



Ch.5 Synchronous Sequential Logic - part D

Finite-state Machine(FSM)

state

주어진 시점에서의 memory에 저장된 contents

- in sequential logic, information from past inputs is stored in memory.(ex. ff)
- 회로에서 접근했던 모든 정보를 contain

Finite-state machine(FSM, FSA, simply a state machine)

주어진 시간 내에 finite한 개수의 state 중 정확히 한 곳에 갈 수 있는 절대적인 machine
(ex. vending machine, elevators)

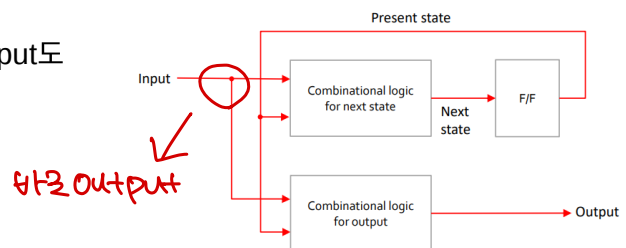
- **transition** : input에 대한 state 이동
- **FSM** : list of states, initial state, and input에 대한 transition

Mealy State Machine

Output- > present state, current input



- input change → output change
 - clock input 바뀌길 기다리지 않고 output도 update



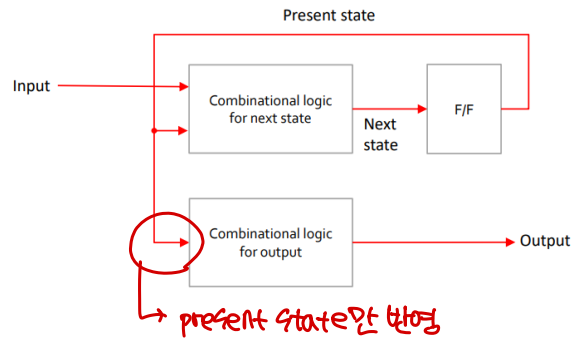
Moore State Machine

Output- > present state

- clock input change → output change

- clock input이 바뀔 때만

output도 update



State Reduction

sequential circuit에서 ff의 개수를 reduction

→ input, output 변화 없이 reduction 해야 함

- (state transition 같음)

equivalent state : 같은 상태에서 정확히 같은 input, output → 두 state는 equivalent

⇒ 하나만 있는 것이 효율적 (don't care ↑) → 필요가 더 간단

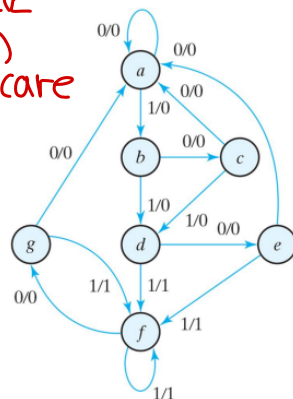
⇒ state가 줄어들면 ff 개수도 줄어듦

- example → 시료에 동일한 transition 이의 다른 transition 쪽 가져올 것

Present State	Next State		Output	
	x = 0	x = 1	x = 0	x = 1
a	a	b	0	0
b	c	d	0	0
c	a	d	0	0
d	e	d	0	1
e	a	d	0	1
f	e	f	0	1
g	a	f	0	1

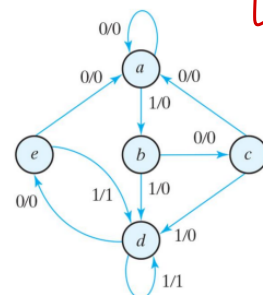
Present State	Next State		Output	
	x = 0	x = 1	x = 0	x = 1
a	a	b	0	0
b	c	d	0	0
c	a	d	0	0
d	e	d	0	1
e	a	d	0	1

* ff: 3개 필요
(000 ~ 111)
1개는 don't care



* ff: 3개 필요
(3개 don't care)

→ 필요가 더 간단



State Assignment

M state는 $\log_2 M$ bits의 코드

state 개수 $\left\{ \begin{array}{l} 1 \sim 2 : ff\ 1개 \\ 3 \sim 4 : ff\ 2개 \\ 5 \sim 8 : ff\ 3개 \\ 9 \sim 16 : ff\ 4개 \dots \end{array} \right.$

Table 5-9
Three Possible Binary State Assignments

⇒ 다양한 방법이 있음

State	Assignment 1 Binary	Assignment 2 Gray code	Assignment 3 One-hot
a	000	000	00001
b	001	001	00010
c	010	011	00100
d	011	010	01000
e	100	110	10000

Table 5-10
Reduced State Table

Present State	Next State		Output	
	x = 0	x = 1	x = 0	x = 1
000	000	001	0	0
001	010	011	0	0
010	000	011	0	0
011	100	011	0	1
100	000	011	0	1

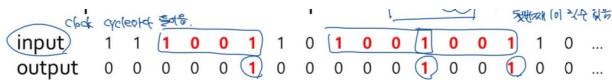
Sequential circuit design ↔ analysis

1. state diagram → state table 만들기 → present state, input | next state, output
2. state table로 state에 binary code 할당(n개의 state → $\log_2 n$ 개의 ff)
3. 각 ff의 input value 찾기(ff excitation table 활용)
4. ff의 input, output equation 찾기
5. 회로 설계

Sequence recognizers

a special kind of sequential circuit that looks for a special bit pattern in some input

→ 지금까지 들어온 input에 대해 'remember' 해야 함.



→ DFA 만들 때
80, 81 ... getting 했던 것처럼 가능한 경우 생각

→ 한 bit씩 check하기에는
상태수 → 너무 많아질림

example - JK ff

"1001"을 recognize

→ output은 1개

1. making a state table

① sequence recognizer 생각해본 것 ⇒ 각 state마다 2개의 arrow

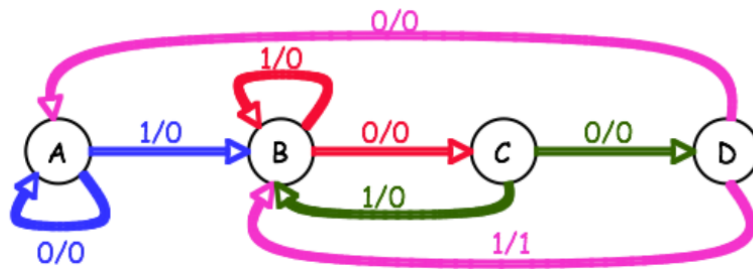
② start state

③ 1만 들어왔을 때 → 1이 들어오면 ①로, 0이면 다음 ②로

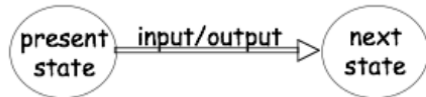
④ 10이 들어왔을 때 → 1이 " ②로, 0이면 ④로

⑤ 100 " → 1이 " ①로, 0이면 ①로

⇒ 100①001 → 처음부터 다시



Remember how the state diagram arrows correspond to rows of the state table:



→ transition 상태 row 상태

Present State	Input	Next State	Output
A	0	A	0
A	1	B	0
B	0	C	0
B	1	B	0
C	0	D	0
C	1	B	0
D	0	A	0
D	1	B	1

2. assigning binary codes to states

2개의 state → 2개의 ff

Present State	Input	Next State	Output
A	0	A	0
A	1	B	0
B	0	C	0
B	1	B	0
C	0	D	0
C	1	B	0
D	0	A	0
D	1	B	1



Present State	Input	Next State	Output
Q ₁ Q ₀	X	Q ₁ Q ₀	Z
0 0	0	0 0	0
0 0	1	0 1	0
0 1	0	1 0	0
0 1	1	0 1	0
1 0	0	1 1	0
1 0	1	0 1	0
1 1	0	0 0	0
1 1	1	0 1	1

3. finding ff input values

ff input 값이 항상

Present State	Input	Next State	Flip flop inputs	Output
Q ₁ Q ₀	X	Q ₁ Q ₀	J ₁ K ₁ J ₀ K ₀	Z
0 0	0	0 0		0
0 0	1	0 1		0
0 1	0	1 0		0
0 1	1	0 1		0
1 0	0	1 1		0
1 0	1	0 1		0
1 1	0	0 0		0
1 1	1	0 1		1

1) ff 종류 고르기

∴ ff 종류에 따라 효율이 달라짐 → 이 예시에서는 JK 두 개 사용

- JK ff input values
 - characteristic table

J	K	Q(t+1)	Operation
0	0	Q(t)	No change
0	1	0	Reset
1	0	1	Set
1	1	Q'(t)	Complement

(여기표)
◦ excitation table

변경해야
채우기 ↓

다 하기에는 번거로움
ex) 0→0
① no change
② reset
둘 중 하나
⇒ J | K
0 | 0
0 | 1
1 | 0
1 | 1
→ don't care
⇒ J | K
0 | 0
0 | 1
1 | 0
1 | 1

Q(t)	Q(t+1)	J	K	Operation
0	0	0	x	No change/reset
0	1	1	x	Set/complement
1	0	x	1	Reset/complement
1	1	x	0	No change/set

⇒ 표 채우기

Present State		Input X	Next State		Flip flop inputs				Output Z
Q ₁	Q ₀		Q ₁	Q ₀	J ₁	K ₁	J ₀	K ₀	
0	0	0	0	0	0	x	0	x	0
0	0	1	0	1	0	x	1	x	0
0	1	0	1	0	1	x	x	1	0
0	1	1	0	1	0	x	x	0	0
1	0	0	1	1	x	0	1	x	0
1	0	1	0	1	x	1	1	x	0
1	1	0	0	0	x	1	x	1	0
1	1	1	0	1	x	1	x	0	1

4. find equations for the ff input, output

: k-map 이용하여 방정식 찾을 ⇒ present state, input 으로부터 k-map

J ₁				1
	Q ₁	x	x	x
				x

$$J_1 = Q_0 X'$$

J ₀			1	x	x
	Q ₁	1	1	x	x
					x

$$J_0 = x + Q_1$$

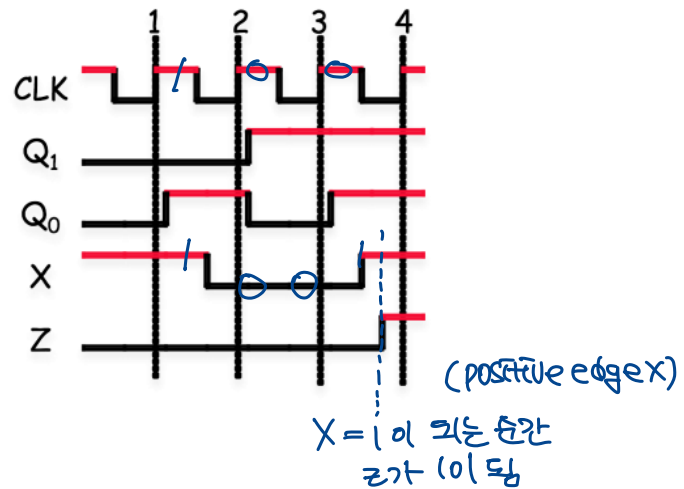
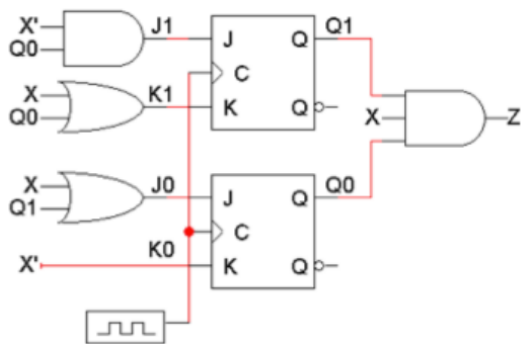
K ₁		x	x	x	x
	Q ₁		1	1	1
					x

$$K_1 = Q_0 + x$$

K ₀		x	x		1
	Q ₁	x	x		1
					x

$$K_0 = x'$$

5. build the circuit



example - D ff

Present State		Input	Next State		Flip-flop inputs		Output
Q ₁	Q ₀		Q ₁	Q ₀	D ₁	D ₀	
0	0	0	0	0	0	0	0
0	0	1	0	1	0	1	0
0	1	0	1	0	1	0	0
0	1	1	0	1	0	1	0
1	0	0	1	1	1	1	0
1	0	1	0	1	0	1	0
1	1	0	0	0	0	0	0
1	1	1	0	1	0	1	1

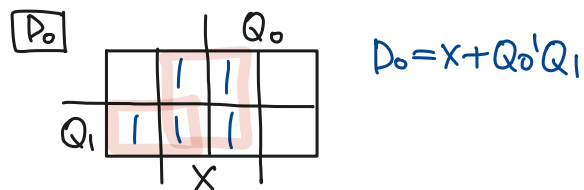
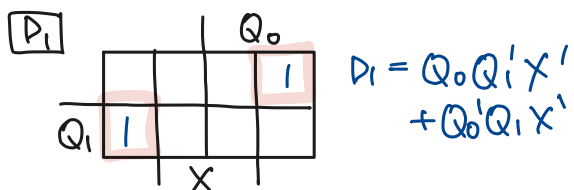
① D ff characteristic table

D	Q(t+1)	operation
0	0	Reset
1	1	Set

② D ff excitation table

Q(t)	Q(t+1)	D	operation
0	0	0	Reset
0	1	1	Set
1	0	0	Reset
1	1	1	Set

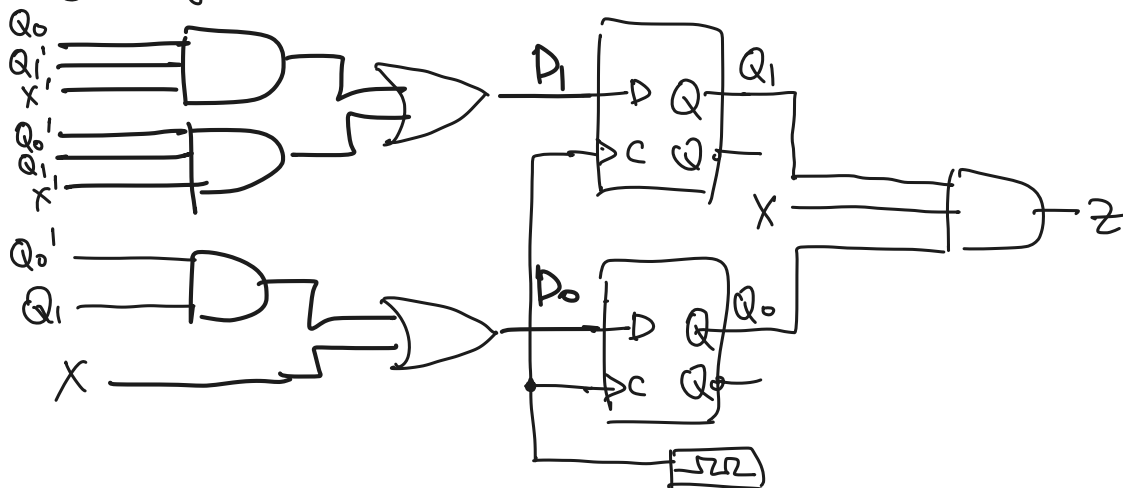
③ D₁, D₀ equation → Kmap



④ Z equation

$$Z = Q_1Q_0X$$

⑤ design circuit

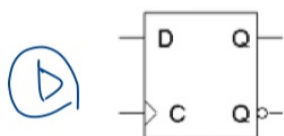


ff comparison

- JK ff : don't care 값이 많아서 간단한 회로를 그릴 수 있음
- D ff : input이 하나만 있어서 input equation을 만드는데 쉬움

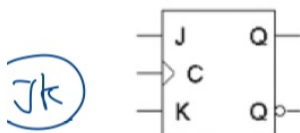
→ 실전에서 D가 더 자주 쓰임.

* excitation table



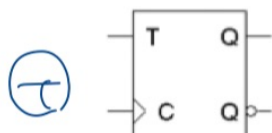
Q(t)	Q(t+1)	D	Operation
0	0	0	Reset
0	1	1	Set
1	0	0	Reset
1	1	1	Set

$$Q(t+1) = D$$



Q(t)	Q(t+1)	J	K	Operation
0	0	0	x	No change/reset
0	1	1	x	Set/complement
1	0	x	1	Reset/complement
1	1	x	0	No change/set

$$Q(t+1) = K'Q(t) + JQ(t)'$$



Q(t)	Q(t+1)	T	Operation
0	0	0	No change
0	1	1	Complement
1	0	1	Complement
1	1	0	No change

$$Q(t+1) = T \oplus Q(t)$$