DMP Model Assignment: Due July 1^{st}, 11:59 PM

In this project you will be using data to determine appropriate values of parameters to use in the Pissarides model of equilibrium unemployment. The data is taken from a recent paper by Elsby, Michaels and Ratner (Sept. 2015 issue of the Journal of Economic Literature). Taking the parameter values estimated from the data, you will then simulate the model and examine its implications for the unemployment rate, vacancy creation, the job filling probability and the job finding probability in response to particular shocks that hit the labour market.

Consider the Pissarides model of equilibrium frictional unemployment that we studied in class. Suppose that the job separation rate, δ, is constant over time. In the version of the model studied in this project, labour productivity will vary across time. In addition, the matching function takes on a Cobb-Douglas form

$$m(u, v) := \chi e^{\xi_t} u^{\alpha} v^{1-\alpha} \tag{1}$$

with[1]

$$\xi' = \rho_\xi \xi + \nu', \qquad \nu \sim N(0, \sigma_\nu^2). \tag{2}$$

so that ξ follows an AR(1) process and the shocks to this AR(1) process are normally distributed with mean zero and variance σ_ν^2. We will also assume that labour productivity is exogenous with $a := e^A$ and

$$A' = \rho_A A + \epsilon', \qquad \epsilon \sim N(0, \sigma_\epsilon^2). \tag{3}$$

In solving the model numerically, we will discretize the state-space for A and ξ so that instead of $A \in \mathbb{R}$, we will have $A \in \mathcal{A}$ where $\mathcal{A} := \{A_1, A_2, ..., A_{n_A}\}$. Similarly, $\xi \in \Xi$ with $\Xi := \{\xi_1, \xi_2, ..., \xi_{n_\xi}\}$. Both A and ξ will evolve following a Markov-chain. Let Π_A denote the matrix of transition probabilities for A while Π_ξ denotes the matrix of transition probabilities for ξ. The typical element of Π_A will be $\pi(A', A)$ which is the probability that given a level of productivity $a = e^A$ in the current period, labour productivity will equal $a' = e^{A'}$ in the following period. There is a similar interpretation for the typical element of Π_ξ, $\pi(\xi', \xi)$.

Letting $\theta_t := \frac{v_t}{u_t}$, $q(\theta_t) := \frac{m(u_t, v_t)}{v_t}$ is the vacancy-filling probability. The job finding probability is then $\theta_t q(\theta_t)$. As is derived in the class notes (and was derived in class

[1] Following convention, let x denote the value of the random variable x in the current period, while x' denotes the value of x in the following period.

lectures) the equilibrium dynamics of this model can be reduced to solving for the dynamics of θ, using the job creation equation

$$\frac{\kappa}{q(\theta)} = \rho \sum_{A' \in \mathcal{A}} \pi(A', A) \sum_{\xi' \in \Xi} \pi(\xi', \xi) \left\{ (1-\beta)e^{A'} - (1-\beta)b + \left[1 - \delta' - \theta' q(\theta')\beta\right] \frac{\kappa}{q(\theta')} \right\}. \tag{4}$$

Once the equilibrium function $\theta(A, \xi)$ is known the dynamics for the unemployment rate can be constructed using the flow equation,

$$u' = u + \delta(1-u) - \theta q(\theta) u. \tag{5}$$

while the equilibrium wage in each period can be backed out using the wage equation,

$$w = \beta e^{A} + (1-\beta)b + \theta q(\theta)\beta \frac{\kappa}{q(\theta)}. \tag{6}$$

1 Empirical Work (30 Marks)

Download the posted Excel spreadsheet "DMPProjectData.xlsx". It contains four spreadsheets with data from the U.S. economy. The first sheet contains data job vacancy data that is constructed by Regis Barnichon. This data is at a monthly frequency running from January 1968 through December 2012. The second sheet contains data on unemployment-to-employment transition probabilities. This data is quarterly. The third sheet contains data on a real output-per-hour index as constructed by the Bureau of Labor Statistics (BLS).

In order to parameterize the stochastic processes for A and ξ you need to use the rouwenhorst.jl Julia code. As was discussed in class (and in the class notes), this code contains an algorithm that discretizes a continuous AR(1) process like those of equations (2) and (3) into a finite number of values for the state variables (ξ and A respectively) along with a Markov-chain transition probability matrix (Π_ξ and Π_A in your case). However, in order to use the rouwenhorst.jl code, you will need to enter the values for ρ_ξ and σ_ξ when constructing the Markov-chain for ξ (and ρ_A and σ_A when constructing the Markov-chain for A).

1. First you will use sheets one and two from the file "DMPProjectData.xlsx" to estimate ρ_ξ and σ_ξ. Start by importing the spreadsheet data into Excel (see the solutions to Julia problem set #1 for an example of using the Julia command "xlsread($\cdot$)" or, possibly better, see the posted code "DataFramesExample.jl" to see how to import data from an Excel workbook into Julia as a DataFrame using the XLSX.jl package. You might have to install the XLSX.jl and DataFrames.jl packages if you have not previously done so). Import the data from sheet #1 and create the variables um and vm which are vectors of monthly unemployment rate data and vacancy rate data. This data has already been deflated by the size of the labour force which is required

to remain consistency with our model. In our model the labour force is always of size one! You will be using this data in conjunction with the quarterly data on u-to-e flows which means that the monthly data must be converted to quarterly data. To do this, created two time series called uq and vq by passing um and vm through the Julia function "monthlytoquarterly.jl". Finally, produce a variable $\theta := \frac{v}{u}$ using this quarterly data.

Next, import the data from sheet #2. Take the second column of the data which is the transition probability that an individual switches from unemployment to employment. From our matching function (1) we have

$$m(u,v) = \chi e^{\xi} u^{\alpha} v^{1-\alpha}.$$

In our model, the transition probability from unemployment to employment is $\frac{m(u,v)}{u}$. If we divide both sides of (1) by u we have

$$\frac{m(u,v)}{u} = \chi e^{\xi} \theta^{1-\alpha}.$$

Taking the logarithm of both sides we have a regression equation

$$\log\left(\frac{m(u,v)}{u}\right) = \log(\chi) + (1-\alpha)\log(\theta) + \xi$$

Note that you have the data for the lefthand side from sheet #2 (but you have to take the logarithm of the data) and you have constructed the data for θ. Set up a matrix X which contains a column of ones in the first column and the $\log(\theta)$ in the second column. Run ordinary least squares (OLS) on this data.[2]. Once you have $\hat{\beta} = [\log(\hat{\chi}), 1-\hat{\alpha}]'$ you can construct the values for ξ as $\xi = Y - X\hat{\beta}$. The intercept will be $\log(\chi)$. You will need to obtain χ from $\log(\chi)$. The other coefficent will be $1-\alpha$ from your matching function. (10 Marks)

2. Now that you have your time-series for $\{\xi_t\}_{t=1}^{T}$ construct a vector of ξ_t and ξ_{t-1}. In this case, the vector for ξ_t is the vector ξ less the first observation while the vector for ξ_{t-1} is the vector ξ dropping the last observation. Now you are ready to run another regression using OLS. Simply regress the vector ξ_t on the regressors ξ_{t-1}. The point estimate will be ρ_ξ that you will use in the model and the standard deviation of your OLS residuals (that you construct using ρ_ξ along with the vectors ξ_t and ξ_{t-1}) will serve as σ_ξ in the Pissarides model. How cool is that?! (10 Marks)

3. Now repeat the same procedure for A. Import the quarterly data from sheet 3 into Julia. You will use column #4 (call this variable "lp") to construct ρ_A and σ_A. This data is your empirical counterpart to A from your model. Before running

[2]Recall from your econometrics classes that $\hat{\beta} = (X'X)^{-1}X'Y$.

any regressions, you will have to take out the upward sloping trend in the data (because this will result in spurious regression results). First create a new variable "lpl" by taking the logarithm of "lp". Using the bpass.jl function, construct the variable $A = \text{bpass}(\text{lpl}, 2, 80)$. The band pass filter is taking the data in "lpl" and removing all fluctuations with frequencies longer than 80 quarters (that is 20 years) and frequencies lower than 2 quarters. This is typically done in studies looking at medium-run dynamics of macroeconomies.

Construct a vector A_t from this data (by dropping the first observation) and a vector A_{t-1} (by dropping the last observation). Run OLS with A_t as the regressand and A_{t-1} as the regressor. The point estimate will serve as your value of ρ_A in the model and the standard deviation of the OLS residuals will be your value of σ_A in your model. (10 Marks)

2 The Quantitative Pissarides Model (70 Marks)

Now solve the Pissarides model as you have done in your homework problem set. You may consult the Julia code that has been posted for the Pissarides model when the job separation rate is stochastic. You will have to choose the value z so that the job finding probability in the stochastic steady state is approximately 0.2635 which is the mean of the observations in the dataset that you used to estimate χ, ρ_ξ and σ_ξ. To calibrate z, parameterize the model using the values in Table 1. Pick a value for z. Solve the model and simulate the model for many periods (say 1000) using the median values for A and ξ (i.e. do not change the values for A and ξ for the simulation of 1000 periods). This simulation results in your economy in its *stochastic steady state* (the exogenous variables are not changing over time but the individuals in the model still face the uncertainty that the exogenous variables can change over time). Look at the value for the job finding probability in the last period of this simulation. If this value is close to 0.2635 then that is a good value for z to use in the rest of the project and record the values of all the variables in the last period of the stochastic steady state simulation. Otherwise, change the value for z and repeat.

1. Solve the model and simulate the stochastic steady state to chose a decent value for z. (20 Marks)

2. Construct the response of the economy for the following exercise. Simulate the model for 40 periods starting all endogenous values at their stochastic steady state values. Set ξ equal to its median state in all periods of the simulation. Set A to be equal to its median value in the first period. Then set A to equal its 16^{th} value (5 values higher than its median value) for the 2^{nd} through 10^{th} periods. Then set A back to its median value for the last 30 periods. Solve for all the values of the unemployment rate, the wage rate, the vacancy rate, the hiring rate, the job finding probability and the job filling probability for all the 40 periods and plot them. These are the

Table 1: Parameters Values for Numerical Example

Parameter	Shimer	Description
ρ	0.99	Subjective Discount Factor
β	0.5	Worker's Bargaining Share
δ	0.0081	Quarterly Job Separation Rate
z	Calibrate	Cost of Creating Job Vacancy
b	0.4	Utility Flow During Unemployment
$\overline{a}$	1	Average Labour Productivity ($\overline{A} = 0$)
χ	OLS	Efficiency Parameter in Matching Function
α	OLS	Curvature in the Cobb-Douglas Matching Function
ρ_a	OLS	Persistence of the Labour Productivity
σ_a	OLS	Volatility of the Labour Productivity
ρ_ξ	OLS	Persistence of the Match Efficiency
σ_ξ	OLS	Volatility of the Match Efficiency
n_A	21	Number of States for the Discretized a Process
n_ξ	15	Number of States for the Discretized a Process

variables' *impulse responses* to the dynamics in A. Give an explanation about why the endogenous variables respond the way that they do in the face of a positive shock to labour productivity. Hint: Look at the job creation equation and the flow equation for the unemployment rate. (15 Marks)

3. Construct the response of the economy for the following exercise. Simulate the model for 40 periods starting all endogenous values at their stochastic steady state values. Set A equal to its median state in all periods of the simulation. Set ξ to be equal to its median value in the first period. Then set ξ to equal its 3^{rd} value (5 values lower than its median value) for the 2^{nd} through 10^{th} periods. Then set ξ back to its median value for the last 30 periods. Solve for all the values of the unemployment rate, the wage rate, the vacancy rate, the hiring rate, the job finding probability and the job filling probability for all the 40 periods and plot them. These are the variables' *impulse responses* to the dynamics in ξ. Give an explanation about why the endogenous variables respond the way that they do in the face of a negative shock to matching efficiency. Hint: Look at the job creation equation and the flow equation for the unemployment rate. (15 Marks)

4. Now that you have the impulse responses in hand, look at Figure 1. The left panel plots a measure of the job finding rate of unemployed workers against a measure of the job filling rate of job vacancies. The right panel is the same data after removing frequencies in the data that are longer than 20 years in length. Look at your impulse response functions. How do these impulse response functions help you understand the contributions of matching efficiency shocks and labour productivity shocks to the dynamics of the (real-world) job finding and job filling probabilities (when viewing the world through the lens of your model). Specifically, think about how the impulse response functions allow you to identify labour productivity shocks and matching efficiency shocks in the real-world data of Figure 1 *if you think about labour markets using this version of the Pissarides model.* (10 Marks)

5. Provide a criticism of the interpretation of the labour productivity variable in your model. Also provide a criticism of the interpretation of the matching efficiency parameter (χe^{ξ}) in your model. How is this model useful in helping you understand the labour market and give one suggestion about how this model can be improved upon. (10 Marks)

IMPORTANT: Create a folder within your Dropbox Folder named "Final Project" and put all the codes required for your final project code to run inside this "Final Project" folder. This includes the Excel workbook with the data, the rouwenhorst.jl code, the bpass.jl code and the fcsolve.jl code. Submit your empirical work in a separate .jl file from your quantitative work. Submit your empirical work in a single .jl file titled "EmpiricalWork.jl". Title your main code for solving the Pissarides model as "MPModel.jl". Submit your written answers in a separate document (typed in Word or some other word processing program). Best if you submit the chatty answers as a .pdf document.

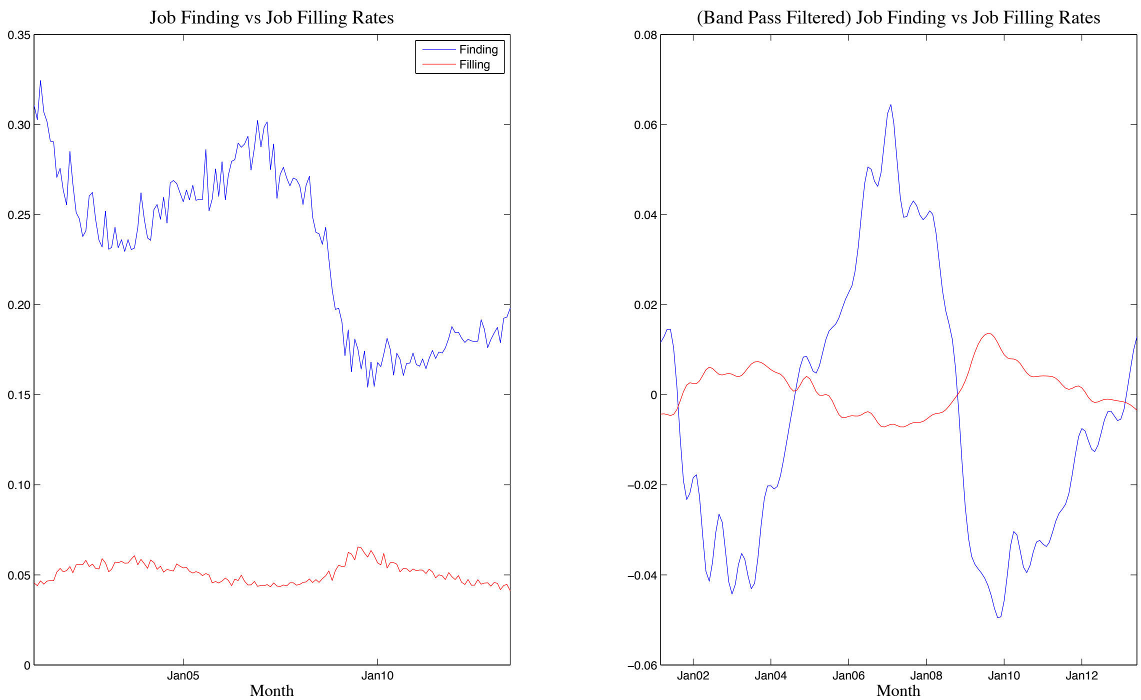

Figure 1: Empirical Job Finding vs Job Filling Probilities

HAVE FUN AND LEARN SOMETHING!!!