Notes on Aiyagari Model (In Progress)

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Abstract

This note details some rough notes for my implementation of an Aiyagari model.

1 An Aiyagari Model

There is a continuum of individuals of unit mass. The period problem faced by individual $I \in [0,1]$ is

$$V(a, l_i) = \max_{c,a'} \left\{ u(c) + \beta \sum_{i=1}^{N} \pi_i V(a', l_j) \right\}$$

subject to the budget constraint $a' = wl_i - c + (1+r)a$. Individual labour supply is exogenous with $l \in \{l_1, ..., l_N\}$ and follows an N-state Markov chain. The transition probability matrix for labour supply is given by $\Pi = [\pi_{j,i}]$, $i, j = 1, ..., N$. Individuals rent their capital to firms at a perfectly competitive price of r units of the consumption good per unit of capital services. Labour is supplied inelastically at the perfectly competitive wage rate w. Assume that individuals are not allowed to borrow so that $a \geq 0$. Following Aiyagari, it is assumed that the values for $\{l_1, ..., l_N\}$ and Π are chosen to approximate the persistence and variance of labour supply that would arise if

$$\ln(l') = \rho \ln(l) + \sigma\sqrt{1-\rho^2}\epsilon', \qquad \epsilon \sim N(0,1).$$

The representative firm purchases capital services and labour services in perfectly competitive markets to maximize profits

$$d = zK^{\alpha}L^{1-\alpha} - wL - rK - \delta K.$$

Market clearing requires $K = \int_0^1 a_I dI$ and $L = \int_0^1 l_I dI$.

Definition 1 *A competitive equilibrium is a set of allocations $\{K, L, (c_I, l_I, a'_I)_{I\in[0,1]}$, along with prices w and r and a distribution of wealth $F(a)$ such that (i) taking prices as given individuals and firms optimize, and (ii) markets clear.*

2 The Quantitative Solution

3 ROUGH STUFF

Obtain an initial guess of the steady state capital stock. Suppose we think of $c_t = \overline{c}$ so that in the intertemporal consumption trade-off we have

$$1 = \beta(1 + \overline{r})$$

Then $\overline{r} = \frac{1}{\beta} - 1$.[1] If $r = zF_K(\overline{K}, L) - \delta$ then with $zF(K, L) = zK^{\alpha}L^{1-\alpha}$ we have

$$\overline{r} + \delta = \alpha z \left(\frac{L}{K}\right)^{1-\alpha}$$

Rearranging we have

$$\overline{K} = \left(\frac{\alpha z}{\overline{r} + \delta}\right)^{\frac{1}{1-\alpha}} L$$

so using our approximation for $\overline{r}$,

$$\overline{K} = \left(\frac{\alpha z}{\frac{1}{\beta} - 1 + \delta}\right)^{\frac{1}{1-\alpha}} L$$

As the wage is $w = (1-\alpha)z\left(\frac{K}{L}\right)^{\alpha}$ we have

$$\overline{w} = (1-\alpha)z\left(\frac{\alpha z}{\overline{r} + \delta}\right)^{\frac{\alpha}{1-\alpha}}$$

Also note that if we know r then we can construct K because L is fixed in this model,

$$\begin{aligned} r &= \alpha z \left(\frac{L}{K}\right)^{1-\alpha} \\ K &= \left(\frac{\alpha z}{r}\right)^{\frac{1}{1-\alpha}} L. \end{aligned}$$

[1] In this context, $\overline{r}$ is equivalent to the subjective discount rate ρ.

What is the borrowing lower bound. If labour hours comes from a grid $l \in \{l_1, ..., l_N\}$ and the equilibrium wage is w,

$$a_{min} = -\frac{wl_{min}}{1 - \frac{1}{1+r}} = \frac{wl_{min}}{\frac{r}{1+r}} = -\frac{(1+r)wl_{min}}{r}$$