A Q-Theory of Banks

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Introduction

- **Four Motivating Facts**
  - Book and market values diverge during crises,
  - The market-to-book ratio predicts future profitability,
  - Book leverage constraints rarely bind strictly even as market leverage fans out during crises
  - Banks delever gradually after net-worth shocks
- **Question**: based on delayed information (loss recognition)
  - How regulation constrains bank risk taking.
  - Implication in accounting books for an ideal regulatory framework.

This Paper

- **Setup.** Heterogenous bank model in continuous time with
  - Market and book-based regulatory constraints.
  - Uninsurable idiosyncratic loan default shocks.
  - Delayed recognition on book losses (zombie loans).
- **Lesson.** Loss-recognition speed as additional policy tool.
- **Microprudential Implication**. Tighter accounting rules vs. Tighter capital regulation
  1. Loss-recognition margin: postponed deleveraging benefit vs. excessive liquidation cost
  2. Optimal policy: slightly relaxed capital requirement and speedier loss recognition.
- **Macroprudential Implication**. Countercyclical accounting standard is more preferred.
  1. Countercyclical Capital Buffer amplifies the credit cycle and overall liquidation risk.
  2. Countercyclical Loss Recognition postpones deleveraging for most affected banks.

Roadmap

- Economic Model
- Positive Analysis
  - Immediate Loss Recognition
  - Delayed Accounting
- Policy Implications
  - Capital Regulation Only
  - Microprudential Regulation
  - Macroprudential Regulation

Environment

- Continuous time $t \in [0, \infty)$. All assets are real. Bank net worth W_t.

- A continuum of risk neutral banks takes deposits D_t at rate r^D and loans L_t at rate r^L.
- Loans are defaultable. Defaults arrives with idiosyncratic Poisson process dN_t at rate σ.
  - A ε fraction of loan defaults in a default event.
  - Defaulted loans become zombie loans Z_t. (If not liquidated, specified later).

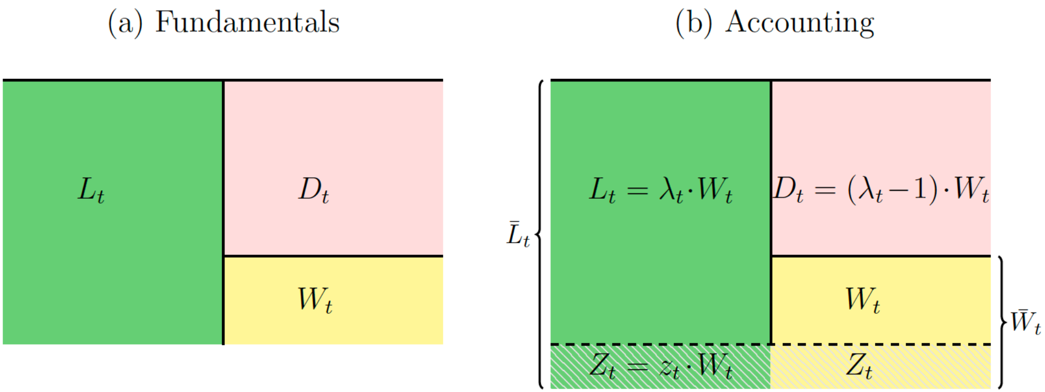
- Given constant dividend rule $C_t = cW_t$, bank chooses leverage to maximize its value.

  In the quantitative model, EZ preference with 0 risk aversion and IES $1/\theta$, C_t is endogenous.

- Regulatory liquidation happens when constraints are violated.

  Liquidated bank recovers $v_o W_t$ net worth.

Balance Sheet Structure

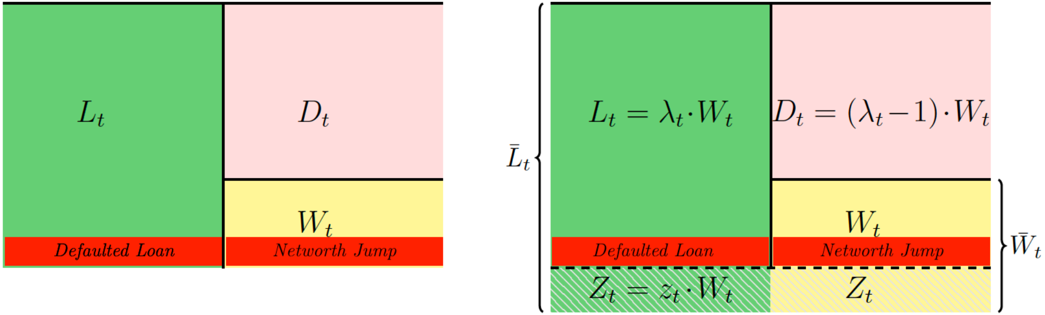


Book equity $\bar{W}_t = W_t + Z_t$; Book loans$\bar{L}_t = L_t + Z_t$;

Book leverage $\bar{\lambda}_t = \bar{L}_t / \bar{W}_t$; Zombie loan ratio: $z_t = Z_t / W_t$.

Balance Sheet Structure when Default Shock Hits Timing

(a) Fundamentals (b) Accounting



$\bar{W}_t = \bar{W}_{t-} - \varepsilon L_{t-}, \bar{L}_t = \bar{L}_{t-} - \varepsilon L_{t-}, W_t = W_{t-} - \varepsilon L_{t-}, L_t = L_{t-} - \varepsilon L_{t-}, Z_{t+} = Z_{t-} + \varepsilon L_{t-}.$

Implication: both fundamental leverage and book leverage go up when default shock hits.

Financial Constraints

Without loan default (normal times):

- Fundamental leverage constraint: $L_t/W_t = \lambda_t \leq \kappa$
- Book leverage constraint: $\bar{L}_t/\bar{W}_t = (\lambda_t + z_t)/(1 + z_t) \leq \Xi$
- The *liquidation boundary*:

$$\lambda \leq \Gamma(z) \equiv \{\Xi + (\Xi - 1)z, \kappa\}$$

Upon default event (shocked times):

- Fundamental leverage constraint: $L_t/W_t = \lambda_{t-}(1-\varepsilon)/(1-\varepsilon\lambda_{t-}) \leq \kappa$
- Book leverage constraint: $\bar{L}_t/\bar{W}_t = (\lambda_{t-}(1-\varepsilon) + z_t)/(1 - \varepsilon\lambda_{t-} + z_{t-}) \leq \Xi$
- The *shadow boundary*:

$$\lambda \leq \Lambda(z) = \min\left\{\frac{\Xi + (\Xi - 1)z}{1 + (\Xi - 1)\varepsilon}, \frac{\kappa}{1 + (\kappa - 1)\varepsilon}\right\}$$

Bank's Optimization

- Banks chooses leverage λ_t, dividend C_t to maximize its value

$$V_0 = \mathbb{E}\left[\int_0^\infty e^{-\rho t} C_t dt\right]$$

subject to: (1) Dividend rule: $C_t = cW_t$, (2) financial constraints, (3) state evolution

$$\begin{aligned} dW_t &= \left[r^L\lambda_t - r^D(\lambda_t - 1) - c\right] W_t dt - \varepsilon\lambda W_t dN_t, \\ dZ_t &= -\alpha Z_t dt + \varepsilon\lambda W_t dN_t. \end{aligned}$$

- Assumptions:
  1. Lending is profitable: $r^L - \sigma\varepsilon \geq r^D$.
  2. Returns are bounded $\rho > r^D + (r^L - r^D)\kappa - c$
  3. Liquidation is costly for self-financed banks: $(\rho - r^L)v_0/c \leq 1 - \varepsilon$
  4. Bank chooses to avoid liquidation, if indifferent between risking and not risking liquidation.

Immediate Loan Loss Recognition $\alpha \to \infty$

- Consider the *laissez-faire* regulation: $\kappa < \Xi$. $\Gamma = \kappa, \lambda = \kappa \cdot (1 + \varepsilon(\kappa - 1))^{-1}$
- Zombie loans $Z_t = 0$, as $\alpha \to \infty$. Scaled solution: $V(W, 0) = vW$. Derivation
- Expression: $v = \int_0^\infty e^{-\rho t} c e^{(\Omega^* - c)t} dt = \frac{c}{\rho - (\Omega^* - c)}$, where

$$\Omega^* = r^D + \max_{\lambda \in [1,\Gamma]} c + (r^L - r^D)\lambda + \sigma \left\{ (1 - \lambda\varepsilon)\mathbb{I}_{\lambda \leq \Lambda} + \frac{v_0}{v}\mathbb{I}_{\lambda > \Lambda} - 1 \right\}$$

- Expected return Ω^* is piecewise linear w.r.t λ.
  $\Rightarrow$ Bank makes binary choice $\{\Gamma, \Lambda\}$ to maximize Ω^*.
- Optimal leverage:

$$\lambda = \begin{cases} \Lambda & \text{if } (r^L - r^D)\kappa - \sigma(1 - \frac{v_o}{v}) \leq \Lambda[(r^L - r^D) - \sigma\varepsilon] \\ \kappa & \text{if } (r^L - r^D)\kappa - \sigma(1 - \frac{v_o}{v}) > \Lambda[(r^L - r^D) - \sigma\varepsilon] \end{cases}$$

Delayed Accounting

- Value function takes form $V(Z, W) = v(z)W$, where $z = Z/W$. Derivation

$$\rho v(z) = c - v_z(z)\alpha z + (v(z) - v_z(z)z) \cdot [\Omega^*(z) - c]$$

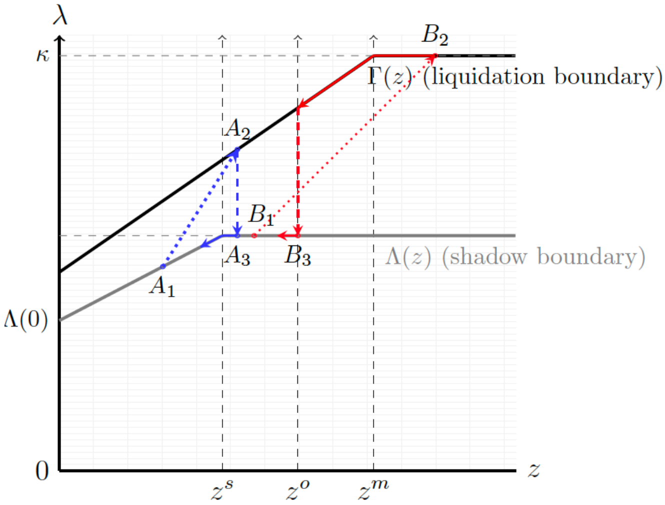
- Optimal leverage $\lambda^*(z)$ solves:

$$\Omega^* = r^* + \max_{\lambda \in [1, \Gamma(z)]} (r^L - r^D)\lambda + \sigma \left\{ \frac{J^v(z, \lambda)}{v(z) - v_z(z)z} \right\}$$

- Value jump upon default shock: $J^v \equiv v(z + J^z)(1 - \varepsilon)\mathbb{I}_{\lambda \leq \Lambda(z)} + v_0 \mathbb{I}_{\lambda > \Lambda(z)} - v(z)$.
- Expected return is also piecewise linear w.r.t. λ.

$$\lambda^*(z) = \begin{cases} \Lambda(z) & \text{if } \Omega(z; \Gamma(z)) \leq \Omega(z; \Lambda(z)) \\ \Gamma(z) & \text{if } \Omega(z; \Gamma(z)) > \Omega(z; \Lambda(z)) \end{cases}$$

Trajectory of z and λ under Delayed Accounting

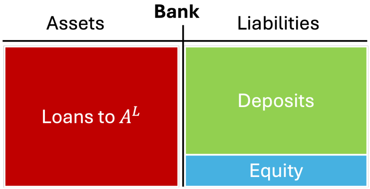


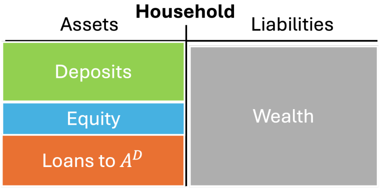
Connection to Facts

- Fact 1: book and market values diverge during crises
  Explanation: Upon loan default, market equity jumps while book equity does not.
- Fact 2: the market-to-book ratio predicts future profitability
  Explanation: Zombie loans are recognized gradually $\rightarrow$ *future book loss predictable.*
- Fact 3: book leverage constraint barely binds Ergodic Distribution
  Explanation: most banks stay at the shadow boundary even if market leverage is high.
- Fact 4: banks delever gradually after net-worth shocks
  Explanation: losses are recognized slowly, banks delever slowly after a negative shock.

Calibration

Policy Implications: Updated Environment





- Two productive sectors (L, D) with AK technology. Productivity: A^L, A^D,$A^L > A^D$
  - Capital is freely mobile across sectors and fully reversible.
  - Investment transforms 1 unit of good to 1 unit of capital. Capital depreciates at rate δ.
- A representative risk-neutral household holds wealth in bank equity and D firms.
- Each bank maximizes its value, lend to L firms and faces idio. Poisson default shock.
- Default destroys ε fraction of loans and ψ fraction of capital. Assume:
  - Economically efficient (individual perspecitve) to allocate capital to L sector: $A^L - \sigma\varepsilon > A^D$.
  - Socially inefficient of bank liquidation: $A^L - \sigma(\varepsilon + (1-\psi)(1-\varepsilon)) > A^D$.
  - Some value left for surviving banks upon liquidation: $v_0 \leq 1 - (1 - \psi(1-\varepsilon))\kappa$.

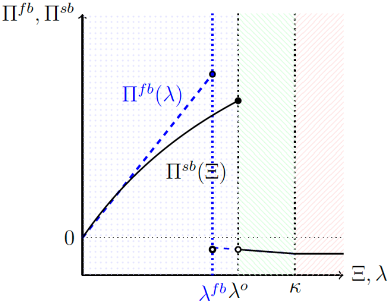
Optimal Capital Requirement with Immediate Accounting

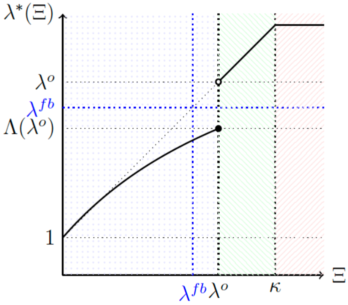
- (First best) Socially optimal leverage λ^{fb} solves:

$$\Pi^{fb} = \max_{\lambda} \left(r^L - r^D - \sigma[\varepsilon + (1-\psi)(1-\varepsilon)\mathbb{I}_{\lambda>\Lambda}] \right) \lambda$$

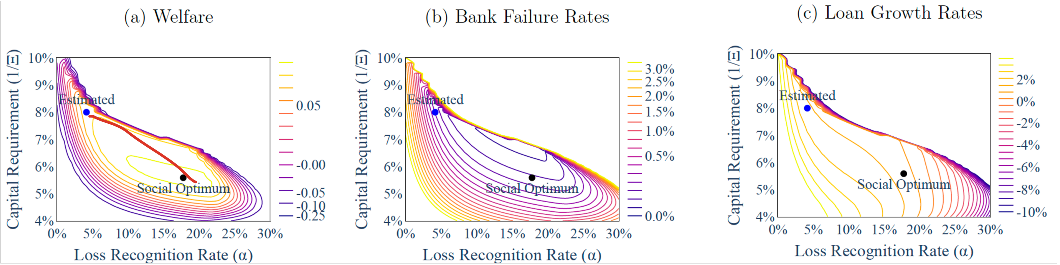
- (Second best) Socially optimal capital requirement Ξ solves (given κ):

$$\Pi^{fb} = \max_{\Xi} \left(r^L - r^D - \sigma[\varepsilon + (1-\psi)(1-\varepsilon)\mathbb{I}_{\lambda^*(\Xi,\kappa)>\Lambda}] \right) \lambda^*(\Xi, \kappa)$$



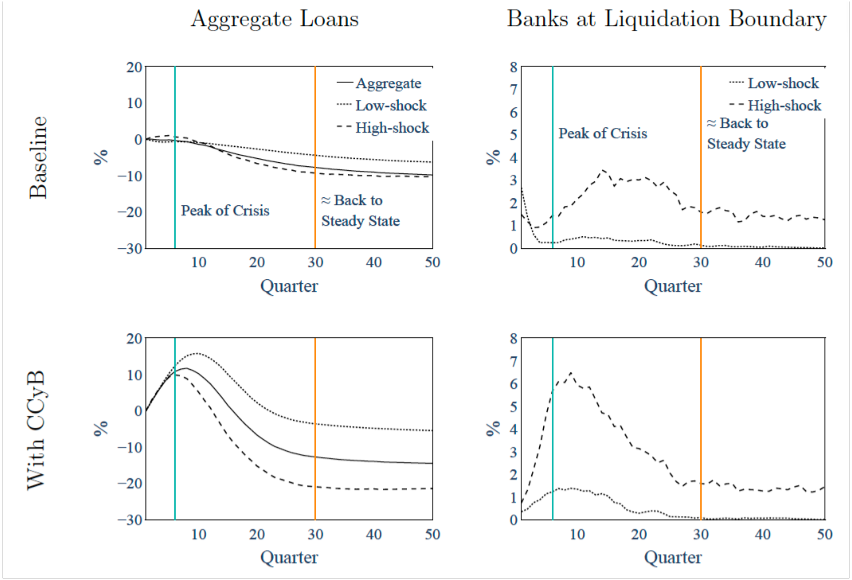


Microprudential Implication: Optimal Regulation with $\{\alpha, \Xi\}$ Planner's Objective

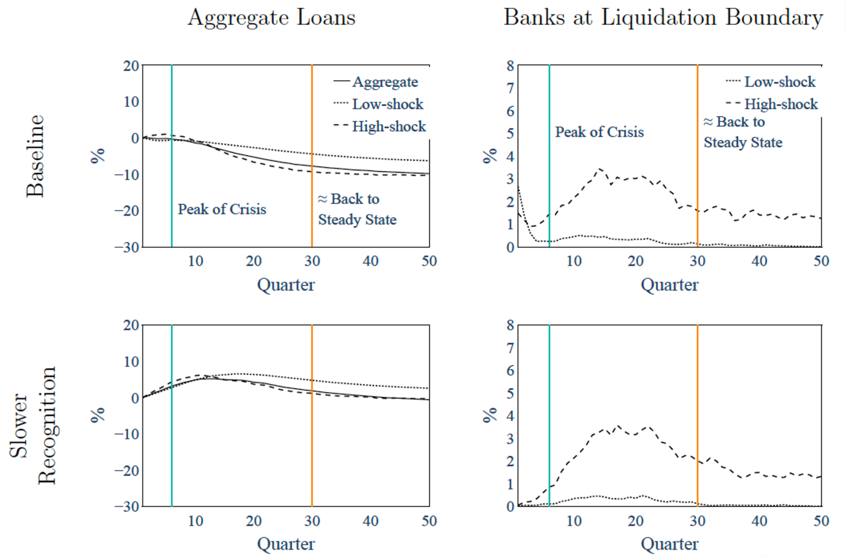


- Results: $\alpha : 17\%$ (optimal) vs. 4.16% (calibrated); $1/\Xi : 5.5\%$ (optimal) vs. 8% (Basel III)
- Moving to $\{\alpha^*, \Xi^*\}$: safer bank system with book values closer to fundamental values less liquidation & social liquidation cost, higher and still safe fundamental & book leverage.

Macroprudential Implications: Countercyclical Capital Buffer



Macroprudential Implications: Countercyclical Accounting Rule

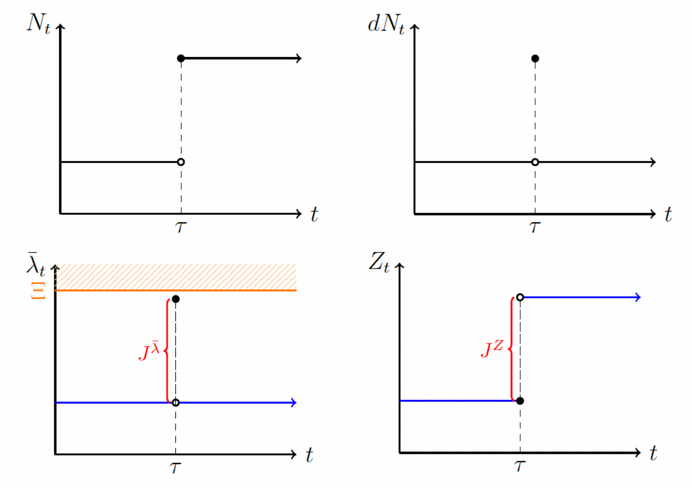


Conclusion

- Built a macroeconomic model with heterogenous banks
  - Distinguishes accounting, fundamental, and market values of bank equity
  - Subject banks to market and book-based constraints
  - Delayed accounting of losses on books.
- Loss accounting: financial fragility vs. growth.
- Comprehensive regulatory: book and market value.
- Future research: (1) real cost of zombie loans, (2) firesale externalities, (3) endogenous accounting standard choice.

Thank you!

Appendix: Timing



Delveraging happens right after jump at τ.

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Appendix: Immediate Accounting

$Z \equiv 0$ for immediate accounting case:

$$\rho V(W,0) = \max_{\lambda \in [1,\Gamma]} cW + V_W(0,W)\mu^W W + \sigma \left[V(W + J^W W, 0)\mathbb{I}_{\lambda \leq \Lambda} + v_0 W \mathbb{I}_{\lambda > \Lambda} - V(0,W) \right]$$

Postulate $V(W,0) = vW$ and plug in $\mu^W = r^L\lambda - r^D(\lambda - 1) - c$:

$$\rho v = \max_{\lambda \in [1,\Gamma]} c + v[r^L\lambda - r^D(\lambda - 1) - c] + \sigma((1 - \lambda\varepsilon)v + v_0\mathbb{I}_{\lambda > \Lambda} - v)$$

Rearrange:

$$v = \max_{\lambda \in [1,\Gamma]} \frac{c}{\rho - [r^L\lambda - r^D(\lambda - 1) - c] - \sigma((1 - \lambda\varepsilon)v + v_0\mathbb{I}_{\lambda > \Lambda} - v)} \equiv \frac{c}{\rho - (\Omega^* - c)}$$

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Appendix: Slow Accounting HJB

The full HJB with state variable W, Z is:

$$\rho V(Z,W) = \max_{\lambda\in[1,\Gamma(Z/W)]} cW + V_Z(Z,W)\mu^Z W + V_W(Z,W)\mu^W W$$
$$+\sigma\left[V(Z+J^Z W, W+J^W W)\mathbb{I}_{\lambda\le\Lambda(Z/W)} + v_0 W\mathbb{I}_{\lambda>\Lambda(Z/W)} - V(Z,W)\right]$$

Postulate $V(Z,W) = v\left(\frac{Z}{W}\right)W \equiv v(z)W$, we have:

$$V_Z(Z,W) = v'(z), V_W(Z,W) = v(z) - zv'(z)$$

$$\rho v(z) = \max_{\lambda\in[1,\Gamma(z)]} c + \mu^W v(z) + (\mu^Z - z\mu^W)v'(z) + \sigma[v(z+J^z)(1-\varepsilon\lambda)\mathbb{I}_{\lambda\le\Lambda(z)} + v_0\mathbb{I}_{\lambda>\Lambda(z)} - v(z)]$$

Plug in $\mu^W = \lambda(r^L - r^D) + r^D - c$, $\mu^Z = -\alpha z$, we get the HJB with single state variable z.

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Appendix: Parameterization

Parameter	Description	Target
	Externally set parameters	
$r^L = 1.01\%$	Loan yield	BHC data: avg. interest income/loans
$r^D = 0.51\%$	Bank debt yield	BHC data: avg. interest expense/debt
$\Xi = 12.5$	Regulatory maximum asset to equity ratio	Capital requirement of 8%
	Jointly determined – estimated	
$\rho = 2.24\%$	Banker's discount rate	Book equity growth rate: 2%
$\rho^I = 3.47\%$	Investor's discount rate	Market-to-book ratio of equity: 1.316
$\theta = 7.94$	Banker's inverse IES	Market leverage IRF
$\varepsilon = 1.12\%$	Loan loss rate in event of default	Mean book leverage
$\alpha = 4.16\%$	Speed of loan loss recognition	Liabilities IRF
$\kappa = 51$	Market-based leverage constraint	Liabilities IRF
$\sigma = 0.115$	Arrival rate of loan default shocks	Mean quarterly net charge-off rate of 0.12%
$v_o = 0.046$	Bank liquidation value	Quarterly bank failure rate of 3.65 basis points

ρ^I is used to compute investor's valuation (same HJB for $v(z)$ with ρ replaced by ρ^I)

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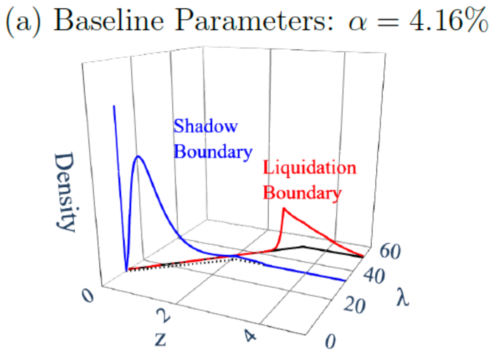
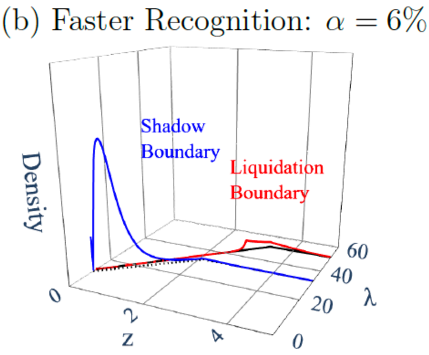
Appendix: Ergodic Distribution





KFE characterizes distribution: (1) banks with $z > 0$ on the shadow boundary, (2) banks with $z > 0$ on the liquidation boundary, (3) reconstructed banks.

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Appendix: Social Planner's Problem

Welfare function:

$$\mathcal{P}^*(\{g_0\}) \equiv \max_{\{\alpha,\Xi\}} \mathcal{P}^*(\alpha, \Xi, \{g_0\}) = \int_0^\infty \int_0^\infty \mathbb{E}\left[\int_0^\infty e^{-\rho t} c W_t dt | W_0 = W, z_0 = z\right] g_0(z, W) dz dW$$

Transformed objective function:

$$\mathcal{P}^*(\{g_0\}) = \max_{\{\alpha,\Xi\}} \int_0^\infty W \int_0^\infty p(z) g_0(z, W) dz dW,$$

where $p(z)$ is the social value of a bank:

$$\rho p(z) = c + p_z(z)\mu^z + p(z)\mu^W + \sigma J^p(z), \qquad \text{and}$$

$$J^p(z) = [p(z + J^z)(1 - \varepsilon\lambda)\mathbb{I}_{\lambda \leq \Lambda(z)} + p(0) + (1 - (\varepsilon + (1 - \psi)(1 - \varepsilon)\lambda))\mathbb{I}_{\lambda > \Lambda(z)} - p(z)]$$

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