Machine Learning for Dynamical Systems

Course Projects

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DMF
Data Analysis and Modeling of Turbulent Flows

Technische Universität Berlin

Course Assessment

Report: approximately 10-12 pages

To be submitted by 30.09.2024 until 23:59 h

Focus on your project work in a structured way, e.g. problem statement, NN architecture, methodology, results, etc. (You can also show bad results and give possible explanations)

Presentation: maximum 10 minutes

To be presented on 16.09.2024

Preliminary results about the project

Plan for the report

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Possible projects

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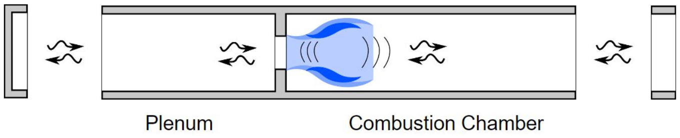
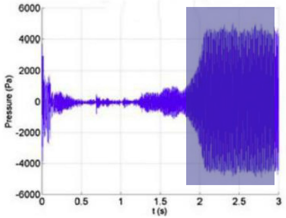
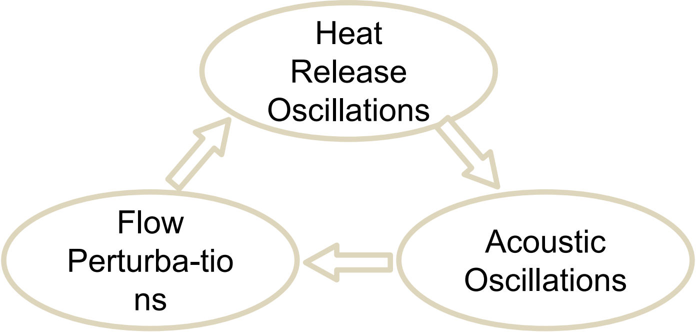
1. **Modelling flame dynamics with neural network**
2. Autoencoder for Kolmogorov Flow
3. Autoencoder for steady state flame images (convolutional, variational)

Thermoacoustic instabilities result from heat release rate oscillations

Thermoacoustic instabilities: High amplitude pressure oscillations in 'lean premixed' combustors



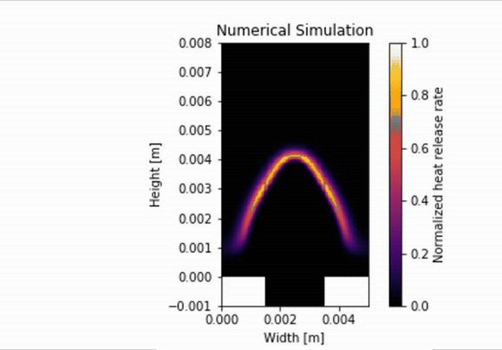


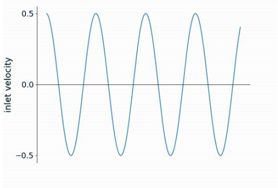


- Prediction requires a coupling between:
- *Acoustic model*
- $\boxed{\text{\textit{Flame model:} } \dot{Q}' = f(u')}$

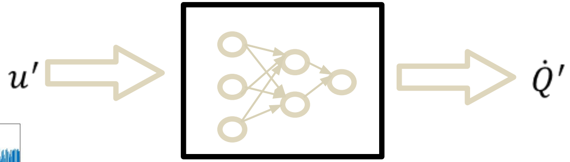
Premixed methane-air laminar slit burner (equivalence ratio of 0.8)

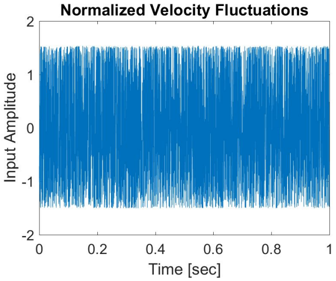
Fully resolved with DNS

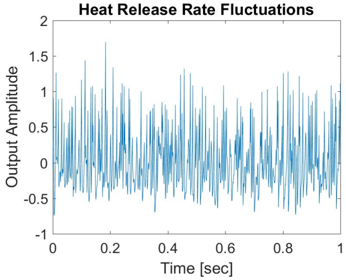




Neural networks trained with result from broadband simulation







Kornilov et al. PROCI (2007)
Tathawadekar et al., PROCI (2020)

Assessment of the trained neural network : Comparison with harmonic forcing

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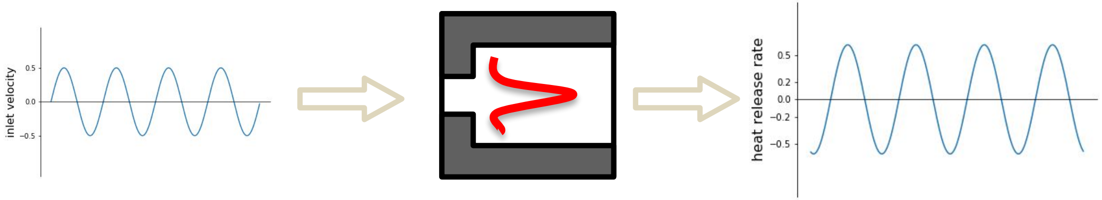
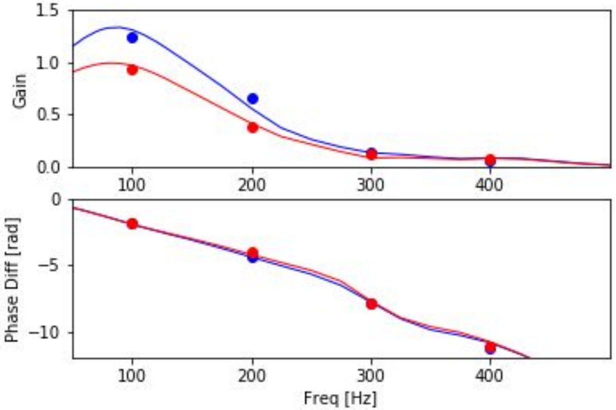
$$F\left(\omega, |u^{'}|\right) = \frac{\dot{Q}^{'}(\omega,|u^{'}|)/\overline{\dot{Q}}}{u^{'}(\omega,|u^{'}|)/\bar{u}}$$





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- Objective: Develop a neural network based model of the flame response
- 3 Training datasets provided for different amplitudes of excitations
- Two approaches possible
  - Recurrent neural network
  - Feedforward NN
- Study of the accuracy of the trained NN depending on the training dataset used
- Study on the impact of the length of the dataset used for training

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Kolmogorov Flow

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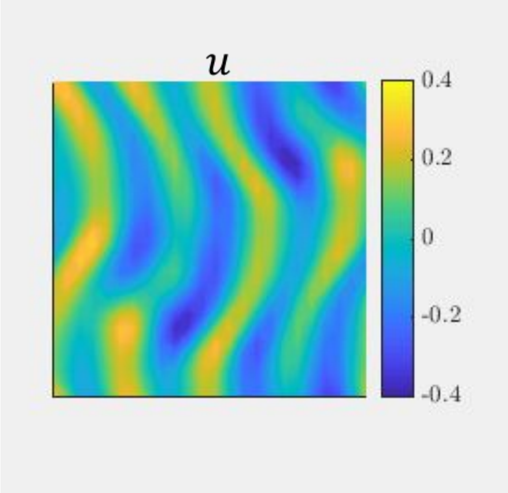
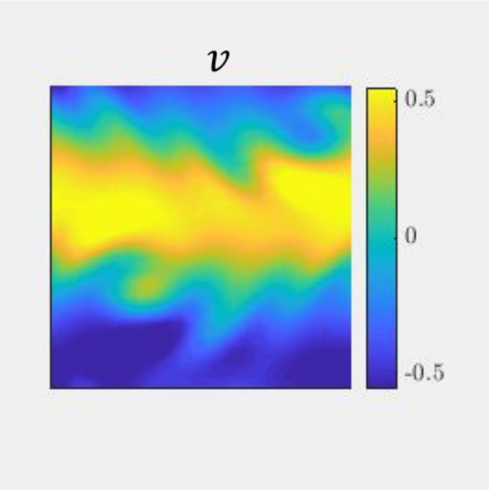
Kolmogorov flow: 2D Navier Stokes subjected to volume for

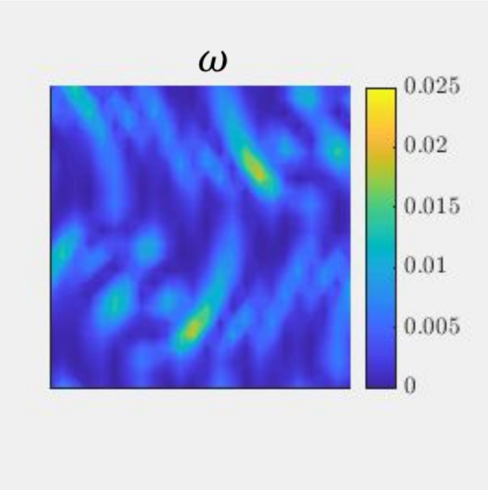
$$\nabla \cdot \boldsymbol{u} = 0$$

$$\partial_t \boldsymbol{u} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} = -\nabla p + \frac{1}{\mathrm{Re}} \Delta \boldsymbol{u} + \boldsymbol{f}$$

$$\boldsymbol{f} = \left(\sin k_f y\,, 0\right), k_f = 4$$

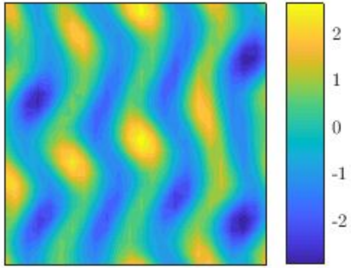


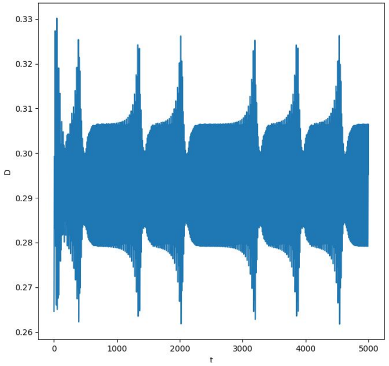


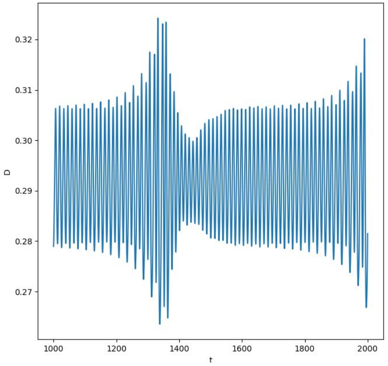


Behaviour of the Kolmogorov flow depends on Reynolds number

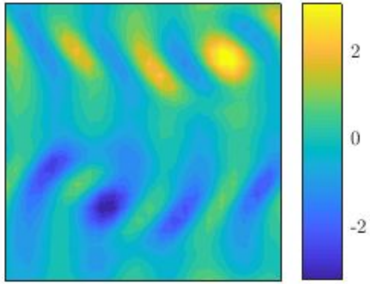
$Re = 20$

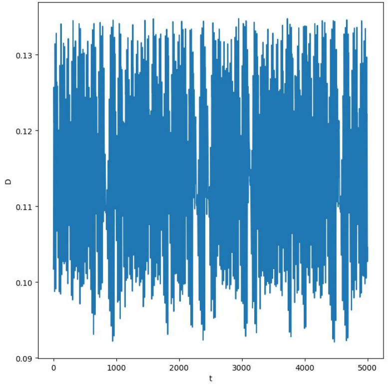


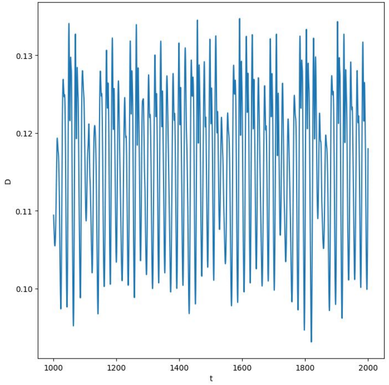




$Re = 30$

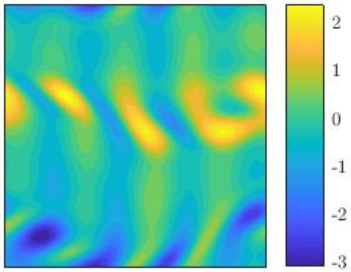


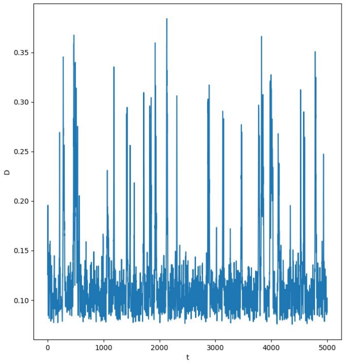


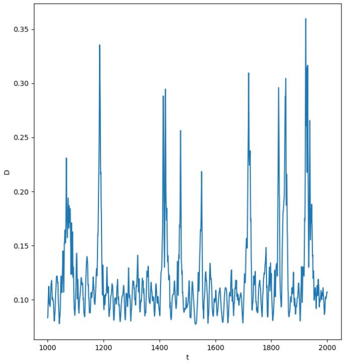


Behaviour of the Kolmogorov flow depends on Reynolds number

$Re = 40$







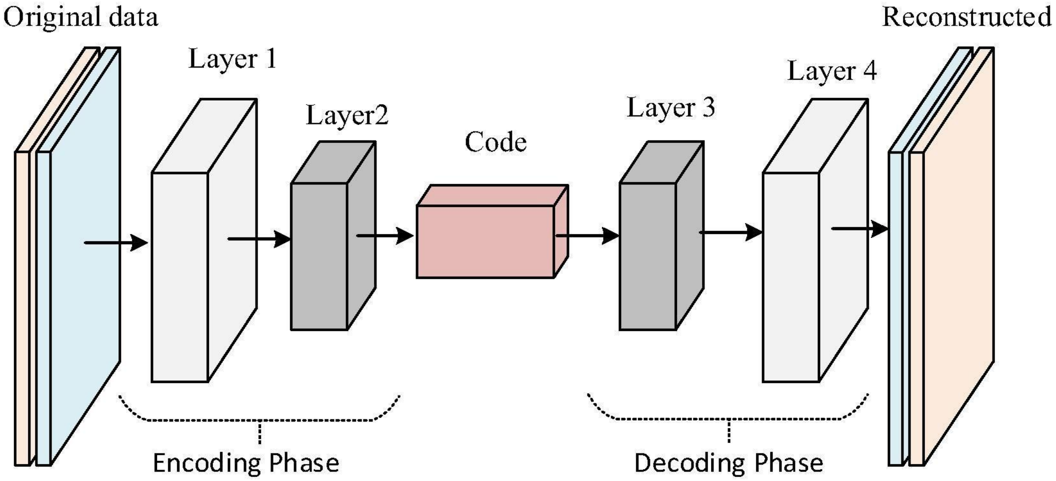
Kolmogorov flow evolves from quasi-periodic to turbulent to turbulent with extreme events.

An application of CNN: Autoencoder for dimensionality reduction

Autoencoder combined dimensionality reduction with reconstruction

Encoder finds the reduced order representation

Once trained, the decoder can be removed



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- Objective: develop an autoencoder to reduce the dimension of the flow information
- Different datasets at different Reynolds numbers (20, 30 and 40)
- Analyse the difference in complexity depending on the Reynolds number
- Assess the accuracy achievable with different sizes of reduced dimension
- Two different approaches possible:
  - Based on feedforward neural network
  - Based on convolutional neural network
- Comparison with the POD approach

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Steady state images for different input conditions

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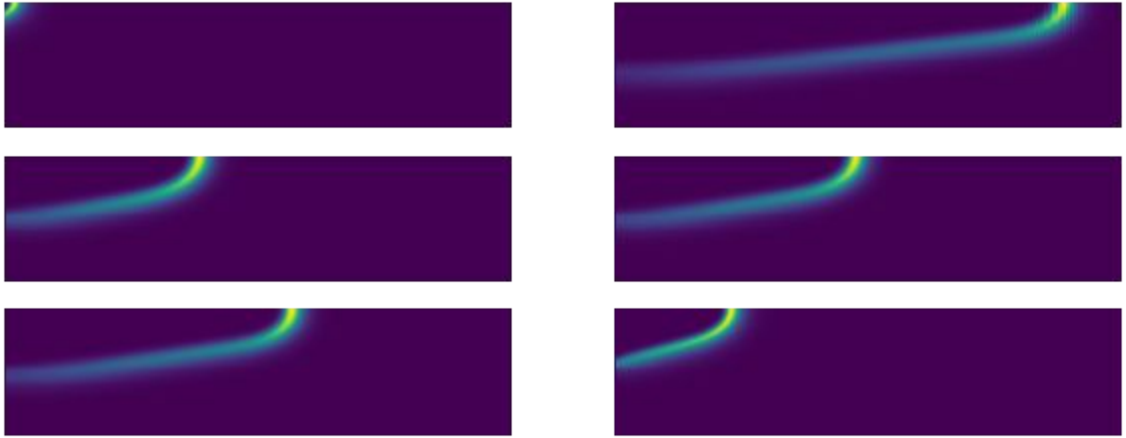
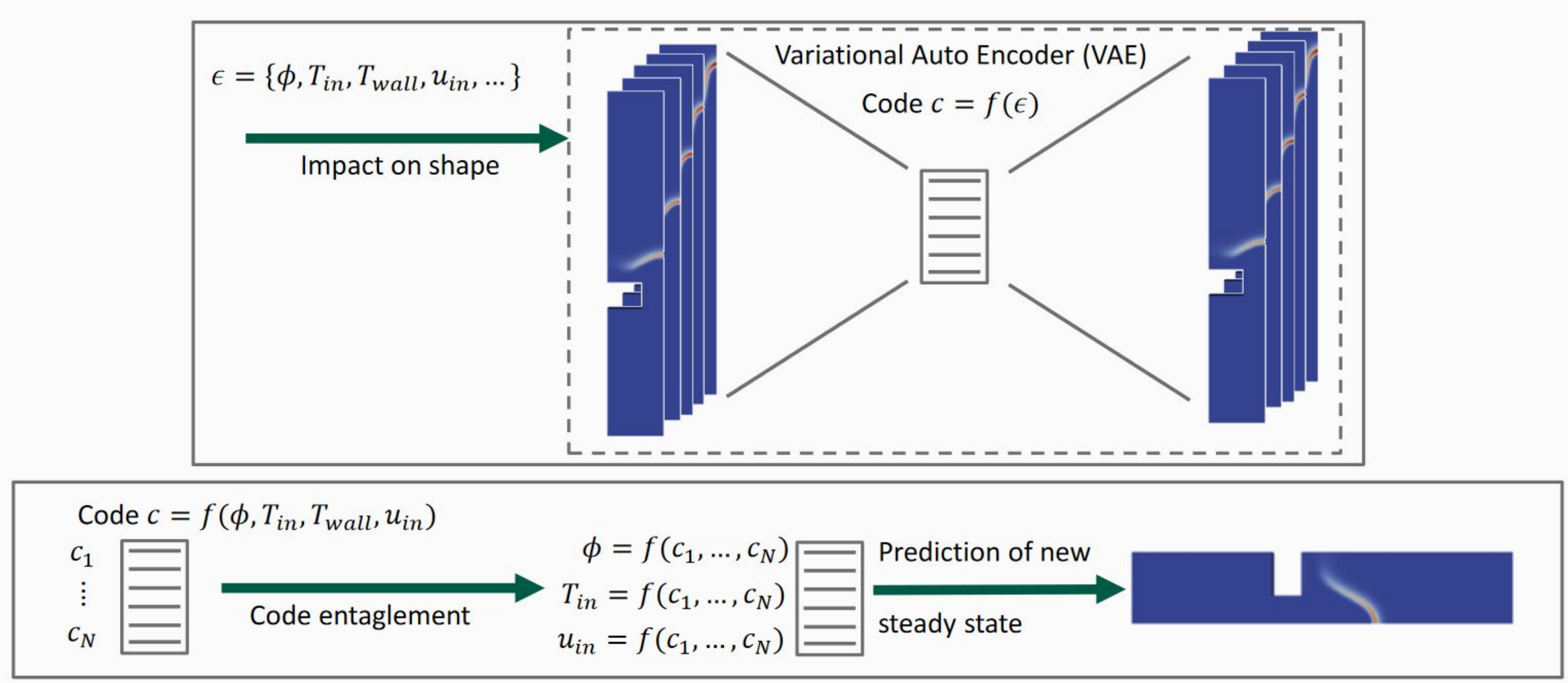
Premixed methane-air laminar slit burner (varying equivalence ratio, input velocity, input temperature, wall temperature)

Fully resolved with DNS



Train autoencoders with the steady state images


$\epsilon = \{\phi, T_{in}, T_{wall}, u_{in}, \dots\}$
Impact on shape
Variational Auto Encoder (VAE)
Code $c = f(\epsilon)$
Code $c = f(\phi, T_{in}, T_{wall}, u_{in})$
c_1
$\vdots$
c_N
Code entaglement
$\phi = f(c_1, \dots, c_N)$
$T_{in} = f(c_1, \dots, c_N)$
$u_{in} = f(c_1, \dots, c_N)$
Prediction of new
steady state

- Objective: Develop a Autoencoder to reduce the dimension of the flow information
- Different datasets at different input parameters
- Assess the accuracy achievable with different sizes of reduced dimension
- Two different approaches possible:
    - Convolutional autoencoder
    - Variational autoencoder
- Develop second neural network to map input parameters to latent space
- Vary the input parameters to predict the latent space and use this latent space to generate steady state images

Important points for the Report

Make sure to have proper labels and legends for all figures

Proper formatting of figures

Provide critical reasoning to your findings/figures

Literature survey and include references in the report

Structure the report well

Pay attention to the language (spellings, grammar, etc.)

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1. **Physics-Informed Neural Network**

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Burgers' equation is a toy-model of turbulence

Simplification of the Navier-Stokes equation:

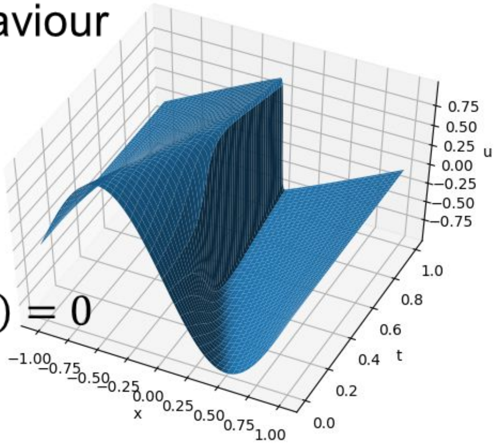
- Neglect the pressure term
- Constant density

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \nu\frac{\partial^2 u}{\partial x^2}\,;\nu = \frac{0.01}{\pi}$$

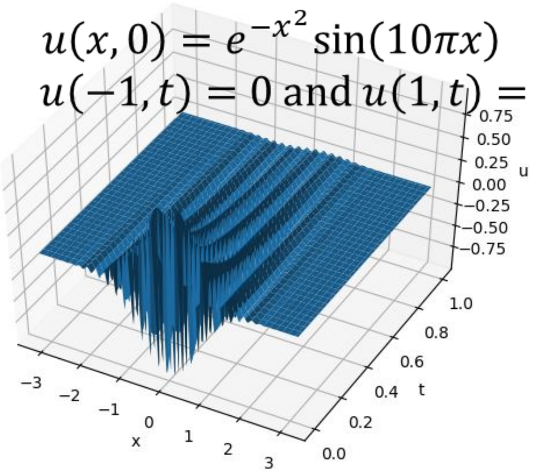
Show some nonlinearity (but is not chaotic).

Can exhibit shock-like behaviour

$u(x,0) = -\sin(\pi x)$
$u(-1,t) = 0$ and $u(1,t) = 0$



$u(x,0) = e^{-x^2}\sin(10\pi x)$
$u(-1,t) = 0$ and $u(1,t) = 0$



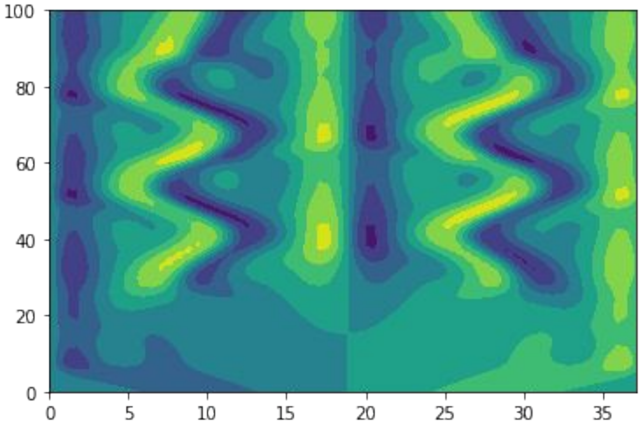
Kuramoto Sivashinsky models diffusion instabilities in flames

The Kuramoto Sivashinsky (KS) equation models diffusion instabilities in laminar flames

$$\frac{\partial u}{\partial t} + \frac{\partial^2 u}{\partial x^2} + \frac{\partial^4 u}{\partial x^4} + u\frac{\partial u}{\partial x} = 0$$

With periodic boundary condition $u(2\pi L, t) = u(0, t)$ and a domain $x \in [0, 2\pi L]$

With initial condition $u(x, 0) = -\sin\left(\frac{x}{2\pi L}\right), L = 6$



- Objective is to develop a Physics-Informed Neural Network that can solve the Burgers' and KS equations
  - 2 Cases for Burgers equation
    - $u(x,0) = -\sin(\pi x)$ with $u(-1,t) = 0$ and $u(1,t) = 0; \nu = 0.01/\pi$
    - $u(x,0) = e^{-x^2}\sin(10\pi x)$ with $u(-\pi,t) = 0$ and $u(\pi,t) = 0$
  - 1 Case for KS equation
    - You will need to modify the physical loss so that you can impose periodic boundary conditions.
  - Study the complexity of the network necessary to obtain a good approximation depending on the complexity of the equation/real solution