



# Ch.1 Digital Systems and Binary Numbers

## Digital systems

### Digital vs Analog

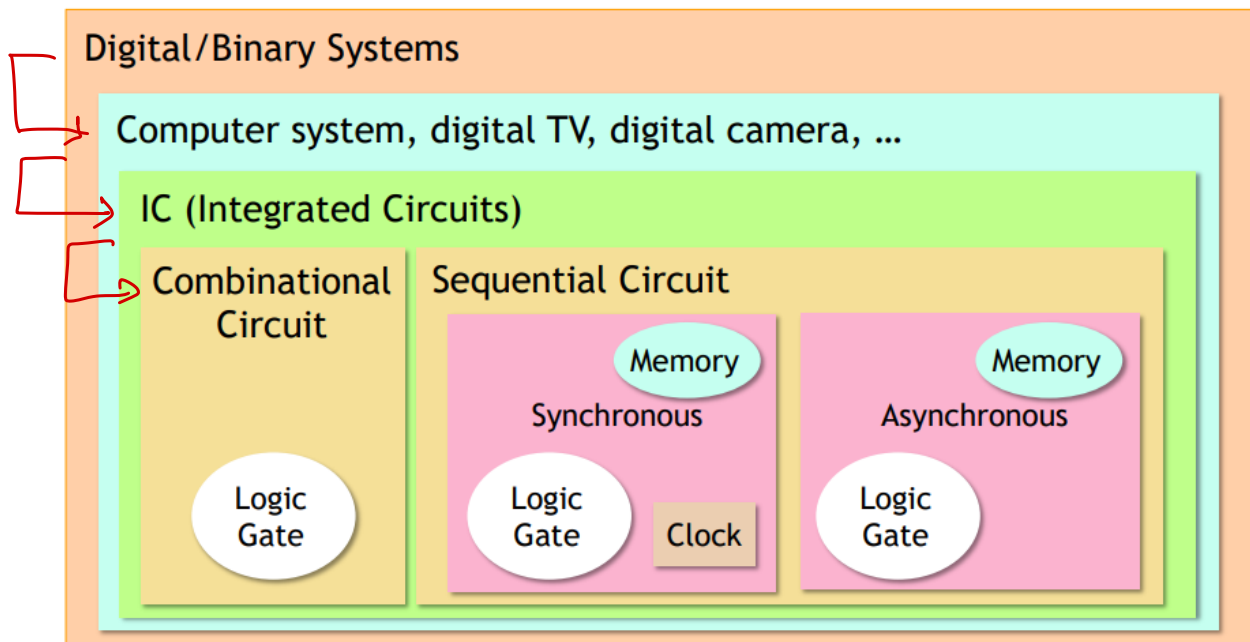
- **Digital** : 한정된 숫자로 이산적인 값을 나타냄
- **Analog** : 무한적인 값들을 셀 수 없는 숫자로 나타냄

### Digits vs Bits

- **Digits** → **discrete**
  - **Bits** → binary digits
- **Digital Computer** → **digital system** → **bits!!**

↳ Binary System을 이해하는데 필요

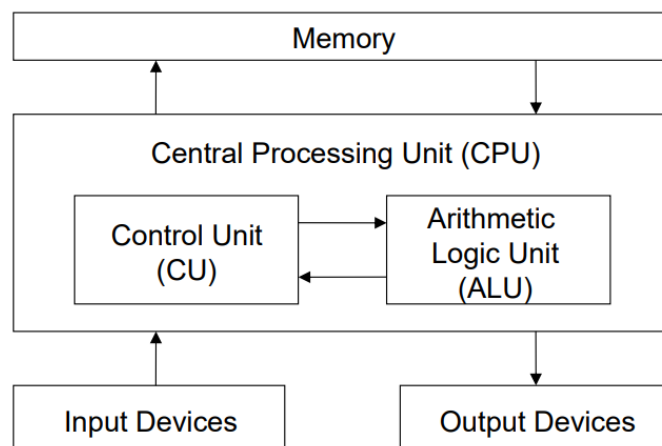
### H/W in Digital Systems



## Digital Computers

- **H/W** : electronic components & electromechanical devices
- **S/W** : programs composed of sequential instructions
  - System S/W → OS, compilers

### Basic Structure



## Number Systems

- **Base(Radix value)** : 기수 / coefficients : 계수
- Binary, Octal, Hexadecimal
- **Base-r**

◦ Base : r

◦ Set of coefficients =  $\{0, 1, 2, \dots, r-1\}$

⇒  $a_2 a_1 a_0 \dots a_{-1} a_{-2}$

$$= r^2 a_2 + r^1 a_1 + r^0 a_0 + r^{-1} a_{-1} + r^{-2} a_{-2}$$

EX

Base : 10

Set of  $r$  : 0, 1, ..., 9

## Binary Number

- **Bit** : a digit in a binary number
- **Byte** : 8bits

- **N bits** → from 0 to  $2^n - 1$

1 Kilo =  $2^{10}$   
1 Mega =  $2^{20}$   
1 Giga =  $2^{30}$   
1 Tera =  $2^{40}$

Numbers with Different Bases

| Decimal<br>(base 10) | Binary<br>(base 2) | Octal<br>(base 8) | Hexadecimal<br>(base 16) |
|----------------------|--------------------|-------------------|--------------------------|
| 00                   | 0000               | 00                | 0                        |
| 01                   | 0001               | 01                | 1                        |
| 02                   | 0010               | 02                | 2                        |
| 03                   | 0011               | 03                | 3                        |
| 04                   | 0100               | 04                | 4                        |
| 05                   | 0101               | 05                | 5                        |
| 06                   | 0110               | 06                | 6                        |
| 07                   | 0111               | 07                | 7                        |
| 08                   | 1000               | 10                | 8                        |
| 09                   | 1001               | 11                | 9                        |
| 10                   | 1010               | 12                | A                        |
| 11                   | 1011               | 13                | B                        |
| 12                   | 1100               | 14                | C                        |
| 13                   | 1101               | 15                | D                        |
| 14                   | 1110               | 16                | E                        |
| 15                   | 1111               | 17                | F                        |

- **Arithmetic Operations(연산)** ← 10진수와 똑같은

## addition

|           |         |
|-----------|---------|
| 0 + 0 = 0 | carry=0 |
| 0 + 1 = 1 | carry=0 |
| 1 + 0 = 1 | carry=0 |
| 1 + 1 = 0 | carry=1 |

sum

## subtraction

|           |          |
|-----------|----------|
| 0 - 0 = 0 | borrow=0 |
| 1 - 0 = 1 | borrow=0 |
| 0 - 1 = 1 | borrow=1 |
| 1 - 1 = 0 | borrow=0 |

difference

① addition

② subtraction

③ multiplication

→ product

### Number base conversion(소수부, 정수부)

integer ⇒ divisions 반복

fraction ⇒ multiplications 반복

12345. 678  
Integer Fraction

### MSB, LSB

2<sup>7</sup> 2<sup>6</sup> 2<sup>5</sup> 2<sup>4</sup> 2<sup>3</sup> 2<sup>2</sup> 2<sup>1</sup> 2<sup>0</sup>  
1 0 0 0 1 0 0 1  
MSB LSB

### Complements

① (r-1)'s complement = (r의 n승-1) - N(Base)

② r's complement = (r의 n승) - N(Base) = (r-1)'s complement + 1

■ 2의 보수 : 첫 1이 나오고 그 뒤로는 반대로 바꿀 것

X = 1010100  
Y = 1000011  
⇒ ① X + (Y의 2의 보수)  
② ① - 2<sup>n</sup>  
③ sum에 음수!

③ Subtraction(작은 거 - 큰 거) ex. 0 - 1 ?

■ M - N (M < N) = M + r의 n승 - N = M + 2의 보수 - r의 n승

ex. +7: 0111

### Signed Binary Numbers

■ positive : 0

■ negative : 1

■ Arithmetic operation → carry out은 버림

① ※ Signed magnitude: changing only the sign bit → 1111  
② ※ Signed-1's complement: complementing all the bits including sign bit → 1000  
③ ※ Signed-2's complement: taking 2's complement of all the bits including sign bit → 1001

→ H/w : leftmost position of the number ⇒ sign bit

→ carry out이 sign-bit position을 때

### Binary Codes(숫자로 된 기호를 할당한다고 생각할 것)

#### Digital System

○ binary numbers 말고 다른 이산적인 요소의 정보를 가지고 있음

■ 이를 binary code로 표현

- code assigning rules

- n-bit binary code → 2의 n승 distinct combinations(0과 1로)
- 각각의 요소가 binary bit combination으로 나타내짐
- 같은 값을 가지는 요소는 x → 모호성을 방지

- Binary Coded Decimal(BCD)**

- 인간과 컴퓨터 간의 정보 교환을 위해선 전환이 필요 → 자연어를 BCD code로 처리
- addition, decimal arithmetic → binary operation과 같음
- 9를 초과하는 것은 meaning이 없다. *ex) "6을 더해서 고쳐줌"*

- ASCII code**

American Standard Code for Information Interchange (ASCII)

| $b_4b_3b_2b_1$ | $b_7b_6b_5$ |     |     |     |     |     |     |     |
|----------------|-------------|-----|-----|-----|-----|-----|-----|-----|
|                | 000         | 001 | 010 | 011 | 100 | 101 | 110 | 111 |
| 0000           | NUL         | DLE | SP  | 0   | @   | P   | ^   | p   |
| 0001           | SOH         | DC1 | !   | 1   | A   | Q   | a   | q   |
| 0010           | STX         | DC2 | "   | 2   | B   | R   | b   | r   |
| 0011           | ETX         | DC3 | #   | 3   | C   | S   | c   | s   |
| 0100           | EOT         | DC4 | \$  | 4   | D   | T   | d   | t   |
| 0101           | ENQ         | NAK | %   | 5   | E   | U   | e   | u   |
| 0110           | ACK         | SYN | &   | 6   | F   | V   | f   | v   |
| 0111           | BEL         | ETB | '   | 7   | G   | W   | g   | w   |
| 1000           | BS          | CAN | (   | 8   | H   | X   | h   | x   |
| 1001           | HT          | EM  | )   | 9   | I   | Y   | i   | y   |
| 1010           | LF          | SUB | *   | :   | J   | Z   | j   | z   |
| 1011           | VT          | ESC | +   | ;   | K   | [   | k   | {   |
| 1100           | FF          | FS  | ,   | <   | L   | \   | l   |     |
| 1101           | CR          | GS  | -   | =   | M   | ]   | m   | }   |
| 1110           | SO          | RS  | .   | >   | N   | ^   | n   | ~   |
| 1111           | SI          | US  | /   | ?   | O   | _   | o   | DEL |

*Handwritten calculation:*  
 4 0100  
 + 8 1000  
 ---  
 +12 1100 ⇒ mean x  
 (+6) 0110  
 ---  
 1 0010  
 2

- ① detect any odd combination errors.
- ② Cannot detect any even combination errors
- **Error-Detecting Code** : to detect the error in data communication or processing.

- parity bit : the total number of 1's either even or odd ⇒ even/odd parity  
 ↳ extra bit

## Binary Storage

- **Binary cell** : a device that possesses two stable states(cpu)

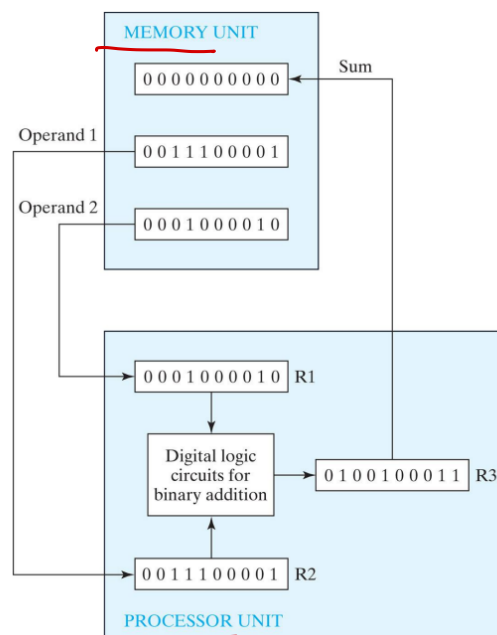
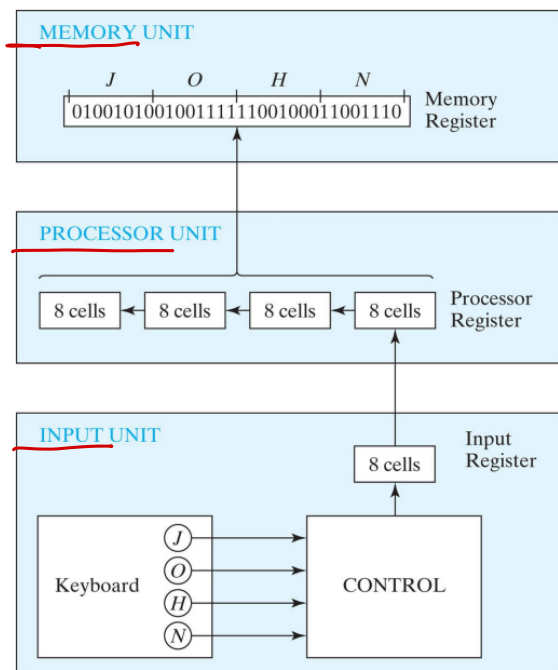
◦ cell input / cell output  
 ↳ receives data and control signals  
 ↳ physical quantity indicating which state the cell is in

## • Registers

- group of n binary cells == an n-bit register
- n-bit register has  $2^n$  states  $\Rightarrow$  all possible n-bit strings
- register state  $\Rightarrow$  value, ASCII 해석될 수 있음
- 직렬(serial), 병렬(parallel) 모두 가능

## • Register Transfer

- registers are interconnected
  - 하나의 register의 content는 다른 것으로 transfer
  - content는 바뀔 수 있음
  - transforming circuit  $\Rightarrow$  data processing or data path element
- 따라서 컴퓨터는 register와 digital logic circuit이 필요



## Binary logic

- deals with variables that have two values  $\Rightarrow \{0,1\}$  (T) (F)
- logic is based on Boolean Algebra operators
- variables denoted by letters of alphabet.

### operations

① and  $\Rightarrow$  논리곱  $\Rightarrow x \cdot y = z$  /  $xy = z$

② or  $\Rightarrow$  논리합  $\Rightarrow x + y = z$

③ not  $\Rightarrow x' = z$   $\overline{x} = z$

| x | y | z |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

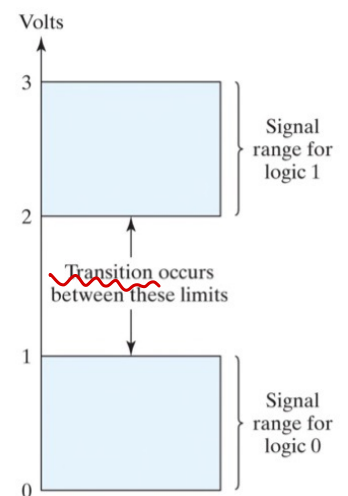
| x | y | z |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

### logic gates

- electronic circuits operate on 1 or more input signals to produce an output signal  $\Rightarrow$  가장 기본적인 단위의 전자회로

① signals  $\Rightarrow$  voltages(전압) or currents

- as 0 or 1
- defined by a range of voltage or current values



② circuit diagrams use graphical symbols for logic gates

- register간의 logic gate들의 연결이 data를 변형할 수 있음
- Logic gates represent interconnections of transistors and other electronic components

③ timing diagrams  $\Rightarrow$  input, output signal 표현

④ multi-input logic gates  $\Rightarrow$  more than one input

|                  |   |   |   |   |   |
|------------------|---|---|---|---|---|
| x                | 0 | 1 | 1 | 0 | 0 |
| y                | 0 | 0 | 1 | 1 | 0 |
| AND: $x \cdot y$ | 0 | 0 | 1 | 0 | 0 |
| OR: $x + y$      | 0 | 1 | 1 | 1 | 0 |
| NOT: $x'$        | 1 | 0 | 0 | 1 | 1 |