

Answer Pages

Question 21 (pushAll) answer:

~~void~~ ~~pushAll~~

void Tree::pushAll(TreeNode * n){

if (n == NULL){ // don't need modify tree, just push data
return; }// push and call recursion
st.push(temp);
pushAll(n->leftChild); }

Question 22 (KStep) answer:

Tree::KStep() {

pushAll(root);

while (st.empty() != true) {

pushAll(st.pop() -> right); }

Question 23 (hasMore) answer:

```
bool Tree::hasMore() {
    treeNode * temp = root;
    if (root != NULL) {
        while (temp->right != NULL) {
            temp = temp->right;
        }
        if (temp->data == st->data) {
            return true;
        }
    }
    if (root == NULL) {
        return true;
    }
    return false;
}
```



Question 24 (step1) answer:

```
int Tree::step1() {
    treeNode * temp = st.pop();
    int out = temp->data;
    return out;
}
```

implementation of $\text{step}(k)$;
 Question 25 (~~step~~ running time) answer:

```
int Tree = Step(int k) {
    treeNode * temp = st.top();
    for (int i = 0, i < k, i++) {
        if (st.isEmpty() != true) {
            temp = st.pop();
            return i;
        }
        if (st.isEmpty() != true) {
            int out = st.top -> data;
            return out;
        }
    }
    return
```

Question 26 answer:

Lower Bound	$O(1)$
Average	$O(\log n)$
Upper Bound Case	$O(n)$



Question 27 (buildPerfectTree) answer:



```

QuadTreeNode * QuadTree :: buildPerfectTree(int k, RGBAPixel p) {
    QuadTreeNode * temp = new QuadTreeNode();
    temp = buildHelper(0, 0, pow(2, k), p, root);
    return temp;
}

int QuadTree::(int num, int it, power) {
    int out = 1;
    for(int i = 0; i <= k; i++) {
        out = out * num;
    }
    return out;
}

QuadTreeNode * QuadTree::buildHelper(int x, int y, int size, RGBAPixel p, QuadTreeNode curr) {
    clear(root);
    if(resolution == 1) {
        curr = new QuadTreeNode();
        curr->element = p;
        buildHelper(x, y, size/2, p, curr->nwChild);
        buildHelper(x+size/2, y, size/2, p, curr->neChild);
        buildHelper(x, y+size/2, p, curr->swChild);
        buildHelper(x+size/2, y+size/2, p, curr->seChild);
    }
}

```

Question 28 (perfectify) answer:

```

void QuadTree::perfectify(int levels) {
    int height = getHeight(root);
    perfectify(level - height, root);
}

int QuadTree::getHeight(QuadTreeNode * curr) {
    int i = 0;
    if(curr == NULL) {
        return i;
    }
    i = 1 + getHeight(curr->nwChild);
}

void QuadTree::perfectify(int num, QuadTreeNode * & curr) {
    if(curr->nwChild == NULL) {
        curr->nwChild = buildHelper(num, p);
        curr->neChild = buildHelper(num, p);
        curr->swChild = buildHelper(num, p);
        curr->seChild = buildHelper(num, p);
        perfectify(num, curr->nwChild);
        perfectify(num, curr->neChild);
    }
}

```

Question 29 (perfectify running time) answer:

```

perfectify(num, curr->swChild);
perfectify(num, curr->seChild);

```

Question 30 answer:

You may answer this question by filling in these blanks, or use the blank space for your own proof/disproof.



Preliminaries Let $H(n)$ denote the maximum height of an n -node AVL tree, and let $N(h)$ denote the minimum number of nodes in an AVL tree of height h . To prove (or disprove!) that $H(n) = \mathcal{O}(\log n)$, we attempt to argue that

$$H(n) \leq 3 \log_2 n, \text{ for all } n$$

Rather than prove this directly, we'll show equivalently that

a) $N(h) \geq \underline{2^{(H(h)/3)}}, (1\text{pt})$

Proof For an arbitrary value of h , the following recurrence holds for all AVL Trees:

b) $N(h) = \underline{1} + \underline{N(h-1)} + \underline{N(h-2)}, (2\text{pt})$

c) and $N(0) = \underline{1}, N(1) = \underline{2}, N(2) = \underline{3}, (2\text{pt})$

We can simplify this expression to the following inequality, which is a function of $N(h-3)$:

d) $N(h) \geq \underline{2} \times \underline{N(h-3)}, (1\text{pt})$

By an inductive hypothesis, which states:

e) $\underline{N(h) \geq 2^{h/3}, N(0) \geq 2^0, N(1) \geq 2^{1/3}, N(2) \geq 2^{2/3}}, (1\text{pt})$

we now have

f) $N(h) \geq \underline{2^{h/3}} = \text{part (a) answer}, (1\text{pt})$

which is what we wanted to show.

Given that $2^0 = 1$, $2^{1/3} \approx 1.25$, and $2^{2/3} \approx 1.58$, what is your conclusion?

Is an AVL tree $\mathcal{O}(\log n)$ or not? (Circle one): (2pt)

YES

NO

Overflow Page

Use this space if you need more room for your answers.

