

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$$v_{clip} = M_{proj} \cdot M_{view} \cdot M_{model} \cdot v$$

projection matrix

view matrix

model matrix

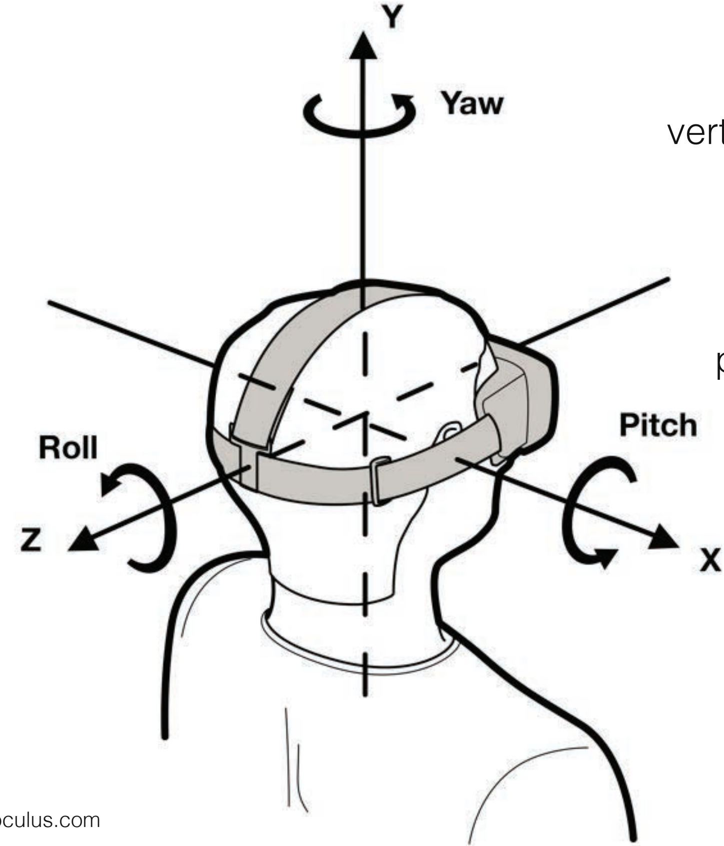
rotation

translation

$$M_{view} = R \cdot T(-eye)$$

view matrix for stereo camera:

$$M_{view}^{stereo} = T\left(\pm\frac{ipd}{2},0,0\right) \cdot R \cdot T(-eye)$$



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from lecture 2:

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$$v_{clip} = M_{proj} \cdot M_{view} \cdot M_{model} \cdot v$$

projection matrix

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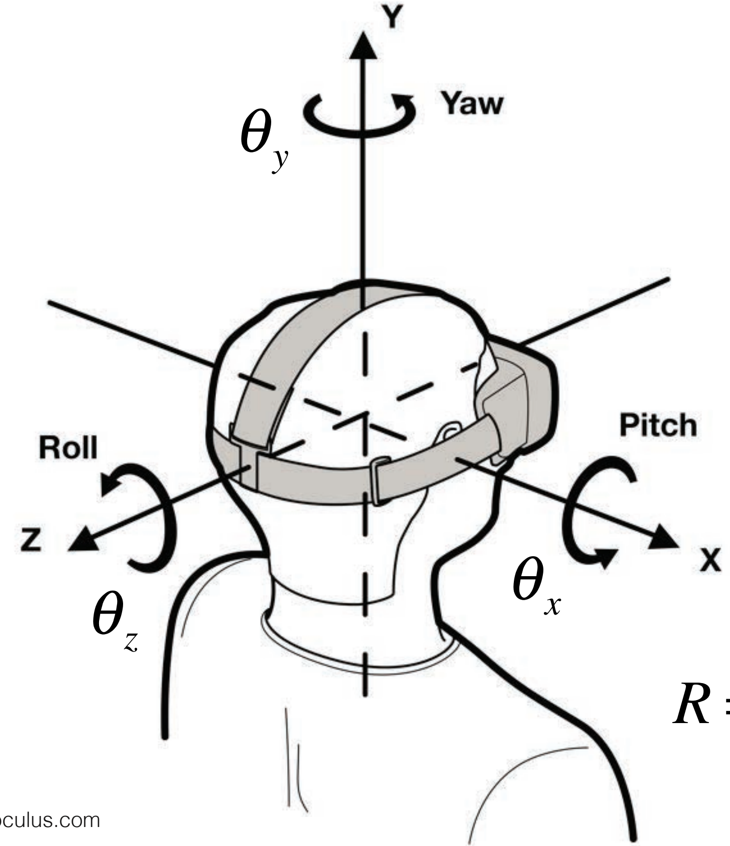
rotation

translation

$$M_{view} = R \cdot T(-eye)$$

<u>sensor/body frame</u>

<u>world/inertial frame</u>

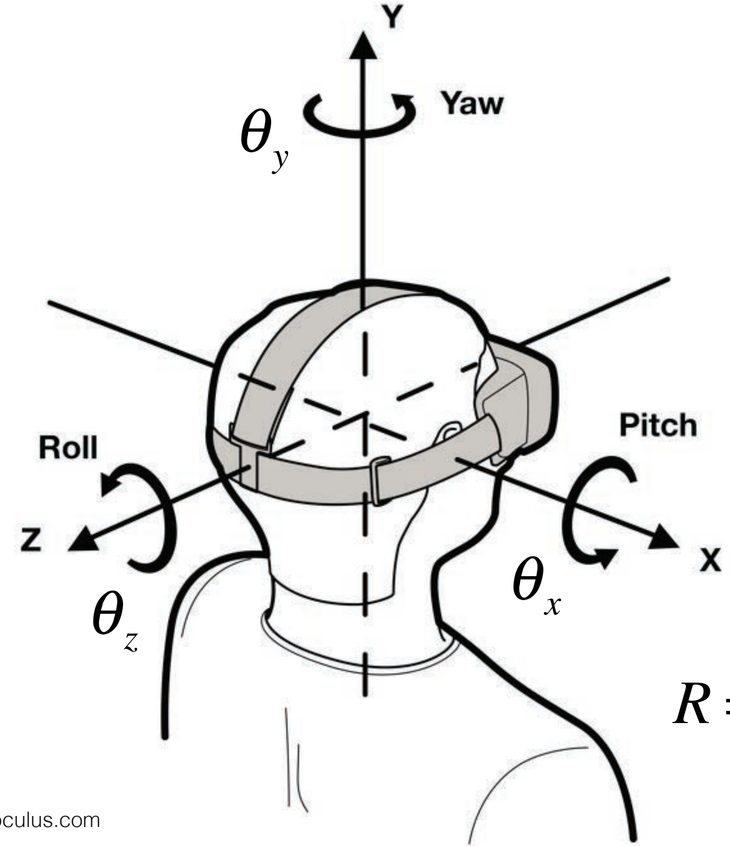


oculus.com

$$M_{view} = \overset{\text{rotation}}{R} \cdot \overset{\text{translation}}{T}\left(-eye\right)$$

$$R = \overset{\text{roll}}{R_z\left(-\theta_z\right)} \cdot \overset{\text{pitch}}{R_x\left(-\theta_x\right)} \cdot \overset{\text{yaw}}{R_y\left(-\theta_y\right)}$$

order of rotations (world to body)



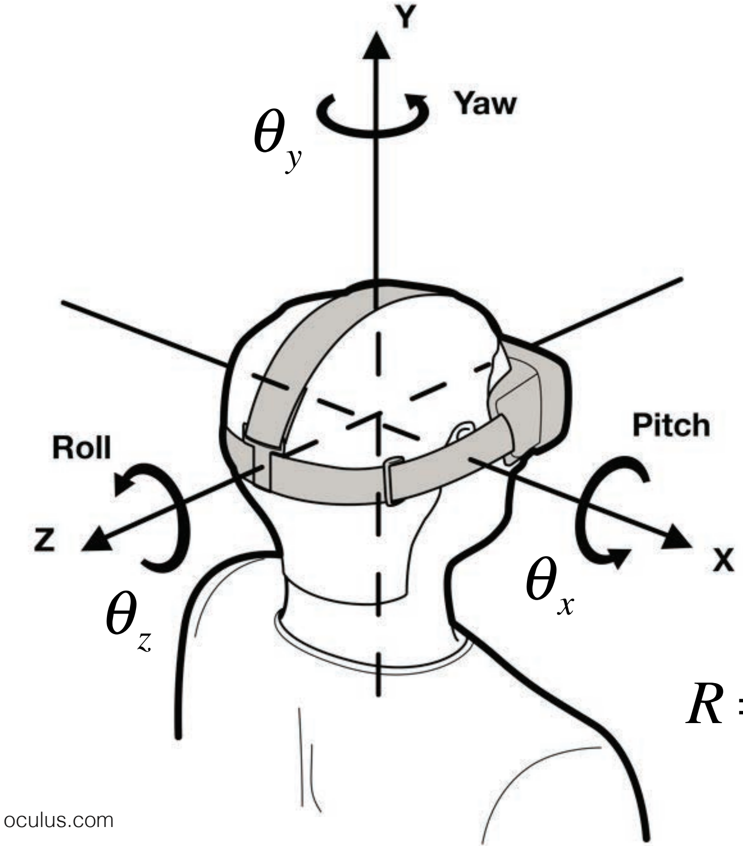
oculus.com

- this representation for a rotation is know as *Euler angles*

- need to specify order of rotation, e.g. yaw-pitch-roll

$$R = \overbrace{R_z(-\theta_z)}^{\text{roll}} \cdot \overbrace{R_x(-\theta_x)}^{\text{pitch}} \cdot \overbrace{R_y(-\theta_y)}^{\text{yaw}}$$

order of rotations (world to body)



ATTENTION!

- Euler angles are usually a <u>terrible idea</u> for orientation tracking with more than 1 axis

- one of several reasons: rotations are not commutative

$$R = \overset{\text{roll}}{R_z\left(-\theta_z\right)} \cdot \overset{\text{pitch}}{R_x\left(-\theta_x\right)} \cdot \overset{\text{yaw}}{R_y\left(-\theta_y\right)}$$

$\longleftarrow$

order of rotations (world to body)

What do Inertial Sensors Measure?

- gyroscope measures angular velocity $\widetilde{\omega}$ in degrees/sec
- accelerometer measures linear acceleration $\tilde{a}$ in m/s^2
- magnetometer measures magnetic field strength $\widetilde{m}$ in uT (micro Tesla) or Gauss → 1 Gauss = 100 uT

What do Inertial Sensors Measure?

- gyroscope measures angular velocity $\tilde{\omega}$ in degrees/sec
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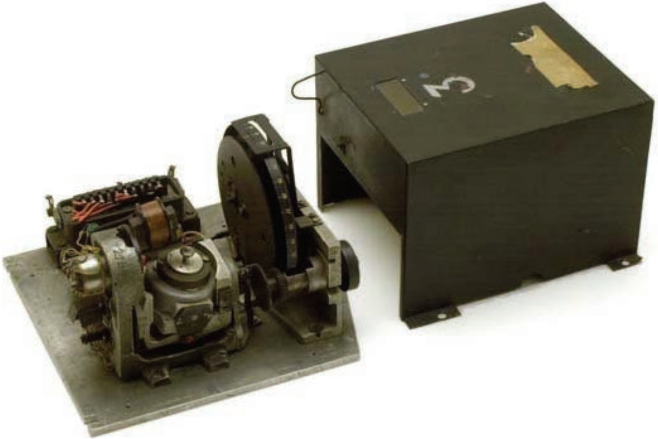
ALL MEASUREMENTS TAKEN IN SENSOR/
BODY COORDINATES!

History of Gyroscopes

- critical for inertial measurements in ballistic missiles, aircrafts, drones, the mars rover, pretty much anything that moves!

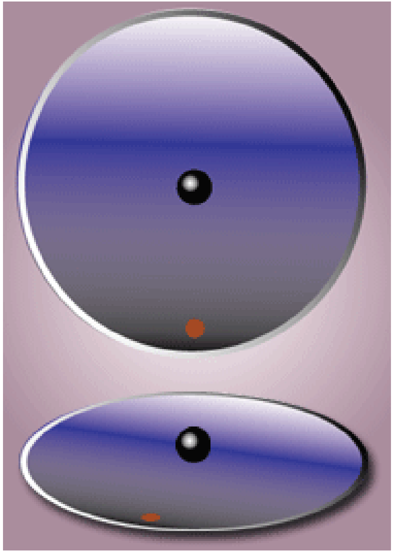


WWII era gyroscope used in the V2 rocket

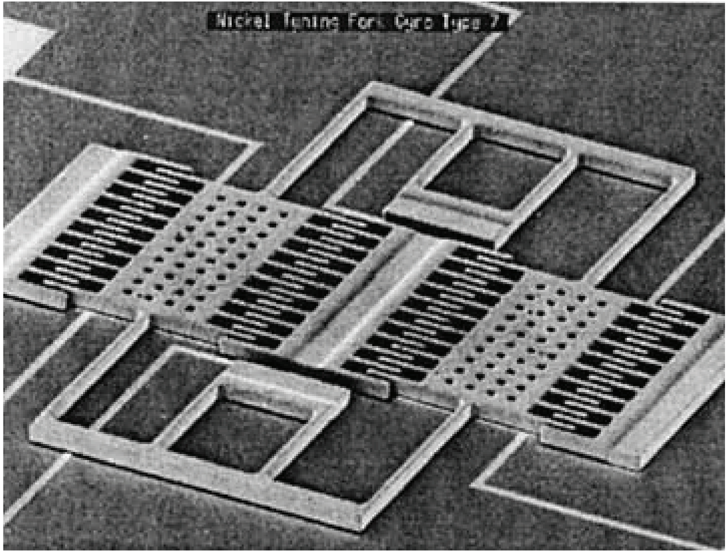
MEMS Gyroscopes

- today, we use microelectromechanical systems (MEMS)

Coriolis Force

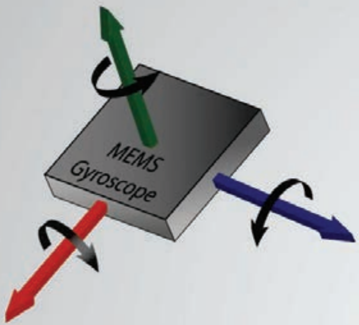


wikipedia



quora.com

MEMS Gyroscope



Gyroscopes

- gyro model: $\widetilde{\boldsymbol{\omega}} = \boldsymbol{\omega} + b + \eta$

Gyroscopes

- gyro model: $\underbrace{\widetilde{\omega} = \omega}_{\text{true angular velocity}} + \underbrace{b}_{\text{bias}} + \underbrace{\eta}_{\text{additive, zero-mean Gaussian noise}}$ $\quad \eta \sim N\left(0, \sigma_{gyro}^2\right)$

Gyroscopes

- gyro model: $\tilde{\omega} = \omega + b + \eta$ $\qquad \eta \sim N\left(0, \sigma^2_{gyro}\right)$

  true angular velocity

  bias

  additive, zero-mean Gaussian noise

- 3 DOF = 3-axis gyros that measures 3 orthogonal axes, assume no crosstalk
- bias is temperature-dependent and may change over time; can approximate as a constant
- additive measurement noise

Gyroscopes

- from gyro measurements to orientation – use Taylor expansion

$$\theta\left(t+\Delta t\right) \approx \theta\left(t\right) + \frac{\partial}{\partial t}\theta\left(t\right)\Delta t + \varepsilon, \quad \varepsilon \sim O\left(\Delta t^2\right)$$

Gyroscopes

- from gyro measurements to orientation – use Taylor expansion

have: angle at last time step

have: time step

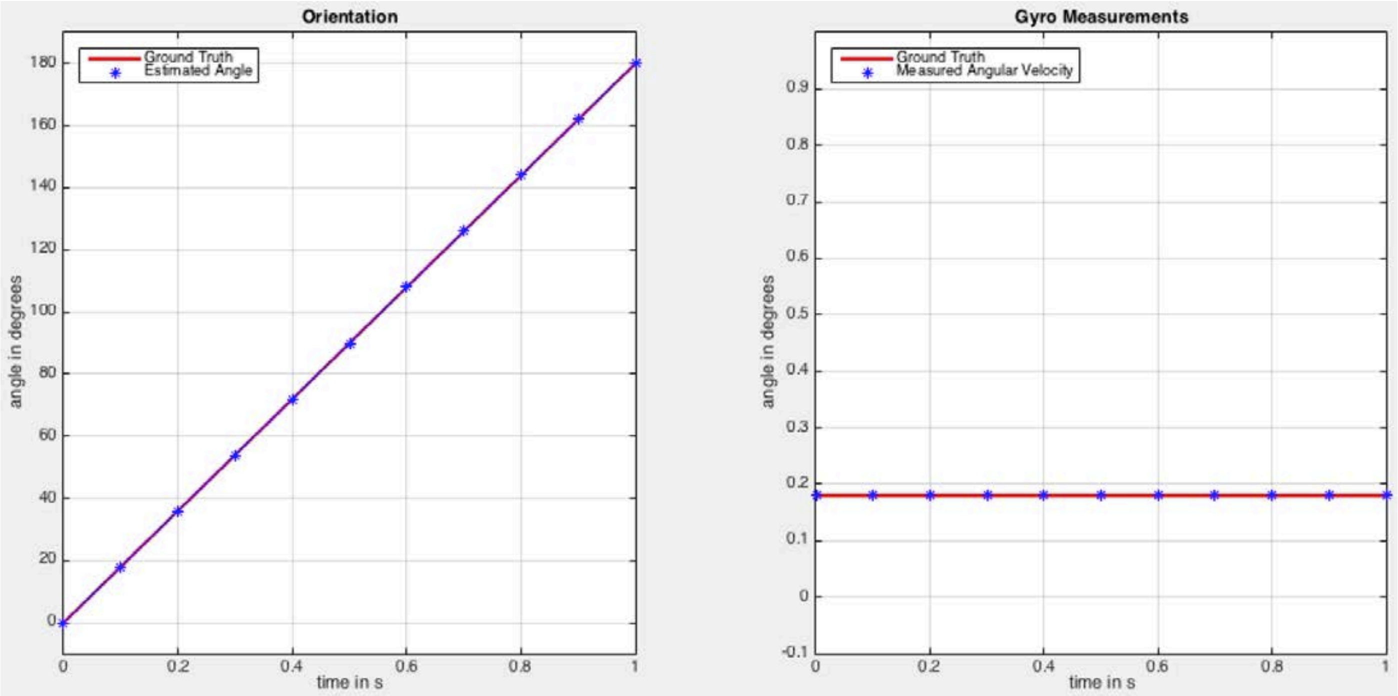
$$\theta(t+\Delta t) \approx \theta(t) + \underbrace{\frac{\partial}{\partial t}\theta(t)}_{=\omega}\Delta t + \varepsilon, \quad \varepsilon \sim O\left(\Delta t^2\right)$$

want: angle at current time step

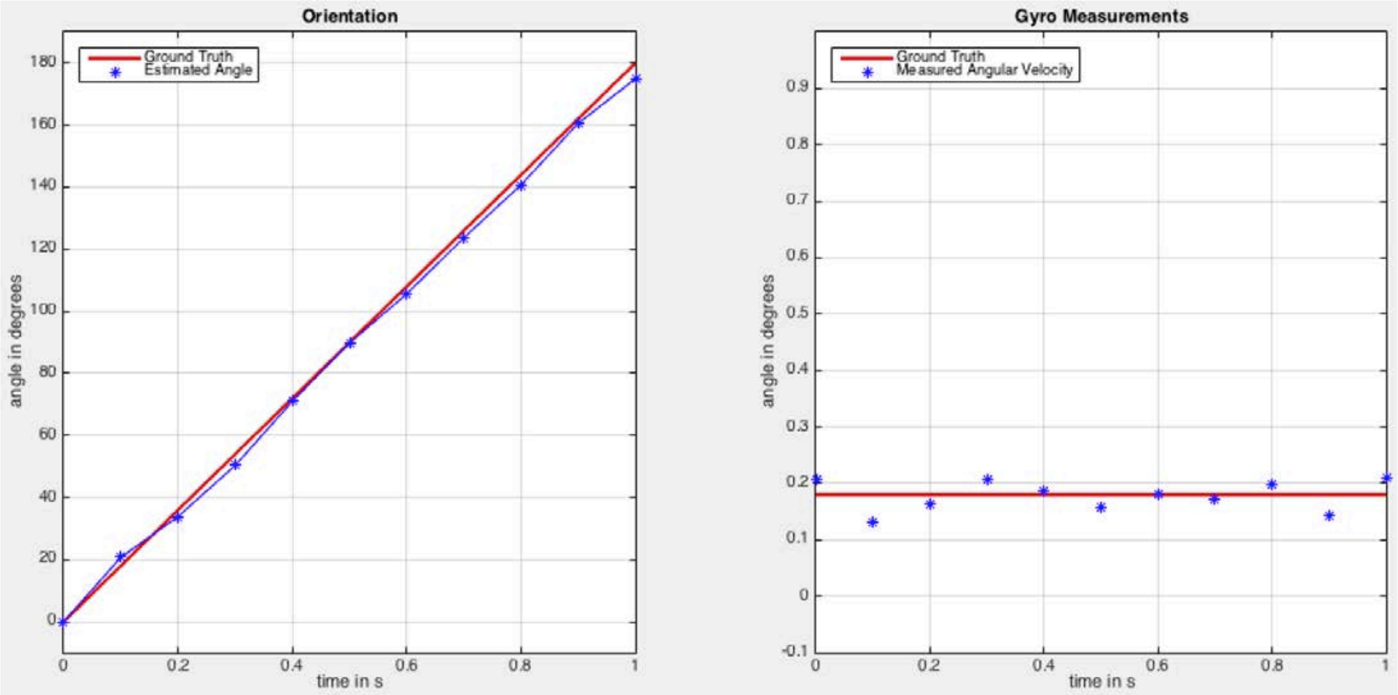
approximation error!

have: gyro measurement (angular velocity)

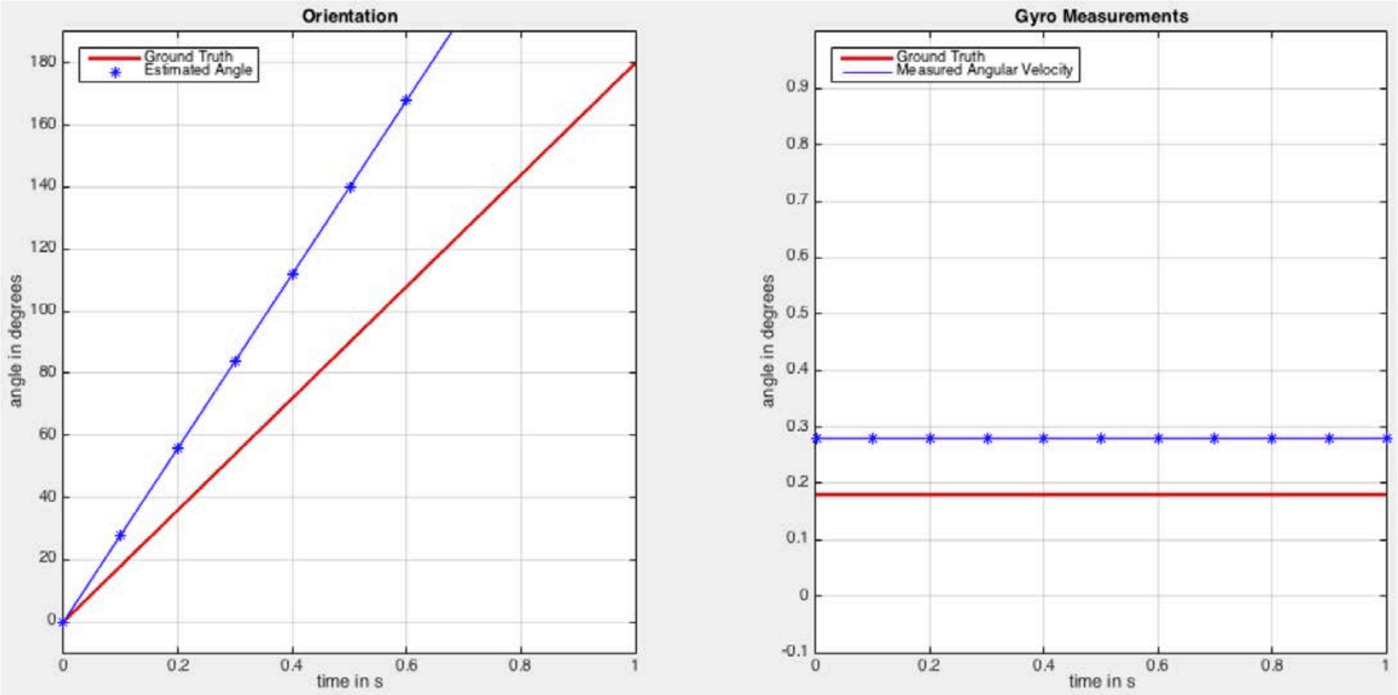
Gyro Integration: linear motion, no noise, no bias



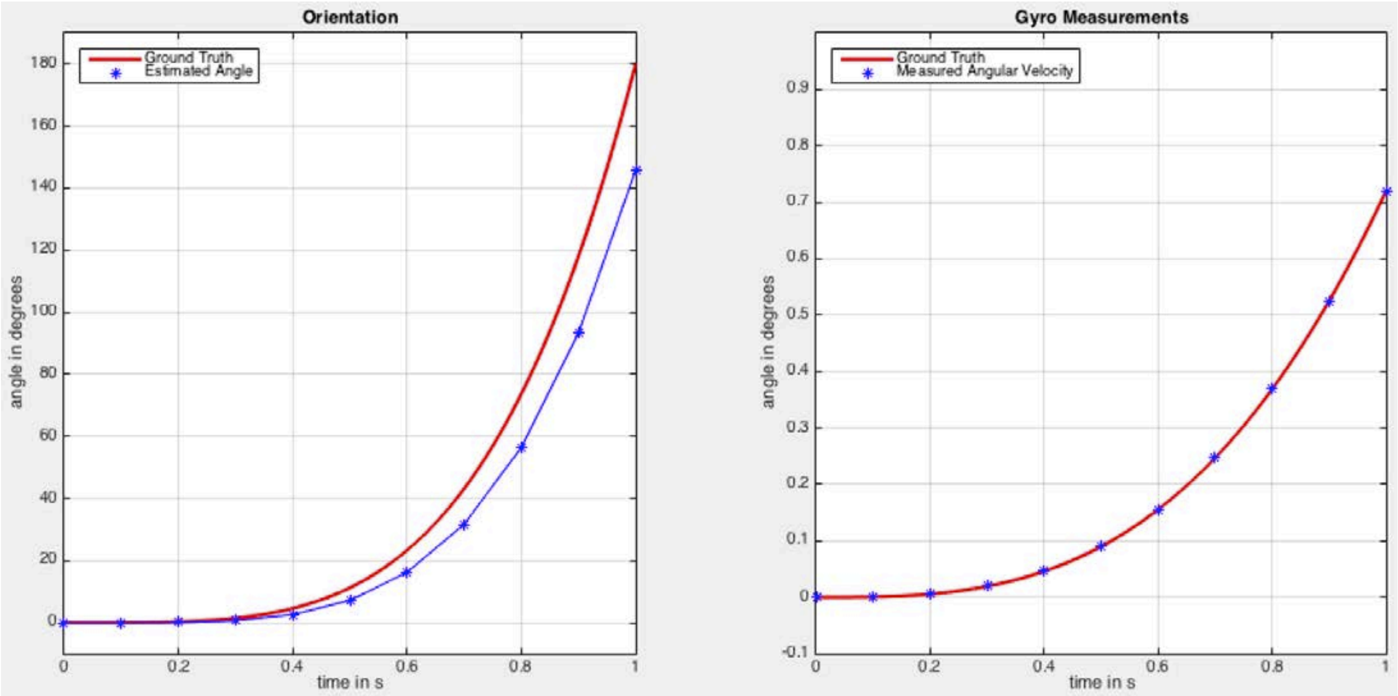
Gyro Integration: linear motion, noise, no bias



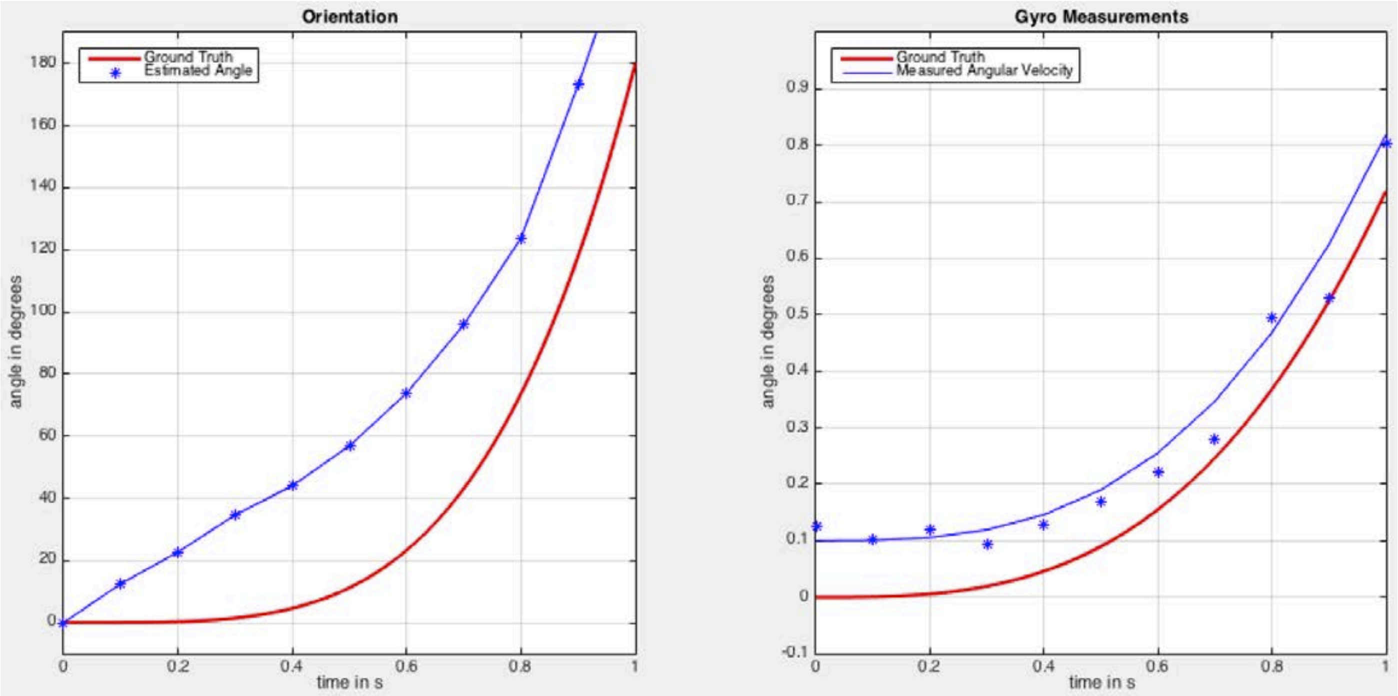
Gyro Integration: linear motion, no noise, <u>bias</u>



Gyro Integration: nonlinear motion, no noise, no bias



Gyro Integration: nonlinear motion, noise, bias



Gyro Integration aka *Dead Reckoning*

- works well for linear motion, no noise, no bias = unrealistic

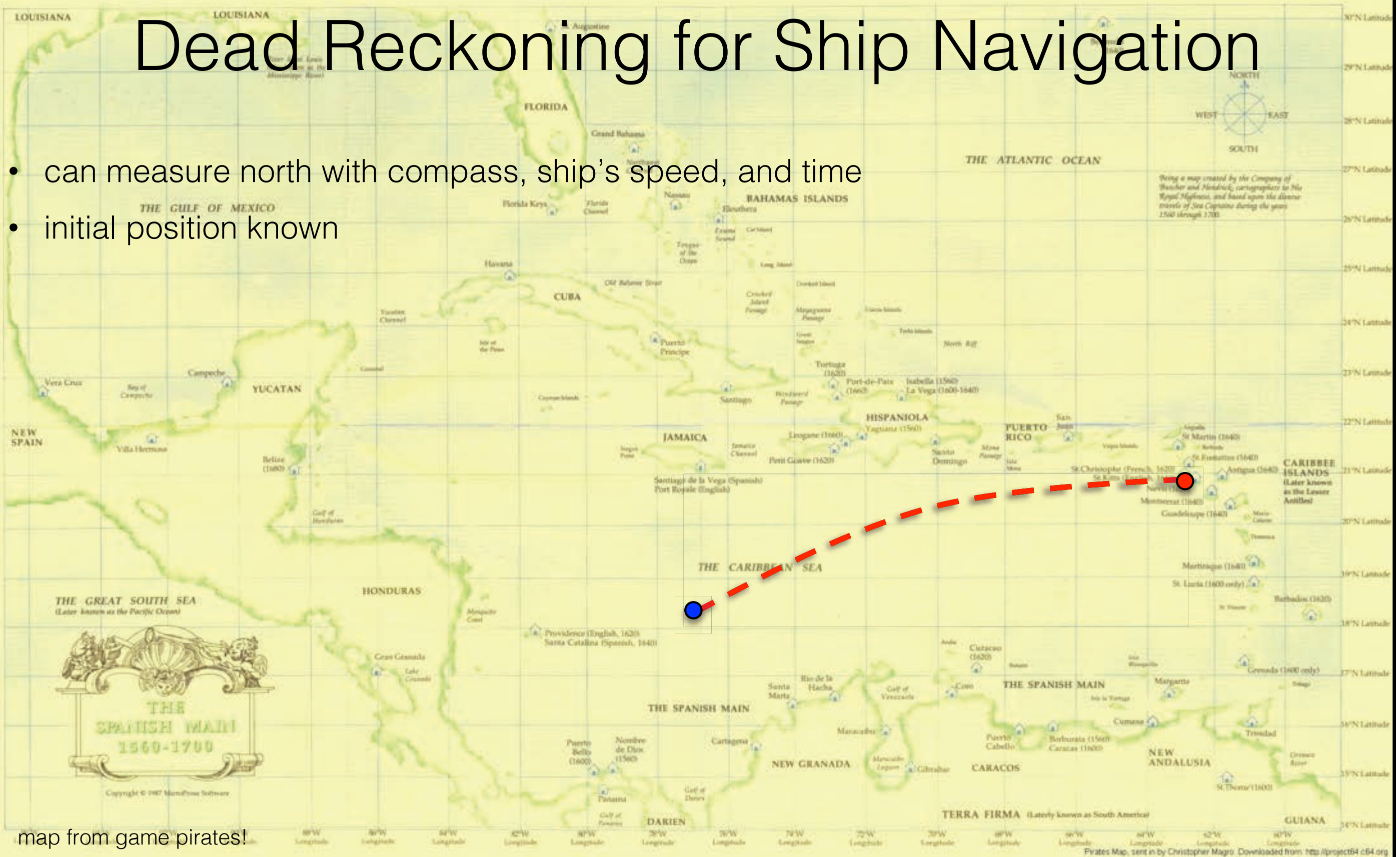
- even if bias is know and noise is zero → <u>drift</u> (from integration)

- bias & noise variance can be estimated, other sensor measurements used to correct for drift (sensor fusion)

- <u>accurate in short term, but not reliable in long term due to drift</u>

Dead Reckoning for Ship Navigation

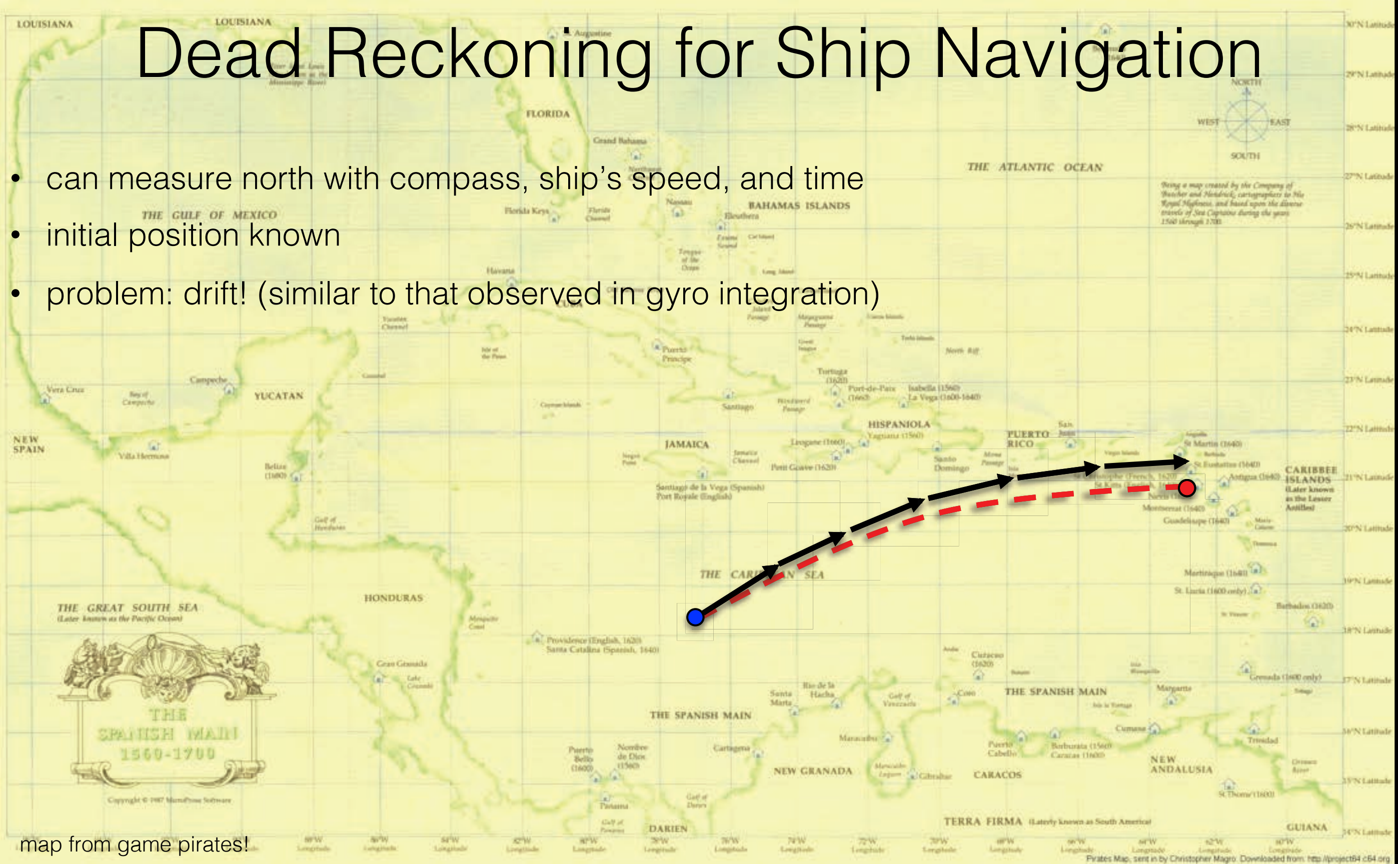
- can measure north with compass, ship's speed, and time
- initial position known



map from game pirates!

Dead Reckoning for Ship Navigation

- can measure north with compass, ship's speed, and time
- initial position known
- problem: drift! (similar to that observed in gyro integration)



map from game pirates!

Gyro Advice

Always be aware of what units you are working with, degrees per second v radians per second!

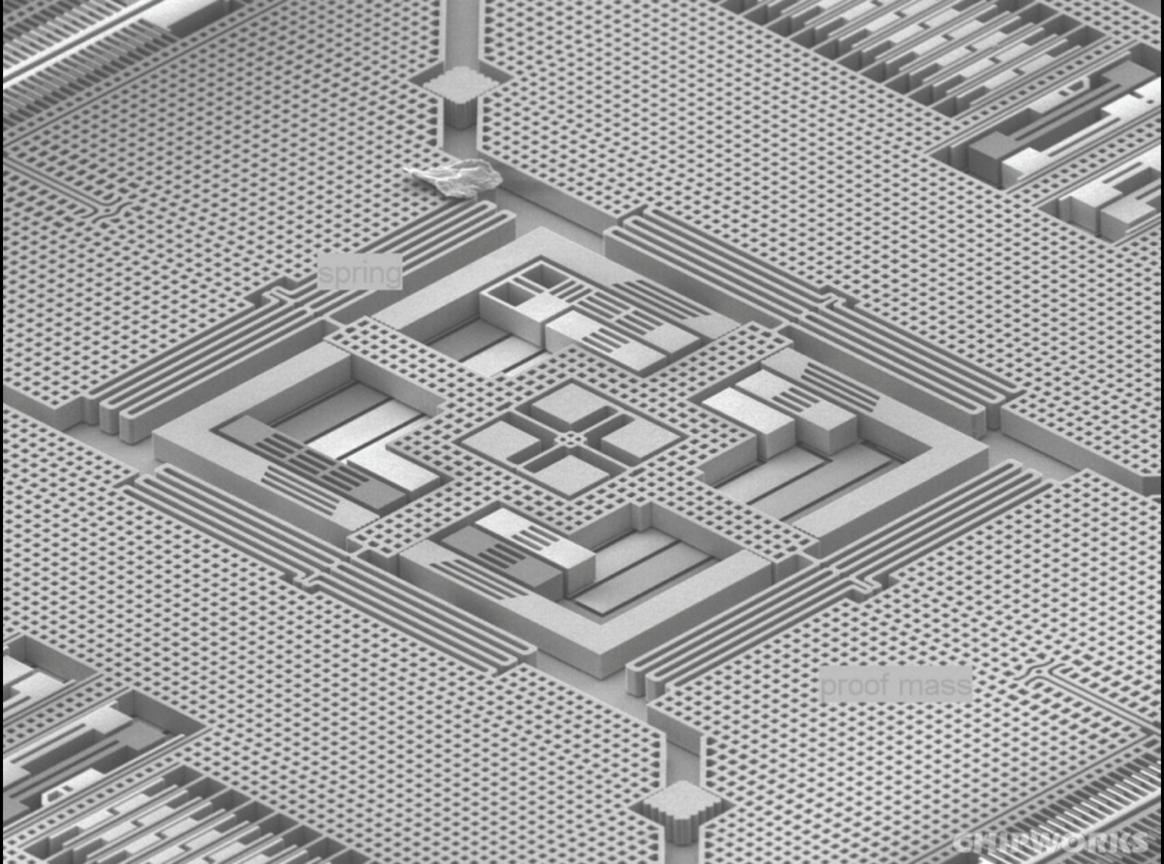
Accelerometers

- measure linear acceleration $\tilde{a} = a^{(g)} + a^{(l)} + \eta, \quad \eta \sim N\left(0, \sigma_{acc}^2\right)$

- without motion: read noisy gravity vector $a^{(g)} + \eta$ pointing UP! with magnitude 9.81 m/s^2 = 1g

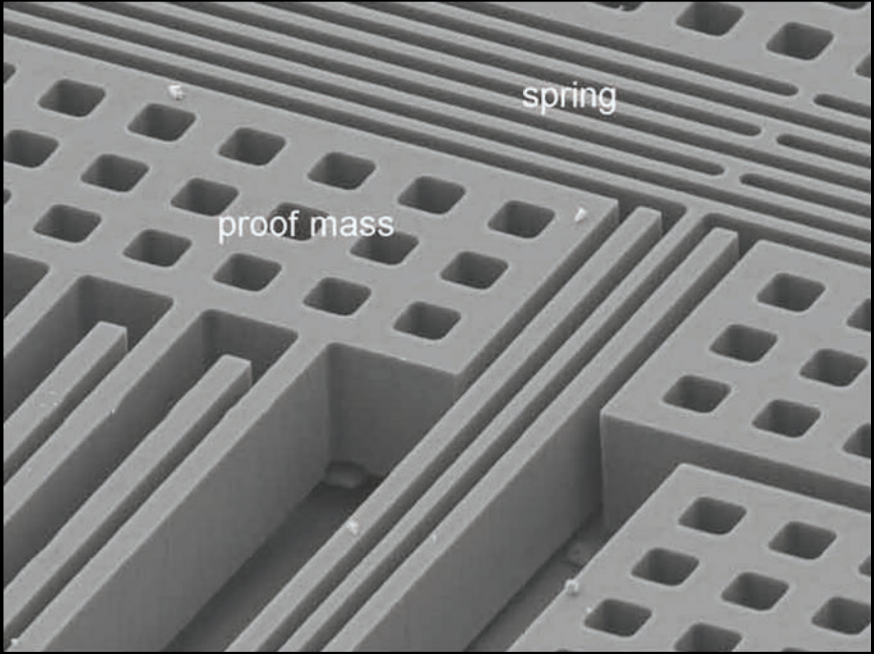
- with motion: combined gravity vector and external forces $a^{(l)}$

chipworks

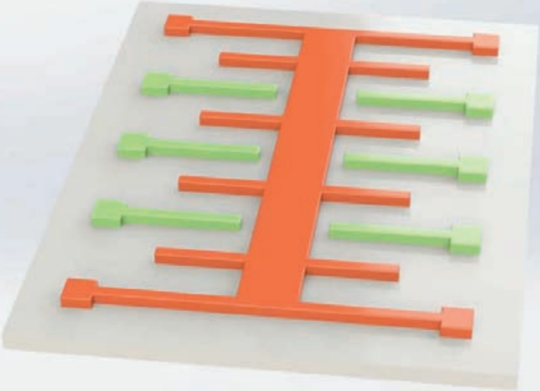


https://stevecurd.wordpress.com/2015/08/25/mems-and-me-how-does-my-fitbit-know-im-walking/

capacitive
plates



MEMS Accelerometer



Accelerometers

- advantages:
  - points up on average with magnitude of 1g
  - accurate in long term because no drift and the earth's center of gravity (usually) doesn't move
- problem:
  - noisy measurements
  - unreliable in short run due to motion (and noise)

- <u>complementary to gyro measurements!</u>

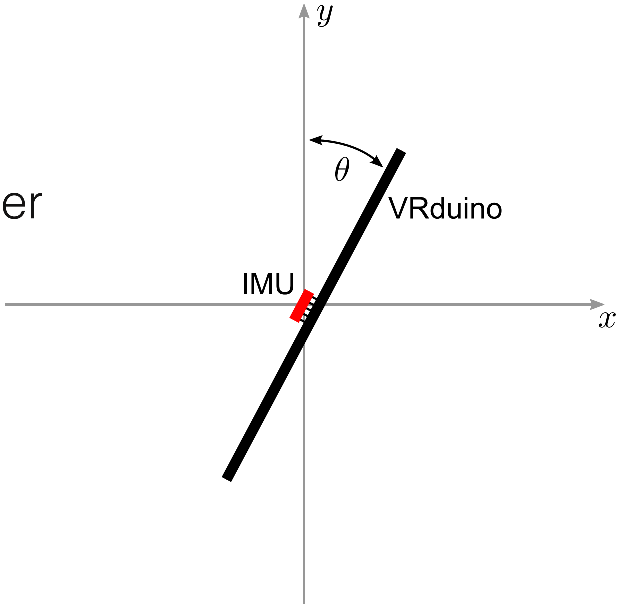
Accelerometers

- fusing gyro and accelerometer data = 6 DOF sensor fusion

- <u>can correct tilt (i.e., pitch & roll) only</u> – no information about yaw

Orientation Tracking in *Flatland*

- problem: track angle $\boldsymbol{\theta}$ in 2D space
- sensors: 1 gyro, 2-axis accelerometer
- goal: understand sensor fusion



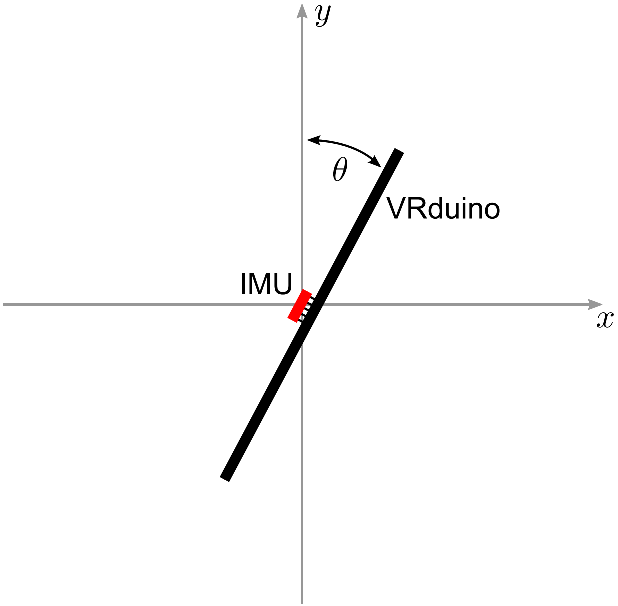
Orientation Tracking in *Flatland*

- gyro integration via Taylor series as

$$\theta_{gyro}^{(t)} = \theta_{gyro}^{(t-1)} + \tilde{\omega}\Delta t$$

- get Δt from microcontroller
- set $\theta_{gyro}^{(0)} = 0$

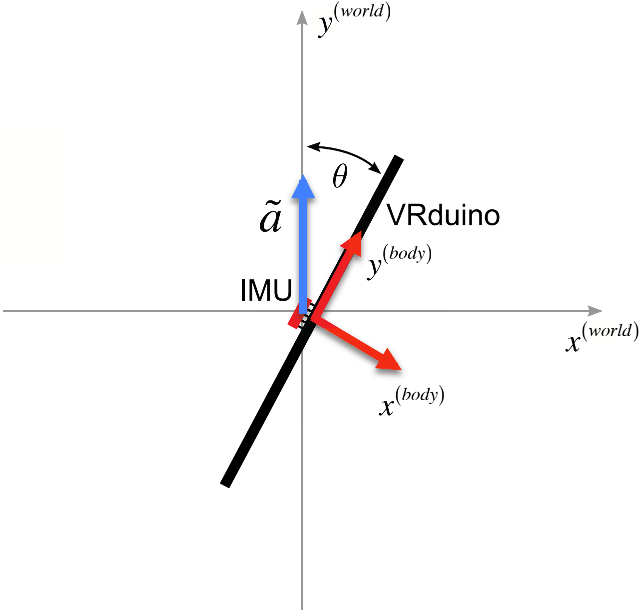
- biggest problem: drift!



Orientation Tracking in *Flatland*

- angle from accelerometer

$$\theta_{acc} = \tan^{-1}\left(\frac{\tilde{a}_x}{\tilde{a}_y}\right)$$

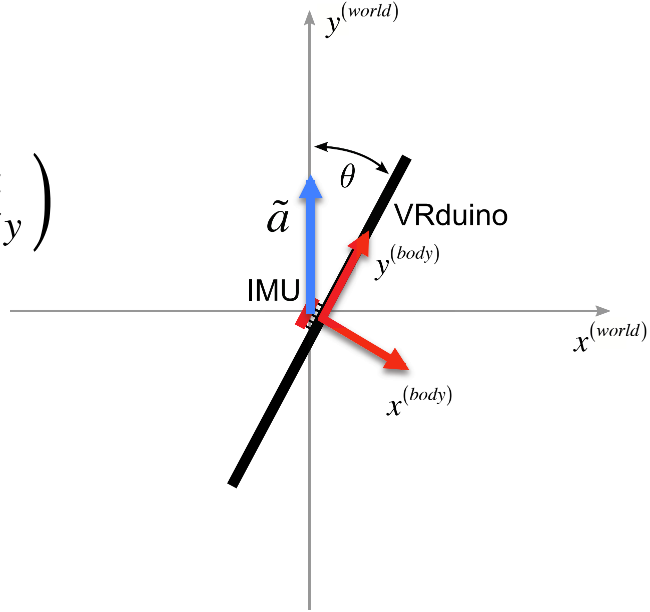


Orientation Tracking in *Flatland*

- angle from accelerometer

$$\theta_{acc} = \tan^{-1}\left(\frac{\tilde{a}_x}{\tilde{a}_y}\right) = \operatorname{atan2}\left(\tilde{a}_x, \tilde{a}_y\right)$$

handles division by 0 and
proper signs, provided by most
programming languages

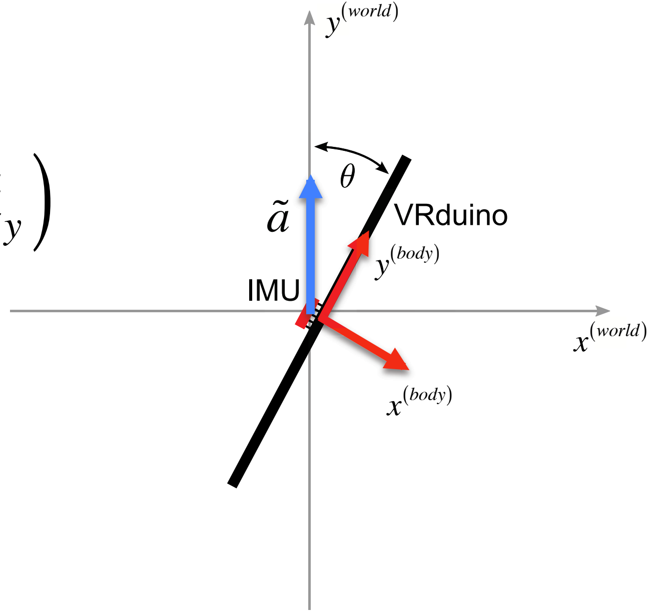


Orientation Tracking in *Flatland*

- angle from accelerometer

$$\theta_{acc} = \tan^{-1}\left(\frac{\tilde{a}_x}{\tilde{a}_y}\right) = \mathrm{atan2}\left(\tilde{a}_x, \tilde{a}_y\right)$$

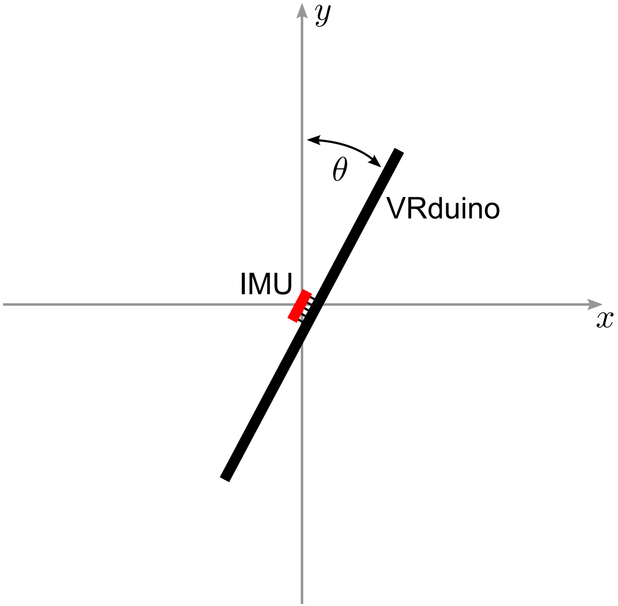
- biggest problem: noise



Orientation Tracking in *Flatland*

- sensor fusion: combine gyro and accelerometer measurements

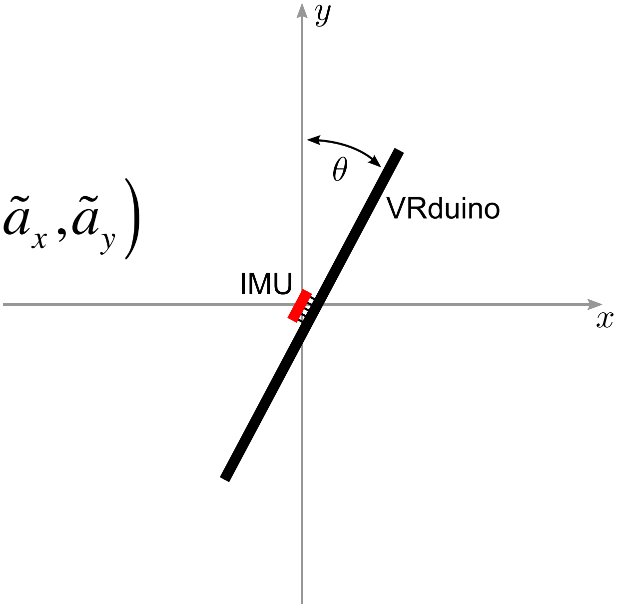
- intuition:
    - remove drift from gyro via high-pass filter
    - remove noise from accelerometer via low-pass filter



Orientation Tracking in *Flatland*

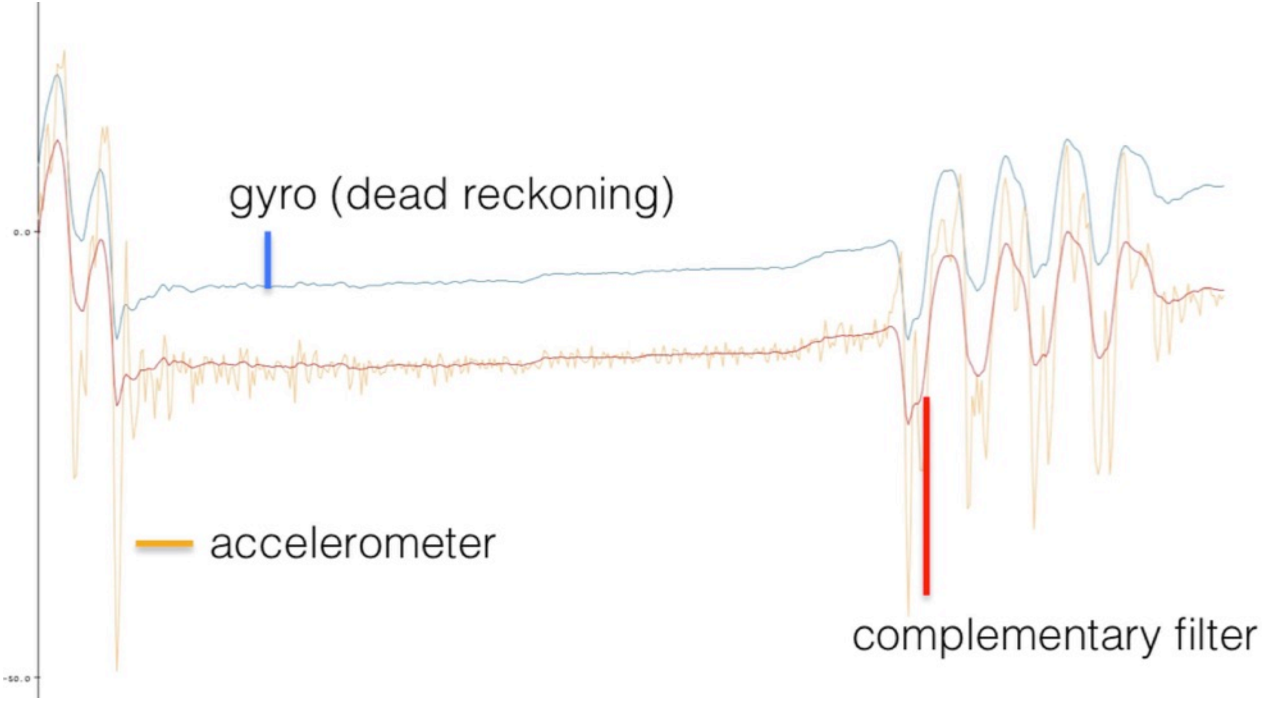
- sensor fusion with complementary filter, i.e. linear interpolation

$$\theta^{(t)} = \alpha\left(\theta^{(t-1)} + \tilde{\omega}\Delta t\right) + (1-\alpha)\,\mathrm{atan2}\left(\tilde{a}_x, \tilde{a}_y\right)$$



- no drift, no noise!

Orientation Tracking in *Flatland*



Pitch and Roll from 3-axis Accelerometer

- problem: estimate pitch and roll angles in 3D, from 3-axis accelerometer
- together, pitch & roll angles are known as *tilt*
- goal: understand tilt estimation in 3D

Pitch and Roll from 3-axis Accelerometer

- use only accelerometer data – can estimate pitch & roll, not yaw
- assume no external forces (only gravity) – acc is pointing UP!

normalize gravity vector in
inertial coordinates

$$\hat{a} = \frac{\tilde{a}}{\|\tilde{a}\|} = R\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = R_z\left(-\theta_z\right)\cdot R_x\left(-\theta_x\right)\cdot R_y\left(-\theta_y\right)\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

normalize gravity vector rotated into
sensor coordinates

Pitch and Roll from 3-axis Accelerometer

- use only accelerometer data – can estimate pitch & roll, not yaw
- assume no external forces (only gravity) – acc is pointing UP!

$$\hat{a} = \frac{\tilde{a}}{\|\tilde{a}\|} = R\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = R_z(-\theta_z)\cdot R_x(-\theta_x)\cdot R_y(-\theta_y)\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} \cos(-\theta_z) & -\sin(-\theta_z) & 0 \\ \sin(-\theta_z) & \cos(-\theta_z) & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(-\theta_x) & -\sin(-\theta_x) \\ 0 & \sin(-\theta_x) & \cos(-\theta_x) \end{pmatrix}\begin{pmatrix} \cos(-\theta_y) & 0 & \sin(-\theta_y) \\ 0 & 1 & 0 \\ -\sin(-\theta_y) & 0 & \cos(-\theta_y) \end{pmatrix}\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

Pitch and Roll from 3-axis Accelerometer

- use only accelerometer data – can estimate pitch & roll, not yaw
- assume no external forces (only gravity) – acc is pointing UP!

$$\hat{a} = \frac{\tilde{a}}{||\tilde{a}||} = \begin{pmatrix} -\cos(-\theta_x)\sin(-\theta_z) \\ \cos(-\theta_x)\cos(-\theta_z) \\ \sin(-\theta_x) \end{pmatrix}$$

Pitch and Roll from 3-axis Accelerometer

- use only accelerometer data – can estimate pitch & roll, not yaw
- assume no external forces (only gravity) – acc is pointing UP!

roll

$$\hat{a} = \frac{\tilde{a}}{\|\tilde{a}\|} = \begin{pmatrix} -\cos(-\theta_x)\sin(-\theta_z) \\ \cos(-\theta_x)\cos(-\theta_z) \\ \sin(-\theta_x) \end{pmatrix} \Rightarrow \frac{\hat{a}_x}{\hat{a}_y} = \frac{-\sin(-\theta_z)}{\cos(-\theta_z)} = -\tan(-\theta_z)$$

$$\boxed{\theta_z = -\mathrm{atan2}(-\hat{a}_x, \hat{a}_y) \text{ in rad} \in [-\pi, \pi]}$$

Pitch and Roll from 3-axis Accelerometer

- use only accelerometer data – can estimate pitch & roll, not yaw
- assume no external forces (only gravity) – acc is pointing UP!

pitch

$$\hat{a} = \frac{\tilde{a}}{\|\tilde{a}\|} = \begin{pmatrix} -\cos(-\theta_x)\sin(-\theta_z) \\ \cos(-\theta_x)\cos(-\theta_z) \\ \sin(-\theta_x) \end{pmatrix} \Rightarrow \frac{\hat{a}_z}{\sqrt{\hat{a}_x^2 + \hat{a}_y^2}} = \frac{\sin(-\theta_x)}{\sqrt{\cos^2(-\theta_x)\underbrace{\left(\sin^2(-\theta_z) + \cos^2(-\theta_z)\right)}_{=1}}}$$

$$= \frac{\sin(-\theta_x)}{\cos(-\theta_x)} = \tan(-\theta_x)$$

Pitch and Roll from 3-axis Accelerometer

- use only accelerometer data – can estimate pitch & roll, not yaw
- assume no external forces (only gravity) – acc is pointing UP!

<u>pitch</u>

$$\hat{a} = \frac{\tilde{a}}{\|\tilde{a}\|} = \begin{pmatrix} -\cos(-\theta_x)\sin(-\theta_z) \\ \cos(-\theta_x)\cos(-\theta_z) \\ \sin(-\theta_x) \end{pmatrix}$$

$$\frac{\hat{a}_z}{\sqrt{\hat{a}_x^2 + \hat{a}_y^2}} = \frac{\sin(-\theta_x)}{\sqrt{\cos^2(-\theta_x)\underbrace{\left(\sin^2(-\theta_z) + \cos^2(-\theta_z)\right)}_{=1}}}$$

$$\theta_x = -\mathrm{atan2}\left(\hat{a}_z, \sqrt{\hat{a}_x^2 + \hat{a}_y^2}\right) \text{ in rad } \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

Pitch and Roll from 3-axis Accelerometer

- use only accelerometer data – can estimate pitch & roll, not yaw
- assume no external forces (only gravity) – acc is pointing UP!

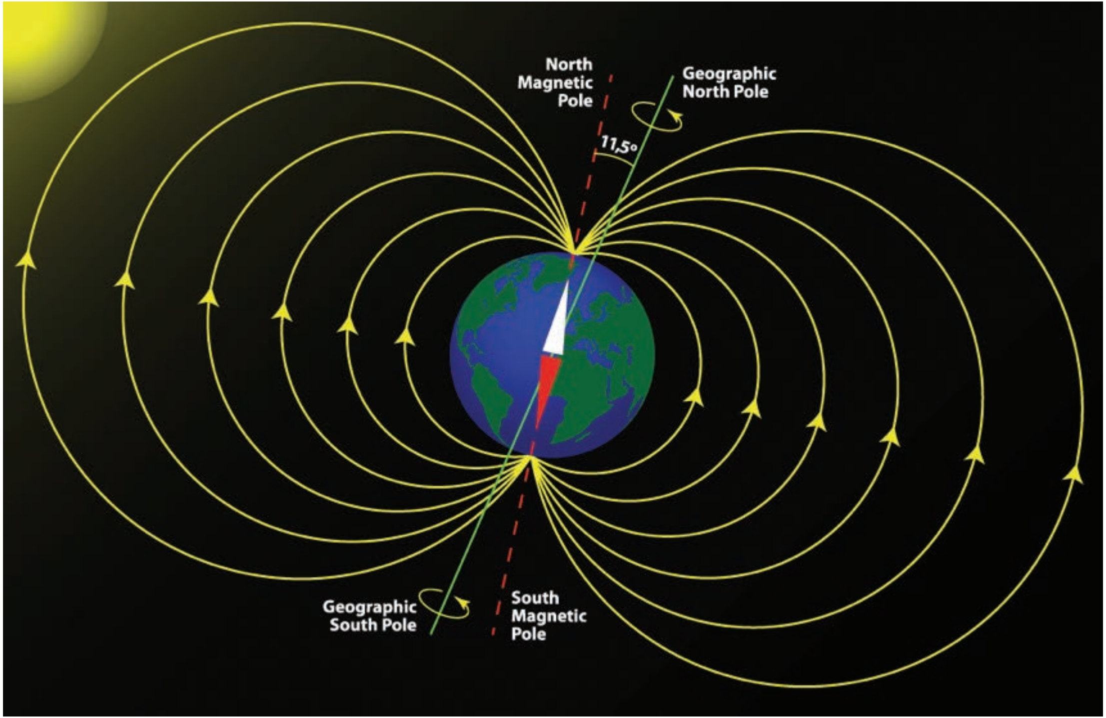
<u>pitch</u>

$$\frac{\hat{a}_z}{\sqrt{\hat{a}_x^2 + \hat{a}_y^2}} = \frac{\sin(-\theta_x)}{\sqrt{\cos^2(-\theta_x)\underbrace{\left(\sin^2(-\theta_z) + \cos^2(-\theta_z)\right)}_{=1}}}$$

$$\boxed{\theta_x = -\mathrm{atan2}\left(\hat{a}_z, \mathrm{sign}\left(\hat{a}_y\right) \cdot \sqrt{\hat{a}_x^2 + \hat{a}_y^2}\right) \text{ in rad} \quad \in [-\pi, \pi]}$$

Magnetometers

wikipedia



lifescience.com

MEMS Magnetometer

Hall Effect

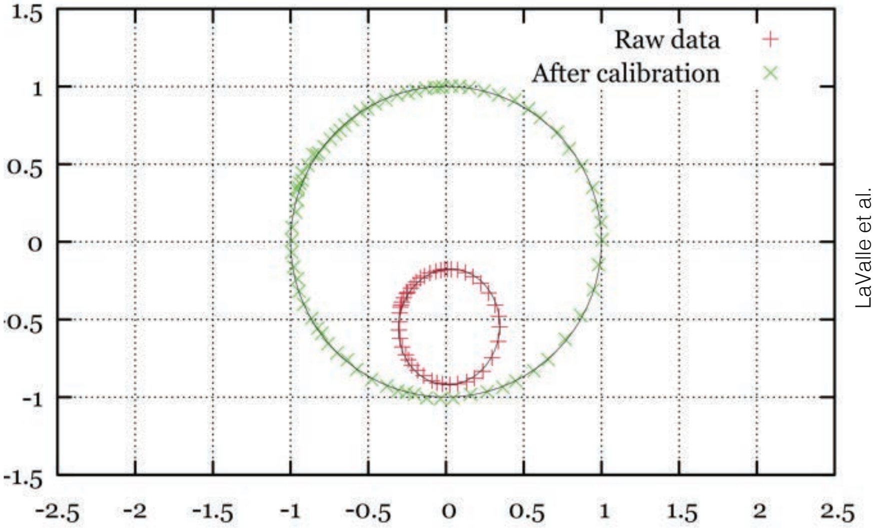
Magneto-resistive effect

Magnetometers

- measure earth's magnetic field in Gauss or uT
- 3 orthogonal axes = vector pointing along the magnetic field
- actual direction depends on latitude and longitude!
- distortions due to metal / electronics objects in the room or in HMD

Magnetometers

difficult to work with magnetometers without proper calibration → we will not use the magnetometer in the HW!



LaValle et al.

Magnetometers

- advantages:
    - complementary to accelerometer – gives yaw (heading)
- problems:
    - affected by metal, distortions of magnetic field
    - need to know location, even when calibrated (e.g. GPS)
- together with gyro + accelerometer = 9 DOF sensor fusion

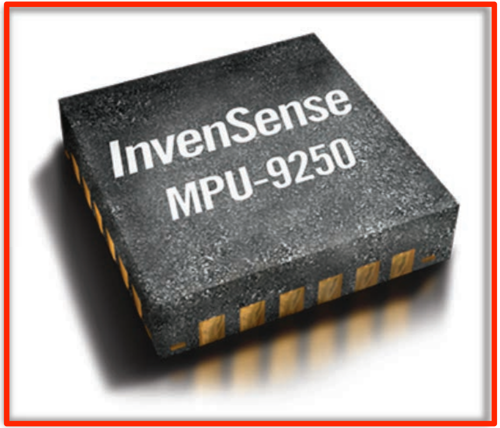
Prototype IMU

- 9 DOF IMU: InvenSense MPU-9250 = updated model of what was in the Oculus DK2
- 3-axis gyro, 3-axis accelerometer, 3-axis magnetometer all on 1 chip (we'll only use gyro and acc, but we'll give you code to read mag if you want to use it in your project)
- interface with I2C (serial bus) from Arduino

Prototype IMU

to host:
serial via USB

I2C

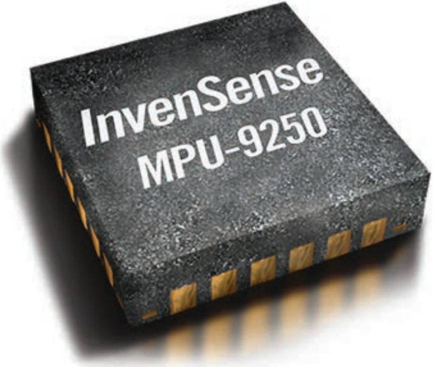


e.g. Arduino

InvenSense MPU-9250

MPU-9250 Specs

- multi-chip module: 1 die houses gyro & accelerometer, the other the magnetometer



- magnetometer: Asahi Kasei Microdevices AK8963 (“3rd party device”)

- 9x 16 bit ADCs for digitizing 9DOF data

MPU-9250 Specs

- gyro modes: ±250, ±500, ±1000, ±2000 °/sec
- accelerometer: ±2, ±4, ±8, ±16 g
- magnetometer: ±4800 uT

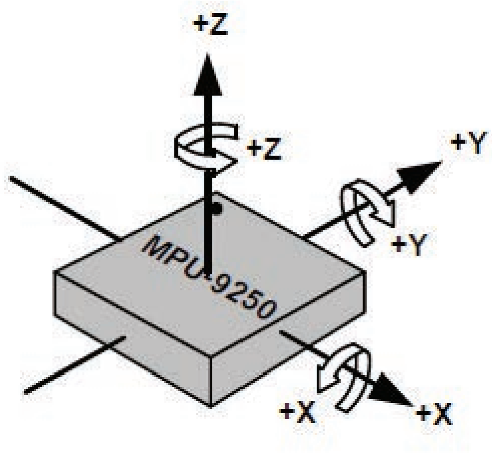
- configure using registers (see specs) via I2C
- also supports on-board Digital Motion Processing™ (DMP™) sorry, we don't have access
- we'll provide starter code for Arduino in lab (easy to use for beginners, not consumer product grade!)

MPU-9250 Specs

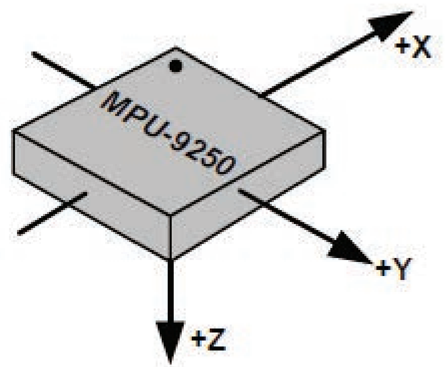
- gyro modes: ±250, ±500, ±1000, ±2000 °/sec
- accelerometer: ±2, ±4, ±8, ±16 g
- magnetometer: ±4800 uT

$$metric\_value = \frac{raw\_sensor\_value}{2^{15}-1} \cdot \max\_range$$

MPU-9250 Coordinate Systems



gyro & accelerometer



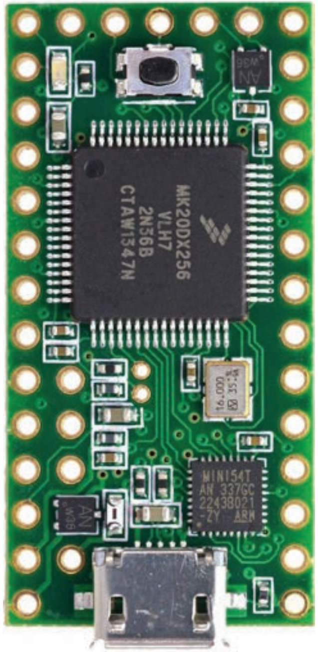
magnetometer

How to read data from IMU

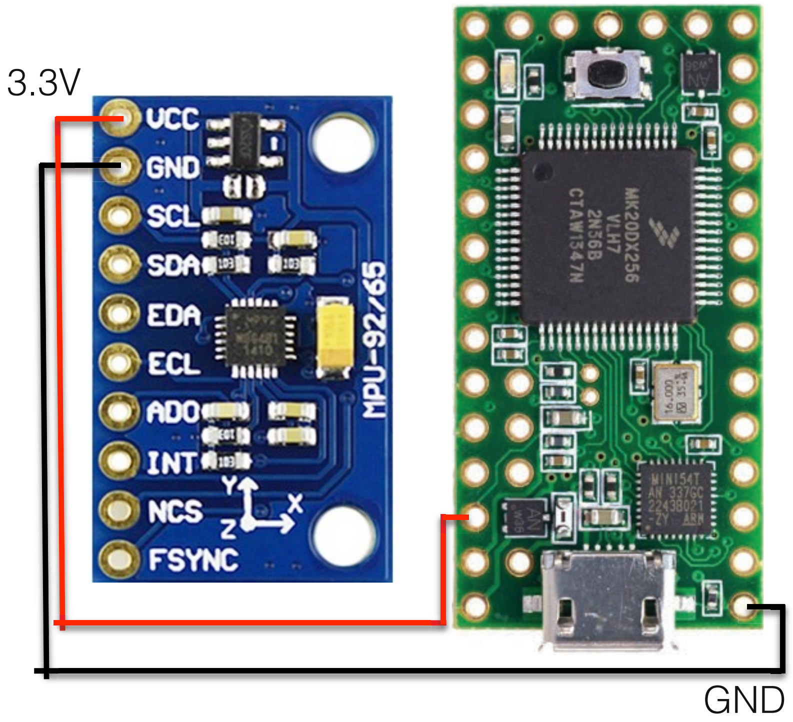
- I2C = serial interface with 2 wires (also see next lab)
- microcontroller to read, we'll use Teensy 3.2, but any Arduino can be used, last year: Metro Mini

- schematics - which pins to connect where
- quick intro to Arduino
- Wire library to stream out data via serial
- serial client using node server

How to read data from IMU

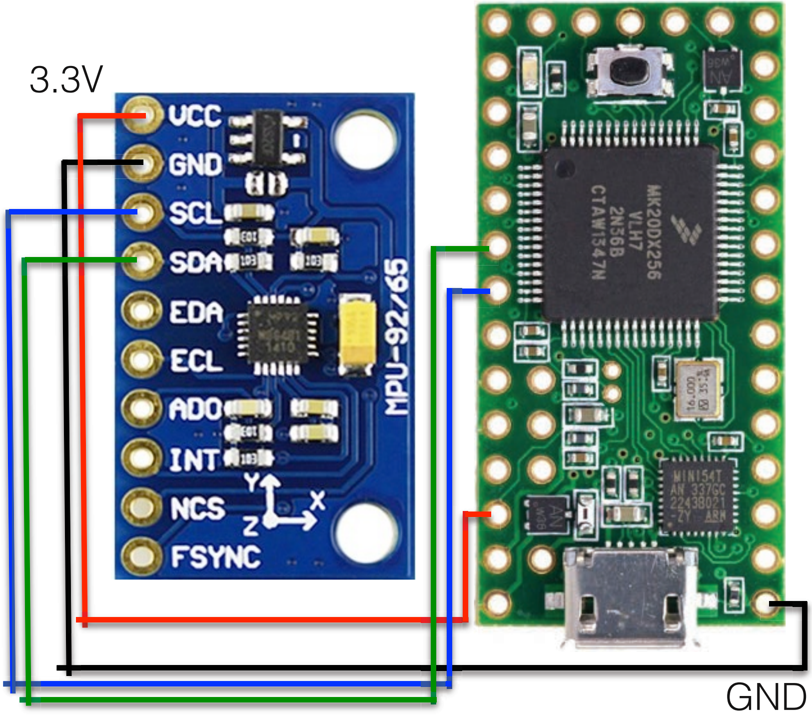


How to read data from IMU



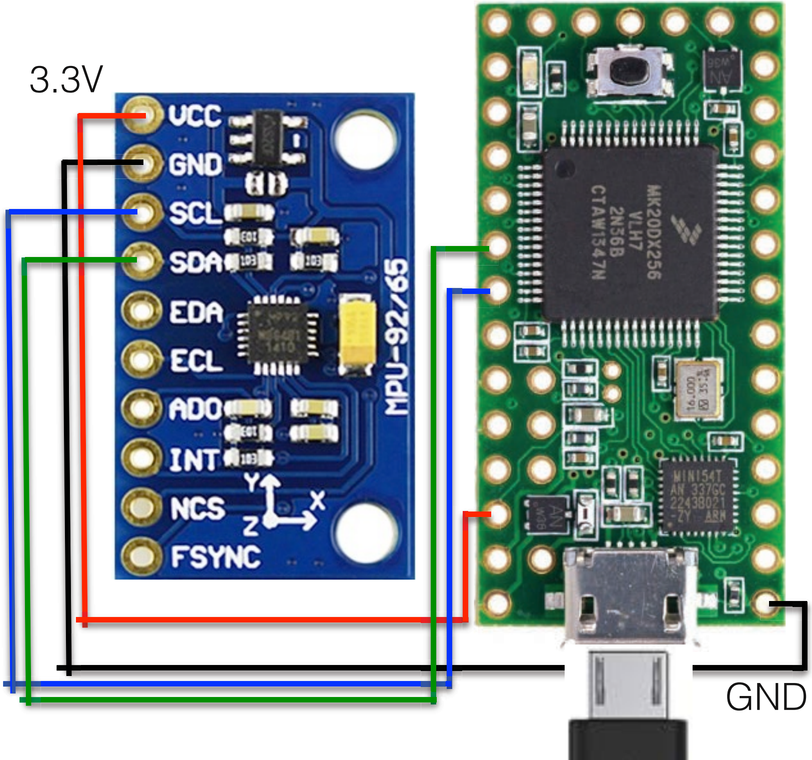
- connect power & ground

How to read data from IMU



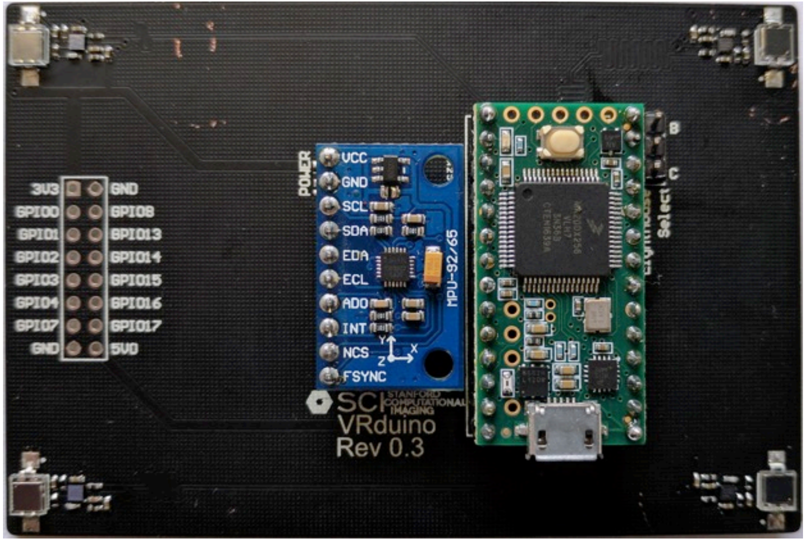
- connect power & ground
- connect I2C clock (SCL,D19) and data (SDA,D18) lines

How to read data from IMU



- connect power & ground
- connect I2C clock (SCL,A5) and data (SDA,A4) lines
- connect micro USB for power and data transfer

VRduino

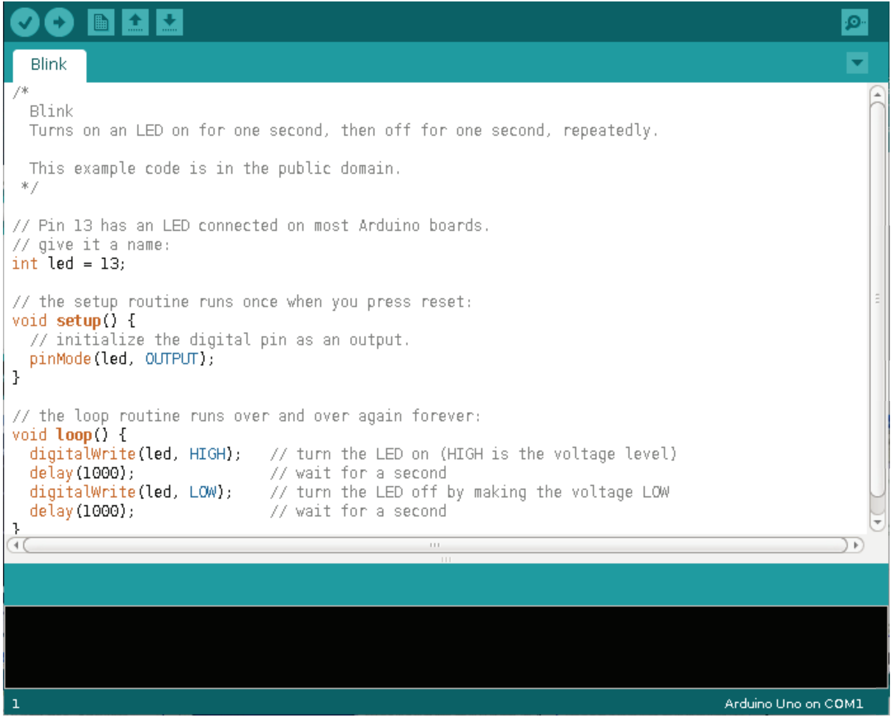


- Teensy 3.2 & IMU already connected through PCB
- also has 4 photodiodes (more details next week)
- GPIO pins for additional sensors or other add-ons

Introduction to Arduino

- open source microcontroller hardware & software
- directly interface with sensors (i.e. IMU) and process raw data
- we will be working with Teensy 3.2 (Arduino compatible)
- use Arduino IDE for all software development, installed on all lab machines
- if you want to install it on your laptop, make sure to get:
    - IDE: https://www.arduino.cc/en/Main/Software
    - Teensyduino: https://www.pjrc.com/teensy/teensyduino.html
    - Wire library (for serial & I2C): http://www.arduino.cc/en/Reference/Wire
    - FTDI drivers: http://www.ftdichip.com/Drivers/VCP.htm

Introduction to Arduino (Random Test Program)



← variable definition

← setup function = initialization

← loop function = runtime callback

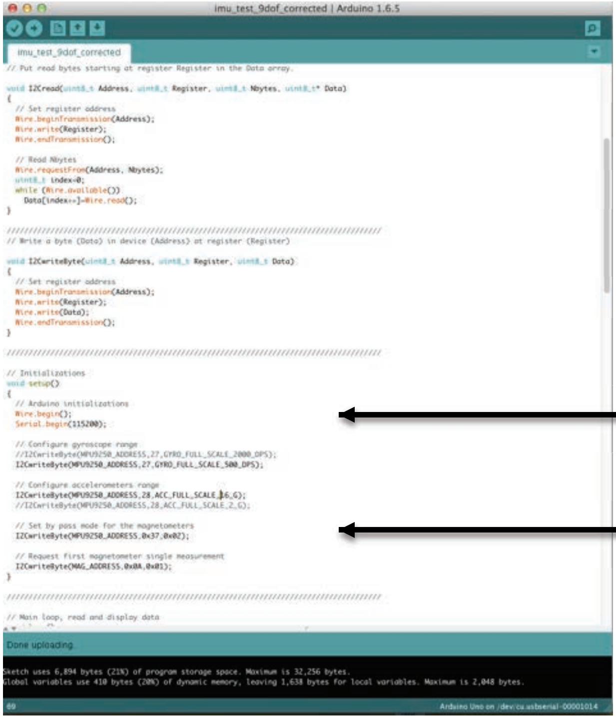
← connected to COM1 serial port

Introduction to Arduino

- need to stream data from Arduino to host PC
- use Wire library for all serial & I2C communication

- use node server to read from host PC and connect to JavaScript (see lab)

Introduction to Arduino



read from I2C (connected to IMU)

write to I2C (connected to IMU)

<u>setup</u> function = one time initialization

open serial connection to communicate with host PC

set registers to configure IMU

Read Serial Data in Windows

- serial ports called COMx (USB serial usually COM3-COM7)
    1. establish connection to correct COM port (choose appropriate baud rate)
    2. read incoming data (in a thread)

Summary

- coordinate systems (world, body/sensor, inertial, transforms)
- overview of inertial sensors: gyroscopes, accelerometers, and magnetometers
- gyro integration aka *dead reckoning*
- orientation tracking in *flatland*
- pitch & roll from accelerometer
- overview of VRduino

Next Lecture

- quaternions and rotations with quaternions
- 6 DOF sensor fusion with quaternions & complementary filtering

Must read: course notes on IMUs!

Additional Information

- D. Sachs “Sensor Fusion on Android Devices: A Revolution in Motion Processing”, Google Tech Talks 2010, Video on youtube.com (https://www.youtube.com/watch?v=C7JQ7Rpwn2k)

- S. LaValle, A. Yershova, M. Katsev, M. Antonov “Head Tracking for the Oculus Rift”, Proc. ICRA 2014

- http://www.chrobotics.com/library

Positional Tracking with Accelerometers

- goal: track 3D position from accelerometer measurements

- question: is that even possible?

Positional Tracking with Accelerometers

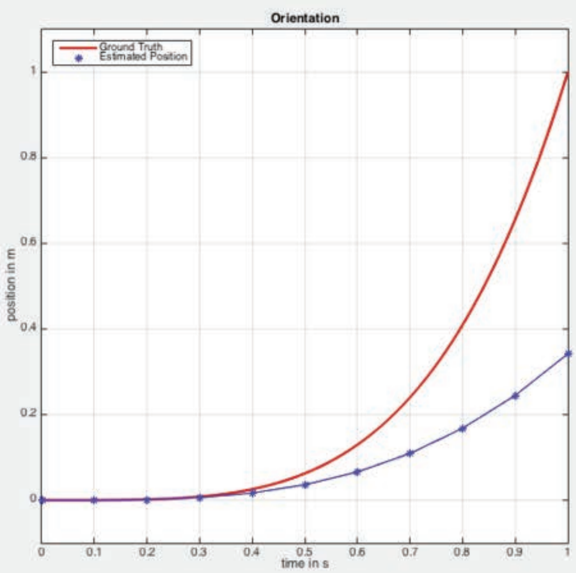
- why not double-integrate acceleration for positional tracking?
- assume you can remove gravity vector (which is not trivial)
- two ODEs to estimate linear velocity and position:

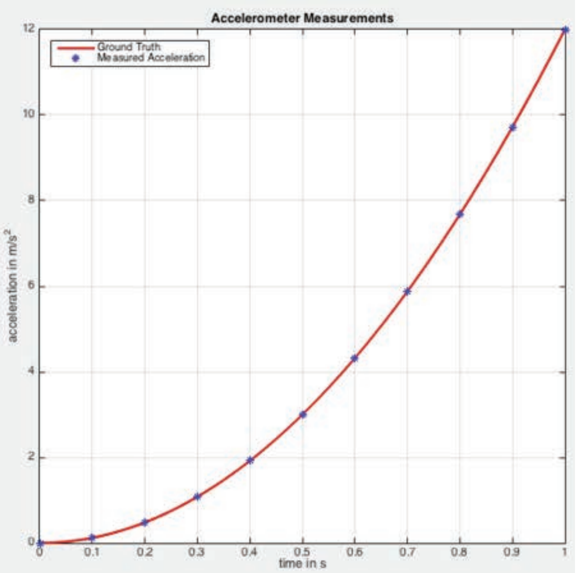
$$\frac{\partial}{\partial t}x(t+1) \approx \frac{\partial}{\partial t}x(t) + \underbrace{\frac{\partial^2}{\partial t^2}x(t)}_{a^{(l)}\text{ from accelerometer}}\Delta t$$

$$x(t+\Delta t) \approx x(t) + \frac{\partial}{\partial t}x(t)\Delta t$$

Positional Tracking with Accelerometers

- error grows quadratically!





Positional Tracking with Accelerometers

- question: is that even possible?
- answer: usually not

- two key challenges:
  1. error in Taylor series for double integration grows quadratically
  2. very difficult to remove gravity from accelerometer measurements