Learn++: An Incremental Learning Algorithm for Supervised Neural Networks

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***Abstract*—We introduce Learn++, an algorithm for incremental training of neural network (NN) pattern classifiers. The proposed algorithm enables supervised NN paradigms, such as the multilayer perceptron (MLP), to accommodate new data, including examples that correspond to previously unseen classes. Furthermore, the algorithm does not require access to previously used data during subsequent incremental learning sessions, yet at the same time, it does not forget previously acquired knowledge. Learn++ utilizes ensemble of classifiers by generating multiple hypotheses using training data sampled according to carefully tailored distributions. The outputs of the resulting classifiers are combined using a weighted majority voting procedure. We present simulation results on several benchmark datasets as well as a real-world classification task. Initial results indicate that the proposed algorithm works rather well in practice. A theoretical upper bound on the error of the classifiers constructed by Learn++ is also provided.**

***Index Terms*—Catastrophic forgetting, classification algorithms, ensemble of classifiers, incremental learning, knowledge acquisition and retention, pattern recognition, supervised neural networks.**

I. Introduction

MACHINE LEARNING offers one of the most cost effective and practical approaches to the design of pattern classifiers for a broad range of pattern recognition applications. The performance of the resulting classifier relies heavily on the availability of a representative set of training examples. In many practical applications, acquisition of a representative training data is expensive and time consuming. Consequently, it is not uncommon for such data to become available in small batches over a period of time. In such settings, it is necessary to update an existing classifier in an incremental fashion to accommodate new data without compromising classification performance on old data. Learning new information without forgetting previously acquired knowledge, however, raises the so-called *stability–plasticity dilemma* [1], one of the fundamental problems in knowledge management (KM): Some information may have to be lost to learn new information, as learning new patterns will tend to overwrite formerly acquired knowledge. The dilemma points out the fact that a completely stable classifier will preserve existing knowledge, but will not accommodate any new information, whereas a completely plastic classifier will learn new information but will not conserve prior knowledge.

A typical approach for learning new information involves discarding the existing classifier, and retraining the classifier using all of the data that have been accumulated thus far. Examples of this approach include common neural network (NN) paradigms, such as multilayer perceptron (MLP), radial basis function (RBF) networks, wavelet networks, and Kohonen networks. This approach, lying on the "stability" end of the spectrum, however, results in loss of all previously acquired information, which is known as *catastrophic forgetting*. Furthermore, this approach may not even be feasible in many applications, particularly if the original data is no longer available. An alternative approach, lying toward the "plasticity" end of the spectrum, involves the use of online training algorithms. However, many existing online algorithms assume rather restricted form of classifiers, such as classifiers that compute conjunctions of Boolean features. Consequently, such algorithms have limited applicability in real-world applications. A third approach to incremental learning is the use of instance-based learners such as nearest neighbor classifiers. However, this approach entails storing all of the data.

Various algorithms suggested in the literature for incremental learning typically use one or a combination of the above-mentioned approaches, and fall somewhere in between the stability–plasticity spectrum. Some of the more recent and prominent of such algorithms are discussed in the next section.

We should also note that the term "incremental learning" has been used rather loosely in the literature, where the term referred to as diverse concepts as incremental network growing and pruning, on-line learning, or relearning of formerly misclassified instances. Furthermore, various other terms, such as constructive learning, lifelong learning, and evolutionary learning have also been used to imply learning new information.

Against this background, precise formulations of the incremental learning problem, characterizations of information requirements of incremental learning, and the establishment of necessary and sufficient conditions for incremental learning need to be established. In this paper, we therefore define an incremental learning algorithm as one that meets the following criteria:

1) It should be able to learn additional information from new data.

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2) It should not require access to the original data, used to train the existing classifier.
3) It should preserve previously acquired knowledge (that is, it should not suffer from catastrophic forgetting).
4) It should be able to accommodate new classes that may be introduced with new data.

An algorithm that possesses these properties would be an indispensable tool for pattern recognition and machine learning researchers, since virtually unlimited number of applications can benefit from such a versatile incremental learning algorithm. The problem addressed in this paper is therefore designing a supervised incremental learning algorithm satisfying all of the above-mentioned criteria.

The rest of this paper is organized as follows. In Section II, we provide an overview of various approaches suggested for incremental learning algorithms, as well as an overview of ensemble-based learning algorithms, which were originally proposed for improving generalization performance of classifiers. In Section III, we show how ensemble-based approaches can be used in an incremental learning setting, and present the Learn++ algorithm in detail. In Section IV, we explain the benchmark and real-world databases used to evaluate the algorithm, along with simulation results obtained on these databases. We also compare the Learn++ performance to that of fuzzy ARTMAP on the real-world database. Finally, in Section V, we summarize our conclusions and point at future research directions.

II. Background

A. Incremental Learning

As mentioned earlier, various algorithms have been suggested for incremental learning, where incremental learning implied different problems. For example, in some cases, the phrase "incremental learning" has been used to refer to growing or pruning of classifier architectures [2]–[4] or to selection of most informative training samples [5]. In other cases, some form of controlled modification of classifier weights has been suggested, typically by retraining with misclassified signals [6]–[12]. These algorithms are capable of learning new information; however, they do not simultaneously satisfy all of the above-mentioned criteria for incremental learning: they either require access to old data, forget prior knowledge along the way, or unable to accommodate new classes. One notable exception is the (fuzzy) ARTMAP algorithm [13], [14], which is based on generating new decision clusters in response to new patterns that are sufficiently different from previously seen instances. This sufficiency is controlled by a user-defined vigilance parameter. Each cluster learns a different hyper-rectangle shaped portion of the feature space in an unsupervised mode, which are then mapped to target classes. Since previously generated clusters are always retained, ARTMAP does not suffer from catastrophic forgetting. Furthermore, ARTMAP does not require access to previously seen data, and it can accommodate new classes. Therefore, ARTMAP fits perfectly into our description of incremental learning.

ARTMAP is a very powerful and versatile algorithm; however, it has its own drawbacks. In many applications, researchers have noticed that ARTMAP is very sensitive to selection of the vigilance parameter, to the noise levels in the training data and to the order in which the training data is presented to the algorithm. Furthermore, the algorithm generates a large number of clusters causing overfitting, resulting in poor generalization performance, if the vigilance parameter is not chosen correctly. Therefore, this parameter is typically chosen in an *ad hoc* manner by trial and error. Various algorithms have been suggested to overcome such difficulties [15]–[20].

Other incremental learning algorithms, such as incremental construction of support vector machine classifiers with provable performance guarantees [21], incremental learning based on reproducible kernel Hilbert spaces [22], or incrementally adding new IF–THEN rules to an existing fuzzy inference system [23] have also been suggested. These algorithms also fit to the incremental learning setting described above, however, they require either precise *a priori* knowledge of data distributions, or an *ad-hoc* selection of a large number of parameters.

B. Ensemble of Classifiers

In this paper, we follow a different approach to the incremental learning problem, and present an algorithm that not only satisfies all criteria mentioned above, but also overcomes the difficulties that are associated with ARTMAP and ARTMAP based classifiers. In essence, instead of generating new cluster nodes for each previously unseen (or sufficiently different) instance, we generate multiple new "weak classifiers" for previously unseen portions of the feature space. This conceptually subtle difference, allows us to develop a fundamentally different incremental learning algorithm that is insensitive to the order of presentation of the training data, or even to the minor adjustments of the algorithm parameters.

Learn++, the proposed incremental learning algorithm described in the next section, was inspired by the **AdaBoost** (*adaptive boosting*) algorithm, originally developed to improve the classification performance of weak classifiers. Schapire showed that for a two class problem, a *weak learner* that can barely do little better than random guessing can be transformed into a *strong learner* that almost always achieves arbitrarily low error rate using a procedure called *boosting* [24]. Freund *et al.* later developed AdaBoost, extending boosting to multiclass and regression problems [25], [26]. In essence, both Learn++ and AdaBoost generate an *ensemble of weak classifiers*, each trained using a different distribution of training samples. The outputs of these classifiers are then combined using Littlestone's majority-voting scheme [27] to obtain the final classification rule. Combining weak classifiers take advantage of the so-called *instability* of the weak classifier. This instability causes the classifiers to construct sufficiently different decision surfaces for minor modifications in their training datasets.

The idea of generating an ensemble of classifiers for improving classification accuracy was formerly introduced by many other researchers. For example, Wolpert suggested combining hierarchical levels of classifiers, using a procedure called *stacked generalization* [28]. Jordan and Jacobs introduced hierarchical mixture of experts (HME), where multiple classifiers were highly trained (hence experts) in different regions of the feature space, and their outputs were then weighted using a gating network [29], [30]. Kitler *et al.* analyzed error sensitivities of various voting and combination schemes [31], whereas Rangarajan *et al.* investigated the capacity of voting systems [32]. Ji and Ma proposed an alternative approach to AdaBoost that generates simple perceptrons of random parameters and then combines the perceptron outputs using majority voting [33], similar to generating an ensemble of classifiers through randomizing the internal parameters of a base classifier, previously introduced by Ali and Pazzani [34]. Ji and Ma give an excellent review of various methods for combining classifiers in [35], whereas Dietterich compares ensemble of classifiers to other types of learners, such as reinforcement and stochastic learners in [36].

There have also been some attempts for using HMEs in an online setting to incrementally learn from incoming data [30], [37], however such attempts have not addressed all of the above mentioned issues of incremental learning, in particular, learning new classes. Consequently, research on combining classifiers have been mostly limited to improving performance of classifiers, rather than incremental learning. This leads us to consider adaptations of ensemble-based methods such as AdaBoost or HME to achieve incremental learning.

III. Ensemble of Classifiers for Incremental Learning: Learn++

Combining ensemble of classifiers in Learn++ is specifically geared toward achieving incremental learning, as described by the criteria mentioned earlier. However, due to their similarities, Learn++ also inherits *performance improvement* properties of AdaBoost, as shown in simulation results. Learn++ is based on the following intuition: Each new classifier added to the ensemble is trained using a set of examples drawn according to a distribution, which ensures that examples that are misclassified by the current ensemble have a high probability of being sampled. In an incremental learning setting, the examples that have a high probability of error are precisely those that are unknown or that have not yet been used to train the classifier.

As mentioned earlier, both AdaBoost and Learn++ generate weak hypotheses and combine them through weighted majority voting of the classes predicted by the individual hypotheses. The hypotheses are obtained by retraining a base classifier (weak learner) using strategically updated distributions of the training database. AdaBoost's distribution update rule is optimized for improving classifier accuracy, whereas Learn++ distribution update rule is optimized for incremental learning of new data, in particular when the new data introduces new classes. In the interest of space, only Learn++, and its major differences from AdaBoost are given below. Details of AdaBoost can be found in [25].

The Learn++ algorithm is given in Fig. 1. Inputs to Learn++ are

1) training data $S_k = [(x_1, y_1), (x_2, y_2), \ldots, (x_m, y_m)]$, where $\boldsymbol{x}_i$ are training instances and y_i are the corresponding correct labels for $i = 1, 2, \ldots, m$ samples randomly selected from the k^{th} database $\boldsymbol{\mathcal{D}_k}$;
2) a weak learning algorithm **WeakLearn**, to be used as the base classifier;
3) an integer T_k, specifying the number of classifiers to be generated.

Recall that a weak learning algorithm is used as a base classifier, to allow sufficiently different decision boundaries to be generated by slightly modified training datasets. Also note that most strong classifiers spend a majority of their training time in fine-tuning the decision boundary. As described below, Learn++ requires each weak learner to generate only a rough estimate of the actual decision boundary, effectively eliminating the costly fine-tuning step, allowing faster training and less over fitting.

Each classifier can be thought of as a hypothesis h from the input space X to the output space Y. Learn++ asks WeakLearn to generate multiple hypotheses using different subsets of the training data S_k, and each hypothesis learns only a portion of the input space. This is achieved by iteratively updating a distribution $D_t, t = 1, 2, \ldots, T_k$ from which training subsets are chosen. The distribution itself is obtained by normalizing a set of weights assigned to each instance based on the classification performance of the classifiers on that instance (Step 1). In general, instances that are difficult to classify receive higher weights to increase their chance of being selected into the next training dataset. The weights $w_1(i)$ for the first iteration are initialized to $1/m$, giving equal likelihood to each instance to be selected into the first training subset, unless there is sufficient reason to initialize otherwise.

At each iteration $t = 1, 2, \ldots, T_k$, Learn++ first dichotomizes S_k into a training subset TR_t and a test subset TE_t according to the current distribution D_t (Step 2), and calls WeakLearn to generate the hypothesis h_t: $X \rightarrow Y$ (Step 3) using the training subset TR_t. The error of h_t on $S_k = TR_t + TE_t$ is defined as (Step 4)

$$\varepsilon_t = \sum_{i:\, h_t(x_i) \neq y_i} D_t(i) \tag{1}$$

which is simply the sum of distribution weights of misclassified instances. If $\varepsilon_t > 1/2$, h_t is discarded and new TR_t and TE_t are selected. That is, the weak hypothesis is only expected to achieve a 50% (or better) empirical classification performance on the S_k. For a binary class problem, this is the least restrictive requirement one could have, since an error of one-half for a binary class problem means random guessing. However, obtaining a maximum error of one-half becomes increasingly difficult as the number of classes increase, since for an N class problem, the error generated by random guessing is $(N-1)/N$. Therefore, the choice of a weak learning algorithm with a minimum classification performance of 50% may not be trivial. However, NN algorithms can easily be configured to simulate weak learners, by modifying their size and error goal parameters. Use of strong learners, on the other hand, are not recommended in algorithms using the ensemble of classifiers approach, since there is little to be gained from their combination, and/or they may lead to over fitting of the data [25], [35].

If, $\varepsilon_t < 1/2$ is satisfied, then the normalized error β_t ($0 < \beta_t < 1$) is computed as

$$\beta_t = \varepsilon_t / 1 - \varepsilon_t. \tag{2}$$

All hypotheses generated in the previous t iterations are then combined using weighted majority voting (Step 5). The voting weights are computed as the logarithms of the reciprocals of normalized errors β. Therefore, those hypotheses that perform

Algorithm Learn++

Input: For each database drawn from $\mathcal{D}_k$ $k=1,2,\ldots,K$

- Sequence of m training examples $S=[(x_1,y_1),(x_2,y_2),\ldots,(x_m,y_m)]$.
- Weak learning algorithm WeakLearn.
- Integer T_k, specifying the number of iterations.

Do for $k=1,2,\ldots,K$:

Initialize $w_1(i) = D(i) = 1/m,\ \forall i$, unless there is prior knowledge to select otherwise.

Do for $t = 1,2,\ldots,T_k$:

1. Set $D_t = \mathbf{w}_t \Big/ \sum_{i=1}^{m} w_t(i)$ so that D_t is a distribution.
2. Randomly choose training TR_t and testing TE_t subsets according to D_t.
3. Call WeakLearn, providing it with TR_t.
4. Get back a hypothesis $h_t : X \rightarrow Y$, and calculate the error of h_t: $\varepsilon_t = \sum_{i:h_t(x_i)\neq y_i} D_t(i)$ on

   $S_t = TR_t + TE_t$. If $\varepsilon_t > ½$, set $t = t-1$, discard h_t and go to step 2. Otherwise,

   compute normalized error as $\beta_t = \varepsilon_t / (1-\varepsilon_t)$.
5. Call weighted majority, obtain the composite hypothesis $H_t = \arg\max_{y\in Y} \sum_{t:h_t(x)=y} \log(1/\beta_t)$,

   and compute the composite error $E_t = \sum_{i:H_t(x_i)\neq y_i} D_t(i) = \sum_{i=1}^{m} D_t(i)[|H_t(x_i)\neq y_i|]$

   If $E_t > ½$, set $t = t-1$, discard H_t and go to step 2.
6. Set $B_t = E_t/(1-E_t)$ (normalized composite error), and update the weights of the instances:

$$w_{t+1}(i) = w_t(i) \times \begin{cases} B_t, & \text{if } H_t(x_i) = y_i \\ 1, & \text{otherwise} \end{cases}$$
$$= w_t(i) \times B_t^{1-[|H_t(x_i)\neq y_i|]}$$

Call weighted majority on combined hypotheses H_t and **Output** the final hypothesis:

$$H_{final} = \arg\max_{y\in Y} \sum_{k=1}^{K} \sum_{t:H_t(x)=y} \log\frac{1}{B_t}$$

Fig. 1. Algorithm Learn++.

well on their own training and test data are given larger voting powers. A classification decision is then made based on the combined outputs of individual hypotheses, which constitutes the *composite hypothesis* H_t

$$H_t = \arg\max_{y\in Y} \sum_{t:\, h_t(x)=y} \log\frac{1}{\beta_t}. \qquad (3)$$

Note that H_t decides on the class that receives the highest total vote from all t hypotheses. The composite error made by H_t is then computed as

$$E_t = \sum_{i:\, H_t(x_i)\neq y_i} D_t(i) = \sum_{i=1}^{m} D_t(i)[|H_t(x_i)\neq y_i|] \qquad (4)$$

on misclassified instances, where $[|\cdot|]$ is 1 if the predicate is true, and 0 otherwise. If $E_t > 1/2$, current h_t is discarded, a new training subset is selected and a new h_t is generated. We note that E_t can only exceed this threshold during the immediate iteration after a new database $\mathcal{D}_{k+1}$ is introduced. At all other times, $E_t < 1/2$ will be satisfied, since all hypotheses h_t that make up the composite hypothesis have already been verified in Step 4 to achieve a minimum of 50% performance on S_k. If $E_t < 1/2$, composite normalized error is computed as

$$B_t = E_t/1 - E_t. \qquad (5)$$

The weights $w_t(i)$ are then updated, for computing the next distribution D_{t+1}, which in turn is used in selecting the next training and testing subsets, TR_{t+1} and TE_{t+1}, respectively.

The distribution update rule constitutes the heart of the algorithm, as it allows Learn++ to learn incrementally

$$w_{t+1}(i) = w_t(i) \times \begin{cases} B_t, & \text{if } H_t(x_i) = y_i \\ 1, & \text{otherwise} \end{cases} \\ = w_t(i) \times B_t^{1-[|H_t(x_i) \neq y_i|]}. \quad (6)$$

According to this rule, if instance x_i is correctly classified by the composite hypothesis H_t, its weight is multiplied by a factor of B_t, which, by its definition, is less than 1. If x_i is misclassified, its distribution weight is kept unchanged. This rule reduces the probability of correctly classified instances being chosen into TR_{t+1}, while increasing the probability of misclassified instances to be selected into TR_{t+1}. If we interpret instances that are repeatedly misclassified as hard instances, and those that are correctly classified as simple instances, the algorithm focuses more and more on hard instances, and forces additional classifiers to be trained with them. Instances coming from previously unseen parts of the feature space, such as those from new classes, can be interpreted as hard instances at the time they are introduced to the algorithm. Note that using the composite hypothesis in (6) makes incremental learning possible particularly when instances from new classes are introduced, since these instances will be misclassified by the composite hypothesis and forced into the next training dataset. The procedure would not work nearly as efficiently, if the weight update rule were based on the performance of the previous h_t only (as AdaBoost does) instead of the composite hypothesis H_t. Apart from the distribution update rule, Learn++ also differs from AdaBoost in definition of training error and the evaluation of individual hypotheses. During each iteration, Learn++ generates an additional test subset (TR_t) on which the training error and hypothesis evaluation are based, whereas AdaBoost computes the individual hypothesis errors on their own training data TE_t only. Finally, since AdaBoost does not compute a composite hypothesis, composite error is also not applicable in AdaBoost.

After T_k hypotheses are generated for each database $\mathcal{D}_k$, the final hypothesis is obtained by the weighted majority voting of all composite hypotheses

$$H_{final} = \arg\max_{y \in Y} \sum_{k=1}^{K} \sum_{t:\, H_t(x)=y} \log \frac{1}{B_t}. \quad (7)$$

Note that while incremental learning is achieved through generating additional classifiers, former knowledge is not lost, since all classifiers are retained. Another important property of Learn++ is its independence of the base classifier used as a weak learner. In particular, it can be used to convert any supervised classifier, originally incapable of incremental learning, to one that can learn from new data.

Fig. 2 conceptually illustrates Learn++ architecture on an example. The dark curve is the decision boundary to be learned and the two sides of the dashed line represent the feature space for two training databases S_1 and S_2, which need not be mutually exclusive. Weak hypotheses are illustrated with simple geometric figures, generated by weak learners $(WL_i,\ i = 1, 2, \ldots 8)$, where h_1 through h_4 are generated due to training with different subsets of S_1, and h_5 through h_8 are generated due to training with different subsets of S_2. Hypotheses decide whether a data point is within the decision boundary. They are hierarchically combined to form composite hypotheses $H_t,\ t = 1, \ldots 8$, which are then combined to form the final hypothesis H_{final}.

Learn++ guarantees convergence on any given training dataset, by reducing the classification error with each added hypothesis. We state the theorem that relates the overall upper error bound of Learn++ to individual errors of each hypothesis, the proof of which is given in the Appendix.

Theorem: The training error of the Learn++ algorithm given in Fig. 1 is bounded above by $E \leq 2^T \prod_{t=1}^{T} \sqrt{E_t \cdot (1 - E_t)}$, where E_t is the error of the tth composite hypothesis H_t. Furthermore, E_t is itself bounded above by $E_t \leq 2^t \prod_{s=1}^{t} \sqrt{\varepsilon_s \cdot (1 - \varepsilon_s)}$, where ε_s is the error of the individual hypothesis h_t.

IV. Experiments With Learn++

The algorithm was tested on various benchmark and real-world databases. Due to space limitations, results on four databases are presented here, with additional results available on the web [38]. Detailed descriptions of each database along with the performance of the algorithm on these databases are explained in the following sections. In all experiments, previously seen data were not used in subsequent stages of learning, and in each case the algorithm was tested on an independent validation dataset that was not used during training. In all cases, we have used a relatively small MLP trained with a large error goal as the base classifier to simulate a weak learner. We note that Learn++ itself is independent of the classifier used. MLP was used since it is the most commonly employed classification algorithm that is not capable of incremental learning without catastrophic forgetting.

Different architectures and error goals were tried to test the algorithm's sensitivity and invariance to minor modifications to parameter selections, including the MLP architecture, mean square error (MSE) goal, and the number of hypotheses generated. The parameters given below are typical representatives of those that have been tried. Furthermore, in order to test the sensitivity of Learn++ to the order of presentation of the data, multiple experiments were performed for all databases, where the order of the datasets introduced to the algorithm at different times were varied. The results for all cases were virtually the same. Average representative performance results are presented below.

A. Optical Digits Database

This benchmark database, obtained from the UCI machine learning repository [39], consisted of 5620 instances of digitized handwritten characters; 1200 instances were used for training and all remaining instances were used for validation. The characters were numerals 0–9, and they were digitized on an 8×8 grid, creating 64 attributes. This dataset was used to evaluate Learn++ on incremental learning without introducing new classes. Fig. 3 shows sample images of this database. The training dataset of 1200 instances were divided into six subsets, $S_1 \sim S_6$, each with 200 instances containing all ten classes

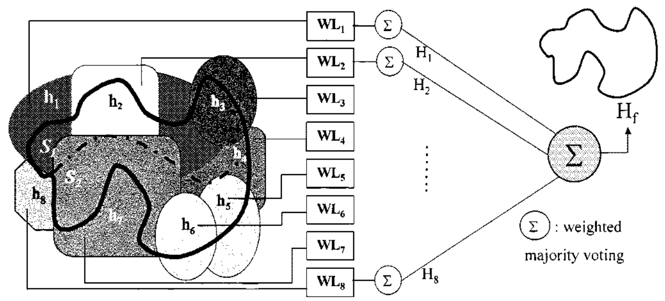

Fig. 2. Combining classifiers for incremental learning.

to be used in six training sessions. In each *training session*, only one of these datasets was used. For each training session k, ($k = 1, 2, \ldots, 6$) 30 weak hypotheses were generated by Learn++. Each hypothesis h_t ($t = 1, 2, \ldots, 30$) of the kth training session was generated using a training subset TR_t and a testing subset TE_t (used to compute hypothesis error), each with 100 instances drawn from S_k. The base classifier was a single hidden layer MLP with 30 hidden layer and ten output nodes with a MSE goal of 0.1. An additional validation set, *TEST*, of 4420 instances was used for validation purposes.

Note that NNs can simulate a weak learner, when their architecture is kept small and their error goal is kept high with respect to the complexity of the particular problem. The relatively high error goal of 0.1 allowed the MLP to serve as a weak learner in this case, as shown in the *Average/learner* column of Table I, which indicates the average performance of individual hypotheses on each database S_k. On average, weak learners performed little over 50%, which improved to over 90% when the hypotheses were combined. This improvement demonstrates the *performance improvement* property of Learn++ (as inherited from AdaBoost) on a given single database. Each column thereafter indicates Learn++ performance on the current and previous training datasets as additional data were introduced. Previous datasets were not used for training in subsequent training sessions, but they were only used to evaluate the algorithm performance on previously seen instances to make sure that previously acquired knowledge was not lost. The last row of Table I shows the classification performance on the validation dataset, which gradually and consistently improved from 82% to 93% as new databases became available, demonstrating the incremental learning capability of the proposed algorithm.

In order to compare the performance of Learn++ to that of a strong learner trained with the entire training data of 1200 instances, various architecture–error goal combinations were tried. An MLP with 50 hidden layer nodes and a 100 times smaller error goal of 0.001 was able to match (and slightly exceed) Learn++ performance, by classifying 95% of the TEST dataset.

B. Vehicle Silhouette Database

Also obtained from the UCI depository, the vehicle silhouette database consisted of 18 features from which the type of a vehicle is determined. The database consisted of 846 instances, which was divided into three training datasets $S_1 \sim S_3$ of 210 instances each, and a validation dataset, *TEST*, of 216 instances in four classes. For each training session $k = 1, 2, 3$, 30 hypotheses were generated using a 30-node single hidden layer MLP with an error goal of 0.1. This particular benchmark database is considered as one of the more difficult databases in the repository, since generalization performances using various algorithms (strong learners) have been in the 65%–80% range [40], [41]. The results are presented in Table II, where the Average/learner column indicates the average performance of a weak hypothesis (a single MLP). We note from the average 62% performance that the chosen MLP architecture and error goal was able to simulate a weak learner. The other columns indicate the Learn++ performance on individual training datasets and on the validation dataset after each of the three training sessions. As seen in Table II, there is a minor and gradual loss of information on the previous training datasets as new datasets are introduced, however, the generalization performance on the validation dataset improved from 78% to 83%. This performance was comparable, or better, than the performance of most algorithms that were trained using the entire data [41].

Fig. 3. Sample images from the optical digits database.

C. Concentric Circles Database

This rather simple synthetic database of concentric rings with two attributes and five classes was generated for testing Learn++ performance on incremental learning when new classes are introduced. Fig. 4 illustrates this database. The database was divided into six training datasets, S_1 through S_6, and a validation dataset *TEST*. S_1 and S_2 had 50 instances from each of the classes 1, 2, and 3; datasets S_3 and S_4 had 50 instances from each of the classes 1, 2, 3, and 4; and datasets S_5, S_6 had 50 instances from each of the classes 1–5. The validation set *TEST* had 500 instances from all five classes. Table III presents the classification performance results. The validation on *TEST* dataset shows steadily increasing generalization performance, indicating the algorithm was able to learn the new information, and the new classes, successfully. Note that larger improvements in the performance are obtained after the third and fifth training sessions, since these training sessions introduced new

TABLE I
TRAINING AND GENERALIZATION PERFORMANCE OF LEARN++ ON OPTICAL DIGITS DATABASE

Inc. Train→ ↓ Dataset	Average / learner	Training 1	Training 2	Training 3	Training 4	Training 5	Training 6
S_1	55%	94%	94%	94%	93%	93%	93%
S_2	53%	---	93.5%	94%	94%	94%	93%
S_3	51%	---	---	95%	94%	94%	94%
S_4	53%	---	---	---	93.5%	94%	94%
S_5	56%	---	---	---	---	95%	95%
S_6	58%	---	---	---	---	---	95%
TEST	**41.3%**	**82%**	**84.7%**	**89.7%**	**91.7%**	**92.2%**	**92.7%**

TABLE II
TRAINING AND GENERALIZATION PERFORMANCE OF LEARN++ ON VEHICLE DATABASE

Inc. Train→ ↓ Dataset	Average / Learner	Training 1	Training 2	Training 3
S_1	62%	93%	82%	79%
S_2	60%	---	86%	78%
S_3	64%	---	---	91%
TEST	**57%**	**78%**	**80.4%**	**83%**

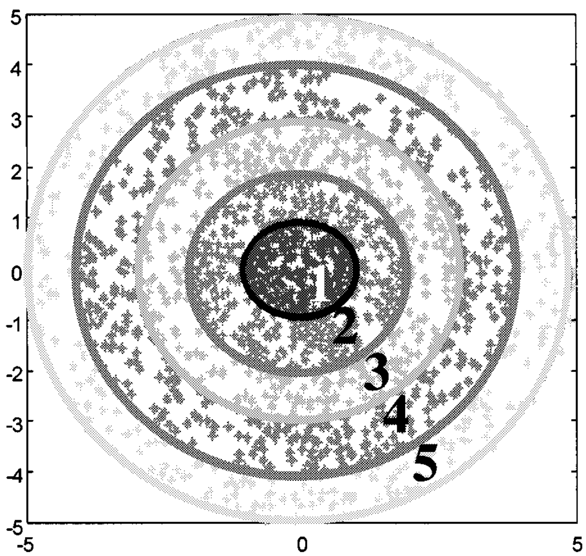

Fig. 4. Circular regions database.

classes that were not available earlier. Similarly, the improvements in the performance after the fourth and sixth training sessions are minor compared to the previous sessions, since these sessions did not introduce new classes. This is also reflected in the number of hypotheses generated during each training session, which are given in parentheses on the first row of the table. Note that when new classes are introduced, the number of hypotheses generated in each session is not the same. The number of hypotheses generated was determined simply by monitoring the classification performance, where each training session was terminated when the performance no longer improved.

The last column titled "Last 7" indicates the Learn++ performance on the last seven hypotheses. Although these hypotheses were trained with a dataset that included all classes, they were not adequate to give satisfactory performance, demonstrating that all hypotheses are required for the final classification.

An alternative set of six datasets was also generated from this database, by changing the order of classes introduced incrementally, to test the algorithm's sensitivity to the order of presentation of the data. The results, which are provided on the web [38], were virtually the same.

Finally, in order to compare the incremental learning performance of Learn++ to that of a strong learner trained on the entire training data, a larger MLP with 50 hidden nodes and an error goal of 0.005 was trained. The performance of this strong learner was 95%, only slightly better than that of Learn++.

D. Gas Sensing Dataset

Learn++ was then implemented on real-world data obtained from a set of six polymer-coated quartz crystal microbalances (QCMs) used to detect volatile organic compounds (VOCs). Detection and identification of VOCs are of crucial importance for environmental monitoring and in gas sensing. Piezoelectric acoustic wave sensors, which comprise a versatile class of chemical sensors, are used for the detection of VOCs. For sensing applications, a sensitive polymer film is cast on the surface of the QCM. This layer can bind a VOC of interest, altering the resonant frequency of the device, in proportion to the added mass. Addition or subtraction of gas molecules from the surface or bulk of an acoustic wave sensor results in a change in its resonant frequency. The frequency change Δf, caused by a deposited mass Δm can be described by $\Delta f = -2.3 \times 10^6 \cdot f^2.(\Delta m/A)$ where f is the fundamental resonant frequency of the bare crystal, and A is the active surface area [42]. The sensor typically consists of an array of several crystals, each coated with a different polymer. This design is aimed at improving identification, hampered by the limited selectivity of individual films. Employing more than one crystal, and coating each with a different partially selective polymer, different responses can be obtained for different gases. The combined response of these crystals can then be used as a signature pattern of the VOC detected.

The gas sensing dataset used in this study consisted of responses of six QCMs to five VOCs, including ethanol (ET), xylene (XL), octane (OC), toluene (TL), and trichloroethelene (TCE). Fig. 5 illustrates sample patterns for each VOC from

TABLE III
TRAINING AND GENERALIZATION PERFORMANCE OF LEARN++ ON CONCENTRIC CIRCLES DATABASE

Inc. Train→ ↓ Dataset	Training1 (10)	Training 2 (10)	Training 3 (13)	Training 4 (3)	Training 5 (15)	Training 6 (7)	Last 7
S_1	98.7%	96.7%	91.4%	91.4%	95.3%	95.3%	41.7%
S_2	---	96.1%	87.1%	85.8%	92.2%	91.6%	40.6%
S_3	---	---	98.3%	98.3%	72%	90.8%	51.5%
S_4	---	---	---	93.6%	77%	88.4%	49.8%
S_5	---	---	---	---	88%	95.2%	60.4%
S_6	---	---	---	---	---	96.4%	53.6%
TEST	**55.6%**	**56.8%**	**73.2%**	**74.4%**	**85.8%**	**89.6%**	52.8%

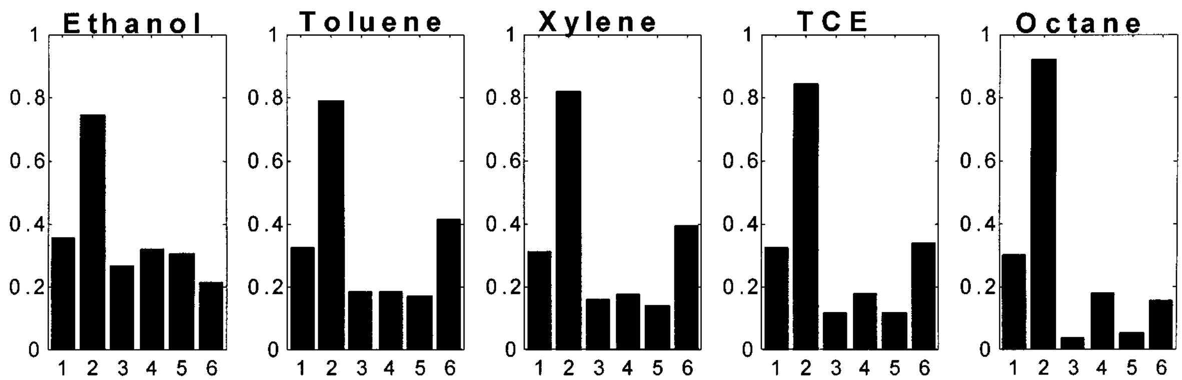

Fig. 5. Sample responses of the six-QCM sensor array to VOCs of interest.

TABLE IV
DATA-CLASS DISTRIBUTION FOR THE GAS SENSING DATABASE

	ETHANOL	TCE	OCTANE	XYLENE	TOLUENE
S_1	**20**	0	**20**	0	**40**
S_2	5	**25**	5	0	5
S_3	5	5	5	**40**	5
TEST	34	34	34	40	62

six QCMs coated with different polymers, where the vertical axis represents normalized frequency change. Note that the patterns from toluene, xylene, and TCE look considerably similar; hence, they are difficult to distinguish from each other. Further details on VOC recognition using QCMs can be found in [42], whereas more information on this dataset, experimental setup for generating the gas sensing signals, and sample patterns are provided on the web [43].

The dataset consisted of 384 six-dimensional patterns, half of which were used for training. Table IV presents the distribution of the datasets, where subsequent datasets are strongly biased toward the new class. Such a distribution results in an even tougher challenge; since the algorithm will no longer have the opportunity to see adequate number of instances from previously introduced classes in the subsequent training sessions. The performance of Learn++ on this dataset is shown in Table V.

The generalization performance of Learn++ on the validation dataset, gradually improving from 61% to 88% as new data was introduced, demonstrates its incremental learning capability even when instances of new classes are introduced in subsequent training sessions. Learn++ performance on this dataset was comparable to that of a strong learner, a two hidden layer MLP of error goal 0.001, trained with the entire training data, which had a classification performance of 90%. Learn++ was able to perform as well as the strong learner, by seeing only a portion of the dataset at a time. This dataset was also presented to the algorithm in a different order, and the resulting performances were virtually the same, implying that the algorithm is not sensitive to the order of presentation of the training data. Furthermore, various minor modifications to the base classifier architecture ($15 \sim 40$ nodes) and error goal ($0.05 \sim 0.2$) also resulted in similar performances, indicating that the algorithm is not very sensitive to minor changes in its parameters. A formal analysis on how much such parameters can be changed without affecting the performance is currently underway.

Finally, Learn++ was also compared to fuzzy ARTMAP on this database. Table VI presents the performance figures of fuzzy ARTMAP on the identical dataset described in Table IV for various values of the vigilance parameter ρ. The classification performance of fuzzy ARTMAP is always 100% on training data, since according to the ARTMAP learning algorithm, convergence is achieved only when all training data are correctly classified. Furthermore, once a pattern is learned, a particular cluster is assigned to it, and future training does not alter this clustering. Therefore, ARTMAP never forgets what it has seen as a training data instance. The improvement in the classification performance of the test data once again demonstrates that ARTMAP is indeed capable of incremental learning. However, fuzzy ARTMAP was indeed sensitive to slight changes in its vigilance parameter, and even its best performance of 83.8% for $\rho = 0.90$ was about 5% points less that than of Learn++.

V. SUMMARY AND DISCUSSION

This paper introduced Learn++, a versatile incremental learning algorithm based on synergistic performance of an ensemble of weak classifiers/learners. Learn++ can learn from new data even when the data introduces new classes.

TABLE V
TRAINING AND GENERALIZATION PERFORMANCE OF LEARN++ ON GAS SENSING DATABASE

Inc. Train→ ↓Dataset	Training1 (10)	Training 2 (11)	Training 3 (11)
S_1	96.2%	77.5%	76.25%
S_2	---	87.5%	82.5%
S_3	---	---	90%
TEST	**60.78%**	**70.1%**	**88.2%**

TABLE VI
FUZZY ARTMAP PERFORMANCE ON THE GAS SENSING DATABASE

Inc. Train→ ↓ Dataset	Training1	Training 2	Training 3
S_1	100%	100%	100%
S_2		100%	100%
S_3			100%
TEST (ρ=*0.85*)	54.9%	68.1%	82.8%
TEST (ρ=*0.90*)	50.5%	67.2%	83.8%
TEST (ρ=*0.95*)	43.6%	59.3%	71.1%

Learn++ does not require access to previously used data during subsequent training sessions, and it is able to retain previously acquired knowledge. Learn++ makes no assumptions as to what kind of weak learning algorithm is to be used. Any weak learning algorithm can serve as the base classifier of Learn++, though the algorithm is optimized for supervised NN-type classifiers, whose weakness can be easily controlled via network size and error goal.

Learn++ is also intuitively simple, easy to implement, and converges much faster than strong learning algorithms. This is because using weak learners eliminates the problem of fine-tuning and over fitting, since each learner only roughly approximates the decision boundary.

Initial results using this algorithm look promising, but there is significant room for improvement and many questions to be answered. The algorithm has two key components, both of which can be improved. The first one is the selection of the subsequent training dataset, which depends on the distribution update rule. AdaBoost depends solely on the performance of individual h_t, whereas Learn++ uses the performance of overall H_t for distribution update. The former guarantees robustness and prevents performance deterioration, whereas the latter allows efficient incremental learning capability when new classes are introduced. An appropriate combination of the two updating schemes might provide optimum performance levels. Initialization of the distribution when a new database is introduced can also be optimized by an initial classification evaluation of the composite hypotheses on the new database.

The second key factor in Learn++ is the hypothesis combination rule. Currently, voting weights are determined based on performances of the hypotheses on their own training data subset. This is suboptimal, since the performance of a hypothesis on a specific subset of the input space does not guarantee the performance of that hypothesis on an unknown instance, which may come from a different subset of the space. This static combination rule can be replaced by a dynamic rule that estimates which hypotheses are likely to correctly classify a given (unknown) instance, based on statistical distance metrics or *a posteriori* probabilities, and determine voting weights accordingly for each instance.

Other issues include selection of algorithm parameters, and using other classifiers as weak learners. The algorithm parameters, such as base classifier architecture, error goal, number of hypotheses to be generated, are currently chosen in a rather *ad hoc* manner. Although the algorithm appears to be insensitive to minor changes in these parameters, a formal method for selecting them would be beneficial. Future work will also include evaluating Learn++ with other classifiers used as weak learners, such as RBF NNs and non-NN-based classification/clustering algorithms.

Finally, the weighted majority voting for combining the hypotheses hints at a simple way of estimating the reliability of the final decision and confidence limits of the performance figures. In particular, if a vast (marginal) majority of h_t agree on the class of a particular instance, then this can be interpreted as the algorithm having high (low) confidence in the final decision. A formal analysis of classifier reliability and confidence intervals of the classifier outputs can be done by computing *a posteriori* probabilities of classifier outputs, which can then be compared to those obtained by using vote count mechanism.

APPENDIX
ERROR BOUND ANALYSIS FOR LEARN++

Theorem: The training error of the Learn++ algorithm given in Fig. 1 is bounded above by $E \le 2^T \prod_{t=1}^{T} \sqrt{E_t \cdot (1-E_t)}$, where E_t is also bounded above by the AdaBoost.M1 error bound $E_t \le 2^t \prod_{s=1}^{t} \sqrt{\varepsilon_s \cdot (1-\varepsilon_s)}$.

Proof: Following a similar approach given in [25], we first show that the above error bound holds for a two-class problem, and then show that a multiclass problem can be reduced to a binary class problem, allowing the same error bound to hold for the multiclass case as well. Let us call the algorithm working on binary problems Learn+ (as opposed to Learn++, which is reserved for the multiclass problem).

In a binary class setting where the two possible values for y are 0 and 1, the equations for error terms and distribution update rules given in Fig. 1 can be simplified as follows. The combined hypothesis is obtained by

$$H_t(x) = \begin{cases} 1, & \text{if} \displaystyle\sum_{t=1}^{T} \log(1/\beta_t) \cdot h_t(x) \ge \tfrac{1}{2} \sum_{t=1}^{T} \log(1/\beta_t) \\ 0, & \text{otherwise} \end{cases} \tag{8}$$

whereas the error for H_t is

$$E_t = \sum_{i:\, H_t(x_i) \ne y_i} D_t(i) = \sum_{i=1}^{m} D_t(i) |H_t(x_i) - y_i| \tag{9}$$

the distribution update rule is given by

$$\begin{aligned} w_{t+1}(i) &= w_t(i) \times \begin{cases} B_t, & \text{if } H_t(x_i) = y_i \\ 1, & \text{otherwise} \end{cases} \\ &= w_t(i) \times B_t^{1-|H_t(x_i) - y_i|} \end{aligned} \tag{10}$$

and the final classification rule for each dataset is

$$H_{final}(x) = \arg\max_{y \in Y} \sum_{t:\, H_t(x)=y} \log \frac{1}{B_t} = \begin{cases} 1, & \text{if} \sum_{t=1}^{T} \log(1/B_t) \cdot H_t(x) \geq \frac{1}{2} \sum_{t=1}^{T} \log(1/B_t) \\ 0, & \text{otherwise.} \end{cases} \tag{11}$$

We define the error of the final hypothesis as sum of the initial weights of the misclassified instances, that is

$$E = \sum_{i:\, H_{final}(x) \neq y_i} D(i). \tag{12}$$

To find and upper bound for E, we analyze the final weights of the instances after T iterations, and associate these weights with the errors committed by combined hypotheses H_t. Note that after T rounds, the final weight for any instance is

$$w_{T+1}(i) = w_1(i) \cdot \prod_{t=1}^{T} B_t^{1-|H_t - y_i|} = D(i) \cdot \prod_{t=1}^{T} B_t^{1-|H_t - y_i|}. \tag{13}$$

The summation over all instances gives

$$\sum_{i=1}^{m} w_{T+1}(i) = \sum_{i=1}^{m} D(i) \cdot \prod_{t=1}^{T} B_t^{1-|H_t - y_i|}. \tag{14}$$

Comparing the sum of weights of all instances to the sum of the weights that are misclassified

$$\sum_{i=1}^{m} w_{T+1}(i) \geq \sum_{i:\, H_{final}(x) \neq y_i}^{m} w_{T+1}(i) = \sum_{i:\, H_{final}(x) \neq y_i}^{m} D(i) \cdot \prod_{t=1}^{T} B_t^{1-|H_t - y_i|}. \tag{15}$$

We now note that the final hypothesis H_{final} will make a mistake on instance i if and only if

$$\sum_{t=1}^{T} \log B_t^{-|H_t(x_i) - y_i|} \geq \sum_{t=1}^{T} \log(B_t)^{-1/2} \tag{16}$$

or alternatively, if and only if

$$\prod_{t=1}^{T} B_t^{-|H_t(x_i) - y_i|} \geq \prod_{t=1}^{T} (B_t)^{-1/2}. \tag{17}$$

Incorporating (17) into (15) for misclassified instances, we obtain

$$\sum_{i=1}^{m} w_{T+1}(i) \geq \sum_{i:\, H_{final}(x) \neq y_i}^{m} D(i) \cdot \prod_{t=1}^{T} B_t \cdot B_t^{-|H_t(x_i) - y_i|} \geq \sum_{i:\, H_{final}(x) \neq y_i}^{m} D(i) \cdot \prod_{t=1}^{T} B_t^{1/2} = E \cdot \prod_{t=1}^{T} B_t^{1/2}. \tag{18}$$

Hence,

$$E \leq \frac{\sum_{i=1}^{m} w_{T+1}(i)}{\prod_{t=1}^{T} B_t^{1/2}} \tag{19}$$

giving us an upper bound for the error of the final hypothesis. However, this upper bound based on the weights of individual instances is of little use, since it is difficult to keep track of the weights of every instance used for each hypothesis. These sum of weights can also be limited by an upper bound, based on the errors of each H_t. Recognizing that $B^{\gamma} \leq 1 - (1 - B) \cdot \gamma$ for $0 < B < 1$, and starting with the sum of the weights of all instances

$$\sum_{i=1}^{m} w_{t+1}(i) = \sum_{i=1}^{m} w_t(i) \cdot B_t^{1-|H_t(x_i) - y_i|} \leq \sum_{i=1}^{m} w_t(i)(1 - (1 - B_t)(1 - |H_t(x_i) - y_i|)). \tag{20}$$

We now define the intermediate variable $\Psi_t(i) = |H_t(x_i) - y_i|$ as the loss of the tth hypothesis on instance i, then the total error of the tth combined hypothesis is

$$E_t = \sum_{i=1}^{m} D_t(i) \cdot |H_t(x_i) - y_i| = \sum_{i=1}^{m} D_t(i) \cdot \Psi_t(i) = \mathbf{D_t} \cdot \Psi_\mathbf{t}. \tag{21}$$

Furthermore, recall from Step 1 of Learn++ algorithm that

$$\mathbf{D}_t = \mathbf{w}_t \Big/ \sum_{i=1}^{m} w_t(i). \tag{22}$$

Substituting (21) and (22) into (20), we obtain

$$\sum_{i=1}^{m} w_{t+1}(i) \leq \sum_{i=1}^{m} w_t(i) - (1 - B_t) \sum_{i=1}^{m} w_t(i)(1 - \Psi_t(i)) \leq \sum_{i=1}^{m} w_t(i) - (1 - B_t) \left(\sum_{i=1}^{m} w_t(i) - \mathbf{w_t} \cdot \Psi_\mathbf{t} \right)$$

from (22), we obtain

$$\leq \sum_{i=1}^{m} w_t(i) - (1 - B_t) \cdot \left(\sum_{i=1}^{m} w_t(i) - \mathbf{D_t} \cdot \Psi_\mathbf{t} \left(\sum_{i=1}^{m} w_t(i) \right) \right)$$

and from (21)

$$\leq \sum_{i=1}^{m} w_t(i) - (1-B_t)\left(\sum_{i=1}^{m} w_t(i) - E_t \cdot \sum_{i=1}^{m} w_t(i)\right)$$
$$\leq \sum_{i=1}^{m} w_t(i) - (1-B_t)\sum_{i=1}^{m} w_t(i)(1-E_t)$$
$$\leq \sum_{i=1}^{m} w_t(i)(1-(1-B_t)(1-E_t)). \tag{23}$$

After T iterations, we obtain

$$\sum_{i=1}^{m} w_{T+1}(i) \leq \prod_{t=1}^{T} 1-(1-B_t)(1-E_t). \tag{24}$$

Substituting (24) into (19), we obtain

$$E \leq \frac{\sum_{i=1}^{m} w_{T+1}(i)}{\prod_{t=1}^{T} B_t^{1/2}} \leq \frac{\prod_{t=1}^{T} 1-(1-B_t)(1-E_t)}{\prod_{t=1}^{T} B_t^{1/2}}$$

or

$$E \leq \prod_{t=1}^{T} \frac{1-(1-B_t)(1-E_t)}{B_t^{1/2}} \tag{25}$$

which gives us an upper bound on the training error in terms of the normalized error and the actual error of the combined hypotheses H_t. Note that no relationship has been assumed between E_t and B_t in this derivation We now find the optimum value for B_t, from (25). Since all terms in (25) are positive, we can take the derivatives individually for each t

$$\frac{\partial\left(\frac{1-(1-B_t)(1-E_t)}{B_t^{1/2}}\right)}{\partial B_t} = 0 \Rightarrow B_t = \frac{E_t}{1-E_t}. \tag{26}$$

Finally, substituting (26) into (25), we obtain

$$E \leq 2^T \prod_{t=1}^{T} \sqrt{E_t(1-E_t)} \tag{27}$$

which is identical in form to that of AdaBoost, except that the errors of individual hypotheses h_t are replaced by the errors of combined hypotheses H_t. Furthermore, since each combined hypothesis is obtained from individual hypotheses much like the final hypothesis is obtained from the combined hypotheses, an identical error analysis can be carried out for each H_t, which obviously will be $E_t \leq 2^t \prod_{s=1}^{t} \sqrt{\varepsilon_s \cdot (1-\varepsilon_s)}$, the error of the AdaBoost algorithm.

So far, we have only shown the error bound for the binary classification problem; however, it is easy to show that the same analysis holds for multiclass problems, by establishing a one-to-one mapping between the binary class and multiclass problems. Again, following a similar approach to that in [25], for each instance in the Learn++ training set (x_i, y_i), we define a Learn+ instance $(\tilde{x}_i, \tilde{y}_i)$ with $\tilde{x}_i =$ some random number, and $\tilde{y}_i = 0$. We also define the initial distribution for Learn+ instances to be the same as Learn++ instances.

For each iteration t we pass the hypothesis $\tilde{H}_t(i) = [|H_t(x_i) \neq y_i|]$ as if **WeakLearn** returns it to Learn+. Note that according to this formulation, if Learn++ misclassifies x_i, then it will return 1 to $\tilde{H}_t(i)$. Since the correct class of the corresponding $\tilde{x}_i$ is 0, (all instances for Learn+ are of class 0 by our previous definition), then $\tilde{H}_t(i)$ misclassifies this instance as well. On the other hand, if Learn++ correctly classifies instance x_i, it will return 0 to $\tilde{H}_t(i)$, and since this is also the correct class for all Learn+ instances, $\tilde{H}_t(i)$ also classifies the corresponding instance $\tilde{x}_i$ correctly. In other words, when the multiclass algorithm makes an error, the binary class algorithm makes an error, and when the multiclass algorithm correctly classifies an instance, so does the binary class algorithm. Since initial distributions for both algorithms were defined to be identical, errors computed by both algorithms will also be identical, hence $\tilde{E}_t = E_t$, $\tilde{B}_t = B_t$, and $\tilde{\mathbf{w}}_t = \mathbf{w}_t$. Therefore, the error of the final hypothesis E will also be identical to that given in (27). •

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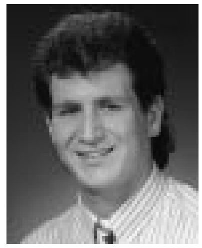

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