



# **Fama-French Three-Factor Model, Carhart Four-Factor Model, and Carhart Four-Factor Plus Mean Reversion Model in Canadian Stock Market**

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## **1. Introduction**

Many investors around the world are attempting to estimate portfolio expected returns. They depend on a variety of models that can calculate expected returns as well as the cost of equity.

Fama–French three-factor model is a model designed by Eugene Fama and Kenneth French in 1992 to estimate expected excess returns. The traditional asset pricing model, formally known as the capital asset pricing model (CAPM), relates the excess returns of a portfolio only to the market factor. Fama and French (1993) added two factors to explain expected excess returns better than the CAPM. The two factors are size factor and value factor, which refer to the outperformance of small versus big companies and the outperformance of high book/market versus low book/market companies, respectively. Carhart (1997) added momentum factor to the three-factor model, which further increased prediction accuracy.

This paper would like to test the performance of these factors when applying to Canadian stocks. During the study, the mean-reversion factor was also added to the research. It is a short-term factor that favors buying if the current market price is less than the average price.

Toronto Stock Exchange is the 8th largest exchange in the world and 3rd in North American by market capitalization.<sup>1</sup> As of January 2021, there are 1,662 listed issuers on the exchange with a combined market capitalization of \$3.4 trillion.<sup>2</sup> The S&P/TSX Composite Index is the benchmark Canadian index, representing roughly 70% of the total market capitalization on the Toronto Stock Exchange (TSX) with about 250 companies included in it.<sup>3</sup> These stocks in the S&P/TSX Composite Index were put in a portfolio as a representation of the Canadian stock market in this paper.

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<sup>1</sup> Toronto Stock Exchange. (2021, April 07). Retrieved from [https://en.wikipedia.org/wiki/Toronto\\_Stock\\_Exchange](https://en.wikipedia.org/wiki/Toronto_Stock_Exchange)

<sup>2</sup> TMX Group Limited. (2021, February 05). TMX Group Equity Financing Statistics – January 2021. Retrieved from <https://tinyurl.com/yxxj6r89>

<sup>3</sup> S&P/TSX Composite Index. (2021, April 08). Retrieved from [https://en.wikipedia.org/wiki/S%26P/TSX\\_Composite\\_Index](https://en.wikipedia.org/wiki/S%26P/TSX_Composite_Index)

Furthermore, this paper decided to study the data between 2013 and 2019. This is due to the fact that many stocks were not issued before 2013 while many stocks behaved normal in 2020 as the pandemic hit the economy.

## **2. Overview of the Canadian Stock Market**

### **2.1 Early Years**

The first set of stock trades in Canada took place in 1832 at the Exchange Coffee House in Montreal.<sup>4</sup> On July 26, 1852, the Association of Brokers was formed by a group of Toronto businessmen. However, no record of the group's transactions survived. On October 25, 1861, the first Canadian exchange, the Toronto Stock Exchange (TSE), was formed by twenty-four brokers gathered at the Masonic Hall, Toronto.<sup>5</sup> In 1874, the Montreal Stock Exchange (MSE) was officially established with the help of Lorn MacDougall and his brothers. By 1910, the value of trades on the Montreal Stock Exchange amounted to about \$2.1 million per day as compared to about \$0.9 million worth of trades on the Toronto Stock Exchange.<sup>6</sup> In the early 20<sup>th</sup> century, two new stock exchanges were formed: the Vancouver Stock Exchange (1906) and the Alberta Stock Exchange (1913).<sup>7</sup> Both exchanges featured mostly small-capitalization, mining, oil and gas exploration, and oil sands stocks.<sup>8</sup> However, the MSE and TSE continued to dominate trading throughout the 20<sup>th</sup> century with TSE driving a large share of the market away from the MSE as a result of a security concern driven by the actions of the separatist group Front de liberation du Québec (including the detonation of a bomb on February 13, 1969) and as a result of a decision by the provincial government of Quebec to make French the language of work in Quebec.<sup>9</sup>

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<sup>4</sup> Kenton, W. (2019, November 22). Montreal Exchange (MX). Retrieved from <https://www.investopedia.com/terms/m/montrealexchange.asp>

<sup>5</sup> Toronto Stock Exchange. (2021, April 07). Retrieved from [https://en.wikipedia.org/wiki/Toronto\\_Stock\\_Exchange](https://en.wikipedia.org/wiki/Toronto_Stock_Exchange)

<sup>6</sup> Montreal Exchange. (2021, April 08). Retrieved from [https://en.wikipedia.org/wiki/Montreal\\_Exchange](https://en.wikipedia.org/wiki/Montreal_Exchange)

<sup>7</sup> Vancouver Stock Exchange. (2021, March 11). Retrieved from [https://en.wikipedia.org/wiki/Vancouver\\_Stock\\_Exchange](https://en.wikipedia.org/wiki/Vancouver_Stock_Exchange)

<sup>8</sup> Alberta Stock Exchange. (2021, March 22). Retrieved from [https://en.wikipedia.org/wiki/Alberta\\_Stock\\_Exchange](https://en.wikipedia.org/wiki/Alberta_Stock_Exchange)

<sup>9</sup> Montreal Exchange. (2021, April 08). Retrieved from [https://en.wikipedia.org/wiki/Montreal\\_Exchange](https://en.wikipedia.org/wiki/Montreal_Exchange)

## 2.2 1999 Reorganization

In 1999, the Vancouver, Alberta, Toronto, and Montreal exchanges worked together to reorganize the Canadian securities market by restructuring along the lines of market specialization. As a result, the Montreal Exchange became the primary hub for derivatives trading, the Toronto Stock Exchange became the primary hub for stock trading, and a new exchange, Canadian Venture Exchange (now, the TSX Venture Exchange) was created to trade shares in smaller companies. Currently, all three exchanges are owned by the TMX Group.

**Toronto Stock Exchange** is a wholly-owned subsidiary of the TMX Group providing trading of senior equities. It is the 8<sup>th</sup> largest exchange in the world and 3<sup>rd</sup> in North American by market capitalization. The exchange lists more mining and oil and gas companies than any other stock exchange in the world.<sup>10</sup> As of January 2021, there are 1,662 listed issuers on the exchange with a combined market capitalization of \$3.4 trillion.<sup>11</sup> The exchange is home to all of Canada's Big Five commercial banks (Canadian Imperial Bank of Commerce, Bank of Montreal, Bank of Nova Scotia, Royal Bank of Canada, and the Toronto-Dominion Bank).<sup>12</sup> The exchange became fully electronic after closing its trading floor in 1997.<sup>13</sup>

**Montreal Exchange** is a derivative exchange facilitating the trading of futures contracts and options on equities, indices, currencies, ETFs, energy, and interest rates.<sup>14</sup> In 1975, the exchange began offering stock options trading, becoming the first Canadian exchange to do so. In 2004, it became the first foreign exchange to provide an American exchange – BOX – with electronic trading systems and support. The continuing rise in exchange's liquidity is largely attributable to a rise

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<sup>10</sup> Toronto Stock Exchange. (2021, April 07). Retrieved from [https://en.wikipedia.org/wiki/Toronto\\_Stock\\_Exchange](https://en.wikipedia.org/wiki/Toronto_Stock_Exchange)

<sup>11</sup> TMX Group Limited. (2021, February 05). TMX Group Equity Financing Statistics – January 2021. Retrieved from <https://tinyurl.com/yxxj6r89>

<sup>12</sup> Toronto Stock Exchange. (2021, April 07). Retrieved from [https://en.wikipedia.org/wiki/Toronto\\_Stock\\_Exchange](https://en.wikipedia.org/wiki/Toronto_Stock_Exchange)

<sup>13</sup> Chen, J. (2021, February 05). Toronto Stock Exchange (TSX). Retrieved from <https://www.investopedia.com/terms/t/toronto-stock-exchange-tsx.asp>

<sup>14</sup> Montreal Exchange. (2021, April 08). Retrieved from [https://en.wikipedia.org/wiki/Montreal\\_Exchange](https://en.wikipedia.org/wiki/Montreal_Exchange)

in trading from different countries, with more than 90% of traders from major cities like New York, London, and Chicago being able to connect directly to the exchange's trading system. The exchange was acquired by the TSX Group in 2007, with the parent company changing its name to TMX Group as a result.<sup>15</sup>

**TSX Venture Exchange**, formerly Canadian Venture Exchange, was formed as a result of a merger between the Vancouver and Alberta stock exchanges as part of the reorganization of the Canadian securities market in 1999.<sup>16</sup> The exchange is now owned and operated by the TMX Group. The exchange primarily focuses on companies that are too small to be listed on the Toronto Stock Exchange. As a result, listed issuers are primarily small-cap companies. There are more than 1,600 companies listed on the exchange of which close to 400 are included in the S&P/TSX Venture Composite Index. The index is primarily composed of mining and traditional energy companies.<sup>17</sup>

### 2.3 Other Exchanges

In addition to the three exchanges formed as a result of the 1999 reorganization, the following exchanges have established a notable presence in the Canadian market:

**NEO Exchange** is a relatively new stock exchange, marketed as Canada's New Stock Exchange. It received a recognition order from the Ontario Securities Commission in 2014 and commenced trading of 45 Toronto Stock Exchange-listed securities in 2015. Over the years, the exchange was able to entice multiple ETFs to changing the listing venue from the Toronto Stock Exchange. It was also able to attract new ETF listings as well as some corporate issuers.<sup>18</sup> The NEO Exchange is now home to over 120 corporate and ETF listings. NEO Exchange now consistently facilitates close to 15% of all volume traded in Canadian-listed companies and roughly 20% of all volume traded in Canadian

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<sup>15</sup> Kenton, W. (2019, November 22). Montreal Exchange (MX). Retrieved from <https://www.investopedia.com/terms/m/montrealexchange.asp>

<sup>16</sup> TSX Venture Exchange. (2021, March 14). Retrieved from [https://en.wikipedia.org/wiki/TSX\\_Venture\\_Exchange](https://en.wikipedia.org/wiki/TSX_Venture_Exchange)

<sup>17</sup> Chen, J. (2019, November 08). TSX Venture Exchange. Retrieved from <https://www.investopedia.com/terms/t/tsxventureexchange.asp>

<sup>18</sup> NEO Exchange. (2021, March 11). Retrieved from [https://en.wikipedia.org/wiki/NEO\\_Exchange](https://en.wikipedia.org/wiki/NEO_Exchange)

ETFs.<sup>19</sup> The exchange aims to attract new listing and order flow by creating a "better listing experience, eliminating predatory market behavior such as high-frequency trading, implementing a unique market-making program to ensure liquidity, and offering real-time market data for all NEO-listed securities." The exchange is owned by Aequis Innovations Inc.<sup>20</sup>

**Canadian Securities Exchange (CSE)** is an alternative stock exchange, offering simplified reporting requirements and reduced barriers to listing. It, therefore, acts as an alternative for micro-cap and emerging companies planning on listing in Canada. The exchange is fully automated and does not have a traditional, physical trading floor.<sup>21</sup> It is based in Toronto and has a branch office in Vancouver. As of January 2021, 636 companies were listed on the exchange. As of December 2020, the total market cap of listed companies was \$36.5 billion. As of 2020, the volume was 26.86 billion shares. The exchange began operations in 2003 and was formally recognized and approved as a stock exchange in 2004.<sup>22</sup>

**ICE Futures Canada** is Canada's largest agricultural derivatives platform. It lists the world's most heavily traded canola contract, as well as futures and options on barley and wheat. The exchange was formally known as Winnipeg Commodity Exchange (WCE) before its parent company was acquired by the Intercontinental Exchange in 2007 with all trading transitioning to the ICE electronic platform by the end of the year. The exchange was the first North American commodities exchange to end all floor-based trading with the last session on December 17, 2004.<sup>23</sup>

**NASDAQ Canada** is a wholly-owned subsidiary of the NASDAQ Stock Market Inc. It was created in 2000 to ensure Canadian investors' immediate trading access to all NASDAQ securities and issuers. The exchange was launched with participation from 10

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<sup>19</sup> NEO Exchange Announces Inclusion in FTSE Russell Indexes. (2021, March 22). Retrieved from [https://www.valdostadailytimes.com/news/business/neo-exchange-announces-inclusion-in-ftse-russell-indexes/article\\_169fb2ab-9a76-5ff8-8e0f-dc80e621da9c.html](https://www.valdostadailytimes.com/news/business/neo-exchange-announces-inclusion-in-ftse-russell-indexes/article_169fb2ab-9a76-5ff8-8e0f-dc80e621da9c.html)

<sup>20</sup> NEO Exchange. (2021, March 11). Retrieved from [https://en.wikipedia.org/wiki/NEO\\_Exchange](https://en.wikipedia.org/wiki/NEO_Exchange)

<sup>21</sup> Hayes, A. (2020, April 8). Canadian Securities Exchange. Retrieved from <https://www.investopedia.com/terms/c/cnq.asp>

<sup>22</sup> Canadian Securities Exchange. (2021, February 02). Retrieved from [https://en.wikipedia.org/wiki/Canadian\\_Securities\\_Exchange](https://en.wikipedia.org/wiki/Canadian_Securities_Exchange)

<sup>23</sup> ICE Futures Canada. (2015, March 16). Retrieved from [http://marketswiki.com/wiki/ICE\\_Futures\\_Canada](http://marketswiki.com/wiki/ICE_Futures_Canada)

Canadian brokerages.<sup>24</sup> Currently, Nasdaq Canada offers three trading books; orders are matched based upon price/broker/time priority.<sup>25</sup> The exchange also offers enhanced connectivity to dark pools.<sup>26</sup> The Nasdaq Canada Index was created at the same time as the launch of the exchange. It is composed of 61 companies. Its ticker is CND.<sup>27</sup>

**ICE NGX (Natural Gas Exchange)** provides electronic trading, central counterparty clearing and data services to the North American natural gas and electricity markets. It is one of the most liquid spots and forward energy markets in North America. The exchange was acquired from the TMX Group by the Intercontinental Exchange (ICE) in 2017.<sup>28</sup>

## 2.4 Dark Pools

Dark pools accounted for about 10% of trading volume in the first half of 2019.<sup>29</sup> The dark pools' share of trading volume was down to about 9% in the first quarter of 2020.<sup>30</sup> The largest dark pool in Canada is MatchNow. It has continued to gain market share despite an overall slight decline in dark pools' share of trading volume. Other major dark pools include Liquidnet, Instinet ICX, and Nasdaq's CXD dark pool, Aequitas NEO-D dark pool. Many exchange operators offer dark order types known locally as gray liquidity. For instance, TSX and CSE offer order types that can access TSX Dark liquidity and CSE Dark liquidity.<sup>31</sup>

**MatchNow**, officially known as TriAct Canada Marketplace LP, is Canada's largest dark pool. It was acquired by the Cboe Global Markets from Virtu Financial in 2020. It handles more than 5 percent of total stock trading and 65 percent of anonymous trading in Canada, where over \$5 billion worth of equities exchange hands each day. The

<sup>24</sup> NASDAQ Canada. (2020, December 23). Retrieved from [https://en.wikipedia.org/wiki/NASDAQ\\_Canada](https://en.wikipedia.org/wiki/NASDAQ_Canada)

<sup>25</sup> Nasdaq Canada. (n.d.). Retrieved from <https://www.nasdaq.com/nasdaqcanada>

<sup>26</sup> Canadian Equities. (n.d.). Retrieved from <https://www.nasdaq.com/solutions/canadian-equities>

<sup>27</sup> NASDAQ Canada. (2020, December 23). Retrieved from [https://en.wikipedia.org/wiki/NASDAQ\\_Canada](https://en.wikipedia.org/wiki/NASDAQ_Canada)

<sup>28</sup> Natural Gas Exchange, Inc. (2018, January 10). Retrieved from [http://www.marketswiki.com/wiki/Natural\\_Gas\\_Exchange,\\_Inc.](http://www.marketswiki.com/wiki/Natural_Gas_Exchange,_Inc.)

<sup>29</sup> Foster, J. (2019, August 19). TMX POV – It's time to rethink Canadian Dark Pools. Retrieved from <https://www.tmx.com/tmx-pov?id=780&year=2019>

<sup>30</sup> Langton, J. (2020, May 25). CBOE's acquisition of Canadian dark pool a positive, Moody's says. Retrieved from <https://www.advisor.ca/news/industry-news/cboes-acquisition-of-canadian-dark-pool-a-positive-moodys-says/>

<sup>31</sup> D'Antona, J. (2016, September 13). Canadian Dark Pools Poised for Growth? Retrieved from <https://www.marketsmedia.com/canadian-dark-pools-poised-growth-itg-dark-pool-ceo/>

platform facilitates buying and selling of about 1.2 billion shares a month, carrying out more than 2 million trades.<sup>32</sup>

### **3. Theoretical Background**

#### **3.1 Fama-French Three-factor model**

Fama and French (1993) found they explained the returns of the diversified portfolio using the three-factor model better than the traditional CAPM model. Although they also tested other factors such as leverage and earnings/price ratio, they concluded that it was the size factor and the value factor that have a closer relationship with the excess return of the portfolios.

Griffin (2002) demonstrated that the Fama and French factors are country-specific and concluded that the local factors explain time-series variation in stock returns better than the global factors. Canadian data needs to be applied to study the Canadian stock market.

The Fama and French three-factor model regression equation is stated as:

$$r_i = \beta_{1,i}r_M + \beta_{2,i}SMB + \beta_{3,i}HML + \mu_i$$

Where:

$r_i$  is the expected excess return

$r_M$  is the excess return on the market portfolio

SMB refers to the size factor, and it is chosen to long 50% of companies with the smallest market capitalizations, and short the 50% of companies with the largest market capitalizations in this paper.

HML refers to the value factor, and it is chosen to long the top 30% of stocks with the highest book to price ratio, and short the bottom 30% of stocks with the lowest book to price ratio in this paper.

$\beta_{n,i}$  are the factor coefficients where  $n=1,2,3$

$\mu_i$  is the idiosyncratic return

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<sup>32</sup> Abdel-Qader, A. (2020, April 08). Cboe Completes Acquisition of Virtu's Dark Pool MatchNow. Retrieved from <https://www.financemagnates.com/institutional-forex/cboe-completes-acquisition-of-virtus-dark-pool-matchnow/>



### 3.2 Carhart Four-factor model

Jegadeesh and Titman (1993) discovered a tendency for good and bad stock performance to persist over several months, referred to as a momentum effect. Carhart (1997) decided to add the momentum factor into the three-factor model when valuing mutual funds.

The Carhart four-factor model regression equation is stated as:

$$r_i = \beta_{1,i}r_M + \beta_{2,i}SMB + \beta_{3,i}HML + \beta_{4,i}UMD + \mu_i$$

Where:

$r_i$  is the expected excess return

$r_M$  is the excess return on the market portfolio

SMB refers to the size factor, and it is chosen to long 50% of companies with the smallest market capitalizations, and short the 50% of companies with the largest market capitalizations in this paper.

HML refers to the value factor, and it is chosen to long the top 30% of stocks with the highest book to price ratio, and short the bottom 30% of stocks with the lowest book to price ratio in this paper.

UMD refers to the momentum factor, and it is chosen to long the top 30% stocks over the time period t-12 months to t-2 months, and short the lowest 30% stocks over the same time period in this paper.

$\beta_{n,i}$  are the factor coefficients where  $n=1,2,3$

$\mu_i$  is the idiosyncratic return

### 3.3 Literature Overview

#### Fama and French 3-factor model:

Fama, E. F., & French, K. R. (1993). Common risk factors in the returns on stocks and bonds. *Journal of Financial Economics*, 33(1), 3-56. doi:10.1016/0304-405x(93)90023-5

**Additional papers related to Fama and French 3-factor model:**

Cakici, N., Fabozzi, F. J., & Tan, S. (2013). Size, value, and momentum in emerging market stock returns. *Emerging Markets Review*, 16, 46-65.  
doi:10.1016/j.ememar.2013.03.001

Fama, E. F., & French, K. R. (1992). The cross-section of expected stock returns. *The Journal of Finance*, 47(2), 427-465. doi:10.1111/j.1540-6261.1992.tb04398.x

Fama, E. F., & French, K. R. (2012). Size, value, and momentum in international stock returns. *Journal of Financial Economics*, 105(3), 457-472.  
doi:10.1016/j.jfineco.2012.05.011

Griffin, J. M. (2002). Are the Fama and French FACTORS global or country specific? *Review of Financial Studies*, 15(3), 783-803. doi:10.1093/rfs/15.3.783

**Carhart 4-factor model:**

Carhart, M. M. (1997). On Persistence in Mutual Fund Performance. *The Journal of Finance*, 52(1), 57-82. doi:10.1111/j.1540-6261.1997.tb03808.x

**Additional papers related to Carhart 4-factor model:**

Jegadeesh, N., & Titman, S. (1993). Returns to Buying Winners and Selling Losers: Implications for Stock Market Efficiency. *The Journal of Finance*, 48(1), 65-91.  
<https://doi.org/10.1111/j.1540-6261.1993.tb04702.x>

Low, R. K., & Tan, E. (2016). The role of analyst forecasts in the momentum effect. *International Review of Financial Analysis*, 48, 67-84.  
doi:10.1016/j.irfa.2016.09.007

**Mean Reversion:**

Ahmed, R. R., Vveinhardt, J., Streimikiene, D., & Channar, Z. A. (2018). Mean reversion in INTERNATIONAL markets: Evidence from G.A.R.C.H. and Half-life volatility models. *Economic Research-Ekonomska Istraživanja*, 31(1), 1198-1217.  
doi:10.1080/1331677x.2018.1456358

Chakraborty, T., & Kearns, M. (2011). Market making and mean reversion. *Proceedings of the 12th ACM Conference on Electronic Commerce - EC '11*.  
doi:10.1145/1993574.1993622

Liew, J., & Roberts, R. (2013). U.S. equity Mean-Reversion Examined. *Risks*, 1(3), 162-175. doi:10.3390/risks1030162

Poterba, J. M., & Summers, L. H. (1988). Mean reversion in stock prices. *Journal of Financial Economics*, 22(1), 27-59. doi:10.1016/0304-405x(88)90021-9

## **4. Methodology**

### **4.1 Time Period**

This research covers the time period from January 2013 to December 2019. When doing regression analysis, if the time span is too large, the regression coefficients can be biased. As some factors in the model are yearly updated, it is necessary to select at least five-year data to construct the factors. To ensure the reliability of the regression analysis of the model, the thesis is limited to 7 years. Because many of the stocks in the research were listed after 2013, the data in use starts from January 2013. Also, to exclude the impact of Covid-19, the research eliminates the data for 2020.

### **4.2 Data and Sources**

The dataset consists of 218 stocks and all of the stocks are in the latest S&P/TSX composite index. Two stocks, AUP.TO and WEED.TO, are eliminated from the analysis as their prices rise rapidly in one day due to shares consolidation and first being traded on the TSX Venture Exchange.

Yahoo Finance is the main data source to provide both market cap data and price-to-book ratio. However, there is missing data in the database and to fill in the missing value, Bloomberg serves as a supplementary data source. As some of the companies' fiscal year ends on 31 March, to keep the time consistent, data from the third quarter is used for these companies.

S&P/TSX Composite Index data and Canada 10-year bond yield are gathered from Yahoo Finance as well. The percent change of the index represents the market return and the 10-year bond yield is used as a proxy for the risk-free rate.

The prices of each stock are from GuruFocus. And the study focuses on the adjusted closing price which is more precise to examine the returns and analyze the past performance. Once the daily prices are prepared, the daily returns are calculated by Python directly.

### **4.3 Fama French Three-Factor Model**

**Rm-Rf**

The S&P/TSX Composite index is used as the market return for all regression processes. The risk-free interest is the Canadian 10-year bond yield issued by the Canadian government. The market return minus the risk-free return on a daily basis, then a time series of Rm-Rf is obtained.

**SMB**

The SMB, small minus big, is the size factor. Stocks in our dataset are ranked by their Capitalization. The stocks in our dataset are divided into two groups: small and big(S and B) based on a limit of 50<sup>th</sup> percentile. The SMB factor measures the historical performance of small companies over big companies.

The formula for SMB factor:

$$SMB = R_{Small} - R_{Big}$$

Where

$R_{Small}$  = Average return of stocks in Small group weighted by their Market Capitalization

$R_{Big}$  = Average return of stocks in Big group weighted by their Market Capitalization

**HML**

The HML, high minus low, is the value factor. All the companies are ranked by their Book-to-Price ratio, and the stocks in our dataset are divided into three groups: low, medium, and high(L, M, and H) based on the breakpoints from the bottom 30 percent, middle 40 percent, and top 30 percent. The HML factor measures the historical performance of the value stocks over the growth stocks.

The formula for HML factor:

$$HML = R_{High} - R_{Low}$$

Where

$R_{High}$  = Average return of stocks in High group weighted by their Market Capitalization

$R_{Low}$  = Average return of stocks in Low group weighted by their Market Capitalization

#### 4.4 Carhart Four-Factor Model

Three of the factors in 4-factor model are the same as three-factor model.  $R_m - R_f$ , SMB and HML are constructed in the same way. And the Carhart model adds a new factor, UMD which means uptrend minus downtrend. To generate UMD factor, the returns between January year  $t-1$  to October year  $t-1$  are calculated as momentum factors for each stocks. After sorting the stocks into uptrend portfolios and downtrend portfolios based on momentum factors, the UMD factor is equal to the difference between the return of uptrend group and downtrend group.

##### UMD

The UMD, up minus down, represents the momentum factor. It assumes that the stocks performed well in the past will continue to perform well in the future. And the assumption indicates the strategy of taking long position in the stocks with uptrend and taking short position in the stocks with downtrend. In the research, the criterion is slightly different from that in Carhart 4-factor model. The stocks are divided into two groups based on their performance over the past 12 months to the past 2 months. The down-trend group includes the stocks with lowest 50% returns between January year  $t-1$  to October year  $t-1$  and the uptrend group includes those with top 50% returns over the same period of time.

After adding UMD factor to the model, 12 portfolios are created as the stocks are classified by their returns over the past  $t-12$  months to  $t-2$  months, book-to-market ratio and market capitalization in year  $t$ . And the 12 portfolios are named as BHU, BMU, BLU, SHU, SMU, SLU, BHD, BMD, BLD, SHD, SMD, SLD. To mention that BHU consists of companies with big market value in year  $t$ , high B/M ratio in year  $t$  and the stocks that generated high returns between January year  $t-1$  to October year  $t-1$ .

The formula for UMD factor:

$$UMD = R_{Uptrend} - R_{Downtrend}$$

To clarify, the returns in the formula are weighted average by the market capitalization of the stock included.

#### 4.5 Five-Factor Model

First four factors in the 4-factor Rm-Rf, SMB and HML are all generated in the same way as for 4-factor model. The fifth factor is MR, mean reversion, which is created for each portfolio constructed in four-factor model. As the mean reversion strategy is valid in short-term, the factor is constructed different from ranking the stocks based on their mean reversion factor. Also, it is not suitable to divide the stocks into different group based on the returns over a short time period since the group will change frequently.

## MR

MR (mean reversion) factor is used to finding whether the stock prices are deviated from mean reversion level in the short run. And it is a signal to buy the stock with negative return over a few days and sell the stocks with positive return over same time period. In this study, to generate the MR factor, the first step is to calculate the normalized return over the past five days for each stock and then sum up the product of the stocks position and return.

The formula for MR factor is:

$$MR_{\text{norm},i} = \frac{R_{\text{past 5 days},i} - \bar{R}_{\text{past 5 days},P}}{Std(\bar{R}_{\text{past 5 days},P})}$$

$$MR_P = \sum_{i=1}^N \frac{MR_{\text{norm},i}}{N - 1} * R_{\text{today},i}$$

in which

$R_{\text{past 5 days},i}$  = Return of stock i over the past 5 days

$R_{\text{today},i}$  = Return of stock i today

$\bar{R}_{\text{past 5 days},P}$  = Average Return of portfolio P over the past 5 days

$N$  = Number of stocks in the portfolio P

To mention that the  $MR_P$  constructed from the formula is the return of mean reversion strategy in dollars.

## 5. Analysis and Result

### 5.1 Portfolio Summary Statistics

Table 5.1 Six-Portfolio Summary Statistics from Fama-French Three-Factor Model

Portfolio	BH	BM	BL	SH	SM	SL
Annual Return	0.1294	0.1306	0.1579	0.2085	0.1747	0.1722
Annual Volatility	0.3968	0.3812	0.3999	0.4287	0.4122	0.404

Table 5.2 Twelve-Portfolio Summary Statistics from Carhart Four-Factor Model

Portfolio	Annual Return	Annual Volatility
BHU	0.1819	0.4137
BMU	0.1413	0.3843
BLU	0.1766	0.4029
SHU	0.2174	0.4451
SMU	0.1785	0.4319
SLU	0.2000	0.4075
BHD	0.1281	0.4159
BMD	0.1208	0.3826
BLD	0.1111	0.4139
SHD	0.2042	0.4359
SMD	0.1616	0.4063
SLD	0.1473	0.4049

According to table 5.2, small size portfolios outperform big size portfolios. It suggests that there is a negative relation between portfolios' mean return and their size for all portfolios. According to table 5.1, small size portfolios also outperform big size portfolios. The relationship suggested in the twelve portfolios is consistent with the relation shown in six portfolios.

According to table 5.2, high value portfolios outperform medium value portfolios, and medium value portfolios outperform low value portfolios except BU and SU. It can be inferred that there is a positive relationship between portfolios' mean return and their value for all portfolios. However, in table 5.1, for portfolios containing big stocks, higher the value of stocks, lower the mean excess return. The result is conflicting to the statistics of twelve portfolios in table 5.2.

According to table 5.2, portfolios with uptrend outperform portfolios with downtrend for all twelve portfolios. Thus, a positive relationship might exist between portfolios' mean

return and their momentum.

## 5.2 Factor Summary Statistics

### Market Excess Return, SMB, HML and UMD factors

Table 5.3 below displays the annualized mean, standard deviation and Sharpe ratio of market excess return, SMB factor, HML factor and UMD factor.

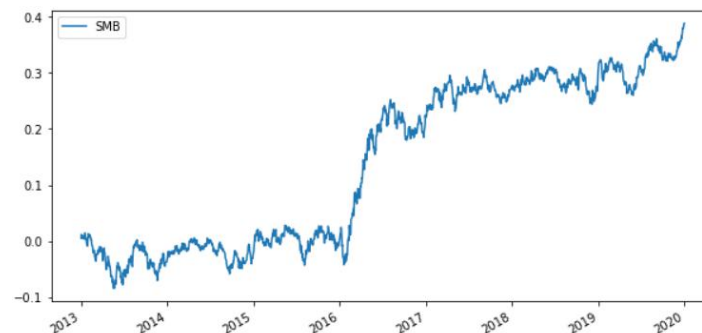
Table 5.3 Summary statistics and Sharpe ratio for market excess return, SMB, HML and UMD

	Annual Return	Annual Volatility	Sharpe Ratio
<b>Market Excess Return</b>	0.1432	0.4632	0.3091
<b>SMB</b>	0.0559	0.0902	0.6196
<b>HML</b>	-0.0121	0.1372	-0.0881
<b>UMD</b>	0.0436	0.0753	0.5797

For the four factors above, the market excess return is much more volatile than the other factors. And the market excess return also has a higher average annual return (14.32%) than the rest. Looking into the last three factors, it shows that HML has a negative mean return (-1.21%) while SMB and UMD have positive mean returns which are 5.59% and 4.36% respectively. What's more, the statistics indicate that in the Canadian market higher book-to-market ratios yield relatively poor earnings on average, companies with small size tend to generate higher returns than big size companies on average and momentum strategy is also effective in the market. The Sharpe ratio shows that the SMB portfolio and UMD portfolio have a relatively high excess return under unit risk.

To further discuss the key factors, the plots of the cumulative return of each factor are displayed below.

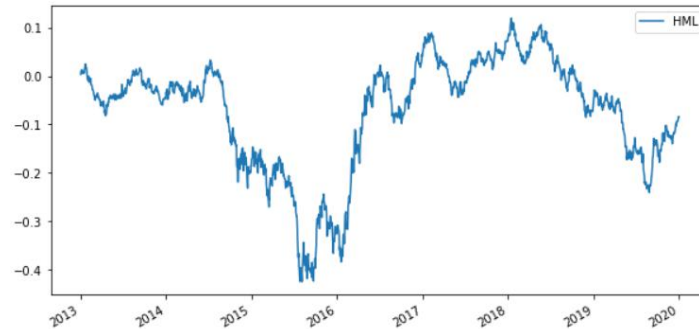
Figure 5.1 Cumulative return of SMB portfolio between 2013 and 2019





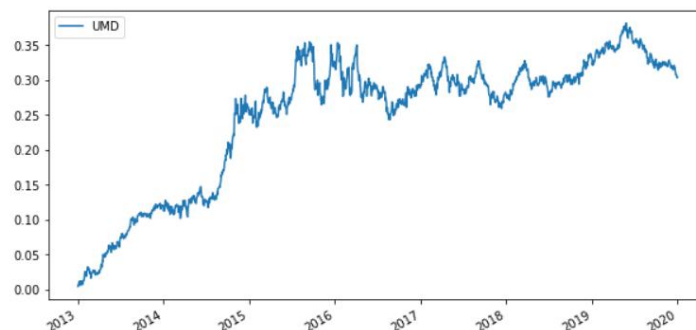
From Figure 5.1, the cumulative return of SMB portfolio was around zero before 2016 and rose rapidly in 2016. After 2016, the cumulative return curve keeps rising at a much slower rate comparing to 2016. The rapid increase in SMB return shows that in the Canadian market the small cap companies significantly outperformed large cap companies in 2016 and the small cap stocks kept increasing in the later 4 years.

Figure 5.2 Cumulative return of HML portfolio between 2013 and 2019



According to Figure 5.2, the curve showed a downward trend from mid-2014 to mid-2015 and from early 2018 to around October 2019. The continuous downward trend in these two periods could be the reason for the negative average return of HML portfolio displayed in Table 5.3. It also implies that during these two periods the companies with low book-to-market ratio beat the companies with high book-to-market ratio. In other words, the growth investment has delivered more returns than value investment in general during these two periods while the value investment outperformed the growth investment in 2016 and 2017.

Figure 5.3 Cumulative return of UMD portfolio between 2013 and 2019



The cumulative return curve of UMD portfolio in Figure 5.3 shows an upward trend over the time period as expected. It is worth noting that the curve fluctuated around a certain

level after 2016 and the momentum strategy seems to be more effective in the Canadian market before 2016 especially in late 2014.

## MR

As the mean reversion factor is constructed for each portfolio respectively, Table 5.4 displays the statistics and Sharpe ratio of mean reversion factor in each portfolio.

Table 5.4 Summary statistics and Sharpe ratio for MR factor

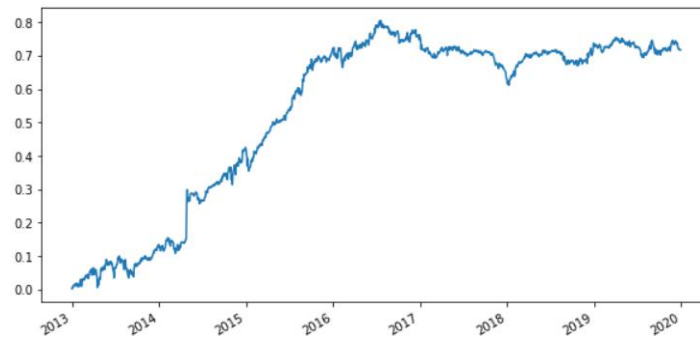
Portfolio	Annual Return	Annual Volatility	Sharpe ratio of MR factor
BHU	0.0352	0.1675	0.2100
BMU	0.0510	0.0845	0.6038
BLU	0.0164	0.0988	0.1656
SHU	0.0653	0.1329	0.4911
SMU	0.1161	0.1469	0.7900
SLU	0.1288	0.1952	0.6599
BHD	-0.0440	0.1680	-0.2621
BMD	-0.0300	0.0820	-0.3662
BLD	0.1030	0.1178	0.8749
SHD	0.1430	0.2765	0.5171
SMD	0.0249	0.1365	0.1823
SLD	0.0383	0.1451	0.2641
All Stocks	0.1023	0.1037	0.9869

Mean reversion factor has generated a positive annual return for most of the portfolio except BHD and BMD. Also, the factor shows relatively high annual returns in SMU, SLU, BLD and SHD which are all above 10%. What's more, the Sharpe ratios of MR factor in these portfolios are also higher than the rest. There is only one exception that though the annual return of MR for BMU is 5.1%, the Sharpe ratio is up to 0.6038 due to the small volatility. And the annual return for all stocks is 10.23% which is higher than that of most portfolios. Also, the Sharpe ratio for the whole portfolio is up to 0.9869 which is higher than all of the separate portfolios. The difference implies that the mean reversion strategy works better on the whole portfolio.

Looking carefully at the cumulative return of MR on the whole portfolio, it shows that before mid-2016, the growth of MR return was quick and could generate lots of revenue from this strategy while the mean reversion strategy was less effective after 2016. To mention that the jump that appeared in 2014 was due to Osisko Mining being acquired by

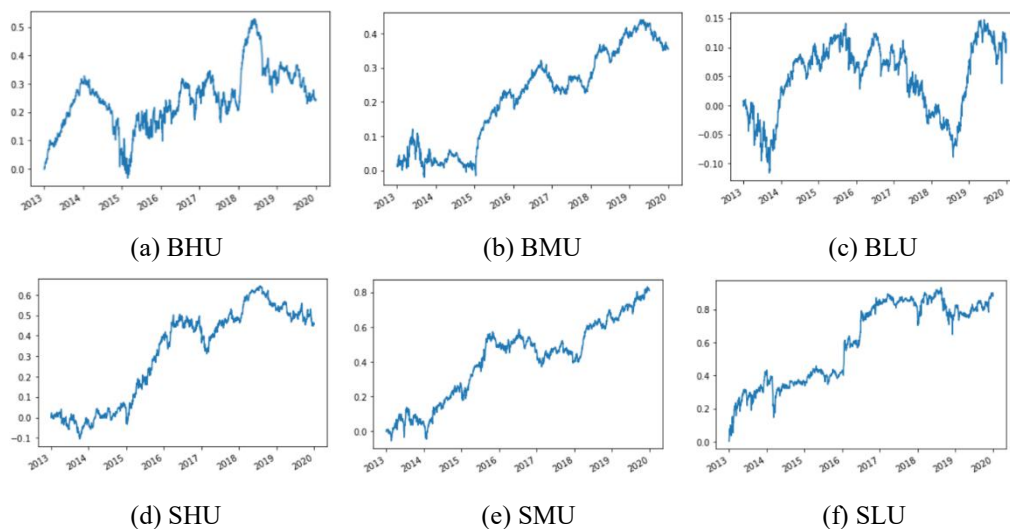
Yamana Gold and Agnico Eagle Mines on April 28 to resist a hostile takeover by Gold Corporation. Overall, the mean reversion factor could contribute to continuous return before 2016 and this positive effect was diminishing during recent 5 years.

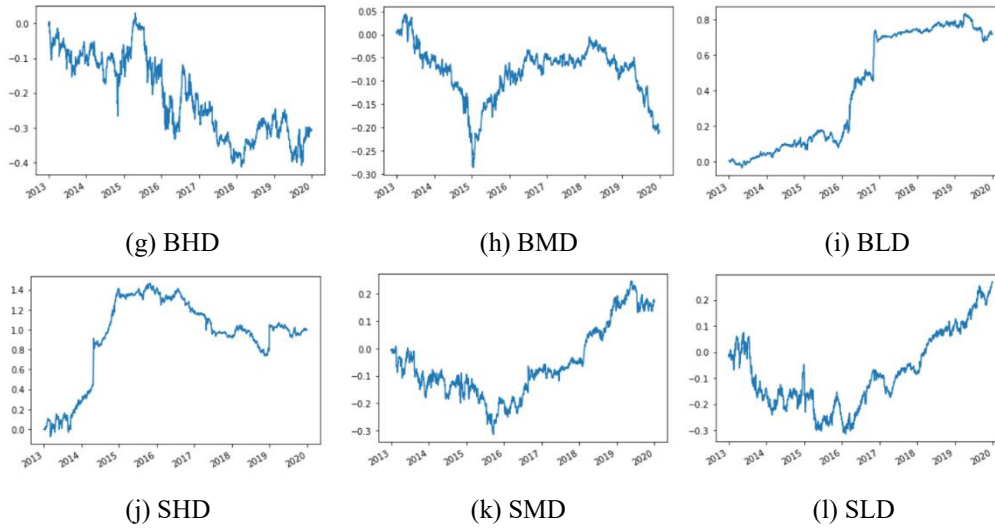
Figure 5.4 Cumulative return of MR for all stocks between 2013 and 2019



From the 12 charts in Figure 5.4, the cumulative returns of the mean reversion factor are increasing in most of the portfolios. However, the trend of portfolio BHD and BMD are negative which implies that the mean reversion strategy is not effective among companies with large cap, relatively high book-to-market ratio and downtrend in the past year. However, the mean reversion strategy still works for those growing companies with large cap and bad performance in the past year according to chart (i). To notice that the upward trend of small cap and uptrend portfolios appeals to be more consistent than others.

Figure 5.5 Cumulative return of MR factor in different portfolios between 2013 and 2019





## 5.3 Regression Results

### 5.3.1 Fama-French Three-Factor Model

The Fama-French three-factor model showed in its regression form

$$r_i - r_f = \alpha_i + \beta_1(r_m - r_f) + \beta_2(SMB) + \beta_3(HML) + \epsilon_i \quad (1)$$

Effects of the three factors on excess portfolio returns are estimated using regression equation (1). Table 5.5 shows the estimation results.

Table 5.5 Fama-French Three-Factor Regression Results

intercept	H	M	L
B	0.0002***	0.0002***	0.0002***
S	0.0003***	0.0002	0.0002
beta1	H	M	L
B	1.0083***	0.9898***	1.0133***
S	1.0116***	0.9997***	0.9834***
SMB	H	M	L
B	-0.1424***	-0.1580***	0.0279***
S	1.0372***	0.800***	0.7141***
HML	H	M	L
B	0.6479***	0.0404***	-0.4261***
S	0.2600***	-0.0585***	-0.3165***

\*\*\*: 1% significance, \*\*: 5% significance, \*: 10% significance.

Table 5.5 shows that only two portfolios, SM and SL, have an alpha close to zero for the significance level of 0.1%. It means that all three factors under the Fama-French Three-

factor model explain the excess return of portfolio SM and SL very well, and the model has explanatory power on four portfolios out of six.

Table 5.5 shows that the Market excess return, size, and value effects are significant for all six portfolios. Results show that the slope market excess return factor is very close to 1 for all six portfolios, and their values are close to each other. It implies that the market excess return factor increases each portfolio's return by approximately the same amount.

All six portfolios have a positive slope for SMB factor except BH and BM, and the slope gets bigger when the portfolio size gets smaller. It means that the size effect is not successfully measured on BH and BM portfolios. Furthermore, the size factor plays an essential role in explaining portfolio returns for big and small size portfolios but it has little effect for medium size portfolios.

### 5.3.2 Carhart Four-Factor Model

The Carhart four-factor model showed in its regression form

$$r_i - r_f = \alpha_i + \beta_1(r_m - r_f) + \beta_2(SMB) + \beta_3(HML) + \beta_4(UMD) + \epsilon_i \quad (2)$$

Effects of the four factors on excess portfolio returns are estimated using regression equation (2). Table 5.6 shows the estimation results.

Table 5.6 Carhart Four-Factor Regression Results

U			
intercept	H	M	L
B	0.0002	0.0002***	0.0003***
S	0.0002	0.0001	0.0002*
market excess return	H	M	L
B	0.9935***	0.9921***	1.0160***
S	1.0301***	1.0219***	0.9829***
SMB	H	M	L
B	-0.0465	-0.1260***	0.0226
S	1.1138	0.9138***	0.7487***
HML	H	M	L
B	0.5908***	0.0578***	-0.2911***
S	0.1014***	0.0374*	-0.2800***
UMD	H	M	L
B	1.0007***	0.3996***	0.4010***
S	0.3087***	0.4192***	0.2221***

D			
intercept	H	M	L
B	0.0003**	0.0002***	0.0002
S	0.0003**	0.0002*	0.0001
market excess return	H	M	L
B	1.0163***	0.9906***	1.0042***
S	1.0003***	0.9800***	0.9809***
SMB	H	M	L
B	0.0874***	-0.1849***	0.1346***
S	1.0992***	0.6422***	0.5936***
HML	H	M	L
B	0.6346***	0.0603***	-0.7395***
S	0.2386***	-0.1270***	-0.2887***
UMD	H	M	L
B	-0.5450***	-0.3357***	-0.9943***
S	-0.4330***	-0.4859***	-0.0463

\*\*\*: 1% significance, \*\*: 5% significance, \*: 10% significance.

Table 5.6 shows that only three portfolios, BMU, BLU, and BMD, have an alpha close to zero for the significance level of 1%. It means that all four factors under the Fama-French Three-factor model explain the excess return of these portfolios very well.

Table 5.6 shows that the Market excess return, size, value, and momentum effects are significant for most portfolios. Both Market excess return and value effects are significant for all twelve portfolios. The size effect is significant for nine portfolios out of twelve for the significance level of 1%. The momentum effect is significant for all portfolios except SLD for the significance level of 1%.

All twelve portfolios have a positive slope for SMB factor except BHU, BMU, and BMD, and the slope gets bigger when the portfolio size gets smaller. It means that the size effect is not successfully measured on these three portfolios which is consistent with the conclusion from the three-factor model.

### 5.3.3 Carhart Four-Factor Model Plus Mean Reversion

The Carhart four-factor model plus mean reversion factor showed in its regression form

$$r_i - r_f = \alpha_i + \beta_1(r_m - r_f) + \beta_2(SMB) + \beta_3(HML) + \beta_4(UMD) + \beta_5(MR) + \epsilon_i \quad (3)$$

Effects of the five factors on excess portfolio returns are estimated using regression equation (3). Table 5.7 shows the estimation results.

Table 5.7 Carhart Four-Factor Plus Mean Reversion Regression Results

U			
intercept	H	M	L
B	0.0002	0.0002***	0.0003***
S	0.0002	0.0001	0.0002*
market excess return	H	M	L
B	0.9935***	0.9921***	1.0159***
S	1.0300***	1.0217***	0.9830***
SMB	H	M	L
B	-0.0491	-0.1260***	0.0209
S	1.1118***	0.9109***	0.7445***
HML	H	M	L
B	0.5908***	0.0578***	-0.2907***
S	0.1004***	0.0382*	-0.2797***
UMD	H	M	L
B	0.9953***	0.3996***	0.3992***
S	0.3077***	0.4192***	0.2221***
MR	H	M	L
B	-0.0381*	0.0005	0.0459***
S	-0.0263	0.0337**	0.0250**

D			
intercept	H	M	L
B	0.0003**	0.0002***	0.0002
S	0.0003*	0.0002*	0.0001
market excess return	H	M	L
B	1.0162***	0.9907***	1.0037***
S	1.0010***	0.9800***	0.9811***
SMB	H	M	L
B	0.0882***	-0.1843***	0.1348***
S	1.0862***	0.6427***	0.5922***
HML	H	M	L
B	0.6354***	0.0599***	-0.7383***
S	0.2415***	-0.1271***	-0.2882***
UMD	H	M	L
B	-0.5432***	-0.3358***	-0.9909***
S	-0.4306***	-0.4853***	-0.0448
MR	H	M	L
B	0.0396***	0.0289***	-0.0346*
S	0.0703***	0.0142	0.0251

\*\*\*: 1% significance, \*\*: 5% significance, \*: 10% significance.

Similar to the Carhart four-factor model, Market excess return, size, value, and momentum effects are significant for most portfolios as shown in Table 5.7. In addition, the beta estimations of these four factors are also very close to the results shown in Table

5.6. However, the mean reversion effect is significant for only four portfolios out of twelve for the significance level of 1%.

### 5.3.4 Adjusted R-Square

Table 5.8 shows the adjusted-R square of each of the six portfolios' regression under the Fama-French three-factor model.

Table 5.8 Adjusted R-squared for each portfolio in three factors model

Adj. R-squared	H	M	L
B	0.9875	0.9939	0.9898
S	0.9776	0.9733	0.9725

The average R-squared value of 0.9824 suggests that three factors can explain a considerable part of the excess return of each portfolio. The R-squared values have a range from 0.9725 to 0.9939.

Table 5.9 shows the adjusted-R square of each of the twelve portfolios' regression under the Carhart four-factor model.

Table 5.9 Adjusted R-squared for each portfolio in four factors model

U			
Adj. R-squared	H	M	L
B	0.8880	0.9896	0.9786
S	0.9352	0.9450	0.9614
D			
Adj. R-squared	H	M	L
B	0.9669	0.9907	0.9511
S	0.9512	0.9492	0.9464

The average R-squared value of 0.9544 suggests that Carhart's four factors can also explain a considerable part of the excess return of each portfolio, but it is not as good as Fama-French three-factor model. The R-squared values have a range from 0.8880 to 0.9907.

Table 5.10 shows the Adjusted-R square of each of the twelve portfolios' regression under the Carhart four-factor plus mean-reversion model.

Table 5.10 Adjusted R-squared for each portfolio in five factors model

U			
Adj. R-squared	H	M	L
B	0.8882	0.9896	0.9787
S	0.9353	0.9452	0.9616

D			
Adj. R-squared	H	M	L
B	0.9671	0.9908	0.9512
S	0.9531	0.9493	0.9464

The average R-squared value of 0.9547 and the range of the R-squared values from 0.8882 to 0.9908 suggest that Carhart's four factors plus mean reversion model has an explanatory power slightly better than the Carhart four-factor model but not as good as Fama-French three-factor model.

#### 5.4 Correlation of Factors

Table 5.11 Factors Correlation

	Market Excess Return	SMB	HML	UMD
Market Excess Return	1	0.1976	-0.0239	-0.0012
SMB	0.1976	1	0.4294	-0.1275
HML	-0.0239	0.4294	1	-0.4509
UMD	-0.0012	-0.1275	-0.4509	1

Table 5.11 shows the correlation between four risk factor portfolios. Market excess return is positively correlated to SMB and negatively correlated to both HML and UMD. UMD is negatively correlated to SMB, but the correlation is not strong. However, the table suggests a positive correlation between HML and SMB and a negative correlation between HML and UMD, and both correlations are relatively stronger.

## **6. Possible Improvements and Suggestions for Future Research**

Two key ideas warrant further investigation: increasing the frequency of rebalancing and adding a sector factor to the model. In the paper, portfolios constructed for HML and SMB factors are rebalanced yearly. While daily and even monthly rebalancing might not be the most appropriate rebalancing frequency for factors such as the value factor,



quarterly rebalancing might be worth investigating. Furthermore, a sector factor can be added to the model. The companies included in the S&P/TSX Composite Index can be assigned to the following eleven sectors: basic materials, communication services, consumer cyclical, consumer defensive, energy, financial services, healthcare, industrials, real estate, technology, and utilities. Following the assignment, a separate factor can be added for each sector with a factor taking a value of one if the company is in the respective sector and zero otherwise.

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