大作业：编程实现NE求解

- 编程实现纳什均衡求解器
- 功能要求
  - 按照指定的输入输出格式读取文件、计算NE、输出结果
  - 给定二人博弈，求解所有MNE
  - 给定多人博弈，求解所有PNE
- 实验环境
  - Python, numpy, scipy
  - 禁止使用除标准库外的其他包
- 提交内容
  - 源代码、输出文件、实验报告
- 截止时间
  - **2023.05.31 23:55 不接受补交**

样例说明

- 输入文件：3×2博弈

```
NFG 1 R "some
lines
of
comments" { "Player 1" "Player 2" } { 3 2 }

1 1 0 2 0 2 1 1 0 3 2 0
```

策略数

收益

		Player 2	
Player 1		1 1	1 1
		0 2	0 3
		0 2	2 0

- 输出文件：2个MNE

```
1,0,0,1,0
1,0,0,0.5,0.5
```

- **注意**：实际上该博弈有无穷多个MNE：

$$\{\lambda x_1 + (1-\lambda)x_2 \mid 0 \leq \lambda \leq 1\}$$

其中$x_1 = \big((1,0,0),(1,0)\big)$，$x_2 = \big((1,0,0),(0.5,0.5)\big)$是该凸集的极点

当有无穷多个MNE时要求输出所有极点

提 示

- 参考资料
  - Algorithmic Game Theory (Chapter 3)
- 常用算法
  - ➢ Support enumeration 实现简单，效率低，能通过大部分测试
  - ➢ Labeled polytopes 实现较复杂，效率较高，能通过所有测试
  - ➢ Lemke-Howson 效率高但只保证一个NE，不宜用于此次作业
  - ➢ 三种算法均在参考资料中有介绍，也可以使用其他算法
- 注意事项
  - **凡发现两份代码重复部分超过30%，两份均记0分**
  - **其他作弊行为，如链接外部工具求解、不求解直接打印答案等，记0分**

Game Theory and Applications (博弈论及其应用)

Chapter 6: Extensive Game

南京大学

高 尉

Recap on the previous chapter

- Strategy game with incomplete information
- Bayes game $G = \{N, \{A_i\}, \{\Theta_i\}, \{u_i\}, p\}$
- Bayes Nash Equilibrium
- How to find Bayes Nash equilibrium

Example

In some situation, players can observe others' strategy before they make decision

- Simple Nim game
  - There are n coins
  - Two players select 1 or
  - The winner is the one tal

Extensive Game

Strategy Game

- Set of players
- Set of strategies
- Payoff functions

Extensive game provides more information

- Sequences of players
- Strategies available at different points in the game

Two variants

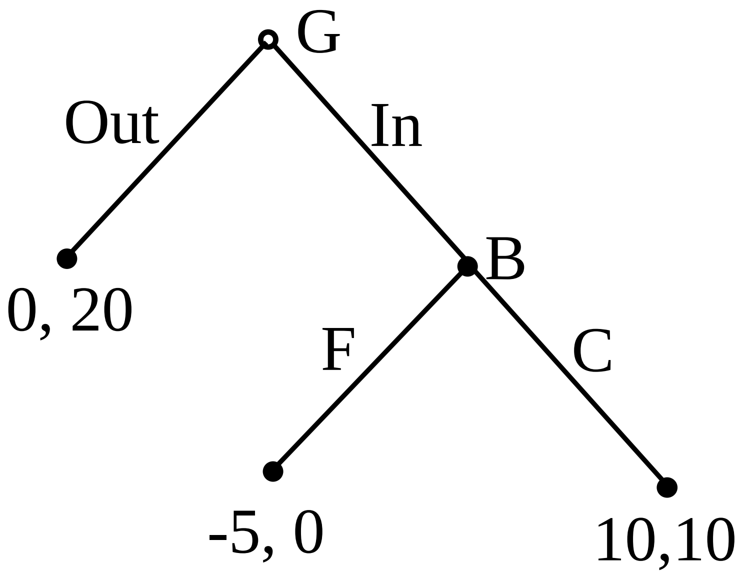
- ✓ perfect information extensive-form games
- ✓ imperfect-information extensive-form games

Entry Game

Google is trying to enter the Chinese market, and Baidu can either fight the entry or cooperate

Game Tree

- node
  - non-terminal node
  - terminal node
- branches
- players
- strategy
- payoff



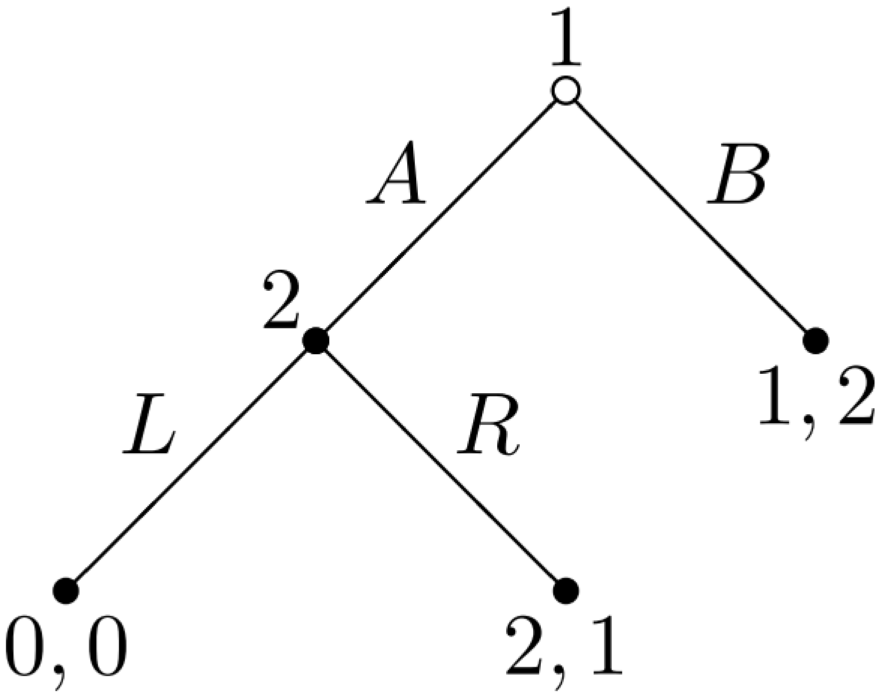
Formal Definition of Extensive Game

An extensive game with perfect information includes

- Players N is the set of N players
- Strategies A is a set of all strategies
- Histories H is a set of strategy sequence (finite or infinite) s.t.
  - The empty sequence $\emptyset \in H$
  - If $a^1 a^2 \dots a^k \in H$ then $a^1 a^2 \dots a^s \in H$ when $s \leq k$
  - If an infinite sequence $\left(a^k\right)_{k=1}^{\infty}$ satisfies $a^1 a^2 \dots a^k \in H$ for each positive k, then $\left(a^k\right)_{k=1}^{\infty} \in H$

History

- H={∅,A,B,AL,AR}
  - ∅
  - A
  - B
  - AL
  - AR



Definition of Extensive Game

An extensive game with **perfect information** is defined by

- Players N is the set of N players
- Strategies A is a set of all strategies
- Histories H is a set of sequence (finite or infinite)
  - Each sequence in H is called a history; each component $a^i \in A$ is a strategy
  - Terminal history $a^1 \ldots a^k \in H$ if $k = +\infty$ or $a^1 \ldots a^{k+1} \notin H$ for any $a^{k+1} \in A$.
  - Terminal history set Z={all terminal histories $a^1 \ldots a^k \in H$}

Definition of Extensive Game

An extensive game with perfect information is defined by

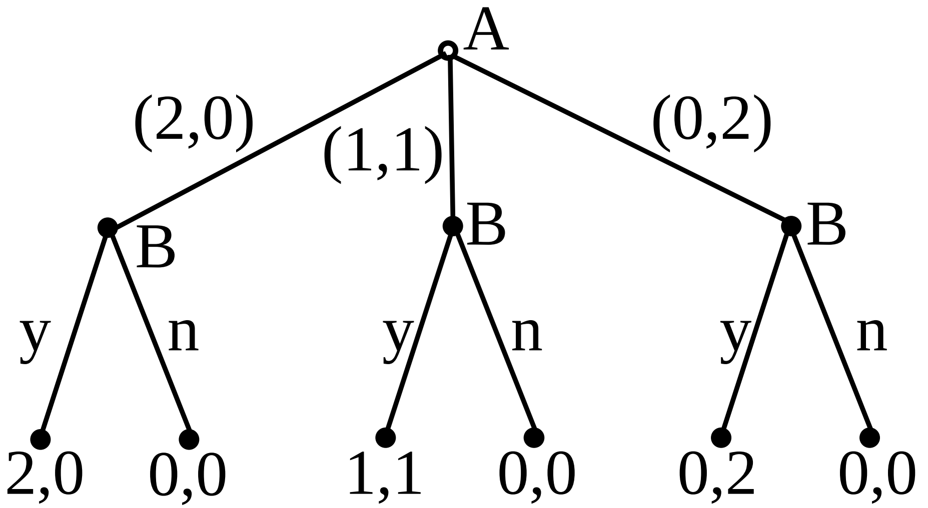
- Players N is the set of N players
- Strategies A is a set of all strategies
- Histories H is a set of sequence (finite or infinite)
- Player function
  - $P: H \setminus Z \rightarrow N$ assigns to **each non-terminal history** a player of N
  - $P(h)$ denotes the player who takes action after the history h
- Payoff function $u_i: Z \rightarrow R$

$$G = \{N, H, P, \{u_i\}\}$$

Ultimatum Game

$G = \{N, H, P, \{u_i\}\}$

N={A,B}



H={∅,(2,0),(1,1),(0,2), ((2,0),y)}

∪{((2,0),n), ((1,1),y), ((1,1),n)}

∪{((0,2),y), ((0,2),n)}

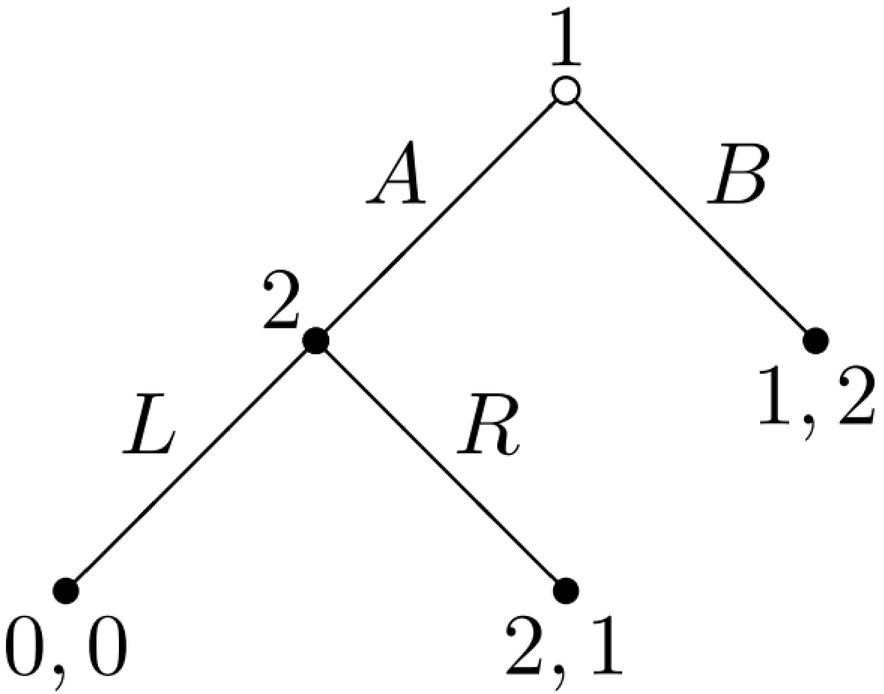
P: P(∅)=A; P((2,0))=B; P((1,1))=B; P((0,2))=B

$$u_1\big((2{,}0), \mathrm{y}\big) = 2, u_1\big((2{,}0), \mathrm{n}\big) = 0, u_1\big((1{,}1), \mathrm{y}\big) = 1, u_1\big((1{,}1), \mathrm{n}\big) = 0$$

$$u_2\big((2{,}0), \mathrm{y}\big) = 0, u_2\big((2{,}0), \mathrm{n}\big) = 0, u_2\big((1{,}1), \mathrm{y}\big) = 1, u_2\big((1{,}1), \mathrm{n}\big) = 0$$

Example

- $G = \{N, H, P, \{u_i\}\}$
- N={1,2}
- H={∅,A,B,AL,AR}
- P: P(∅)=1; P(A)=2
- $u_1(B) = 1, u_1(AL) = 0, u_1(AR) = 2$
- $u_2(B) = 2, u_2(AL) = 0, u_2(AR) = 1$



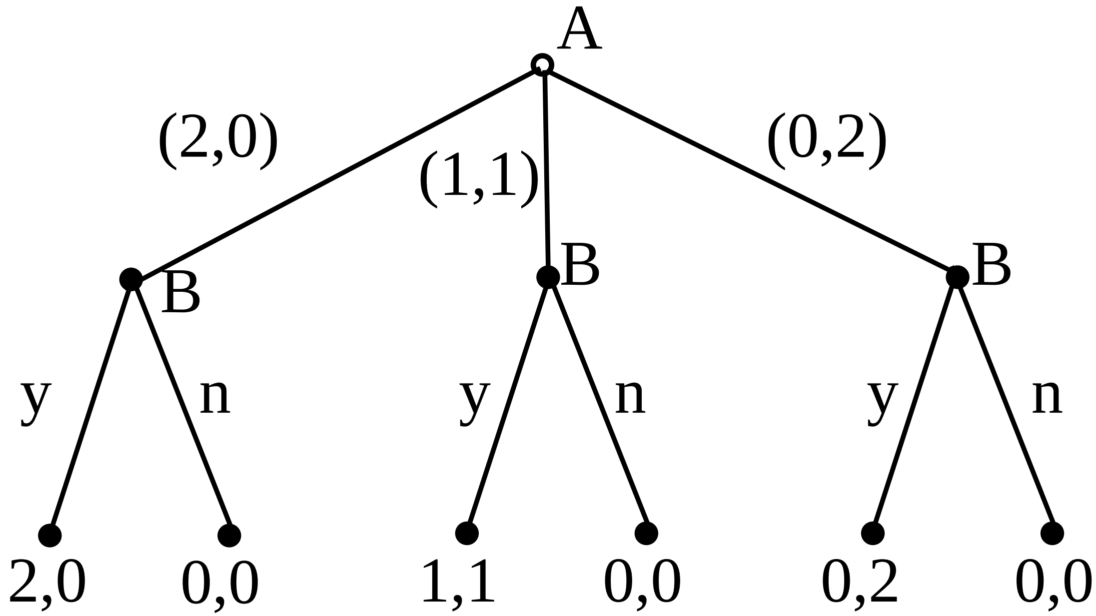
Pure strategies

Definition: Given game $G = \{N, H, P, \{u_i\}\}$, the pure strategy for player i is given by the cross product

$$\times_{h \in H} \{a^s : (h, a^s) \in H, p(h) = i\}.$$

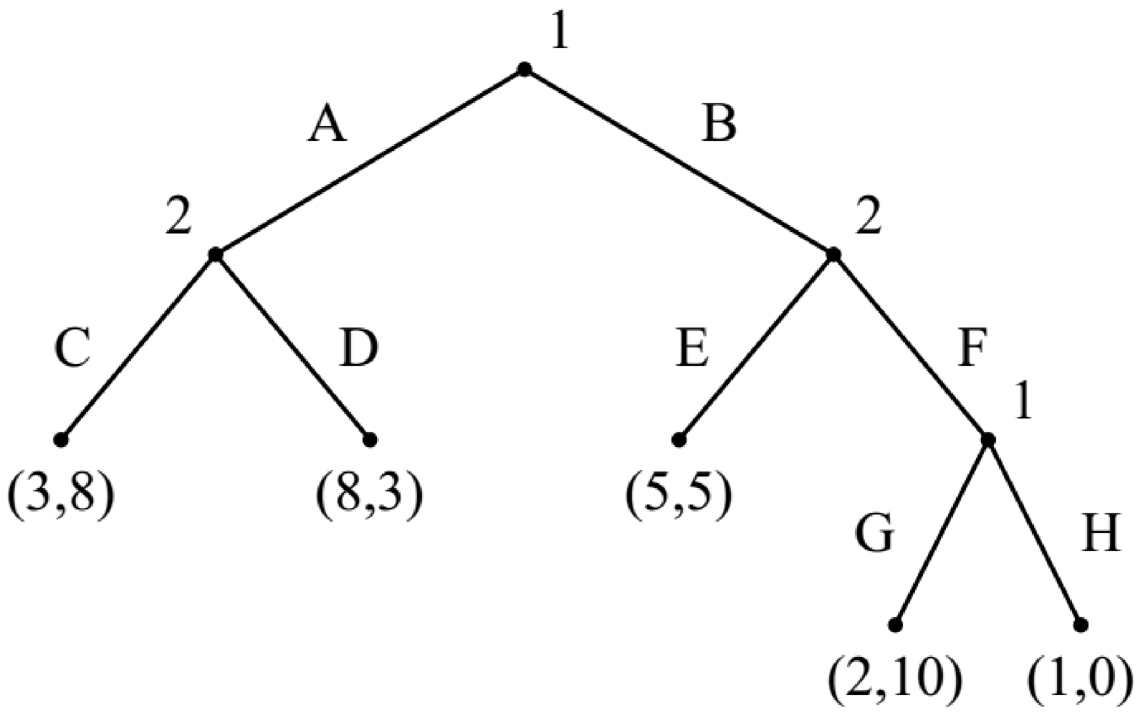
A pure strategy for a player is a complete specification of which deterministic action to take at every node belonging to that player.

Pure Strategies



- How many pure strategies for each player?
- Player A: {(2,0),(1,1),(0,2)}
- Player B: {yyy,yyn,yny,ynn,nyy,nyn,nny,nnn}

Pure Strategy Example



What are the pure strategies for players 1 and 2?

Nash Equilibrium

Based on the definition of pure strategy, we can define

- Mixed strategies
- Best response
- Nash equilibrium

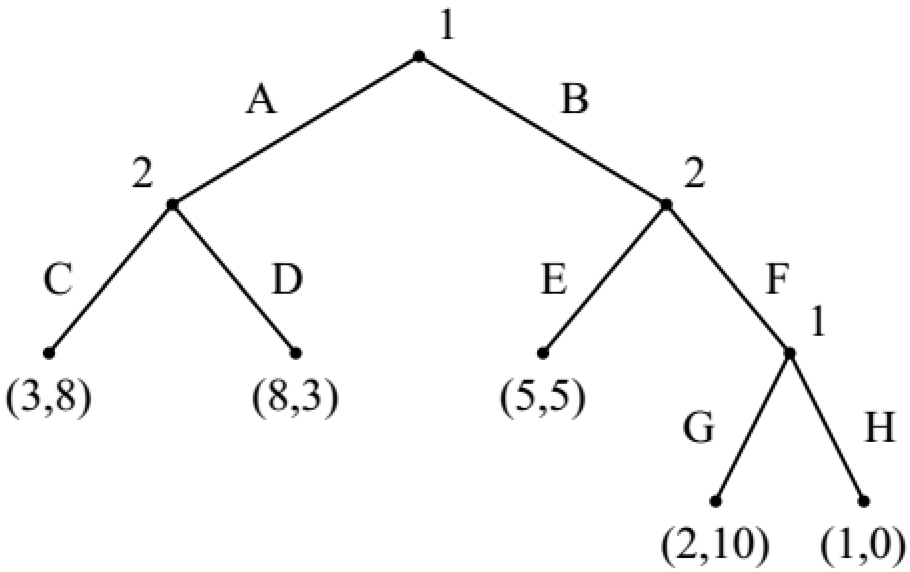
Given extensive $G = \{N, H, P, \{u_i\}\}$, an strategy outcome $a^* = (a_1^*, a_2^*, \ldots, a_N^*)$ is a **Nash equilibrium** if and only if

$$u_i(a_i^*, a_{-i}^*) \geq u_i(a_i, a_{-i}^*) \text{ for every } a_i \text{ of player } i$$

How to find Nash Equilibrium: Induced strategy game

Induced Strategy Game

Every extensive game can be converted to a strategy game



	CE	CF	DE	DF
AG	$3,8$	$3,8$	$8,3$	$8,3$
AH	$3,8$	$3,8$	$8,3$	$8,3$
BG	$5,5$	$2,10$	$5,5$	$2,10$
BH	$5,5$	$1,0$	$5,5$	$1,0$

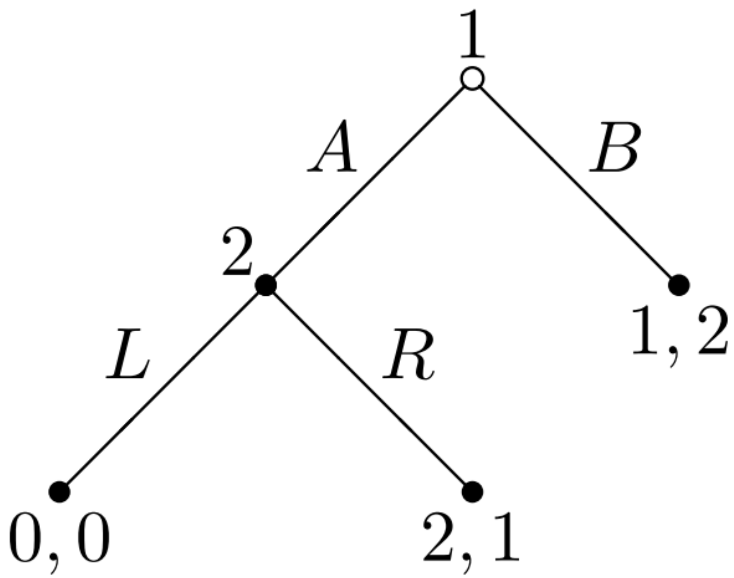
Remark: This conversion is not reverse

Kuhn Theorem (1953)

Theorem Every extensive game with perfect information has at least one Pure Strategy Nash Equilibrium (PSNE).

Proof Constructive proof will be introduced later.

Example



	L	R
A	0 0	2 1
B	1 2	1 2

Nash Equilibria are (B,L) and (A,R)

- (B,L) is a Nash equilibrium: if player 2 select L, then player 1 select B, and vice verse.
- Is (B,L) reasonable?

(B,L) is an non-credible threat.

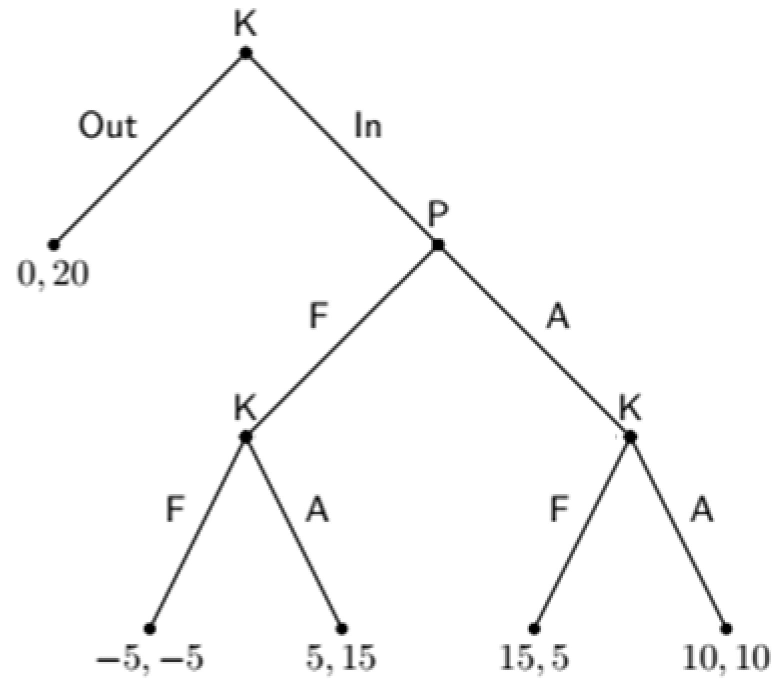
Subgame (子博弈)

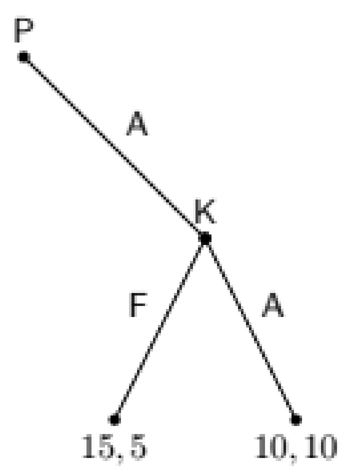
Definition A subgame is a set of nodes, strategies and payoffs, following from a single node to the end of game.

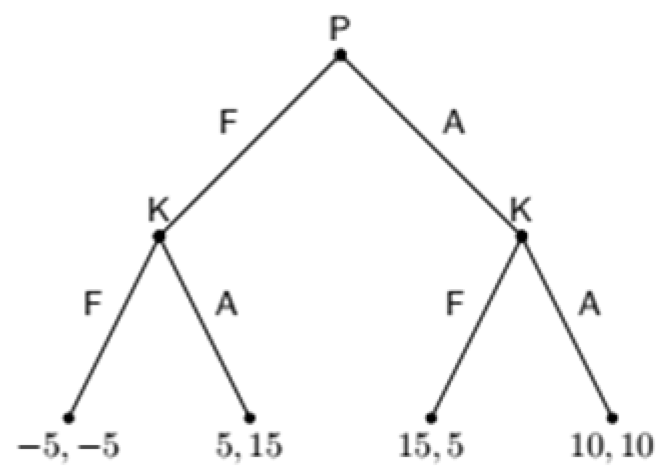
A subgame is a part of the game tree such that

- It starts at a single strategy node
- It contains every successor to this node
- It contains all information in every successor

Example


K
Out
In
0, 20
P
F
A
K
K
F
A
F
A
−5, −5
5, 15
15, 5
10, 10


P
A
K
F
A
15, 5
10, 10


P
F
A
K
K
F
A
F
A
−5, −5
5, 15
15, 5
10, 10

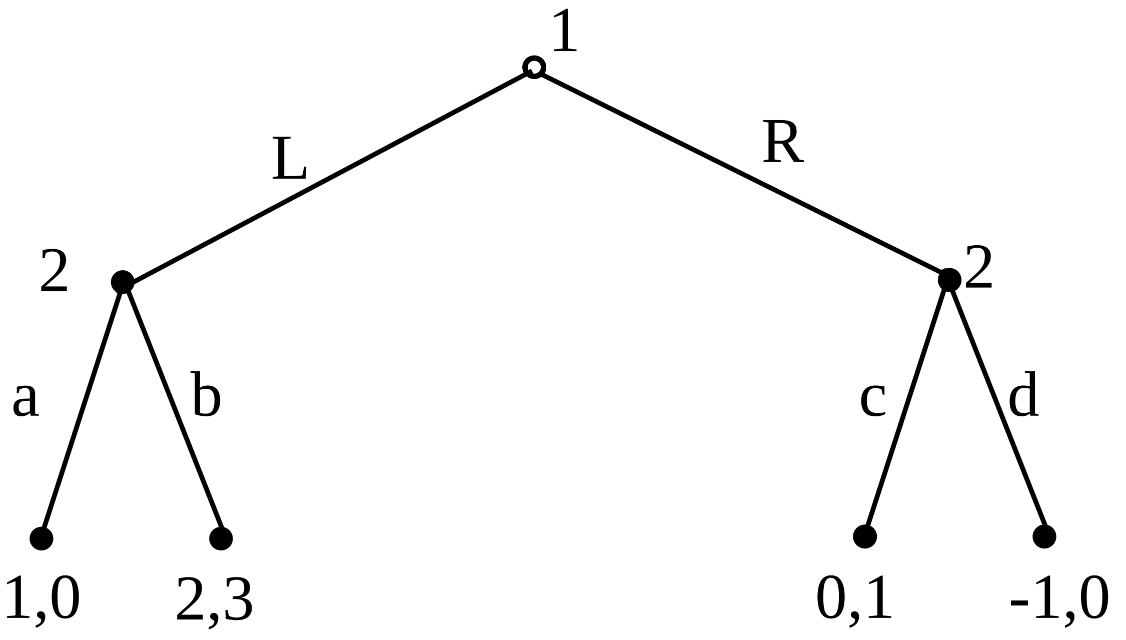
Subgame Perfect Equilibrium

Definition An outcome is $a = (a_1^*, a_2^*, \dots, a_N^*)$ is a subgame perfect (子博弈完美) if it is Nash Equilibrium in every subgame

- Subgame perfect is a Nash Equilibrium
- This definition rules out “non-credible threat”

Theorem Every extensive game with perfect information has a subgame perfect.

Example



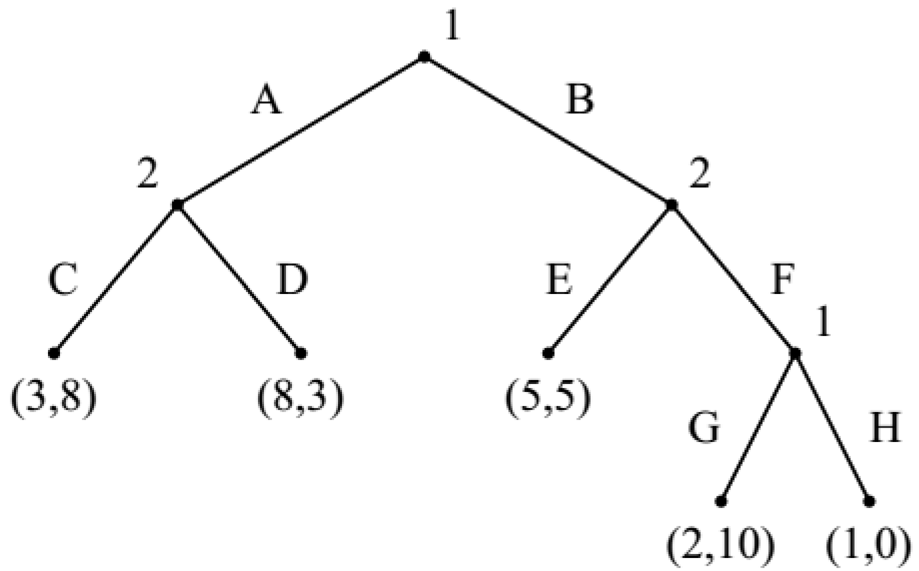
How to find Nash Equilibrium

How to find the subgame perfect?

Summaries

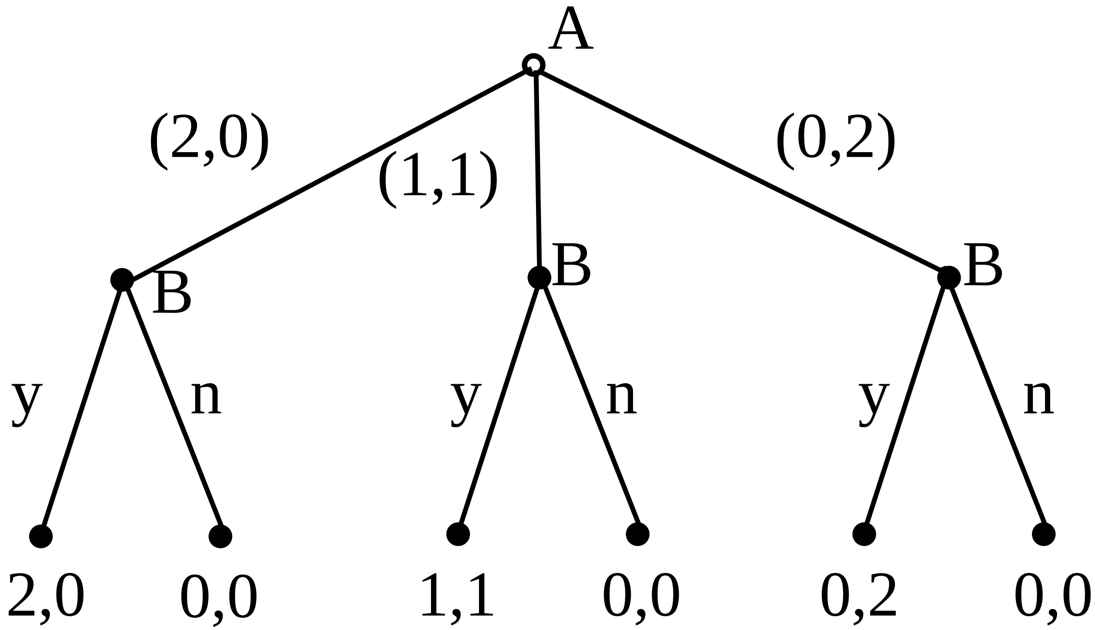
- Formal definition of extensive game
- Pure strategy for each player and Nash Equilibrium
- How to find Nash Equilibrium
- Subgame
- Subgame Perfect

Homework



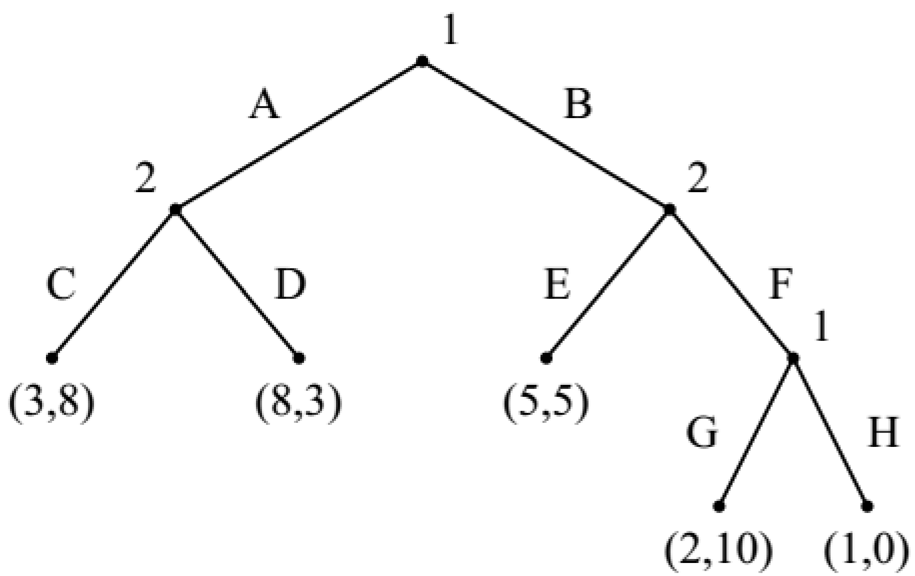
How to formalize $G = \{N, H, P, \{u_i\}\}$

Homework



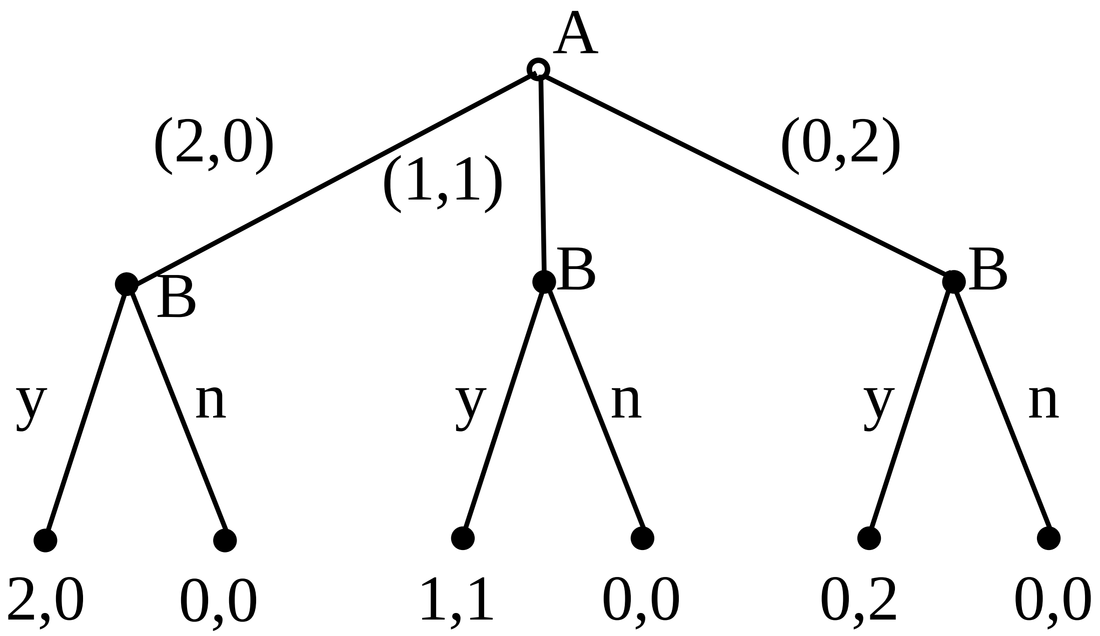
Find all Nash Equilibria for ultimatum game?

Homework



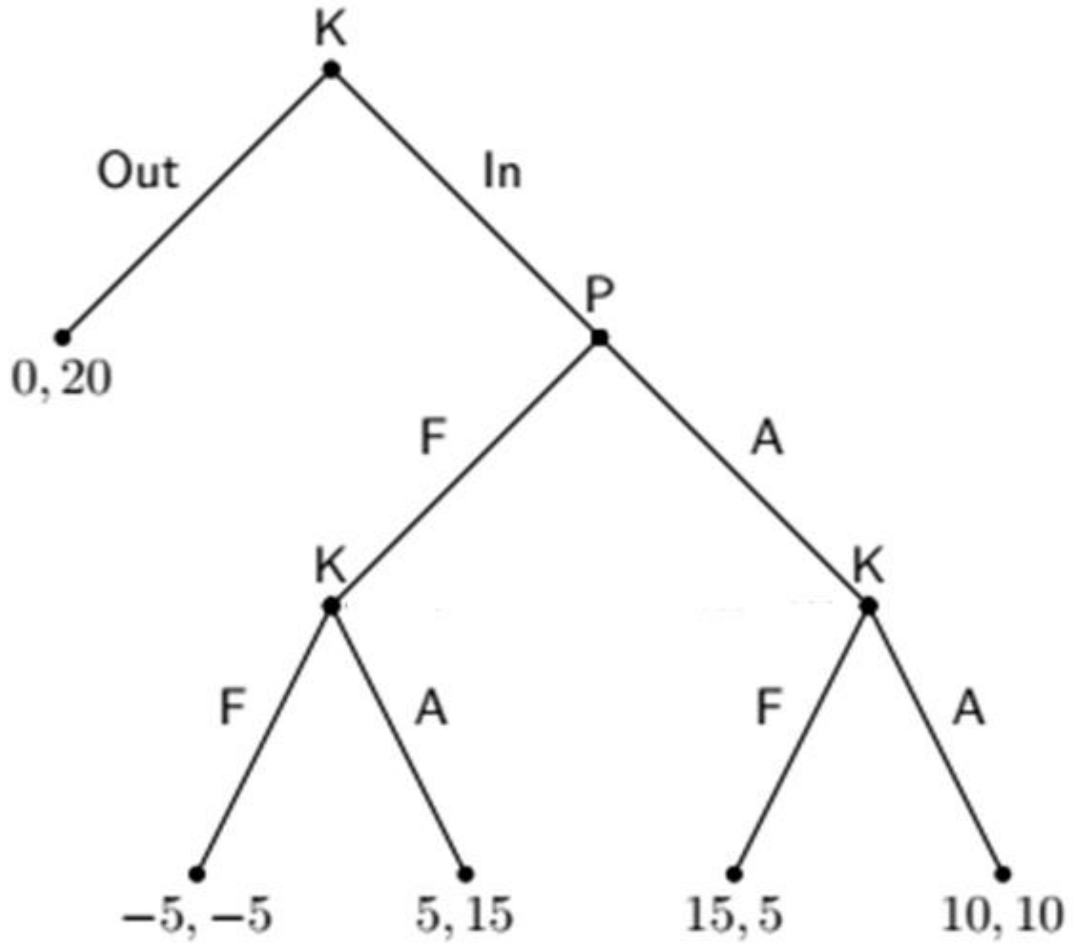
How to find NE and subgame perfect?

Homework



How to find the subgame perfect?

Homework



How to find the subgame perfect?